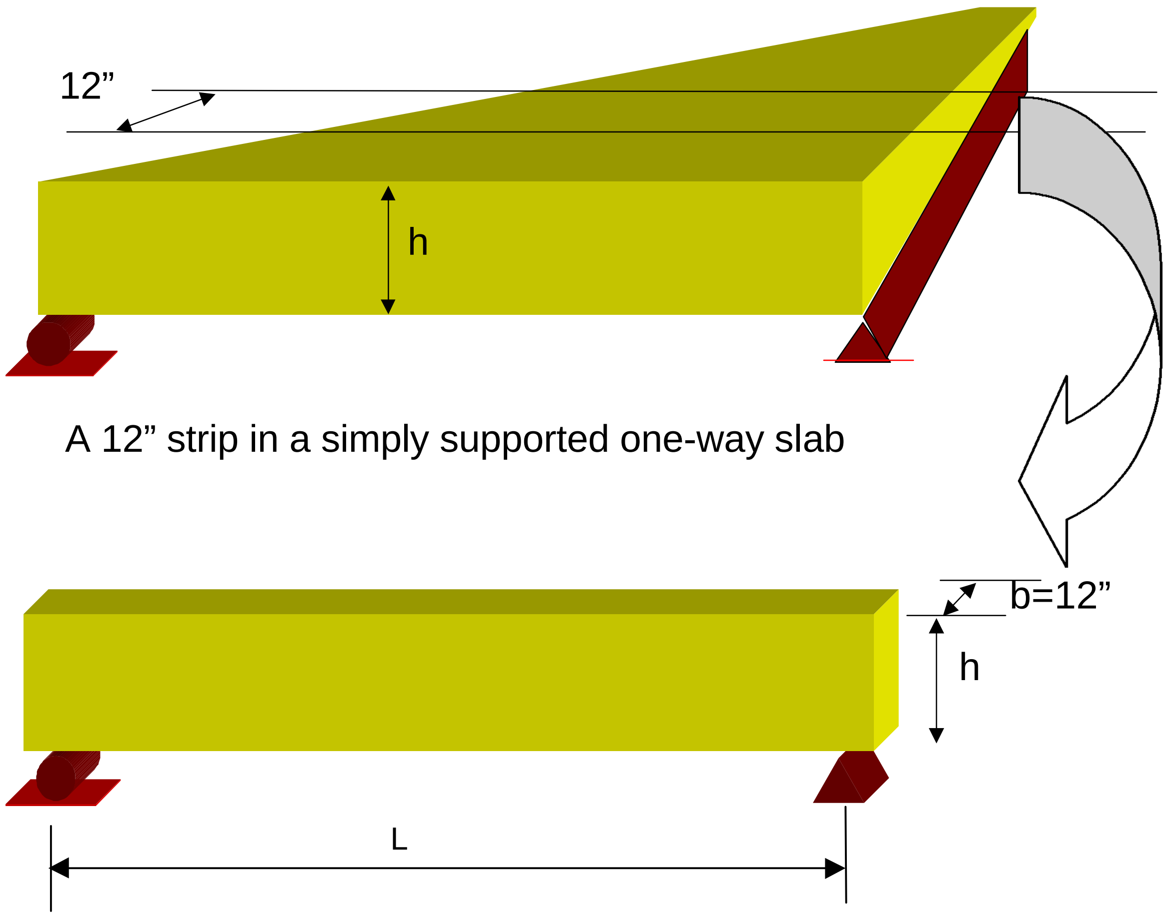
Chapter - 3

Design of Rectangular Beams and One-way Slabs



A 12” strip in a simply supported one-way slab

Two Methods:

1. Allowable stress design or working stress design (WSD)
2. Strength Design or Ultimate-strength design (USD)

Strength Design or Ultimate-strength design (USD)

With strength design method, the working dead load (DL) and live load (LL) are multiplied by certain load factors (equivalent to safety factors), and the resulting values are called **FACTORED LOADS.**

According to ACI 318-02:

Factored load, U=1.4DL ; U = 1.2 DL + 1.6 LL

U = 1.2 DL + 1.6 LL + 0.8 W; U=1.2DL +1.0 LL + 1.6 W

U = 0.9 DL + 1.6 W

(Where, W= wind load)

Similarly for other loads combinations.

Capacity Reduction Factors, φ ACI 318-02

φ = 0.90 for bending in reinforced concrete

φ = 0.75 for shear and torsion

φ = 0.65 for bearing on concrete

Balanced Steel Ratio

A beam that has a balanced steel ratio is one for which the tensile steel will theoretically start to yield and the compression concrete reach its ultimate strain (0.003) at exactly the same load.

