

# ENDS 231

## ARCHITECTURAL STRUCTURES I

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LECTURE NOTE SET  
Spring 2008



by

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## ENDS 231. Architectural Structures I

**Instructor:** Prof. Anne B. Nichols  
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**Office Hours:** 1-2 pm MW  
10:00-11:30 am TR  
(and by appointment M-R)

**Prerequisites:** ENDS 106; MATH 142 or equivalent (linear algebra and calculus); PHYS 201

**Catalogue Description:** Introduction to the physical principles that govern classical statics and strengths of materials through the design of timber and steel components of architectural structures; computer applications.

**Goals:** ENDS 231 is the study of structural design concepts that influence the development of architectural space and form. In all engineering construction, the component parts of a structure must be assigned definite physical sizes, constructed of specific materials and designed to resist various load combinations. The course is divided into two parts: Statics and Strength of Materials. **Statics** is the branch of mechanics that involves the study of external forces and the effects of these forces on bodies or structural systems in equilibrium (at rest or moving with a constant velocity). **Strength of Materials** involves analytical methods for determining the strength, stiffness (deformation characteristics), and stability of the various load-carrying members. Members are designed for specific materials using current national design specifications.

**Objective:** To understand the significance, assumptions, applications, and limitations of the basic principles of Statics and Strength of Materials as they apply to the design and analysis of structural members and simple connections.

**Text:** Statics and Strength of Materials –Foundations for Structural Design, Onouye, (2005)  
Pearson - Prentice Hall, ISBN 0-13-111837-4

**Reference:** ACI 318-02 Code and Commentary  
AISC 3<sup>rd</sup> ed. Load and Resistance Factor Design  
AISC 9<sup>th</sup> ed. Allowable Stress Design  
National Design Specifications for Wood

**Timetable:** CREDIT 3.0 (2:2) 3:55-4:45 pm Lecture T,R  
(section 501) 4:45-5:35 pm Lab T,R

**Grading:** The levels listed for graded work (projects, quizzes, exams) and pass-fail work (assignments) *must be met or exceeded* to earn the course letter grade:

<i>Letter Grade</i>	<i>Graded work</i>	<i>Pass-fail work</i>
A	A average (90-100%)	Pass for 90% to 100% of assignments
B	B average (80-89%)	Pass for 83% to 100% of assignments
C	C average (70-79%)	Pass for 75% to 100% of assignments
D	D average (60-69%)	Pass for 65% to 100% of assignments
F	F average (<59%)	Pass for 0% to 100% of assignments

*Graded work:* This typically constitutes 10 quizzes, a learning portfolio (worth 1.5 quizzes) and a final exam (worth 4 quizzes). This equates to proportions of approximately 64.5% to quizzes, 9.7% to the learning portfolio, and 25.8% to the final exam.

*Pass/fail work:* This constitutes all practice assignments and projects, each with a value of 1 unit. Criteria for passing is *at least* 75% completeness and correctness along with every problem attempted. Percent effort expected for a problem in a practice assignment is provided on the assignment statement. This is considered a lab course and the assignments **are required work** with credit given for competency. The work is necessary to apply the material and prepare for the quizzes and exam. It is expected that this work will be completed with assistance or group participation, but all *graded* work is only by the individual.

- Policy: 1) Attendance:** Necessary. Required.\* And subject to University Policy. See Part I Section 7 in Texas A&M University Student Rules: <http://student-rules.tamu.edu/> Absences related to illness or injury must be documented according to <http://shs.tamu.edu/attendance.htm> *including* the Explanatory Statement for Absence from class for 3 days or less. Doctors visits not related to immediate illness or injury are not excused absences.
- 2) Lecture, Lab and Textbook:** The lecture slide shows that correspond to the Handouts (see #3) are to be viewed prior to lecture which will be reserved for review of the full lecture and text reading. Lab will consist of problem solving requiring the textbook. The lecture shows are available on the class web page, class folder (see #3), and Vista (see #7). Attendance is required for both lecture and lab.
- 3) Notes:** The notes and related handouts are available on the class web page at [http://archone.tamu.edu/faculty/anichols/index\\_files/courses/ends231/index.html](http://archone.tamu.edu/faculty/anichols/index_files/courses/ends231/index.html), on Vista (see #7) or in the class folder on \\Xavier\classes\ENDS231501. A full set can be purchased from the TEES copy center located on the second floor of Wisenbaker Engineering Research Lab. They are listed under Anne Nichols, ENDS 231. **COSC 321 notes are NOT EQUIVALENT.**
- 4) Assignments:** Due as stated on the assignment statements. One late assignment will be allowed without excuse turned in no later than one week after the due date. All other assignments and projects will receive no credit if late. Assignments with incorrect formatting will be penalized.
- | <i>Format:</i> |      |        |
|----------------|------|--------|
| Date           | Name | Course |
| Given:         |      |        |
| Find:          |      |        |
| Solution:      |      |        |
| :              |      |        |
- 5) Quizzes:** Quizzes will be given at any time during the period. Make-up quizzes without an excuse will not be given. Practice quizzes will be posted electronically.
- 6) Grader:** Hidekazu Takahashi.... ([wish-takahashi@tamu.edu](mailto:wish-takahashi@tamu.edu))
- 7) Vista:** Vista is a web course tool for posting, reading messages and replying as well as recording scores and is accessed with your neo account. This will be used to post questions and responses by class members and the instructor, for posting scores and for e-mail. It can be accessed at <http://elearning.tamu.edu/>
- 8) Final Exam:** The final exam will be comprehensive, and is officially scheduled for **1:00-3:00 PM, Tuesday, May 6.**
- 9) Other Resources:** The Student Learning Center provides tutoring in math and physics. See their schedule at <http://slc.tamu.edu/tutoring.shtml>



- 10) Aggie Honor Code:** "An Aggie does not lie, cheat, or steal or tolerate those who do."  
The University policy will be strictly enforced. See Part I Section 20 in Texas A&M University Student Rules: <http://student-rules.tamu.edu/> Plagiarism (deliberate misrepresentation of someone else's work as your own) will be treated strictly according to University policy as outlined by the Office of the Aggie Honor System: <http://www.tamu.edu/aggiehonor/>
- 11) The American with Disabilities Act (ADA)** is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring accommodation, please contact the Department for Student Life, Services for Students with Disabilities, in Cain Hall or call 845-1637. Also contact Prof. Nichols at the beginning of the semester.

### Learning Objectives:

- 1) The student will be able to read a text or article about structural technology, identify the key concepts and related equations, and properly apply the concepts and equations to appropriate structural problems (**relevance**). The student will also be able to define the answers to key questions in the reading material. The student will be able to evaluate their own skills, or lack thereof, with respect to reading and comprehension of structural concepts, **clarity** of written communication, reasonable determination of **precision** in numerical data, and **accuracy** of computations.
- 2) The student will be able to read a problem statement, interpret the structural wording in order to identify the concepts and select equations necessary to solve the problem presented (**significance**). The student will be able to identify common steps in solving structural problems regardless of the differences in the structural configuration and loads, and apply these steps in a clear and structured fashion (**logic**). The student will draw upon existing mathematical and geometrical knowledge to gather information, typically related to locations and dimensions, provided by representational drawings or models of structural configurations, and to present information, typically in the form of plots that graph variable values. The student will be able to draw representational structural models and diagrams, and express information provided by the figures in equation form. The student will compare the computational results in a design problem to the requirements and properly decide if the requirements have been met. The student will take the corrective action to meet the requirements
- 3) The student will create a structural model with a computer application based on the concepts of the behavior and loading of the structural member or assemblage. The student will be able to interpret the modeling results and relate the results to the solution obtained by manual calculations.
- 4) The student will be able to articulate the physical phenomena, behavior and design criteria which influence structural space and form. (**depth**) The student will be able to identify the structural purpose, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies. (**breadth**) The student will create a physical structure or structures using non-traditional building materials, considering material and structural behavior, in order to demonstrate the behavior and limitations of a variety of structural arrangements.

- 5) The student will interact and participate in group settings to facilitate peer-learning and teaching. In addition, the student will be able to evaluate the comprehension of concepts, clarity of communication of these concepts or calculations, and the precision and accuracy of the data used in the computations in the work of their peers.

<b>Lecture</b>	<b>Text Topic</b>	<b>Articles/ Problems</b>
1.	Basic Concepts and Principles	<b>Read*:</b> Text Ch. 1, Appendix B; note sets 1.1, 1.2 & 1.3
2.	Forces	<b>Read:</b> Text 5.1, 2.1–2.3; note set 2 <b>Solve:</b> Assignment 1
3.	Moments	<b>Read:</b> Text 2.4; note set 3
4.	Equilibrium of a Particle	<b>Read:</b> Text 3.1; note set 4 <b>Solve:</b> Assignment 2
5.	Free Body Diagrams Support Conditions	<b>Read:</b> Text 3.2, 4.3; note set 5
6.	Introduction to Trusses Method of Joints	<b>Read:</b> Text 4.1 (89-97); note set 6 (first part) <b>Solve:</b> Assignment 3
7.	Design Project Reviews	<b>Project due</b>
8.	Trusses – Method of Sections	<b>Read:</b> Text 4.1 (98-110); note set 6 (rest) <b>Reference:</b> note set 7 <b>Quiz 1</b>
9.	Pinned Frames & Hinged Arches	<b>Read:</b> Text 4.2; note set 9 <b>Solve:</b> Assignment 4
10.	Distributed Loads on Beams, Concentrated Loads and Load Tracing	<b>Read:</b> Text 3.3, 5.2-5.3; note set 10 <b>Quiz 2</b>
11.	Structural Properties of Areas - Centroids	<b>Read:</b> Text 7.1; note set 11 <b>Solve:</b> Assignment 5
12.	Structural Properties of Areas – Moment of Inertia	<b>Read:</b> Text 7.2-7.4; note set 12 <b>Quiz 3</b>
13.	Beam Forces	<b>Read:</b> Text 8.1-8.2; note set 13 <b>Solve:</b> Assignment 6
14.	Shear and Bending Moment Diagrams	<b>Read:</b> Text 8.3-8.4; (note set 13) <b>Reference:</b> note set 14 <b>Quiz 4</b>
15.	Material Properties – Stress & Connections	<b>Read:</b> Text 6.1; note set 15 <b>Solve:</b> Assignment 7

<b>Lecture</b>	<b>Text Topic</b>	<b>Articles/ Problems</b>
16.	Material Properties – Strain, Strength and Elasticity	<b>Read:</b> Text 6.2-6.3; note set 16 <b>Quiz 5</b>
17.	Torsion Stress and Thermal Strain	<b>Read:</b> Text 6.4; note set 17 <b>Solve:</b> Assignment 8
18.	Stresses in Beams – Bending	<b>Read:</b> Text 9.1-9.2; Appendix C.1; note set 18 (first part) <b>Quiz 6</b>
19.	Design Project Reviews	<b>Project due</b>
20.	Stresses in Beams –Shear & Connectors	<b>Read:</b> Text 9.3-9.4; Appendix C.2; note set 18 (rest) <b>Solve:</b> Assignment 9
21.	Beam Deformation & Design	<b>Read:</b> Text 9.5-9.6; Appendix C.3; note set 21 <b>Quiz 7</b>
22.	LRFD Steel Design – Beams	<b>Read:</b> note set 22 <b>Solve:</b> Assignment 10
23.	Stability of Structures & Design	<b>Read:</b> Text 10.1-10.2; note set 23 <b>Quiz 8</b>
24.	Column Design – Wood, Steel & LRFD Steel	<b>Read:</b> Text 10.3-10.4; note set 24 <b>Solve:</b> Assignment 11
25.	Design of Eccentrically Loaded Columns	<b>Read:</b> Text 10.5; note set 25 <b>Quiz 9</b>
26.	Tension Members and Connections - Steel	<b>Read:</b> note set 26 <b>Solve:</b> Assignment 12
27.	Rigid and Braced Frames	<b>Read:</b> note set 27.1 <b>Reference:</b> note set 27.2 <b>Quiz 10</b>
28.	Review	<b>Read:</b> note set 28 <b>Learning Portfolio due</b>
	Final Exam Period	<b>Exam</b>

\*Note: Materials in the Class Note Set not specifically mentioned above are provided as references or aids.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
JANUARY			1	2	3	4	5
	6	7	8	9	10	11 last day to register	12
	13	14 classes begin	15 <b>Lect 1</b>	16	17 <b>Lect 2</b>	18 last day to add/drop	19
	20	21 King Holiday	22 <b>Lect 3</b>	23	24 <b>Lect 4</b> #1 due	25	26
	27	28	29 <b>Lect 5</b>	30	31 <b>Lect 6</b> #2 due	1	2
FEBRUARY	3	4	5 <b>Lect 7</b> project	6	7 <b>Lect 8</b> Quiz 1	8	9
	10	11	12 <b>Lect 9</b> #3 due	13	14 <b>Lect 10</b> Quiz 2	15	16
	17	18	19 <b>Lect 11</b> #4 due	20	21 <b>Lect 12</b> Quiz 3	22	23
	24	25	26 <b>Lect 13</b> #5 due	27	28 <b>Lect 14</b> Quiz 4	29	1
MARCH	2	3 mid-term grades due	4 <b>Lect 15</b> #6 due	5	6 <b>Lect 16</b> Quiz 5	7	8
	9	10	11	12 Spring Break	13	14	15
	16	17	18 <b>Lect 17</b> #7 due	19	20 <b>Lect 18</b> Quiz 6	21 Reading Day	22
	23	24	25 <b>Lect 19</b> project	26	27 <b>Lect 20</b> #8 due	28	29
	30	31	1 <b>Lect 21</b> Quiz 7 and last day to Q-drop	2	3 <b>Lect 22</b> #9 due	4	5
APRIL	6	7	8 <b>Lect 23</b> Quiz 8	9	10 <b>Lect 24</b> #10 due and pre-registration begins (to 25 <sup>th</sup> )	11	12
	13	14	15 <b>Lect 25</b> Quiz 9	16	17 <b>Lect 26</b> #11 due	18	19 Muster
	20	21	22 <b>Lect 27</b> Quiz 10	23	24 <b>Lect 28</b> #12 & portfolio due	25	26
	27	28 (dead day) Monday classes	29 (dead day) Friday classes	30 Reading	1 Days	2 Final exams	3
MAY	4	5	6 1-3pm 231 FINAL	7	8	9 Commencement (and Saturday)	10
	11	12 Grades due	13	14	15	16	17

## ENDS 231. Student Understandings

- 1) I understand that there are intellectual standards in this course and that I am responsible for monitoring my own learning. \_\_\_\_\_
- 2) I understand that the class will focus on practice, not on lecture. \_\_\_\_\_
- 3) I understand that I am responsible for preparing for lecture with the assigned reading and lecture show by internalizing key concepts, recognizing key questions, and evaluating what makes sense and what doesn't make sense to me. \_\_\_\_\_
- 4) I understand that I will be held regularly responsible for assessing my own work using criteria and standards discussed in class. \_\_\_\_\_
- 5) I understand that if at any time in the semester I feel unsure about my "grade", I may request and assessment from the instructor. \_\_\_\_\_
- 6) I understand that there are 12 practice assignments, one due every week during the bulk of the semester. \_\_\_\_\_
- 7) I understand that there are group projects and I will be responsible to take an active part in advancing the work of the group. \_\_\_\_\_
- 8) I understand that I will occasionally be required to assess the work of my classmates in an objective manor using the same criteria and standards used to assess my own work. \_\_\_\_\_
- 9) I understand that there are 10 graded quizzes, one given every week during the bulk of the semester. \_\_\_\_\_
- 10) I understand that there is a final exam in the course. \_\_\_\_\_
- 11) I understand that I must do a Learning Portfolio, which is a self-evaluation that makes my "case" for receiving a particular grade using criteria provided in class and citing evidence from my work across the semester. \_\_\_\_\_
- 12) I understand that the work of the course requires Consistent classroom attendance and active participation. \_\_\_\_\_
- 13) I understand that I will regularly be required to demonstrate that I have prepared for lecture. \_\_\_\_\_
- 14) I understand that the class will not be graded on a curve. I understand that it is theoretically possible for the whole class to get an A or an F. \_\_\_\_\_
- 15) I understand the basis of the final grade as outlined in the syllabus. \_\_\_\_\_
- 16) I understand that since the final grade is based on percentages from graded work and competency on assignments as outlined in the syllabus, that the minimum level of both must be satisfied to obtain the letter grade. The criteria for assignments that are considered "passing" is outlined in the syllabus section on Learning Objectives. \_\_\_\_\_