Under-reinforced Beams

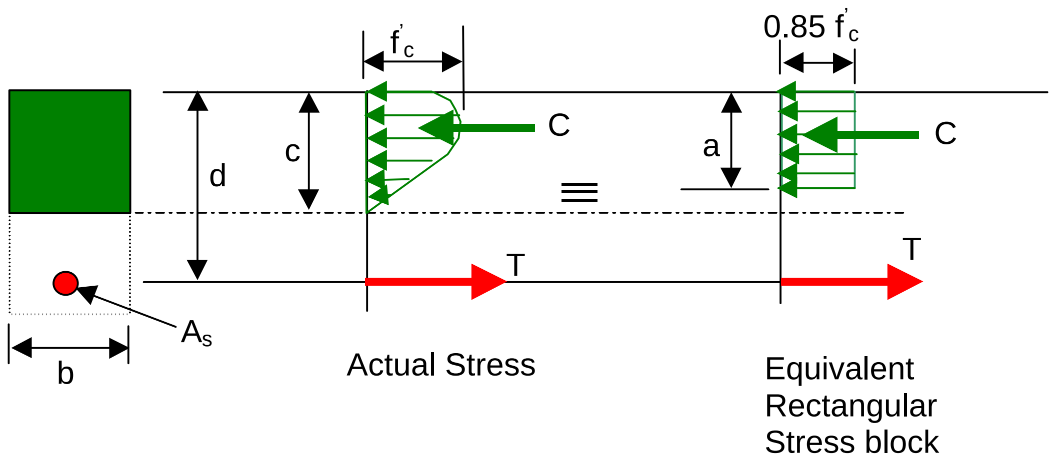
If a beam is under-reinforced and the ultimate load is approached, the steel will begin to yield even though the compression concrete is still under-stressed (strain < 0.003).

Over-reinforced Beams

If a beam should be over-reinforced, the steel will not yield before compression concrete failure (concrete strain = 0.003).

Over-reinforcing is a situation to be avoided if at all possible, and the ACI code, by limiting the percentage of tensile steel that may be used in a beam, ensures the design of under-reinforced beams and thus the ductile type of failures that provide adequate "running time".

Design of Rectangular Beams and One-way Slabs (Continued)



$a = \beta_1 c$

Where $\beta_1 = 0.85$ for $f_c' \leq 4{,}000$ psi, and decreases by 0.05 for every 1,000 psi above 4,000 psi. β_1 cannot be less than 0.65

$C = T$

$\Rightarrow 0.85\ f_c' \times a \times b = A_s \times f_y$

$\Rightarrow a = (A_s \times f_y)/(0.85\ f_c' \times b)$

$\Rightarrow a = (\rho \times f_y \times d)/(0.85\ f_c')$, where steel ratio, $\rho = A_s/(b \times d)$

Because the reinforcing steel is limited to an amount such that it will yield before the concrete reaches its ultimate strength, the nominal moment, M_n can be written as:

$M_n = T\ (d - 0.5a) = A_s\ f_y\ (d - 0.5a)$

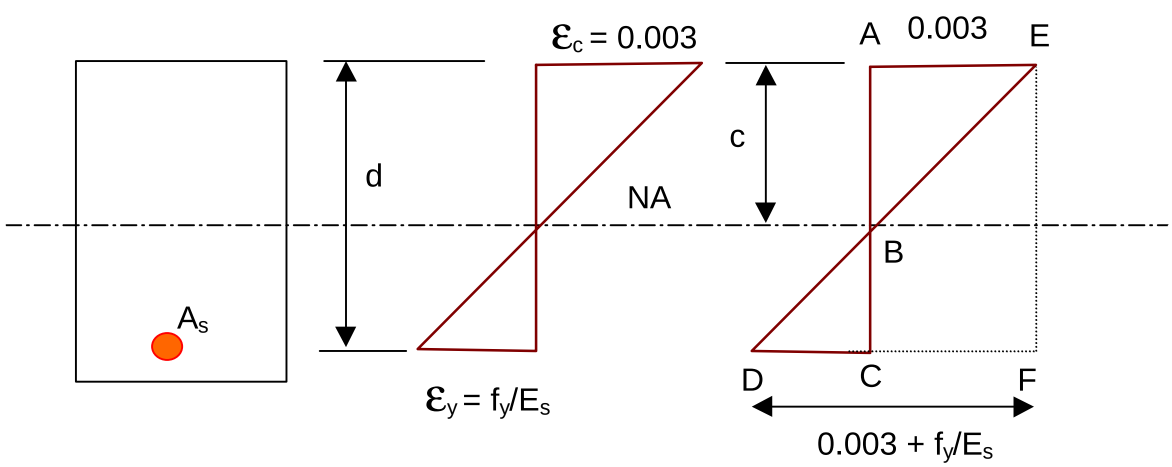
The usable flexural strength or design strength, ϕM_n is

$\phi M_n = \phi[\ A_s\ f_y\ (d - 0.5a)]$

Substituting $a = (\rho \times f_y \times d)/(0.85\ f_c')$,

$\phi M_n = \phi\ \rho\ f_y\ b\ d^2\ [1 - (0.59\ \rho\ f_y/f_c')]$

BALANCED STEEL RATIO, ρ_b



Strain Diagram

The NA is located by the triangular strain relationships, and is obtained by evaluating the similar triangles (ΔABE and ΔEDF), and is

AB/EF = AE/DF

$c / d = 0.003/[0.003 + f_y/E_s]$

Since, $E_s = 29 \times 10^6$ psi

$\Rightarrow$ c= 0.003 d /[0.003 + fy/29x10^6]