NAME (sign and print) \_\_\_\_\_

\_\_\_\_\_

DATE \_\_\_\_\_

## List of Symbol Definitions

- a* long dimension for a section subjected to torsion (in, mm);  
acceleration (ft/sec<sup>2</sup>, m/sec<sup>2</sup>)
- A* area, often cross-sectional (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>)
- A<sub>e</sub>* net effective area, equal to the total area ignoring any holes (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>) (*see A<sub>net</sub>*);
- A<sub>g</sub>* gross area, equal to the total area ignoring any holes (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>)
- A<sub>net</sub>* net effective area, equal to the gross area subtracting any holes (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>) (*see A<sub>e</sub>*)
- A<sub>p</sub>* bearing area (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>)
- A<sub>throat</sub>* area across the throat of a weld (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>)
- A<sub>web</sub>* web area in a steel beam equal to the depth x web thickness (in<sup>2</sup>, ft<sup>2</sup>, mm<sup>2</sup>, m<sup>2</sup>)
- ASD* Allowable Stress Design
- b* width, often cross-sectional (in, ft, mm, m);  
narrow dimension for a section subjected to torsion (in, mm);  
number of truss members
- b<sub>f</sub>* width of the flange of a steel beam cross section (in, mm)
- c* distance from the neutral axis to the top or bottom edge of a beam (in, mm, m);  
distance from the center of a circular shape to the surface under torsional shear strain (in, mm, m)
- c<sub>i</sub>* distance from the center of a circular shape to the inner surface under torsional shear strain (in, mm, m)
- c<sub>o</sub>* distance from the center of a circular shape to the outer surface under torsional shear strain (in, mm, m)
- c<sub>1</sub>* coefficient for shear stress for a rectangular bar in torsion
- c<sub>2</sub>* coefficient for shear twist for a rectangular bar in torsion
- CL*,  $\phi$  center line
- C* compression label;  
compression force (lb, kips, N, kN)
- C<sub>b</sub>* modification factor for moment in ASD & LRFD steel beam design,  $C_b = 1$  for simply supported beams (0 moments at the ends)
- C<sub>c</sub>* column slenderness classification constant for steel column design
- C<sub>D</sub>* load duration factor for wood design
- C<sub>F</sub>* size factor for wood design
- C<sub>m</sub>* modification factor for combined stress in steel design
- C<sub>M</sub>* wet service factor for wood design
- C<sub>p</sub>* column stability factor for wood design
- C<sub>t</sub>* temperature factor for wood design

$d$	depth, often cross-sectional (in, mm, m); perpendicular distance from a force to a point in a moment calculation (in, mm, m)
$d_x$	difference in the x direction between an area centroid ( $\bar{x}$ ) and the centroid of the composite shape ( $\hat{x}$ ) (in, mm)
$d_y$	difference in the y direction between an area centroid ( $\bar{y}$ ) and the centroid of the composite shape ( $\hat{y}$ ) (in, mm)
$D$	diameter of a circle (in, mm, m); dead load for LRFD design
$DL$	dead load
$e$	eccentric distance of application of a force (P) from the centroid of a cross section (in, mm)
$E$	modulus of elasticity (psi; ksi, kPa, MPa, GPa); earthquake load for LRFD design
$f$	symbol for stress (psi, ksi, kPa, MPa)
$f_a$	calculated axial stress (psi, ksi, kPa, MPa)
$f_b$	calculated bending stress (psi, ksi, kPa, MPa)
$f_c$	calculated compressive stress (psi, ksi, kPa, MPa)
$f_{cr}$	calculated column stress based on the critical column load $P_{cr}$ (psi, ksi, kPa, MPa)
$f_t$	calculated tensile stress (psi, ksi, kPa, MPa)
$f_p$	calculated bearing stress (psi, ksi, kPa, MPa)
$f_x$	combined stress in the direction of the major axis of a column (psi, ksi, kPa, MPa)
$f_v$	calculated shearing stress (psi, ksi, kPa, MPa)
$f_y$	yield stress (psi, ksi, kPa, MPa)
$F$	force (lb, kip, N, kN); capacity of a nail in shear (lb, kip, N, kN); symbol for allowable stress in design codes (psi, ksi, kPa, MPa)
$F_a$	allowable axial stress (psi, ksi, kPa, MPa)
$F_b$	allowable bending stress (psi, ksi, kPa, MPa)
$F'_b$	allowable bending stress for combined stress for wood design (psi, ksi, kPa, MPa)
$F_c$	allowable compressive stress (psi, ksi, kPa, MPa)
$F_{connector}$	resistance capacity of a connector (lb, kips, N, kN)
$F_{cE}$	intermediate compressive stress for ASD wood column design dependant on material (psi, ksi, kPa, MPa)
$F'_c$	allowable compressive stress for ASD wood column design (psi, ksi, kPa, MPa)
$F^*_c$	intermediate compressive stress for ASD wood column design dependant on load duration (psi, ksi, kPa, MPa)

$F'_e$	allowable buckling stress for combined bending steel design (psi, ksi, kPa, MPa)
$F_t$	allowable tensile stress (psi, ksi, kPa, MPa)
$F_v$	allowable shear stress (psi, ksi, kPa, MPa); allowable shear stress in a welded connection
$F_x$	force component in the x coordinate direction (lb, kip, N, kN)
$F_y$	force component in the y coordinate direction (lb, kip, N, kN); yield stress (psi, ksi, kPa, MPa)
$F_u$	ultimate stress a material can sustain prior to failure (psi, ksi, kPa, MPa)
$F.S.$	factor of safety
$g$	acceleration due to gravity, 32.17 ft/sec <sup>2</sup> , 9.807 m/sec <sup>2</sup>
$G$	shear modulus (psi; ksi, kPa, MPa, GPa)
$h$	depth, often cross-sectional (in, ft, mm, m); sag of a cable structure (ft, m)
$I$	moment of inertia (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$\bar{I}$	moment of inertia about the centroid (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$I_c$	moment of inertia about the centroid (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$I_{min}$	minimum moment of inertia of $I_x$ and $I_y$ (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$I_x$	moment of inertia with respect to an x-axis (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$I_y$	moment of inertia with respect to a y-axis (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$J, J_o$	polar moment of inertia (in <sup>4</sup> , mm <sup>4</sup> , m <sup>4</sup> )
$k$	kips (1000 lb); shape factor for plastic design of steel beams, $M_p/M_y$
$kg$	kilograms
$kN$	kiloNewtons (10 <sup>3</sup> N)
$kPa$	kiloPascals (10 <sup>3</sup> Pa)
$K$	effective length factor with respect to column end conditions
$K_{cE}$	material factor for wood column design
$\ell$	length (in, ft, mm, m); cable span (ft, m)
$lb$	pound force
$L$	length (in, ft, mm, m); live load for LRFD design
$L_b$	unbraced length of a steel beam in LRFD design (in, ft, mm, m)
$L_c$	maximum unbraced length of a steel beam in ASD design for maximum allowed bending stress (in, ft, mm, m)
$L_e$	effective length that can buckle for column design (in, ft, mm, m)
$L_r$	roof live load in LRFD design



$L_p$	maximum unbraced length of a steel beam in LRFD design for full plastic flexural strength (in, ft, mm, m)
$L_r$	maximum unbraced length of a steel beam in LRFD design for inelastic lateral-torsional buckling (in, ft, mm, m)
$L_u$	maximum unbraced length of a steel beam in ASD design for reduced allowed bending stress (in, ft, mm, m)
$LL$	live load
$LRFD$	Load and Resistance Factor Design
$m$	mass (lb-mass, g, kg); meters
$mm$	millimeters
$M$	moment of a force or couple (lb-ft, kip-ft, N-m, kN-m); bending moment (lb-ft, kip-ft, N-m, kN-m)
$M_A$	moment value at quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
$M_B$	moment value at half point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
$M_C$	moment value at three quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
$M_n$	nominal flexure strength with the full section at the yield stress for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
$M_p$	(also $M_{ult}$ ) internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)
$M_u$	maximum moment from factored loads for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)
$M_{ult}$	(also $M_p$ ) internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)
$M_y$	internal bending moment when the extreme fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)
$M_1$	smaller end moment used to calculate $C_m$ for combined stresses in a beam-column (lb-ft, kip-ft, N-m, kN-m)
$M_2$	larger end moment used to calculate $C_m$ for combined stresses in a beam-column (lb-ft, kip-ft, N-m, kN-m)
$MPa$	megaPascals ( $10^6$ Pa or $1 \text{ N/mm}^2$ )
$n$	number of truss joints, nails or bolts
$n.a.$	neutral axis (axis connecting beam cross-section centroids)
$N$	Newtons ( $\text{kg}\cdot\text{m}/\text{sec}^2$ ); bearing-type connection with bolt threads included in shear plane
$O$	point of origin
$p$	pitch of nail spacing (in, ft, mm, m)
$P$	force, concentrated (point) load (lb, kip, N, kN); axial load in a column or beam-column (lb, kip, N, kN)

$P_{cr}$	critical (failure) load in column calculations (lb, kip, N, kN)
$P_n$	nominal load strength capacity for LRFD design (lb, kip, N, kN)
$P_u$	maximum load from factored loads for LRFD design (lb, kip, N, kN)
$Pa$	Pascals ( $N/m^2$ )
$q$	shear flow (lb/in, kips/ft, N/m, kN/m)
$Q$	first moment area used in shearing stress calculations ( $in^3$ , $mm^3$ , $m^3$ )
$Q_{connected}$	first moment area used in shearing stress calculations for built-up beams ( $in^3$ , $mm^3$ , $m^3$ )
$Q_x$	first moment area about an x axis (using y distances) ( $in^3$ , $mm^3$ , $m^3$ )
$Q_y$	first moment area about an y axis (using x distances) ( $in^3$ , $mm^3$ , $m^3$ )
$r$	radius of a circle (in, mm, m); radius of gyration (in, mm, m)
$r_o$	polar radius of gyration (in, mm, m)
$r_x$	radius of gyration with respect to an x-axis (in, mm, m)
$r_y$	radius of gyration with respect to a y-axis (in, mm, m)
$R$	force, reaction or resultant (lb, kip, N, kN); radius of curvature of a beam (ft, m); rainwater or ice load for LRFD design; generic design quantity (force, shear, moment, etc.) for LRFD design
$R_n$	generic nominal capacity (force, shear, moment, etc.) for LRFD design
$R_u$	generic maximum quantity (force, shear, moment, etc.) from factored loads for LRFD design
$R_x$	reaction or resultant component in the x coordinate direction (lb, kip, N, kN)
$R_y$	reaction or resultant component in the y coordinate direction (lb, kip, N, kN)
$s$	length of a segment of a thin walled section (in, mm)
$s.w.$	self-weight
$S$	section modulus ( $in^3$ , $mm^3$ , $m^3$ ); snow load for LRFD design; allowable strength per length of a weld for a given size (lb/in, kips/in, N/mm, kN/m)
$S_{required}$	section modulus required to not exceed allowable bending stress ( $in^3$ , $mm^3$ , $m^3$ )
$S_x$	section modulus with respect to the x-centroidal axis ( $in^3$ , $mm^3$ , $m^3$ )
$S_y$	section modulus with respect to the y-centroidal axis ( $in^3$ , $mm^3$ , $m^3$ )
$SC$	slip critical bolted connection
$S4S$	surface-four-sided
$t$	thickness (in, mm, m)
$t_f$	thickness of the flange of a steel beam cross section (in, mm, m)
$t_w$	thickness of the web of a steel beam cross section (in, mm, m)
$T$	tension label; tensile force (lb, kip, N, kN); torque (lb-ft, kip-ft, N-m, kN-m); throat size of a weld (in, mm)

$V$	shearing force (lb, kip, N, kN)
$V_n$	nominal shear strength capacity for LRFD beam design (lb, kip, N, kN)
$V_u$	maximum shear from factored loads for LRFD beam design (lb, kip, N, kN)
$w$	(also $\omega$ ) load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m)
$W$	weight (lb, kip, N, kN); total load from a uniform distribution (lb, kip, N, kN); wind load for LRFD design
$x$	a distance in the x direction (in, ft, mm, m)
$\bar{x}$	the distance in the x direction from a reference axis to the centroid of a shape (in, mm)
$\hat{x}$	the distance in the x direction from a reference axis to the centroid of a composite shape (in, mm)
$X$	bearing-type connection with bolt threads excluded from shear plane
$y$	a distance in the y direction (in, ft, mm, m); distance from the neutral axis to the y-level of a beam cross section (in, mm)
$\bar{y}$	the distance in the y direction from a reference axis to the centroid of a shape (in, mm)
$\hat{y}$	the distance in the y direction from a reference axis to the centroid of a composite shape (in, mm)
$Z$	plastic section modulus of a steel beam (in <sup>3</sup> , mm <sup>3</sup> )
'	symbol for feet
"	symbol for inches
#	symbol for pounds
$\alpha$	coefficient of thermal expansion (/°C, /°F); angle, in a math equation (degrees, radians)
$\beta$	angle, in a math equation (degrees, radians)
$\delta$	elongation (in, mm)
$\delta_p$	elongation due to axial load (in, mm)
$\delta_s$	shear deformation (in, mm)
$\delta_T$	elongation due to change in temperature (in, mm)
$\Delta$	beam deflection (in, mm); an increment
$\Delta_{LL}$	beam deflection due to live load (in, mm)
$\Delta_{max}$	maximum calculated beam deflection (in, mm)
$\Delta_{TL}$	beam deflection due to total load (in, mm)
$\Delta T$	change in temperature (°C, °F)
$\varepsilon$	strain (no units)
$\varepsilon_t$	thermal strain (no units)

$\phi$	diameter symbol; angle of twist (degrees, radians); resistance factor in LRFD steel design
$\phi_b$	resistance factor for flexure in LRFD steel design
$\phi_c$	resistance factor for compression in LRFD steel design
$\phi_t$	resistance factor for tension in LRFD steel design
$\phi_v$	resistance factor for shear in LRFD steel design
$\lambda_c$	design constant for slenderness evaluation for steel columns in LRFD design
$\mu$	Poisson's ratio
$\gamma$	specific gravity of a material (lb/in <sup>3</sup> , lb/ft <sup>3</sup> , N/m <sup>3</sup> , kN/m <sup>3</sup> ); angle, in a math equation (degrees, radians); shearing strain; load factor in LRFD design
$\gamma_D$	dead load factor in LRFD steel design
$\gamma_L$	live load factor in LRFD steel design
$\theta$	angle, in a trig equation (degrees, radians); slope of the deflection of a beam at a point (degrees, radians)
$\pi$	pi
$\rho$	radial distance (in, mm)
$\sigma$	engineering symbol for normal stress (axial or bending)
$\tau$	engineering symbol for shearing stress
$\Sigma$	summation symbol
$\omega$	(also w) load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m)

**THE DEACON'S MASTERPIECE**  
**Or, the Wonderful "one-Hoss Shay"**  
**A Logical Story**  
**By Oliver Wendell Homes**

HAVE you heard of the wonderful one-hoss-shay,  
That was built in such a logical way  
It ran a hundred years to a day,  
And then, of a sudden, it--ah, but stay  
I'll tell you what happened without delay,  
Scaring the parson into fits,  
Frightening people out of their wits,--  
Have you ever heard of that, I say?

Seventeen hundred and fifty-five,  
Georgius Secundus was then alive,--  
Snuffy old drone from the German hive;  
That was the year when Lisbon-town  
Saw the earth open and gulp her down,  
And Braddock's army was done so brown,  
Left without a scalp to its crown.  
It was on the terrible earthquake-day  
That the Deacon finished the one-hoss-shay.

Now in building of chaises, I tell you what,  
There is always somewhere a weakest spot,--  
In hub, tire, felloe, in spring or thill,  
In panel, or crossbar, or floor, or sill,

In screw, bolt, thoroughbrace,--lurking still,  
Find it somewhere you must and will,--  
Above or below, or within or without,--  
And that's the reason, beyond a doubt,  
A chaise breaks down, but doesn't wear out.

But the Deacon swore (as Deacons do,  
With an "I dew vum," or an "I tell yeou,"  
He would build one shay to beat the taown  
'n' the keounty 'n' all the kentry raoun';  
It should be so built that it couldn' break daown!  
--"Fur," said the Deacon, "t 's mighty plain  
Thut the weakes' place mus' stan' the strain;

'n' the way t' fix it, uz I maintain,  
Is only jest  
T' make that place uz strong uz the rest."

So the Deacon inquired of the village folk  
Where he could find the strongest oak,  
That couldn't be split nor bent nor broke,--

That was for spokes and floor and sills;  
He sent for lancewood to make the thills;  
The crossbars were ash, from the straightest trees,  
The panels of whitewood, that cuts like cheese,  
But lasts like iron for things like these;  
The hubs of logs from the "Settler's ellum,"  
Last of its timber,--they couldn't sell 'em,  
Never an axe had seen their chips,  
And the wedges flew from between their lips  
Their blunt ends frizzled like celery-tips;  
Step and prop-iron, bolt and screw,  
Spring, tire, axle, and linchpin too,  
Steel of the finest, bright and blue;  
Thoroughbrace bison-skin, thick and wide;  
Boot, top, dasher, from tough old hide  
Found in the pit when the tanner died.

That was the way he "put her through."  
"There!" said the Deacon, "naow she'll dew."

Do! I tell you, I rather guess  
She was a wonder, and nothing less!

Colts grew horses, beards turned gray,  
Deacon and deaconess dropped away,  
Children and grandchildren--where were they?  
But there stood the stout old one-hoss-shay  
As fresh as on Lisbon-earthquake-day!

THE DEACON'S MASTERPIECE

EIGHTEEN HUNDRED;--it came and found  
The Deacon's Masterpiece strong and sound.  
Eighteen hundred increased by ten;--  
"Hahnsum kerridge" they called it then.  
Eighteen hundred and twenty came;--  
Running as usual; much the same.  
Thirty and forty at last arrive,  
And then come fifty, and FIFTY-FIVE.

Little of all we value here  
Wakes on the morn of its hundredth year  
Without both feeling and looking queer.

In fact, there's nothing that keeps its youth  
So far as I know, but a tree and truth.  
(This is a moral that runs at large;  
Take it.--You 're welcome.--No extra charge.)

FIRST OF NOVEMBER,--the Earthquake-day.--  
There are traces of age in the one-hoss-shay--  
A general flavor of mild decay,  
But nothing local, as one may say.  
There couldn't be,--for the Deacon's art  
Had made it so like in every part  
That there wasn't a chance for one to start.  
For the wheels were just as strong as the thills,  
And the floor was just as strong as the sills,  
And the panels just as strong as the floor,  
And the whippetree neither less nor more,  
And the back-crossbar as strong as the fore,  
And spring and axle and hub encore,  
And yet, as a whole, it is past a doubt  
In another hour it will be worn out!

First of November, 'Fifty-five!  
This morning the parson takes a drive.  
Now, small boys, get out of the way!  
Here comes the wonderful one-hoss-shay,  
Drawn by a rat-tailed, ewe-necked bay.  
"Huddup!" said the parson. --Off went they.

The parson was working his Sunday's text,--  
Had got to fifthly, and stopped perplexed  
At what the--Moses--was coming next.  
All at once the horse stood still,  
Close by the meet'n'-house on the hill

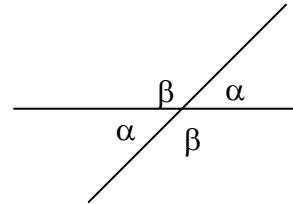
--First a shiver, and then a thrill,  
Then something decidedly like a spill,--  
And the parson was sitting upon a rock,  
At half-past nine by the meet'n'-house clock,--  
Just the hour of the Earthquake shock!

--What do you think the parson found,  
When he got up and stared around?  
The poor old chaise in a heap or mound,  
As if it had been to the mill and ground!  
You see, of course, if you 're not a dunce,  
How it went to pieces all at once,--  
All at once, and nothing first,--  
Just as bubbles do when they burst.

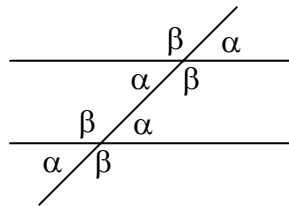
End of the wonderful one-hoss-shay.  
Logic is logic. That's all I say.

## Math for Structures I

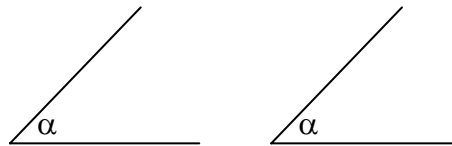
1. Parallel lines never intersect.
2. Two lines are *perpendicular* (or *normal*) when they intersect at a right angle =  $90^\circ$ .
3. *Intersecting* (or *concurrent*) lines cross or meet at a point.
4. If two lines cross, the opposite angles are identical:



5. If a line crosses two parallel lines, the intersection angles with the same orientation are identical:



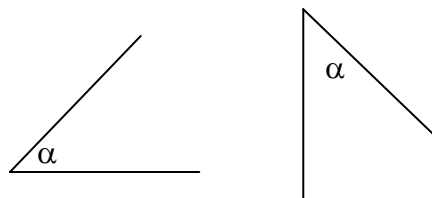
6. If the sides of two angles are parallel and intersect in the same fashion, the angles are identical.



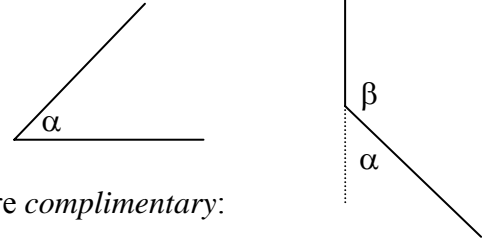
7. If the sides of two angles are parallel, but intersect in the opposite fashion, the angles are *supplementary*:  $\alpha + \beta = 180^\circ$ .



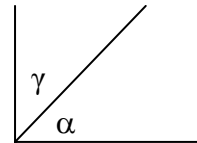
8. If the sides of two angles are perpendicular and intersect in the same fashion, the angles are identical.



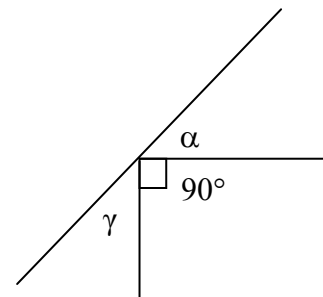
9. If the sides of two angles are perpendicular, but intersect in the opposite fashion, the angles are *supplementary*:  $\alpha + \beta = 180^\circ$ .



10. If the side of two angles bisects a right angle, the angles are *complimentary*:  $\alpha + \gamma = 90^\circ$ .

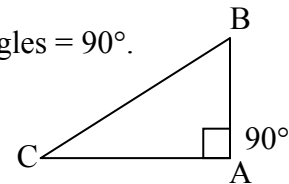


11. If a right angle bisects a straight line, the remaining angles are *complimentary*:  $\alpha + \gamma = 90^\circ$ .



12. The sum of the interior angles of a triangle =  $180^\circ$ .

13. For a right triangle, that has one angle of  $90^\circ$ , the sum of the other angles =  $90^\circ$ .

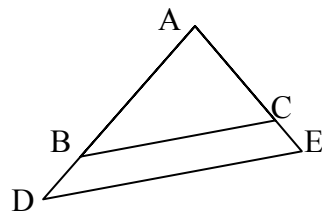


14. For a right triangle, the sum of the squares of the sides equals the square of the hypotenuse:

$$AB^2 + AC^2 = CB^2$$

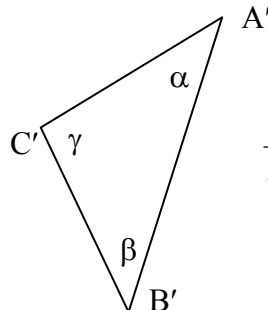
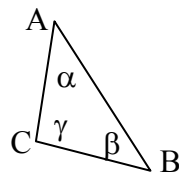
15. Similar triangles have identical angles in the same orientation. Their sides are related by:

Case 1:



$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

Case 2:



$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}$$

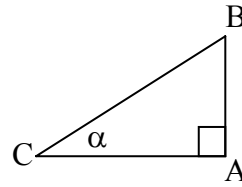


16. For right triangles:

$$\sin = \frac{\text{opposite side}}{\text{hypotenuse}} = \sin \alpha = \frac{AB}{CB}$$

$$\cos = \frac{\text{adjacent side}}{\text{hypotenuse}} = \cos \alpha = \frac{AC}{CB}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}} = \tan \alpha = \frac{AB}{AC}$$



(SOHCAHTOA)

17. If an angle is greater than  $180^\circ$  and less than  $360^\circ$ , *sin* will be less than 0.

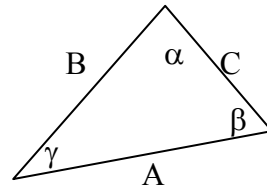
If an angle is greater than  $90^\circ$  and less than  $270^\circ$ , *cos* will be less than 0.

If an angle is greater than  $90^\circ$  and less than  $180^\circ$ , *tan* will be less than 0.

If an angle is greater than  $270^\circ$  and less than  $360^\circ$ , *tan* will be less than 0.

18. LAW of SINES (any triangle)

$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$



19. LAW of COSINES (any triangle)

$$A^2 = B^2 + C^2 - 2BC \cos \alpha$$

20. Surfaces or areas have dimensions of width and length and units of length *squared* (ex.  $\text{in}^2$  or inches x inches).

21. Solids or volumes have dimension of width, length and height or thickness and units of length *cubed* (ex.  $\text{m}^3$  or  $\text{m} \times \text{m} \times \text{m}$ )

22. Algebra: If  $a \cdot b = c \cdot d$  then it can be rewritten:

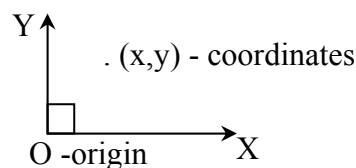
$$a \cdot b + k = c \cdot d + k \quad \text{add a constant}$$

$$c \cdot d = a \cdot b \quad \text{switch sides}$$

$$a = \frac{c \cdot d}{b} \quad \text{divide both sides by } b$$

$$\frac{a}{c} = \frac{d}{b} \quad \text{divide both sides by } b \cdot c$$

23. Cartesian Coordinate System



## 24. Solving equations with one unknown:

$$1^{\text{st}} \text{ order polynomial: } 2x - 1 = 0 \dots \quad 2x = 1 \dots \quad x = \frac{1}{2}$$

$$ax + b = 0 \dots \quad x = \frac{-b}{a}$$

2<sup>nd</sup> order polynomial

$$ax^2 + bx + c = 0 \dots \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{two answers (radical cannot be negative)}$$

$$x^2 - 1 = 0 \dots \quad (a = 1, b = 0, c = -1) \quad x = \frac{-0 \pm \sqrt{0^2 - 4(-1)}}{2 \cdot 1} \dots \quad x = \pm 1$$

## 25. Solving 2 linear equations simultaneously:

Equations can be added and factored to eliminate one variable:

ex:	$2x + 3y = 8$		$2x + 3y = 8$
	$4x - y = 2$	multiply both sides by 3	$12x - 3y = 6$
		and add	$14x + 0 = 14$
		simplify	$x = 1$
		put $x=1$ in an equation for $y$	$2 \cdot 1 + 3y = 8$
		simplify	$3y = 6$
			$y = 2$

## 26. Derivatives of polynomials

$$y = \text{constant} \quad \frac{dy}{dx} = 0$$

$$y = x \quad \frac{dy}{dx} = 1$$

$$y = ax \quad \frac{dy}{dx} = a$$

$$y = x^2 \quad \frac{dy}{dx} = 2x$$

$$y = x^3 \quad \frac{dy}{dx} = 3x^2$$

27. The minimum and maximum of a function can be found by setting the derivative = 0 and solving for the unknown variable.

28. Calculators (and software) process equations by an “order of operations”, which typically means they process functions like exponentials and square roots before simpler functions such as + or -. BE SURE to specify with parenthesis what order you want, or you’ll get the wrong answers. It is also important to have degrees set in your calculator for trig functions.

For instance, Excel uses – for sign (like -1) first, then will process exponents and square roots, times and divide, followed by plus and minus. If you type  $4 \times 10^2$  and really mean  $(4 \times 10)^2$  you will get an answer of 400 instead of 1600.

## Numerical Computations

from Statics and Strength of Materials, 5<sup>th</sup> ed. Morrow & Kokernak, Prentice Hall, 2004

### Accuracy

The accuracy of a numerical value is often expressed in terms of the number of *significant digits* that the value contains. What are significant digits? Any nonzero digit is considered significant; zeroes that appear to the left or right of a digit sequence are used to locate the decimal point and are not considered significant. Thus the numbers 0.00345, 3.45, 3450, and 3,450,000 all contain three significant digits represented by the sequence 3–4–5. Zeroes bounded on both sides by nonzero digits are also significant; 0.0005067, 5.067, 50.67, and 506,700 each contain four significant digits, as represented by the numerical sequence 5–0–6–7.

The accuracy of a solution can be no greater than the accuracy of the data on which the solution is based. For example, the length of one side of a right triangle may be given as 20 ft. Without knowing the possible error in the length measurement, it is impossible to determine the error in the answer obtained from it. We will usually assume that the data are known with an accuracy of 0.2 percent. The possible error in the 20-ft length would therefore be 0.04 ft.

To maintain an accuracy of approximately 0.2 percent in our calculations, we will use the following practical rule: use four digits to record numbers beginning with 1 and three digits to record numbers beginning with 2 through 9. Thus a length of 19 ft becomes 19.00 ft, a length of 20 ft becomes 20.0 ft, and a length of 43 ft becomes 43.0 ft.

You will notice one exception to this rule throughout the text: values of the trigonometric functions are traditionally written to four decimal places, and that practice will be followed here, not for increased accuracy, but to clarify the computations used in worked examples.

### Rounding Off Numbers\*

If the data are given with greater accuracy than we wish to maintain (see Fig. 1.1), the following rules may be used to round off their values:

1. When the digit dropped is greater than 5, increase the digit to the left by 1. *Example:* 23.56 ft becomes 23.6 ft.
2. When the digit dropped is less than 5, drop it without changing the digit to the left. *Example:* 23.34 ft becomes 23.3 ft.
3. When the digit dropped is 5 followed only by zeros, increase the digit to the left by 1 only if it becomes even. If the digit to the left becomes odd, drop the 5 without changing the digit to the left. *Example:* 23.5500 ft rounded to three numbers becomes 23.6 ft, and 23.4500 ft becomes 23.4 ft. (This practice is often referred to as the *round-even rule*.)

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\*American Society of Mechanical Engineers (ASME) *Orientation and Guide for Use of SI (Metric) Units*, 9th edition, 1982, p 11. By increasing the digit to the left for a final 5 followed by zeros only if the digit becomes even, we are dividing the rounding process evenly between increasing the digit to the left and leaving the digit to the left unchanged.

## Calculators

Electronic calculators and computers are widely available for use in engineering. Their speed and accuracy make it possible to do difficult numerical computations in a routine manner. However, because of the large number of digits appearing in solutions, their accuracy is often misleading. As pointed out previously, the accuracy of the solution can be no greater than the accuracy of the data on which the solution is based. Care should be taken to retain sufficient digits in the intermediate steps of the calculations to ensure the required accuracy of the final answer. Answers with more significant digits than are reasonable should not be recorded as the final answer. An accuracy greater than 0.2 percent is rarely justified.

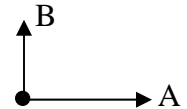
## Problem Solving, Units and Numerical Accuracy

### Problem Solution Method:

1.            Inputs  
              Outputs  
              “Critical Path”             $\Rightarrow$              $\left. \begin{array}{l} \underline{\text{GIVEN:}} \\ \underline{\text{FIND:}} \\ \underline{\text{SOLUTION}} \end{array} \right\} \text{ on graph paper}$
  2. Draw simple diagram of body/bodies & forces acting on it/them.
  3. Choose a reference system for the forces.
  4. Identify key geometry and constraints.
  5. Write the basic equations for force components.
  6. Count the equations & unknowns.
  7. SOLVE
  8. “Feel” the validity of the answer. (Use common sense. Check units...)
- 

Example:      Two forces, A & B, act on a particle. What is the resultant?

1. GIVEN: Two forces on a particle and a diagram with size and orientation



FIND: The “resultant” of the two forces

SOLUTION:

2. Draw what you know (the diagram, any other numbers in the problem statement that could be put on the drawing....)
3. Choose a reference system. What would be the easiest? Cartesian, radian?
4. Key geometry: the location of the particle as the origin of all the forces  
Key constraints: the particle is “free” in space
5. Write equations:       $\text{size of } A^2 + \text{size of } B^2 = \text{size of resultant}$   
                                  $\sin \alpha = \frac{\text{size of } B}{\text{size of } A+B}$
6. Count:                Unknowns: 2, magnitude and direction  $\leq$  Equations: 2  $\therefore$  can solve
7. Solve: graphically or with equations
8. “Feel”: Is the result bigger than A and bigger than B? Is it in the right direction? (like A & B)

Units

Units	Mass	Length	Time	Force
SI	kg	m	s	$N = \frac{kg \cdot m}{s^2}$
Absolute English	lb	ft	s	$Poundal = \frac{lb \cdot ft}{s^2}$
Technical English	$slug = \frac{lb_f \cdot s^2}{ft}$	ft	s	lb <sub>force</sub>
Engineering English	lb	ft	s	lb <sub>force</sub>
	$lb_{force} = lb_{(mass)} \times 32.17 \frac{ft}{s^2}$			
gravitational constant	$g_c = 32.17 \frac{ft}{s^2}$	(English)		
	$g_c = 9.81 \frac{m}{s^2}$	(SI)		
conversions (pg. vii)	$1 \text{ in} = 25.4 \text{ mm}$ $1 \text{ lb} = 4.448 \text{ N}$			

Numerical Accuracy

Depends on 1) accuracy of data you are given  
 2) accuracy of the calculations performed

*The solution CANNOT be more accurate than the less accurate of #1 and #2 above!*

DEFINITIONS: *precision* the number of significant digits  
*accuracy* the possible error

*Relative error* measures the degree of accuracy:

$$\frac{\text{relative error}}{\text{measurement}} \times 100 = \text{degree of accuracy (\%)}$$

For engineering problems, accuracy *rarely* is less than 0.2%.

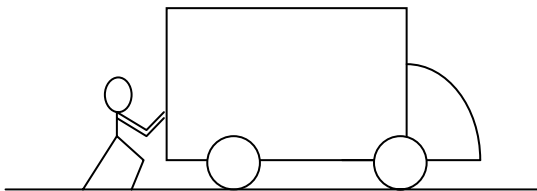
## Forces and Vectors

### Characteristics

- Forces have *a point of application* – tail of vector  
*size* – units of lb, K, N, kN  
*direction* – to a reference system, sense indicated by an arrow
- Classifications include: *Static & Dynamic*
- Structural types separated primarily into *Dead Load* and *Live Load* with further identification as wind, earthquake (seismic), impact, etc.