$\Rightarrow$ **c = 87,000 d / [87,000 + f_y]** , where f_y in psi

Since a = $(\rho \times f_y \times d)/(0.85 f_c')$

Therefore, **c = $a/\beta_1 = (\rho \times f_y \times d)/(0.85\ \beta_1\ f_c')$ = 87,000 d / [87,000 + f_y]**

Since, this is a balanced steel ratio ρ_b condition, change ρ by ρ_b

Therefore, **balanced steel ratio**,

$$\rho_b = [(0.85\ \beta_1\ f_c') / f_y] \times [87{,}000 / (87{,}000 + f_y)]$$

Maximum permissible steel ratio, ρ_{max}

An over-reinforced beam will fail in a brittle manner while an under-reinforced beam will fail in a ductile manner. To attempt to make sure that only ductile failures can occur the ACI code limits the maximum steel ratio as following:

$$\rho_{max} = 0.75\ \rho_b$$

$$\rho_{max} = 0.75[(0.85\ \beta_1\ f_c') / f_y]x[87{,}000 / (87{,}000 + f_y)]$$

Minimum permissible steel ratio, ρ_{min}

If the ultimate resisting moment of a beam section is less than its cracking moment capacity, the beam section will fail immediately when a crack occurs. ACI provides a minimum steel ratio, ρ_{min}

$\rho_{min} = (3\sqrt{f_c'}) / f_y$, **but not less than 200/fy**

Minimum slab thickness:

Use TABLE 9.5(a) of ACI 318/318R-97

Example-1: One-way simply supported slab, Span length, L= 10 ft.; Normal weight concrete (unit weight = 145 pcf), 60 Grade steel, f_y = 60,000. Minimum slab thickness, h = L/20 = 10' x12"/20 = 6". [Note: L=span length in inches].

Example-2: One-way simply supported slab, Span length, L= 10 ft.; Normal weight concrete (unit weight = 145 pcf), 40 Grade steel, f_y=40,000 psi. Minimum slab thickness, h = L/20 x [0.4 + f_y/100,000] = 10' x12"/20 [0.4 + 40,000/100,000] = 6" x 0.8 = 4.8". [Note: L=span length in inches].

Rectangular Reinforced Concrete Beam Analysis

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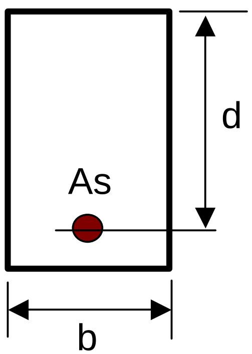

d=	20	in
b=	16	in
As	4.74	sqin

fc' =	3	ksi
fy =	60	ksi

ρ = As/(b.d) = 0.01481

FROM TABLE A-5 (Page 462)

ρ max=	0.0160
ρ min=	0.0033

ρ <= ρ max — N → NOT ACCEPTED - ACI CODE

Y ↓

ρ >= ρ min — N → NOT ACCEPTED - ACI CODE

Y ↓

PERMISSIBLE DESIGN MOMENT STRENGTH

$$\phi Mn = \phi.\rho.f_y.b.d^2\,[1 - (0.59\,\rho.f_y/f_c')]$$

ϕMn =

4224.43	k-in
352.04	k-ft

β_1 = 0.85 for $f_c' \leq 4{,}000$ psi, and decreases by 0.05 for every 1,000 psi above 4,000 psi. β_1 cannot be less than 0.65; β_1 = 0.85

Put fc' & fy in psi (NOT IN KSI)

$\rho_{max} = 0.75\ \rho_b$

$= 0.75\ \{(0.85\ \beta_1\ f_c') / fy\}.\{87000 / (87000 + f_y)\}$ = 0.0160

$\rho_{min} = 3\sqrt{f_c'} / f_y >= 200/f_y$ = 0.0033

Rectangular Reinforced Concrete Beam Design

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$\varphi M_n = \varphi\ \rho\ f_y\ b\ d^2\ [1 – (0.59\ \rho\ f_y/f_c')]$

For design problem, put $\varphi M_n = M_u$

Select ρ (values in between ρ_{max} and ρ_{min}), and find d.

$d = \sqrt{M_u/[\varphi\ \rho\ f_y\ b\{1 – (0.59\ \rho\ f_y/f_c')\}]}$

Select d, and ρ find.
Check $\rho_{min} <= \rho <= \rho_{max}$.
If not, select another d, and repeat.

Method 1

Calculate $\bar{k} = M_u / [\varphi\ b\ d^2]$

Look at Tables A-7 through A-11, and find ρ

USE Course Website Tables 5 through 8

EXAMPLE

Method 2

$M_u = \varphi\ \rho\ f_y\ b\ d^2\ [1 – (0.59\ \rho\ f_y/f_c')]$

Solve the quadratic equation of ρ. You will get two values of ρ.
Accept the value of ρ which satisfies $\rho_{min} <= \rho <= \rho_{max}$.
If none of the values satisfies, you need to change d.

One-Way Slab Design

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