### Rigid Body

- *Ideal* material that doesn't deform
- Forces on rigid bodies can be *internal* – within or at connections  
or *external* – applied
- Rigid bodies can *translate* (move in a straight line)  
or *rotate* (change angle)

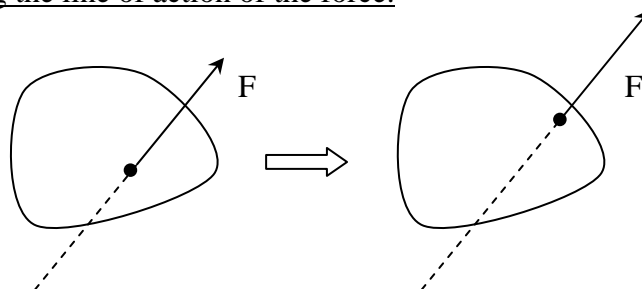


- Weight of truck is external (gravity)
- Push by driver is external
- Reaction of the ground on wheels is external

If the truck moves forward: *it translates*

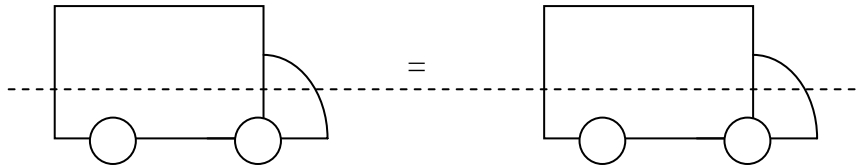
If the truck gets put up on a jack: *it rotates*

- *Transmissibility*: We can replace a force at a point on a body by that force on another point on the body along the line of action of the force.



External conditions haven't changed

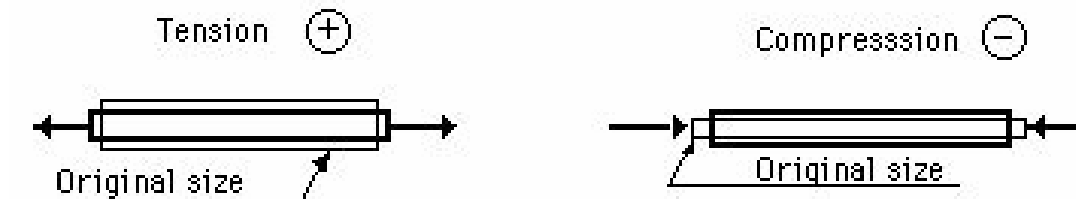
For the truck:



- The same external forces will result in the same conditions for motion
- Transmissibility applies to EXTERNAL forces. INTERNAL forces respond differently when an external force is moved.
- DEFINITION: *2D Structure* - A structure that is flat and may contain a plane of symmetry. All forces on this structure are in the same plane as the structure.

### Internal and External

- *Internal forces* occur within a member or between bodies within a system
- *External forces* represent the action of other bodies or gravity on the rigid body



### Force System Types

- *Collinear* – all forces along the same **line**
- *Coplanar* – all forces in the same **plane**
- *Space* – out there

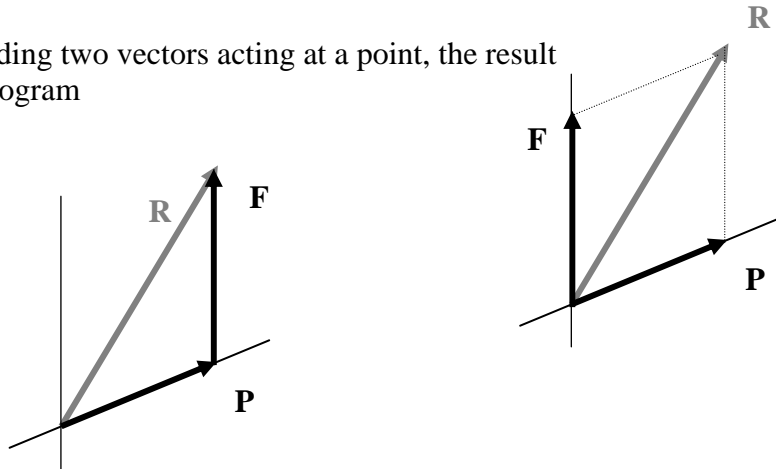
Further classification as

- *Concurrent* – all forces go through the same **point**
- *Parallel* – all forces are **parallel**



**Graphical Addition**

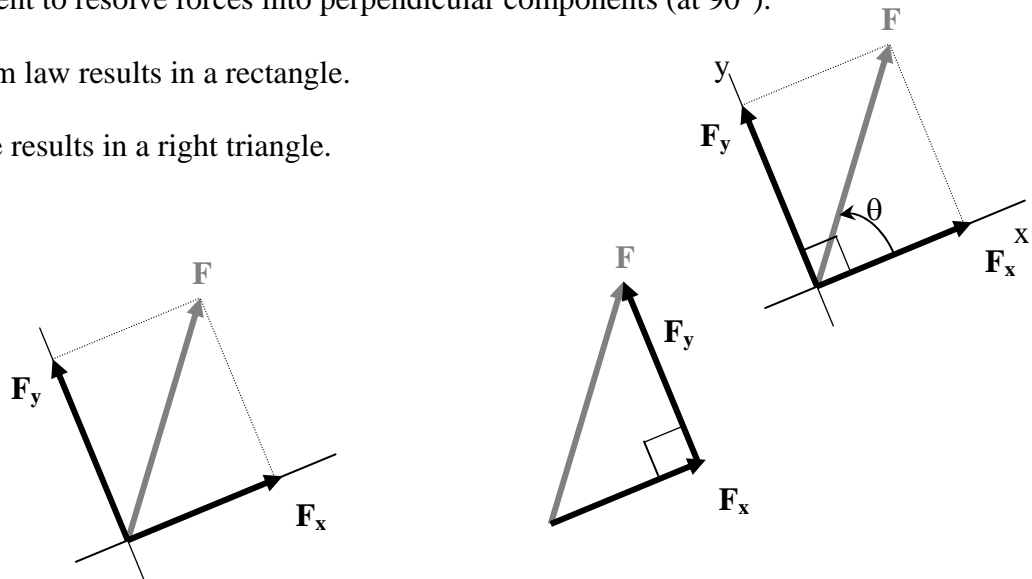
- *Parallelogram law*: when adding two vectors acting at a point, the result is the **diagonal** of the parallelogram
- The *tip-to-tail* method is another graphical way to add vectors.



- With **3 (three)** or more vectors, successive application of the parallelogram law will find the resultant *OR* drawing all the vectors **tip-to-tail** in any order will find the resultant.

**Rectangular Force Components and Addition**

- It is convenient to resolve forces into perpendicular components (at 90°).
- Parallelogram law results in a rectangle.
- Triangle rule results in a right triangle.



$\theta$  is: *between x & F*

$F_x = F \cdot \cos\theta$

$F_y = F \cdot \sin\theta$

$F = \sqrt{F_x^2 + F_y^2}$

$\tan\theta = \frac{F_y}{F_x}$

} magnitudes are *scalar* and can be negative  
 $F_x$  &  $F_y$  are *vectors* in x and y direction

When  $90^\circ < \theta < 270^\circ$ ,  $F_x$  is *negative*

When  $180^\circ < \theta < 360^\circ$ ,  $F_y$  is *negative*

When  $0^\circ < \theta < 90^\circ$  and  $180^\circ < \theta < 270^\circ$ ,  $\tan\theta$  is *positive*

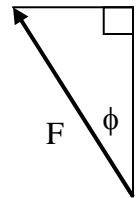
When  $90^\circ < \theta < 180^\circ$  and  $270^\circ < \theta < 360^\circ$ ,  $\tan\theta$  is *negative*

- Addition (analytically) can be done by adding all the  $x$  components for a **resultant**  $x$  component and adding all the  $y$  components for a resultant  $y$  component.

$$R_x = \sum F_x, \quad R_y = \sum F_y \quad \text{and} \quad R = \sqrt{R_x^2 + R_y^2} \quad \tan \theta = \frac{R_y}{R_x}$$

**CAUTION:** An interior angle,  $\phi$ , between a vector and *either* coordinate axis can be used in the trig functions. BUT *No sign* will be provided by the trig function, which means **you** must give a sign and determine if the component is in the  $x$  or  $y$  direction.

*For example,  $F \sin \phi = \text{opposite side}$ , which should be negative in  $x$ !*

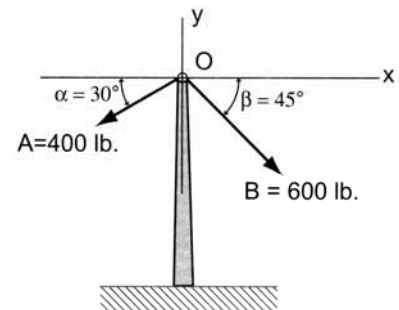


### Example 1 (page 9)

#### Example Problem 2.2

A utility pole supports two tension forces  $A$  and  $B$  with the directions shown. Using the parallelogram law and the tip-to-tail methods, determine the resultant force for  $A$  and  $B$  (magnitude and direction).

Scale: 1" = 200 lb.



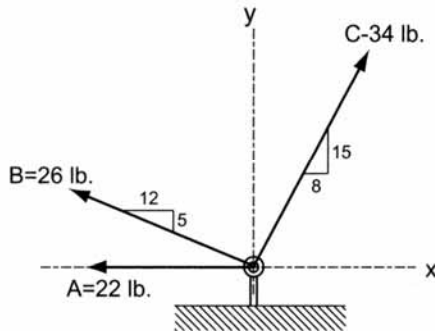
#### Steps:

1. **GIVEN:** Write down what's given (drawing and numbers).
2. **FIND:** Write down what you need to find. (resultant graphically)
3. **SOLUTION:**
4. Draw the 400 lb and 600 lb forces to scale with tails at  $O$ . (If the scale isn't given, you must choose one that fits on your paper, ie. 1 inch = 200 lb.)
5. Draw parallel reference lines at the ends of the vectors.
6. Draw a line from  $O$  to the intersection of the reference lines
7. Measure the length of the line
8. Convert the line length by the scale into pounds (by multiplying by the force measure and dividing by the scale value, ie  $X \text{ inches} * 200 \text{ lb} / 1 \text{ inch}$ ).

**Alternate solution:**

4. Draw one vector
5. Draw the other vector at the TIP of the first one (away from the tip).
6. Draw a line from 0 to the tip of the final vector and continue at step 7

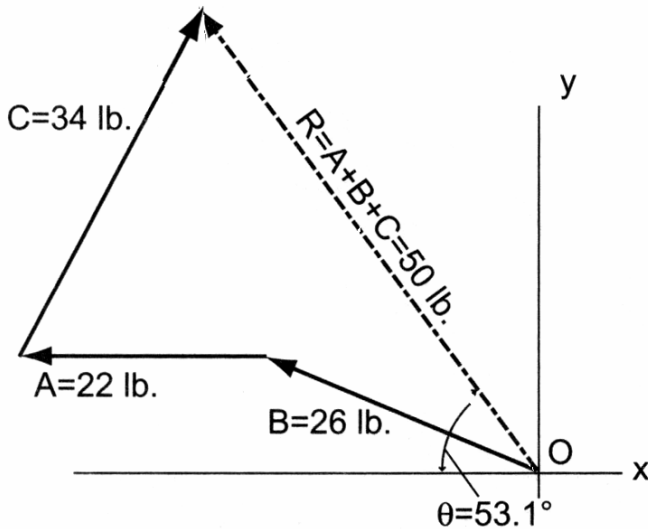
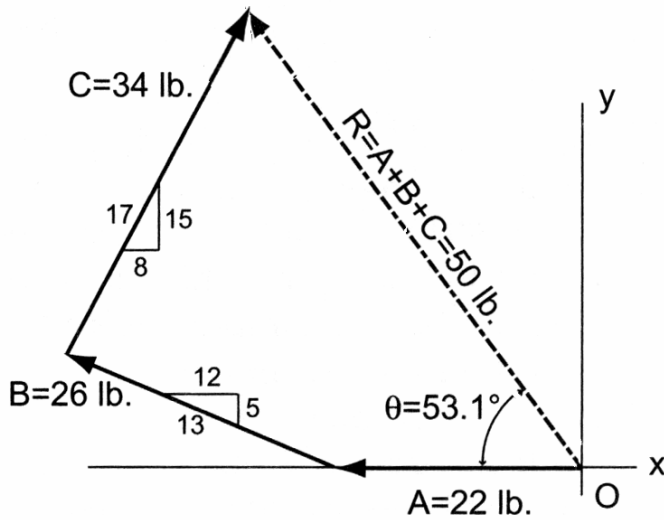
Example 2 (pg 12)

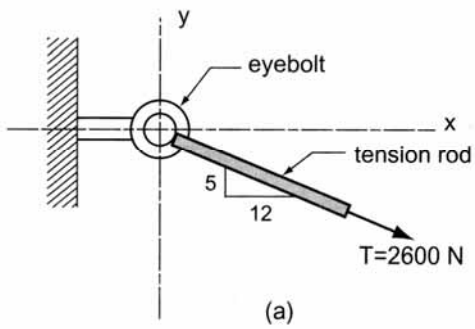


**Example Problem 2.4**

A tent stake is subjected to three pulling forces, as shown in Figure 2.18. Using the graphical tip-to-tail method, determine the resultant of forces *A*, *B*, and *C* (magnitude and direction).

Suggested scale: 1.5 mm = 1 lb. or 1 mm = 2/3 lb.



Example 3 (pg 16)**Example Problem 2.7**

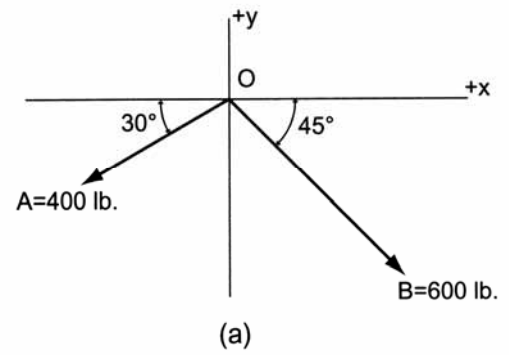
A large eyebolt (Figure 2.24) is used in supporting a canopy over the entry to an office building. The tension developed in the support rod is equal to 2600 newtons. Determine the rectangular components of the force if the rod is at a 5 in 12 slope.

Also determine the embedment length,  $L$ , if the anchor can resist 500 N for every cm of embedment.

Example 4 (pg 19) Determine the resultant vector analytically with the component method.

**Example Problem 2.9 (Figure 2.29)**

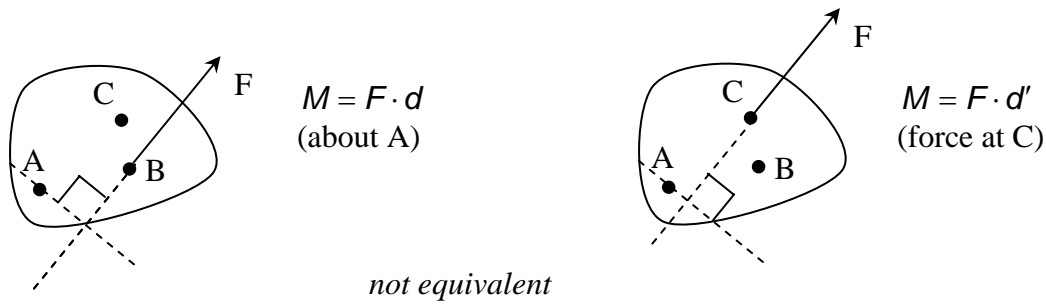
This is the same problem as Example Problem 2.2, which was solved earlier using the graphical methods.



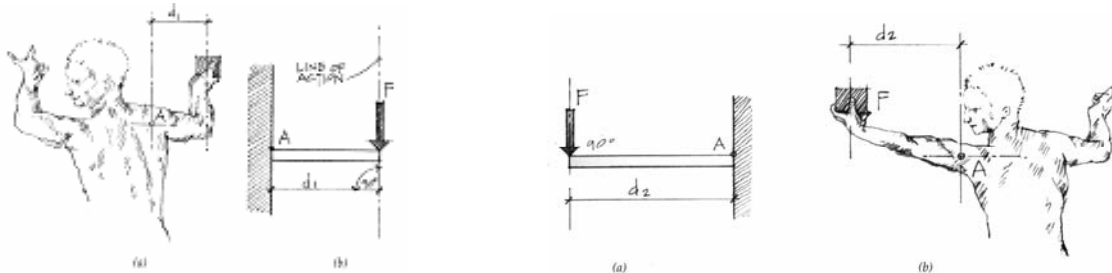
## Moments

### Moment of a Force About an Axis

- Two forces of the same size and direction acting at different points *are not equivalent*. They may cause the same **translation**, but they cause different **rotation**.
- DEFINITION: *Moment* – A moment is the tendency of a force to make a body rotate about an axis. It is measured by  $F \cdot d$ , where  $d$  is the distance **perpendicular** to the line of action of the force and through the axis of rotation.

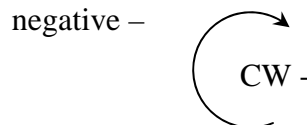
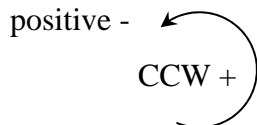


- For the same force, the bigger the **lever arm (or moment arm)**, the bigger the moment magnitude, i.e.  $M_A = F \cdot d_1 < M_A = F \cdot d_2$

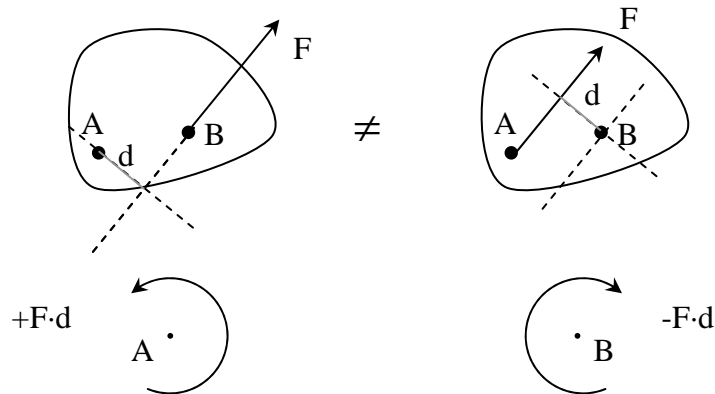


- Units:                      SI:                      N·m, KN·m  
     Engr. English: lb-ft, kip-ft

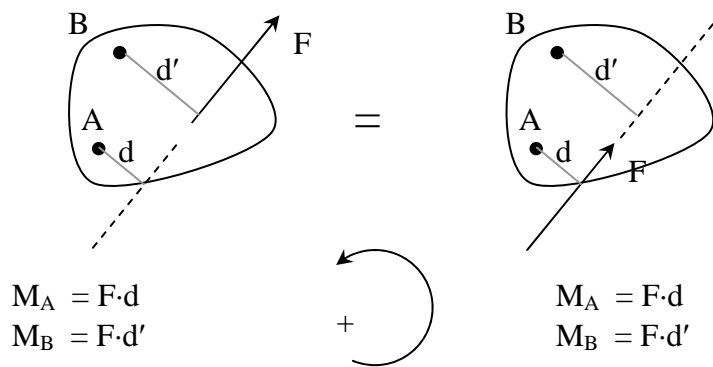
- Sign conventions: Moments have magnitude *and* rotational direction:



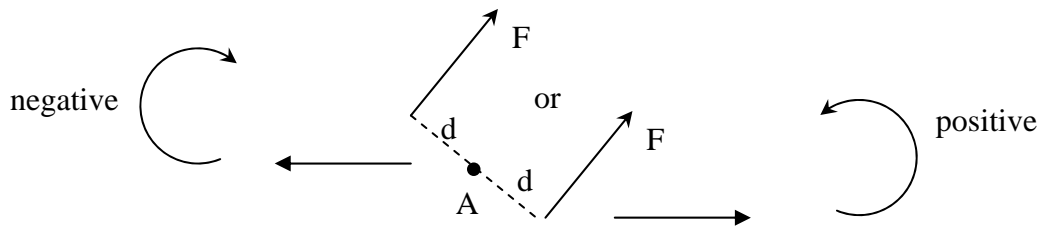
- Moments can be added as scalar quantities when there is a sign convention.



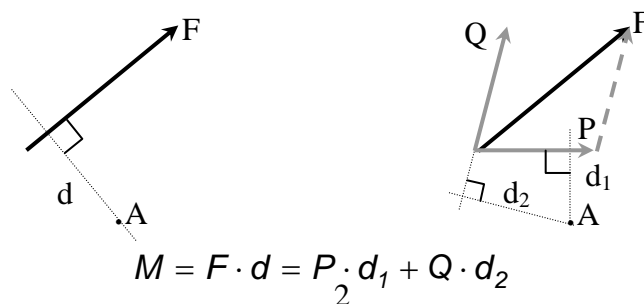
- Repositioning a force along its line of action results in the same moment about any axis.



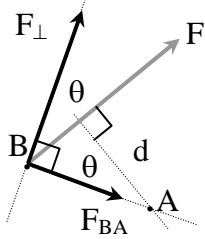
- A force is completely defined (except for its exact position on the line of action) by  $F_x$ ,  $F_y$ , and  $M_A$  about  $A$  (size and direction).
- The *sign* of the moment is determined by which side of the axis the force is on.



- Varignon's Theorem:* The moment of a force about any axis is equal to the sum of moments of the components about that axis.



- **Proof 1:** Resolve  $F$  into components along line  $BA$  and perpendicular to it ( $90^\circ$ ).



$d$  from  $A$  to line  $AB = 0$

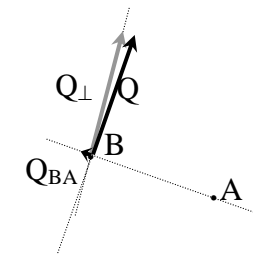
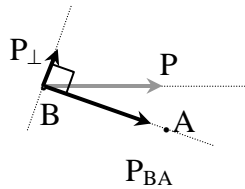
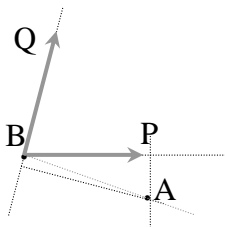
$$d \text{ from } A \text{ to } F_{\perp} = d_{BA} = \frac{d}{\cos \theta}$$

$$F_{BA} = F \sin \theta$$

$$F_{\perp} = F \cos \theta$$

$$\sum M = -F \cdot d = -F_{BA} \cdot 0 - F_{\perp} \cdot d_{BA} = -F \cos \theta \cdot \frac{d}{\cos \theta} = -F \cdot d$$

- **Proof 2:** Resolve  $P$  and  $Q$  into  $P_{BA}$  &  $P_{\perp}$ , and  $Q_{BA}$  &  $Q_{\perp}$ .



$d$  from  $A$  to line  $AB = 0$

$$M_{A \text{ by } P} = -P_{\perp} \cdot d_{BA}$$

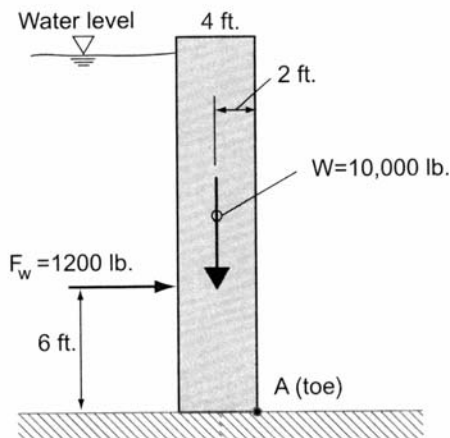
$$M_{A \text{ by } Q} = -Q_{\perp} \cdot d_{BA}$$

$$\sum M = -P_{\perp} \cdot d_{BA} + (-Q_{\perp} \cdot d_{BA})$$

and we know  $d_{BA}$  from Proof 1, and by definition:  $P_{\perp} + Q_{\perp} = F_{\perp}$ . We know  $d_{BA}$  and  $F_{\perp}$  from above, so again  $M = -F_{\perp} \cdot d_{BA} = -F \cdot d$

- By choosing component directions such that  $d = 0$  to one of the components, we can simplify many problems.

**Example 1 (pg 24)**



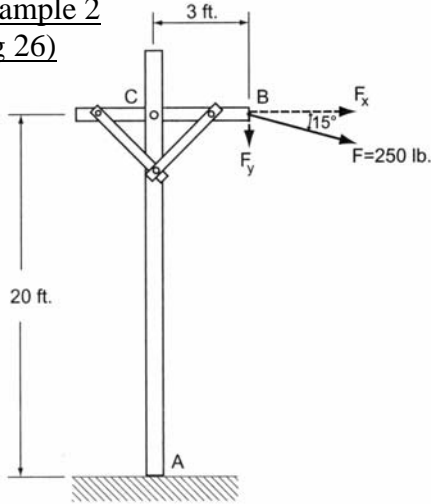
**Example Problem 2.13 (Figure 2.35)**

A 1-foot-wide slice of a 4-foot-thick concrete gravity dam weighs 10,000 pounds and the equivalent force due to water pressure behind the dam is equal to 1200 pounds. The stability of the dam against overturning is evaluated about the "toe" at  $A$ .

Determine the resultant moment at  $A$  due to the two forces shown. Is the dam stable?



Example 2  
(pg 26)

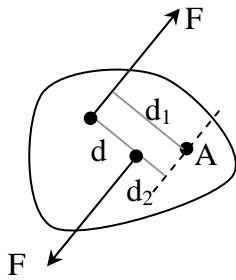


**Example Problem 2.15 (Figure 2.38)**

The same problem from Example Problem 2.14 will be solved using the principle of moments. (Figure 2.36)  
(Moment at A)

**Moment Couples**

- *Moment Couple*: Two forces with equal magnitude, parallel lines of action and opposite sense tend to make our body rotate even though the sum of forces is 0. The sum of the moment of the forces about any axis *is not* zero.

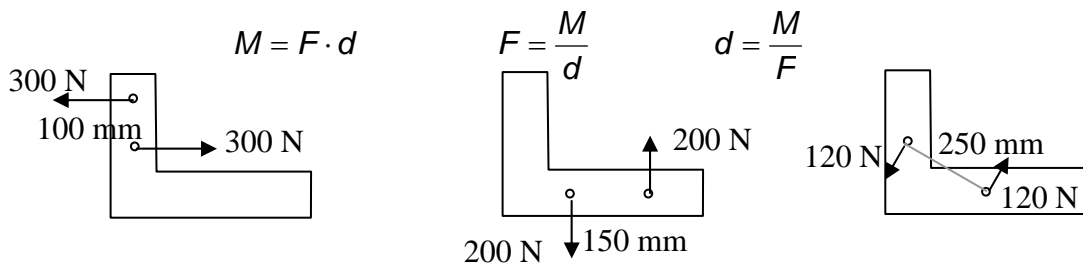


$$\sum M = F \cdot d_2 - F \cdot d_1 = M$$

$$M = F(d_2 - d_1)$$

$$M = -F \cdot d: \text{ moment of the couple (CW)}$$

- M does *not depend* on where A is. M depends on the perpendicular distance between the line of action of the parallel forces.
- M for a couple (defined by F and d) is a constant. And the sense (+/-) is obtained by observation.
- Just as there are equivalent moments (other values of F and d that result in M) there are equivalent couples. The magnitude is the same for different values of F and resulting d or different values of d and resulting F.



**Equivalent Force Systems**

- Two systems of forces are equivalent if we can transform one of them into the other with:
  - 1.) replacing *two forces on a point* by their **resultant**
  - 2.) resolving a *force* into two components
  - 3.) canceling two equal and opposite forces on a point
  - 4.) attaching two equal and opposite forces to a point
  - 5.) moving a force along its line of action'
  - 6.) replacing a force and moment on a point with a force on another (specific) point
  - 7.) replacing a force on point with a force and moment on another (specific) point

\* based on the parallelogram rule and the principle of transmissibility
- The size and direction are important for a moment. The location on a body doesn't matter because couples with the same moment will have the same effect on the rigid body.

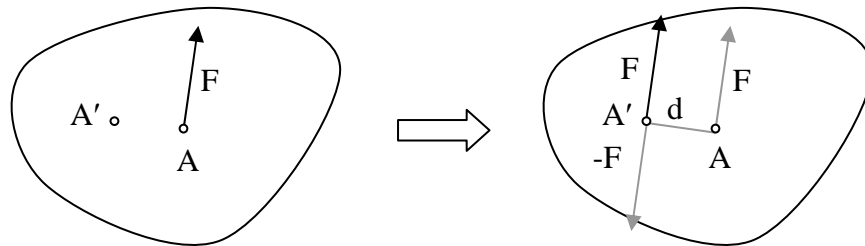
**Addition of Couples**

- Couples can be added as *scalars*.
- Two couples can be *replaced* by a single couple with the magnitude of the algebraic sum of the two couples.

**Resolution of a Force into a Force and a Couple**

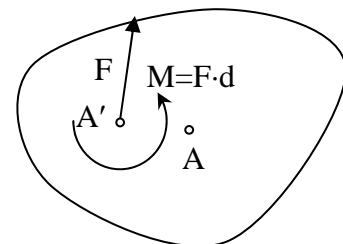
- The equivalent action of a force on a body can be reproduced by that force and a force couple:

If we'd rather have  $F$  acting at  $A'$  which isn't in the line of action, we can instead add  $F$  and  $-F$  at  $A'$  with no change of action by  $F$ . Now it becomes a couple of  $F$  separated by  $d$  and the force  $F$  moved to  $A'$ . The size is  $F \cdot d = M$



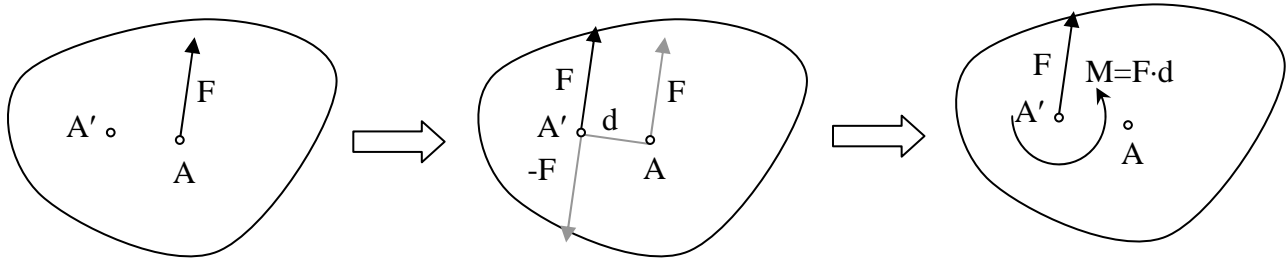
The couple can be represented by a moment symbol:

- Any force can be replaced by itself at another point and the moment equal to the force multiplied by the distance between the original line of action and *new* line of action.



**Resolution of a Force into a Force and a Moment**

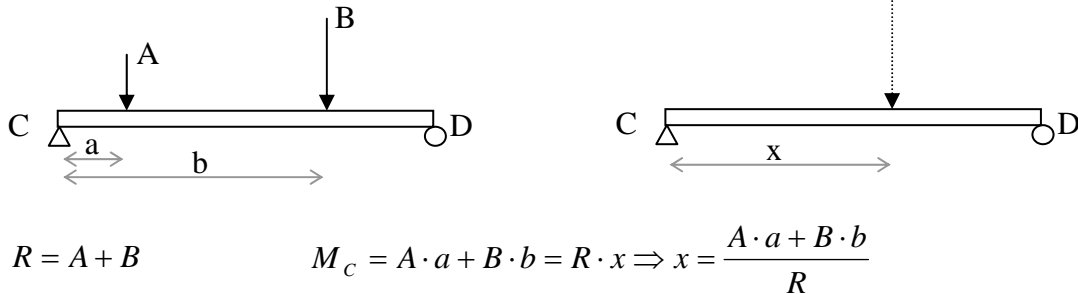
- *Principle:* Any force  $F$  acting on a rigid body (say the one at  $A$ ) may be moved to any given point  $A'$ , provided that a couple  $M$  is added: the moment  $M$  of the couple must equal the moment of  $F$  (in its original position at  $A$ ) about  $A'$ .



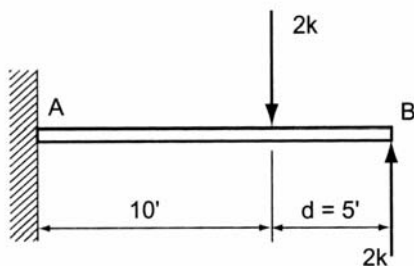
- **IN REVERSE:** A force  $F$  acting at  $A'$  and a couple  $M$  may be combined into a single resultant force  $F$  acting at  $A$  (a distance  $d$  away) where the moment of  $F$  about  $A'$  is equal to  $M$ .

**Resultant of Two Parallel Forces**

- Gravity loads act in one direction, so we may have parallel forces on our structural elements. We know how to find the resultant **force**, but the *location* of the resultant must provide the equivalent total moment from each individual force.



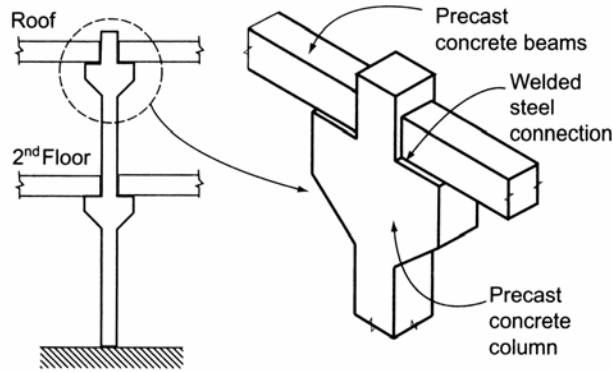
Example 3 (pg 19)



**Example Problem 2.19**

The cantilevered beam shown in Figure 2.43 is subjected to two equal and opposite forces as shown. Determine the resultant moment  $M_A$  at the beam support and the moment  $M_B$  at the free end.

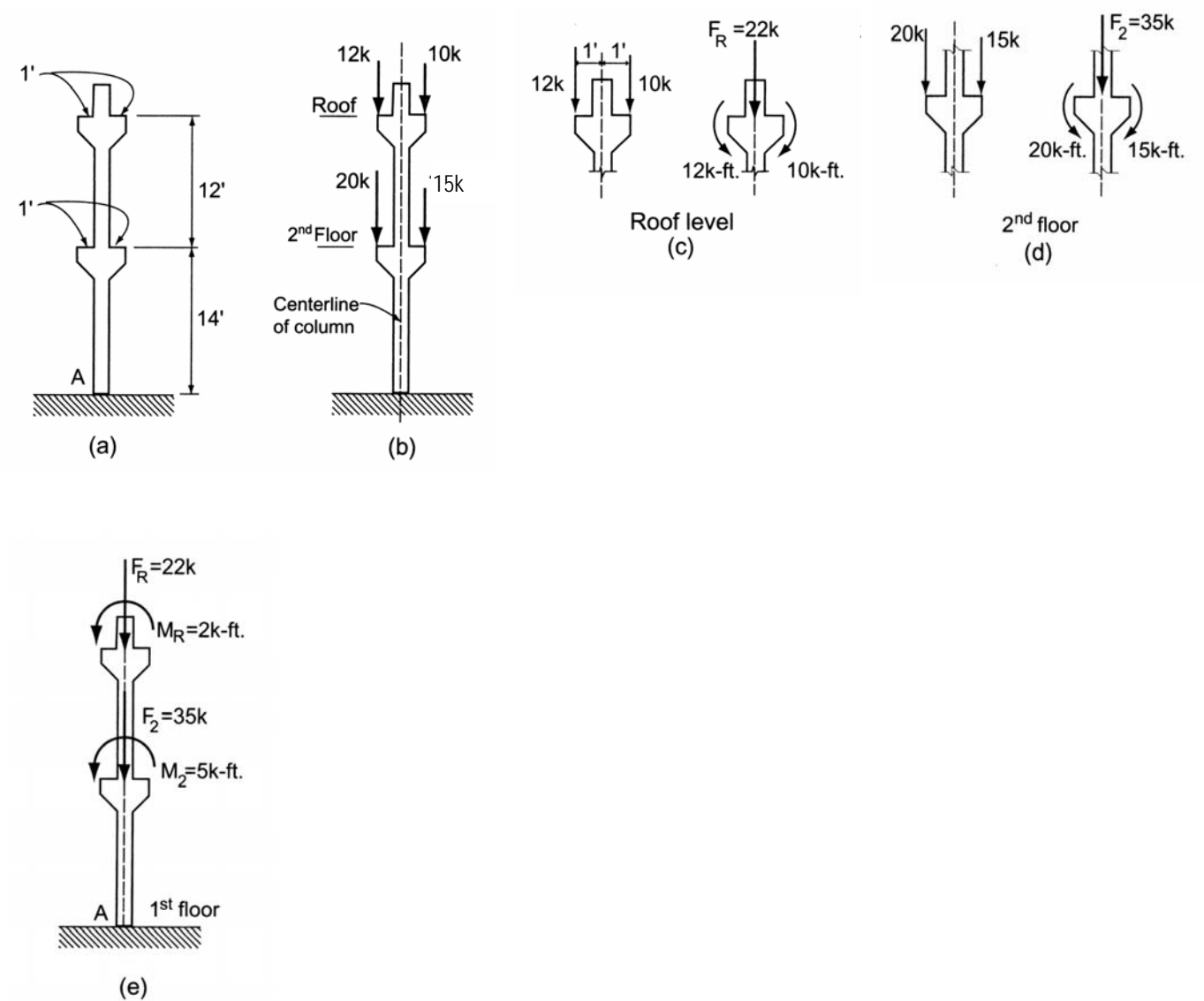
Example 4 (pg 34)



Example Problem 2.22 (Figures 2.49 and 2.50)

A major, precast-concrete column supports beam loads from the roof and second floor as shown. Beams are supported by seats projecting from the columns. Loads from the beams are assumed to be applied one foot from the column axis.

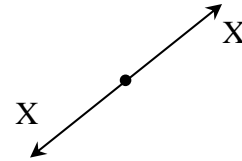
Determine the equivalent column load condition when all beam loads are shown acting through the column axis.



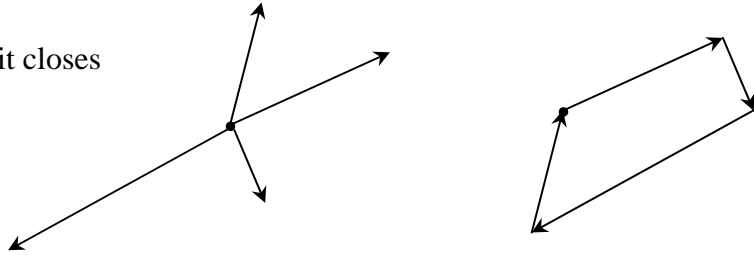
## Equilibrium of a Particle

- EQUILIBRIUM is the state where the resultant of the forces on a particle is *zero*.

ex: 2 forces of same size, opposite direction



ex: 4 forces, polygon rule shows that it closes



- Analytically:  $R_x = \sum F_x = 0$        $R_y = \sum F_y = 0$  (scalar addition)  
 $M = \sum M = 0$       (always true when the forces run through the point)
- NEWTON'S FIRST LAW: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

### Collinear Force System

- *All forces act along the same line.* Only one equilibrium equation is needed:  $\sum F_{(in-line)} = 0$
- Equivalently:  $R_x = \sum F_x = 0$  and  $R_y = \sum F_y = 0$   
 We know that  $\Sigma M$  has to equal 0 for no rotation.

### Concurrent Force System

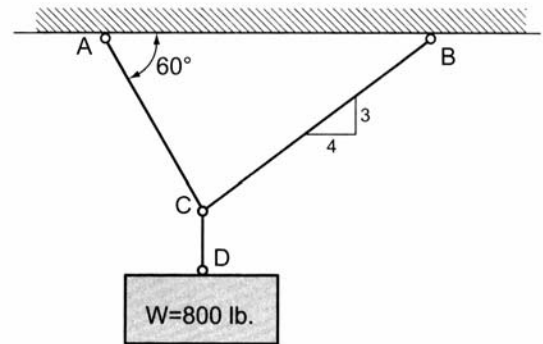
- *All forces act through the same point.* Only two equilibrium equations are needed:  
 $R_x = \sum F_x = 0$  and  $R_y = \sum F_y = 0$   
 We know that  $\Sigma M$  has to equal 0 for no rotation.
- FREE BODY DIAGRAM (aka FBD): Sketch of a significant isolated particle of a body or structure showing all the forces acting on it. Forces can be from
  - externally applied forces
  - weight of the rigid body
  - reaction forces or constraints
  - externally applied moments
  - moment reactions or constraints
  - forces developed within a section member

- How to solve when there are more than three forces on a free body:
  1. *Resolve all forces into  $x$  and  $y$  components using known and unknown forces and angles. (Tables are helpful.)*
  2. *Determine if any unknown forces are related to other forces and write an equation.*
  3. *Write the two equilibrium equations (in  $x$  and  $y$ ).*
  4. *Solve the equations simultaneously when there are the same number of equations as unknown quantities. (see math handout)*
- Common problems have unknowns of:
  - 1) **magnitude of force**
  - 2) **direction of force**

### Example 1 (pg 49)

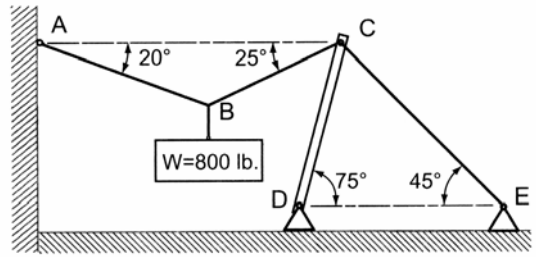
#### Example Problem 3.1: Equilibrium of a Particle

Two cables, shown in Figure 3.8, are used to support a weight  $W = 800$  lb., suspended at concurrent point  $C$ . Determine the tension developed in cables  $CA$  and  $CB$  for the system to be in equilibrium. Solve this problem analytically and check the answer graphically.



Example 2 (pg 56)**Example Problem 3.5**

A compound cable system supports a weight  $W = 800$  lb. at point  $B$ , as shown in Figure 3.18. Cable  $BA$  is attached to a wall support at  $A$  and concurrent point  $C$  is supported by a compression strut  $DC$ . Determine all of the cable forces and the compression in strut  $DC$ .

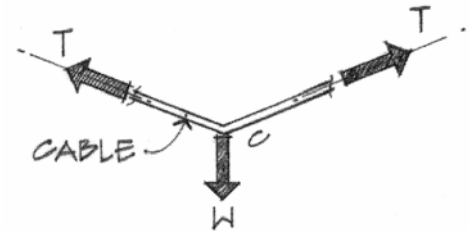


- **CABLES:** have the same tension all along the length if they are not cut. The force *magnitude* is the same everywhere in the cable *even if it changes angles*. Cables **CANNOT** be in **compression**. (They flex instead.)
- **CABLE STRUCTURES:**

*High-strength steel* is the most common material used for cable structures because it has a high strength to weight ratio.

Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

Cables with a single load have a **concurrent** force system. It will only be in equilibrium if the cable is **symmetric**.

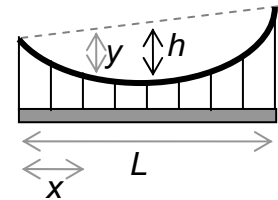


The forces anywhere in a *straight segment* can be resolved into x and y components of  $T_x = T \cos \theta$  and  $T_y = T \sin \theta$ .

The shape of a cable having a *uniform distributed load* is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h(Lx - x^2) / L^2$$

where y is the vertical distance from the straight line from cable start to end  
 h is the vertical sag (maximum y)  
 x is the distance from one end to the



location of y

L is the horizontal span.

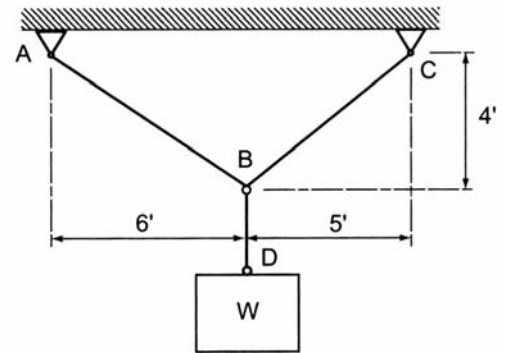
$$L_{total} = L(1 + \frac{8}{3} \frac{h^2}{L^2} - \frac{32}{5} \frac{h^4}{L^4})$$

where  $L_{total}$  is the total length of parabolic cable  
 h and L are defined above.



Example 3 (pg 55)**Example Problem 3.4**

Determine the maximum weight  $W$  that can be safely supported by the cable system shown in Figure 3.15 if cables  $AB$  and  $BC$  have a breaking strength of 500 pounds each.



## Equilibrium of Rigid Bodies

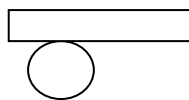
- *Definition:* Equilibrium is the state when all the external forces acting on a rigid body form a system of forces equivalent to zero. There will be no rotation or translation. The forces are referred to as balanced.

$$R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0 \quad \text{AND} \quad \sum M = 0$$

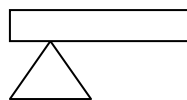
- It is **ABSOLUTELY NECESSARY** to consider all the forces acting on a body (applied directly and indirectly) using a **FREE BODY DIAGRAM**. Omission of a force would ruin the conditions for equilibrium.

### FREE BODY DIAGRAM STEPS:

1. Determine the free body of interest. (What body is in equilibrium?)
  2. Detach the body from the ground and all other bodies (“free” it).
  3. Indicate all external forces which include:
    - action on the free body by the **supports & connections**
    - action on the free body by other bodies
    - the weigh effect (=force) of the free body itself (force due to gravity)
  4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting *on the body* not by the body.
  5. Dimensions/angles should be included for moment computations and force computations.
  6. Indicate the unknown angles, distances, forces or moments, such as those reactions or constraining forces where the body is supported or connected. (*Text uses hashes on the unknown forces to distinguish them.*)
- *Reactions* can be categorized by the type of connections or supports. A reaction is a force with known line of action, or a force of unknown direction, or a moment. The line of action of the force or direction of the moment is directly related to the motion that is prevented.



prevents motion:  
**up and down**



prevents motion:  
**vertical & horizontal**



prevents:  
**rotation & translation**

**Reactions and Support Connections**

Structural Analysis, 4<sup>th</sup> ed., R.C. Hibbeler

**Table 2-1 Supports for Coplanar Structures**

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) light cable weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) rollers rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5) smooth pin or hinge			Two unknowns. The reactions are two force components.
(6) slider fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7) fixed support			Three unknowns. The reactions are the moment and the two force components.

- The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc...) *If the sense isn't obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was *opposite* the actual sense. ***DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.***
- With the 3 equations of equilibrium, there can be no more than 3 unknowns. ***COUNT THE NUMBER OF UNKNOWN REACTIONS.***

Example 1  
(similar to ex. on pg 65)

500 lb known

check:

reactions for the pin-type support at A =  $A_x$  &  $A_y$

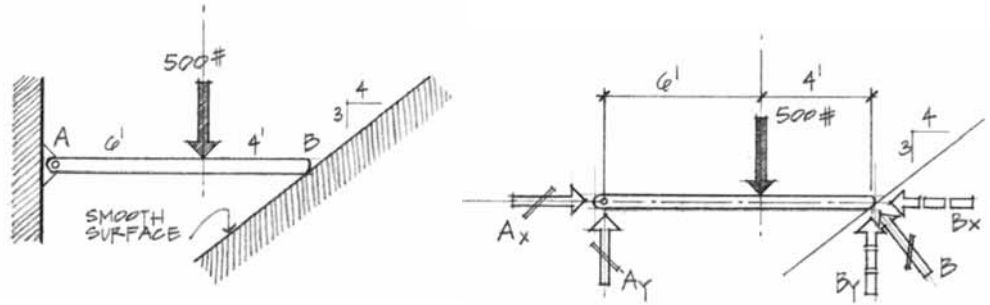
reactions and components for the smooth surface at B =  $B$  (perpendicular to ground only)

# equations = 3

procedure:

Write summation of forces in x and y and set = 0.

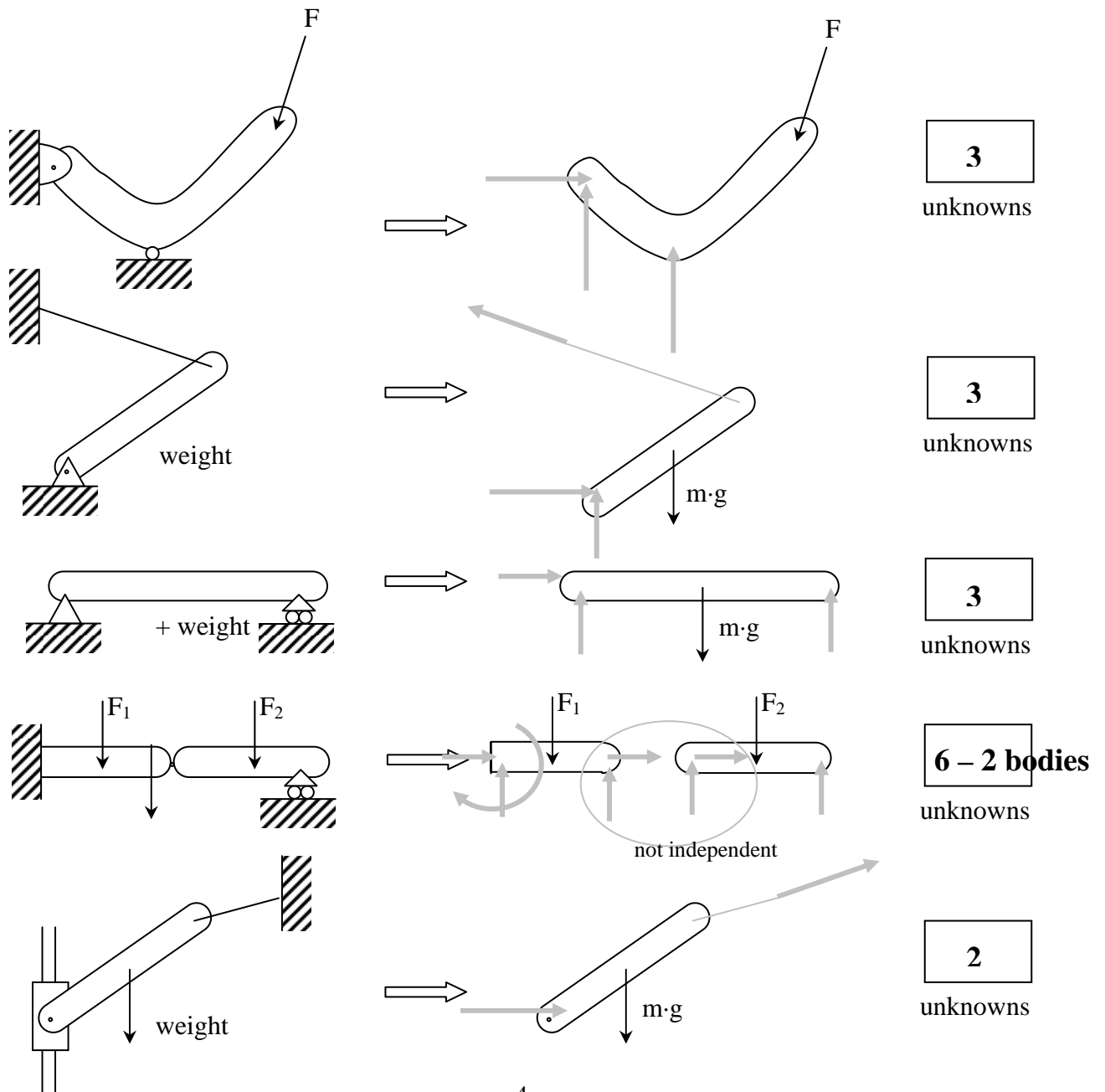
Choose a place to take a moment. Summing moments at **A** means that  $A_x$ ,  $A_y$  and  $B_x$  have moment arms of *zero*.



- The general rule is to sum at point where there are the most unknown reactions which usually results in one unknown left in the equation. This “point” could also be where two lines of action intersect.
- More than one moment equation can be used, *but it will not be unique*. Only 3 equations are unique. Variations:

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum M_1 = 0 \quad \text{or} \\ \sum F_x = 0 & \sum M_1 = 0 & \sum M_2 = 0 \quad \text{or} \\ \sum M_1 = 0 & \sum M_2 = 0 & \sum M_3 = 0 \end{array}$$

Recognizing support unknowns in FBD's

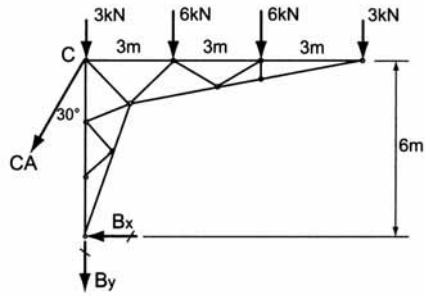
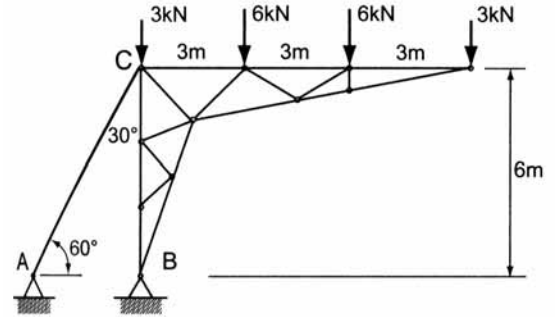


Example 2 (pg 63)

**Example Problem 3.7**

A cantilevered, stadium-type truss supports roof loads as shown in Figure 3.28.

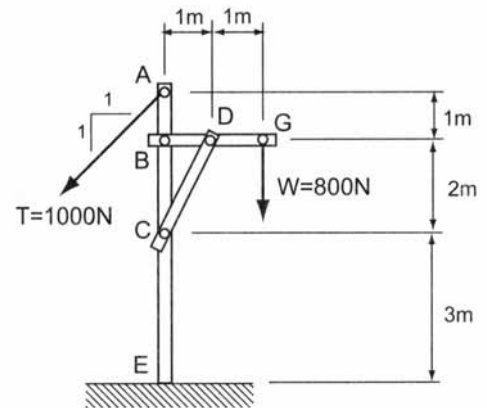
A guying cable  $CA$  and a hinge support at  $B$  are provided for stability and equilibrium. Draw a FBD of the truss and solve for the support reaction  $B$  and the cable tension in  $CA$ . Neglect the truss weight.



Example 3 (pg 67)

**Example Problem 3.10**

A utility pole is embedded firmly at the base and supports the two applied loads as shown in Figure 3.31a. Draw a FBD of the pole and determine the support reactions generated at the base  $E$  in response to the loading.

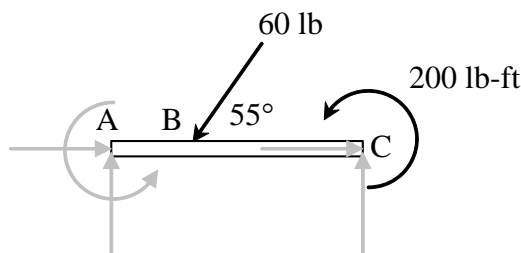
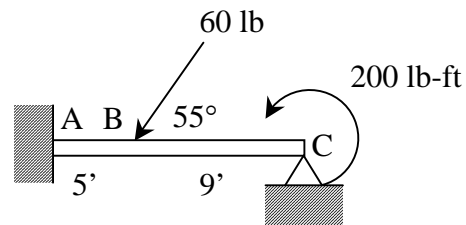


**Static Indeterminacy and Improper Constraints**

- *Definition:* A completely constrained rigid body has the same number of unknown reactions as number of equilibrium equations and cannot move under the loading conditions. The reactions are statically determinate.
- *Definition:* Statically indeterminate reactions appear on a rigid body when there are more unknown reactions than the number of equilibrium equations. The reactions that cannot be solved for are statically indeterminate. The degree of indeterminacy is the number of additional equations that would be needed to solve, i.e. one more = 1<sup>st</sup> degree, 2 more = 2<sup>nd</sup> degree...

Example of Static Indeterminacy:

Find the reactions on the cantilever when a pin is added at C

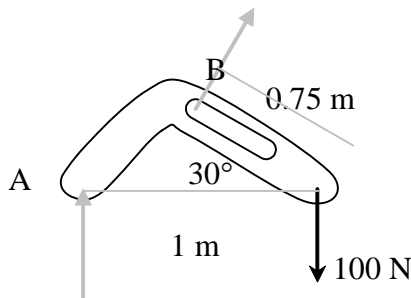
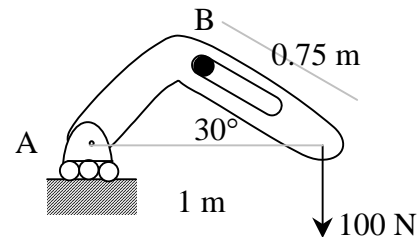


With 5 unknowns, two won't be solvable. (statically indeterminate to the 2<sup>nd</sup> degree)

- *Definition:* When the support conditions provide the same or less unknown reactions as the equations of equilibrium *but allow the structure to move (not equilibrium)*, the structure is considered partially constrained. This occurs when the reactions must be either **concurrent** or **parallel**.

Example of Partial Constraints:

Find the reactions when the pin support at A changes to a roller



If  $\Sigma F$  has to equal 0, the x component must be 0, meaning  $B = 0$ .  
 A would have to equal 100 N, but then  $\Sigma M$  wouldn't be 0.

- The condition of at most as many unknown reactions as equilibrium equations is necessary for static determinacy, but isn't sufficient. *The supports must completely constrain the structure.*
- We'd like to avoid partial or improper constraint in the design of our structures. However, some structures with these types of constraints may not collapse. They may move. Or they may require advanced analysis to find reaction forces.

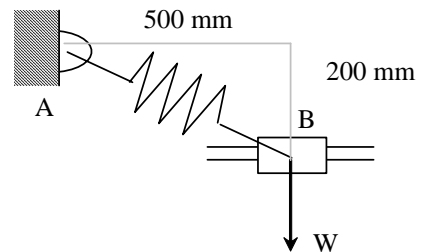
Example of Partial Constraints and Static Indeterminacy:

Find the weight and reactions when the sleeve track is horizontal

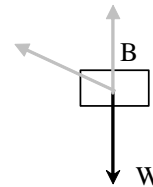
$k = 5 \text{ N/mm}$

$k(\Delta l) = F$  by spring

length of unstretched spring = 450 mm

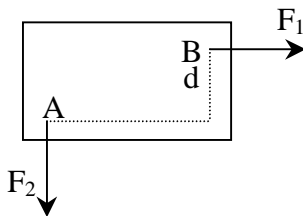


For  $\Sigma F$  to equal 0, the spring force must be 0 (x component = 0) meaning it *can't* be stretched if there is no movement

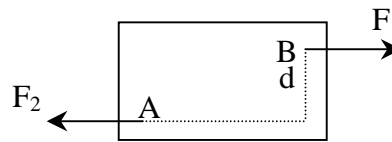


**Rigid Body Cases:**

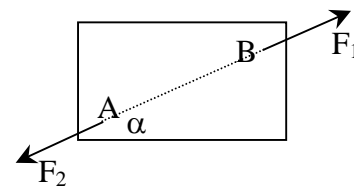
1. Two-force body: Equilibrium of a body subjected to two forces on two points requires that those forces be **equal** and **opposite** and act in the same line of action.



(A)



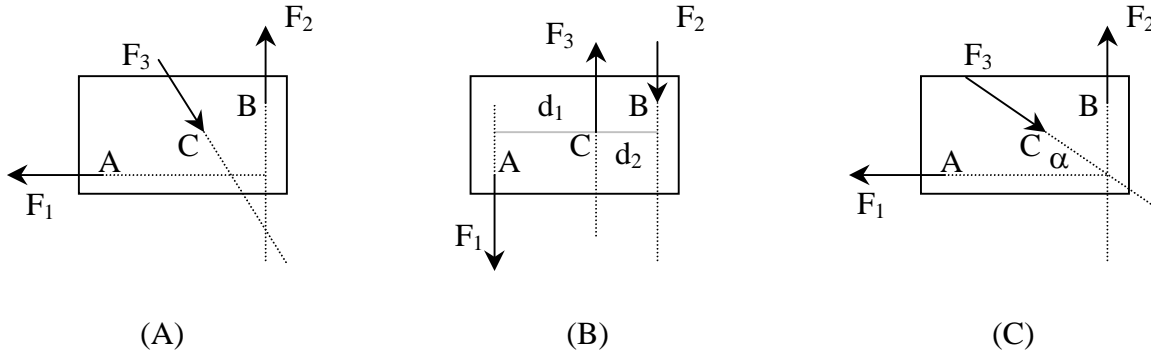
(B)



(C)

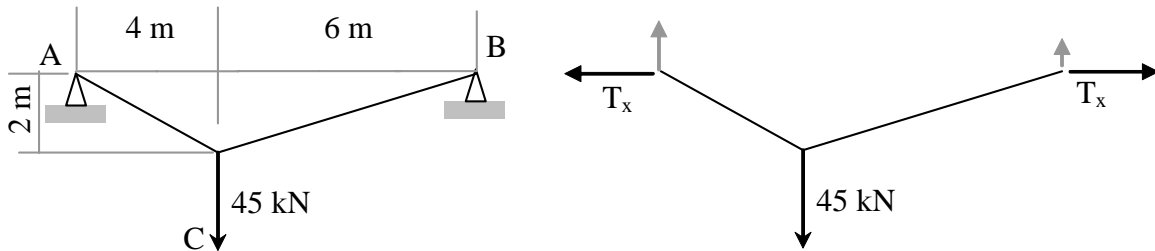


2. Three-force body: Equilibrium of a body subjected to three forces on three points requires that the line of action of the forces be concurrent (intersect) or parallel AND that the resultant equal zero.



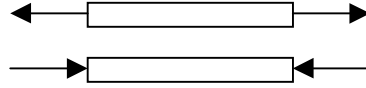
### Cables with Several Concentrated Loads or Fixed Geometry

- In order to completely constrain cables, the number of unknown support reactions *will be more* than the available number of equilibrium equations. We can solve because we have additional equations from geometry due to the **slope** of the cable.
- The tension in the cable IS NOT the same everywhere, but the horizontal component in a cable segment WILL BE.



## Truss Structures

- A truss is made up of straight two-force members connected at its ends. The triangular arrangement produces stable geometry. Loads on a truss are applied at the joints only.
- Joints are pin-type connections (resist translation, not rotation).
- Forces of action and reaction on a joint must be equal and opposite.
- Members in TENSION are being pulled.
- Members in COMPRESSION are being squeezed.
- External forces act on the joints.

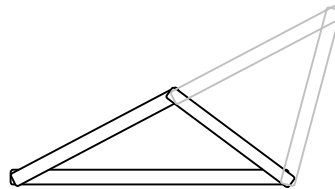


- Truss configuration:

Three members form a rigid assembly with **3 (three)** connections.

To add members and still have a rigid assembly, **2 (two)** more must be added with one connection between.

For rigidity:  $b = 2n - 3$ , where  $b$  is number of members and  $n$  is number of joints



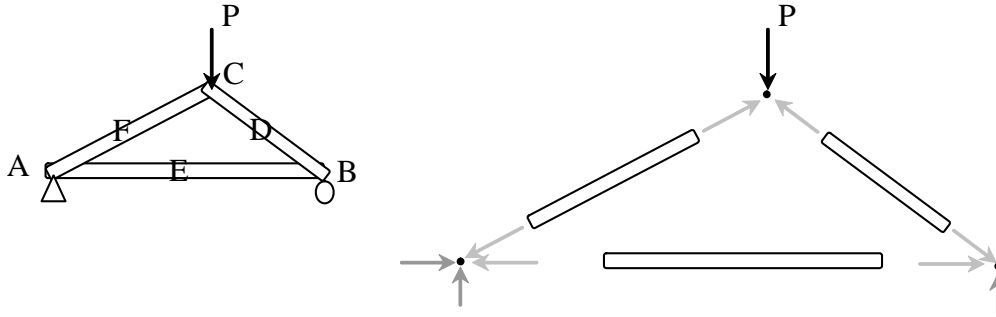
### Method of Joints

- The method takes advantage of the conditions of equilibrium at each joint.
1. Determine support reaction forces.
  2. Draw a FBD of each member AND each joint
  3. Identify geometry of angled members
  4. Identify zero force members and other special (easy to solve) cases
  5. Each pin is in equilibrium ( $\sum F_x = 0$  and  $\sum F_y = 0$  for a concurrent force system)
  6. Total equations =  $2n = b+3$  (one force per member + 3 support reactions)

Advantages: Can find every member force

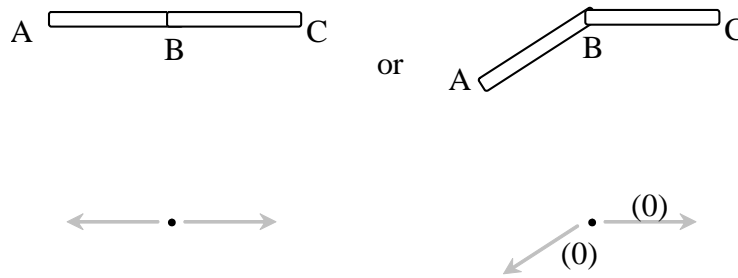
Disadvantages: Lots of equations, easy to lose track of forces found.

*Tools available:* Tip-to-tail method for 3 joint forces must close  
Analytically, there will be at most 2 unknowns with 2 equilibrium equations.



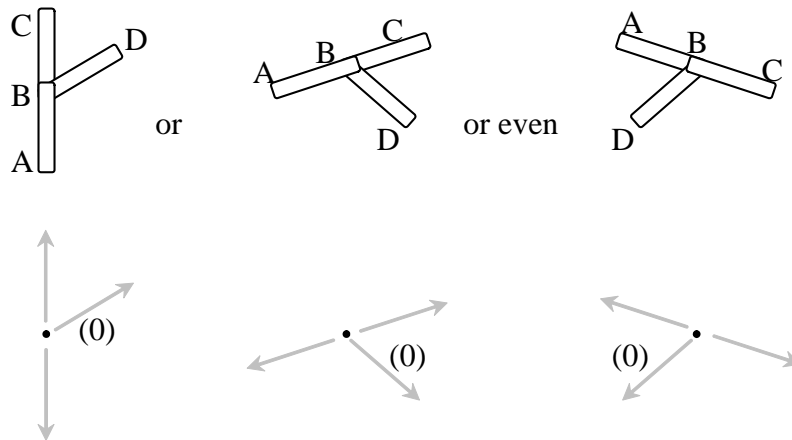
Joint Configurations (special cases to recognize for faster solutions)

Case 1) Two Bodies Connected



$F_{AB}$  has to be **equal (=)** to  $F_{BC}$

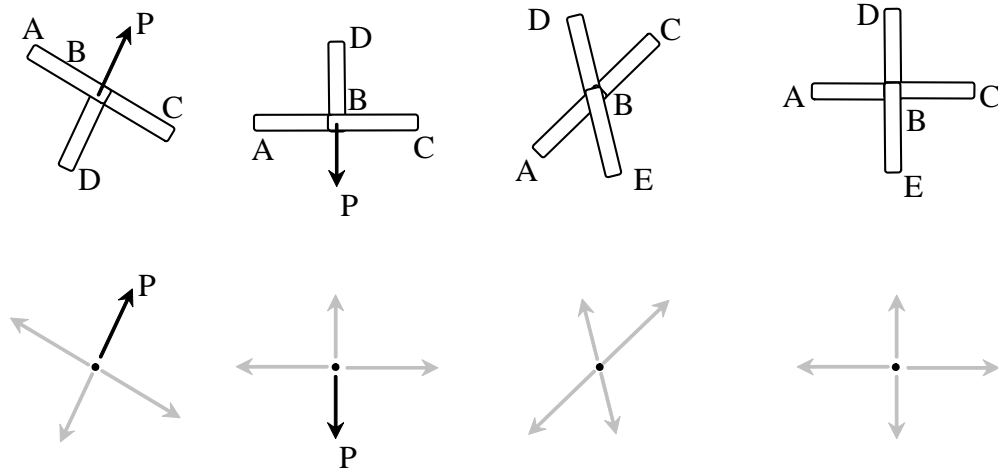
Case 2) Three Bodies Connected with Two Bodies in Line



$F_{AB}$  and  $F_{BC}$  have to be equal, and  $F_{BD}$  has to be **0 (zero)**.

Case 3) Three Bodies Connected and a Force – 2 Bodies aligned & 1 Body and a Force are Aligned

Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned



$F_{AB}$  has to equal  $F_{BC}$ , and  $[F_{BD}$  has to equal  $P$ ] or  $[F_{BD}$  has to equal  $F_{BE}]$

### Graphical Analysis

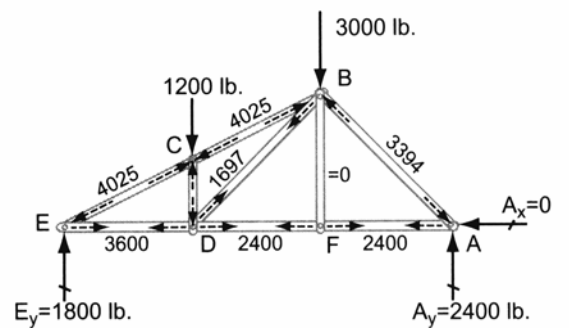
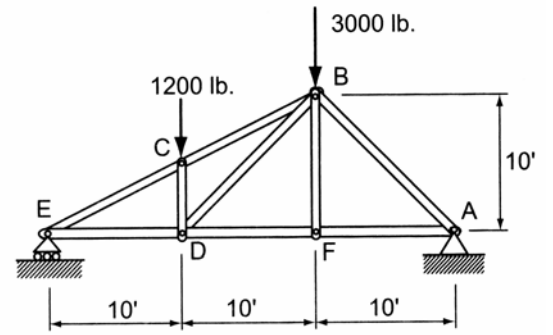
The method utilizes what we know about force triangles and plotting force magnitudes *to scale*.

1. Draw an accurate form diagram of the truss at a convenient scale with the loads and support reaction forces.
2. Determine the support reaction forces.
3. Working clockwise and from left to right, apply interval notation to the diagram, assigning capital letters to the spaces between external forces and numbers to internal spaces.
4. Construct a load line to a convenient scale of length to force by using the interval notation and working clockwise around the truss from the upper left plotting the lengths of the vertical and horizontal loads.
5. Starting at a left joint where we know there are fewer than three forces, we draw reference lines in the direction of the unknown members so that they intersect. Label the intersection with the number of the internal space.
6. Go to the next joint (clockwise and left to right) with two unknown forces and repeat for all joints. The diagram should close.
7. Measure the line segments and apply interval notation to determine their sense: Proceeding clockwise around the joint, follow the notation. The direction toward the joint is compressive. The direction away from the joint is tensile.

Example 1 (pg 90)

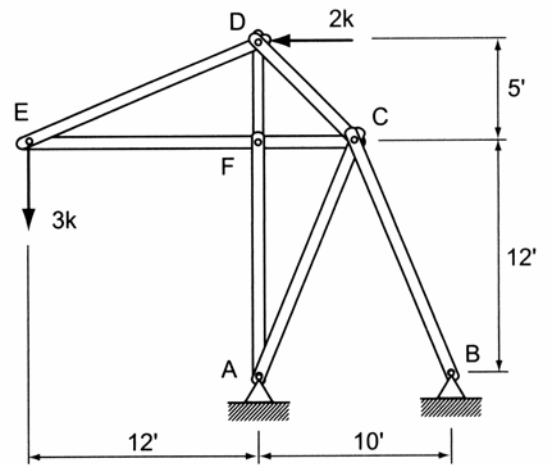
Example Problem 4.1 (Method of Joints)

An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. Determine the support reactions at each end, then, using the method of joints, solve for all member forces. Summarize the results of all member forces on a FBD (this diagram is referred to as a *force summation diagram*).



Example 2 (pg 93)**Example Problem 4.2 (Method of Joints)**

A simple stadium truss (Figure 4.12) is loaded with two forces as shown. Solve for the support reactions at *A* and *B*, then, using the method of joints, determine all of the member forces.

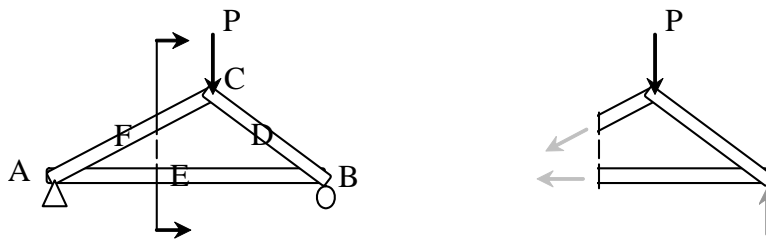


Method of Sections (relies on internal forces being in equilibrium with external forces on a section)

1. Determine support reaction forces.
2. Cut a section in such a way that force action lines intersect and no more than 3 members are cut.
3. Solve for equilibrium. Sum moments about an intersection of force lines of action

Advantages: Quick when you only need one or two forces (only 3 equations needed)

Disadvantages: Not always easy to find a place to cut a section or see where force lines intersect



- Compound Truss: A truss assembled of simple trusses and additional links. It has  $b=2n-3$ , is statically determinate, rigid and completely constrained with a pin and roller. It can be identified by triangles with pins in the middle of some sides.

• Statically Indeterminate Trusses:

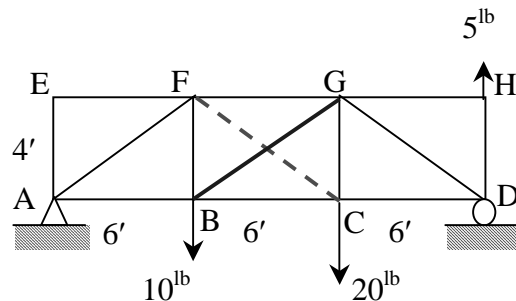
Occur if there are more members than equations for all the joints

OR if there are more reaction supports unknowns than 3

- Diagonal Tension Counters: Crossed bracing of cables or slender members is commonly used in bridge trusses, buildings and towers. These trusses look indeterminate, but can be solved statically because the bracing cannot hold a compressive force. The members are excluded in the analysis.

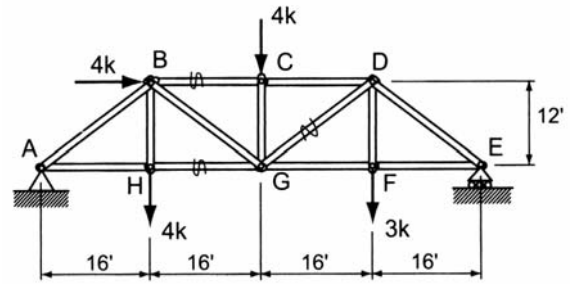
Method:

1. Determine support reaction forces.
2. Cut a section in such a way that the tension counters are exposed.
3. Solve for force equilibrium in y with one counter. If the value is positive (in tension), this is the solution.
4. Solve for force equilibrium in y with the other counter.



Example 3 (pg 99)**Example Problem 4.3 (Method of Sections)**

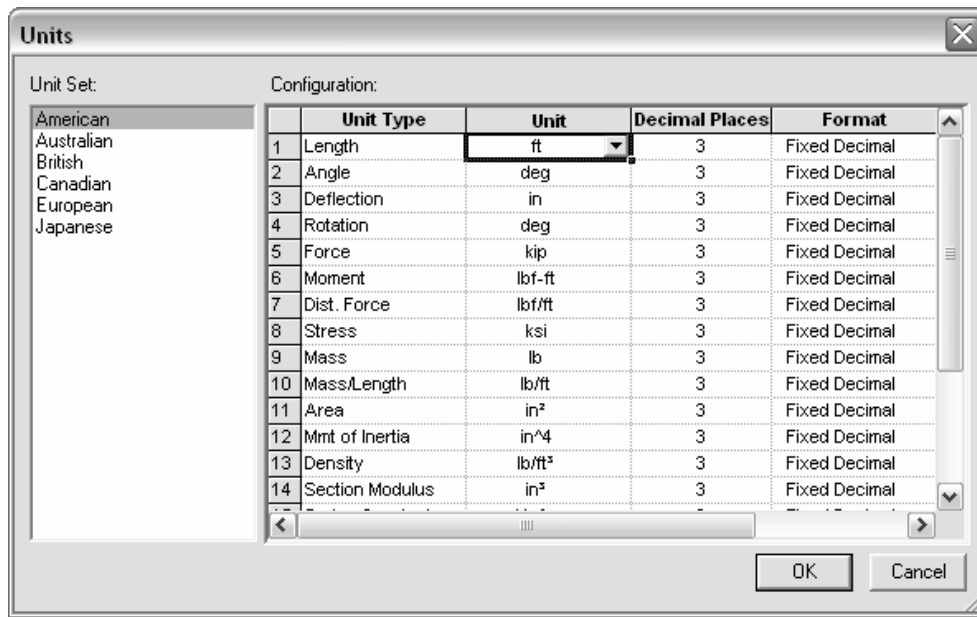
A 64-foot parallel chord truss (Figure 4.30) supports horizontal and vertical loads as shown. Using the method of sections, determine the member forces  $BC$ ,  $HG$ , and  $GD$ .



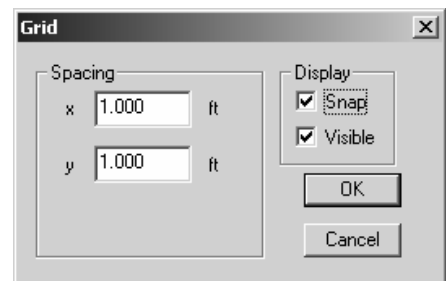


### Truss Analysis using Multiframe4D

1. The software is on the computers in the college computing lab (<http://thelab.tamu.edu>) in Programs under the Windows Start menu. ~~Multiframe4D is under the COSC menu.~~
2. There is a tutorial in the Help menu (Chapter 1 – 2D Tutorial) that lists the tasks and order in greater detail. The first task is to define the unit system:
  - Choose Units... from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.



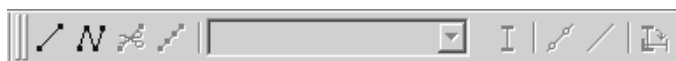
3. To see the scale of the geometry, a grid option is available:
  - Choose Grid... from the View menu



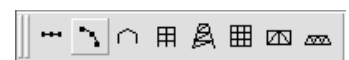
4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:



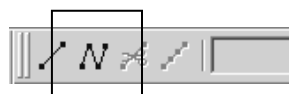
The Member toolbar shows ways to create members:



The Generate toolbar has convenient tools to create typical structural shapes.

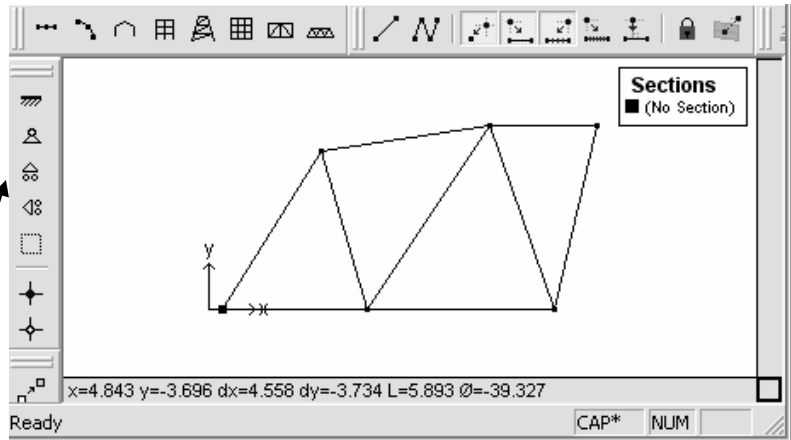


- To create a truss, use the add connected members button:



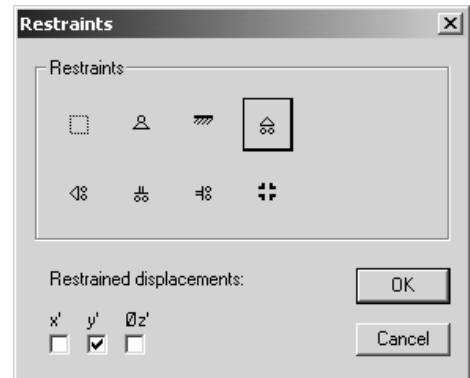
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing. Continue to use the add connected members button. Any time the cursor is over an existing joint, the joint will be highlighted by a red circle.

- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.



- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint... menu (right click).

NOTE: If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.



The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

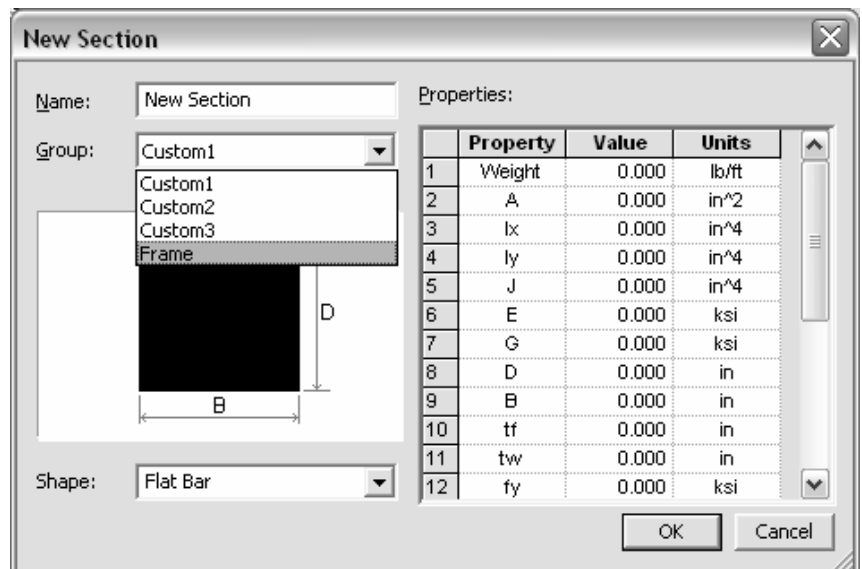


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like  $t_f$ ,  $t_w$ , etc. refer to wide flange sections.

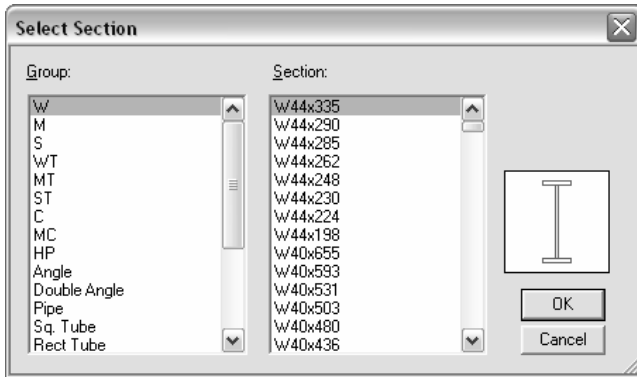
- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

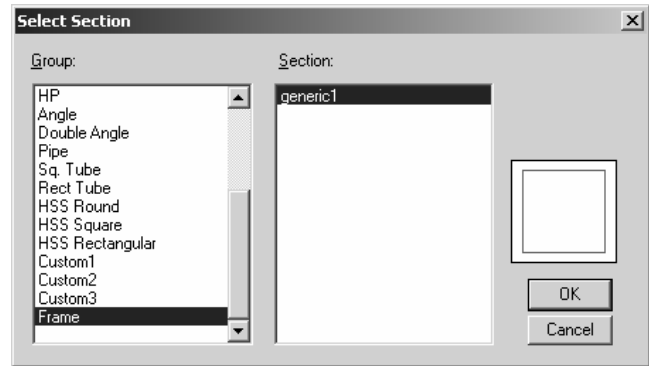


- Choose the group name and section name:

(STANDARD SHAPES)



(CUSTOM)



7. In order for Multiframe4D to recognize that the truss members are two-force bodies, all members must be highlighted and assigned pinned ends with the Pinned Ends button on the Member toolbar:



8. The truss geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:



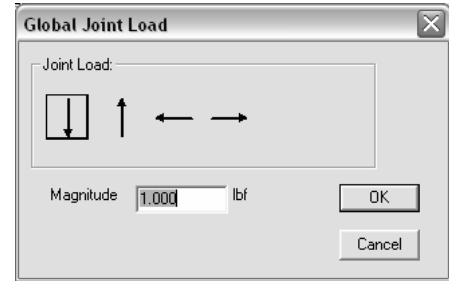
9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force in the local coordinate system (last two buttons).



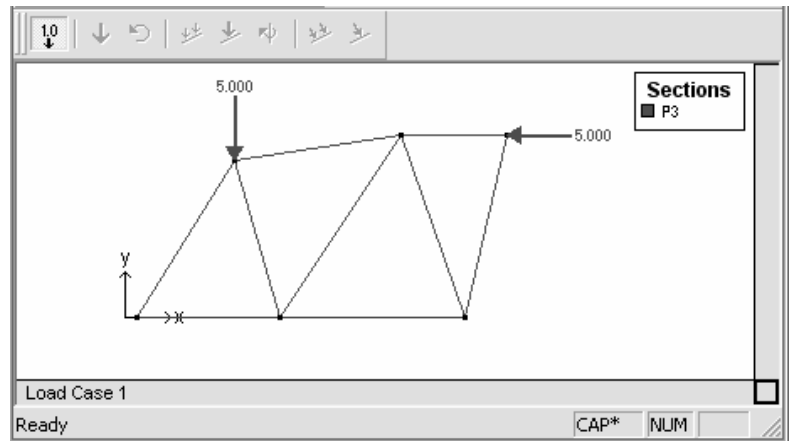
- Choose the joint to be loaded (drag) and select the load type (here shown for point loading):



- Choose the direction by the arrow shown. There is no need to put in negative values for downward loading.
- Enter the values of the load



NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values. Multiframe2D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).



10. In order to run the analysis after the geometry, member properties and loading has been defined:

- Choose Analyze Linear from the Case menu

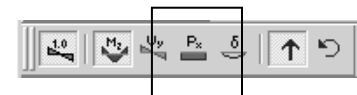
11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:



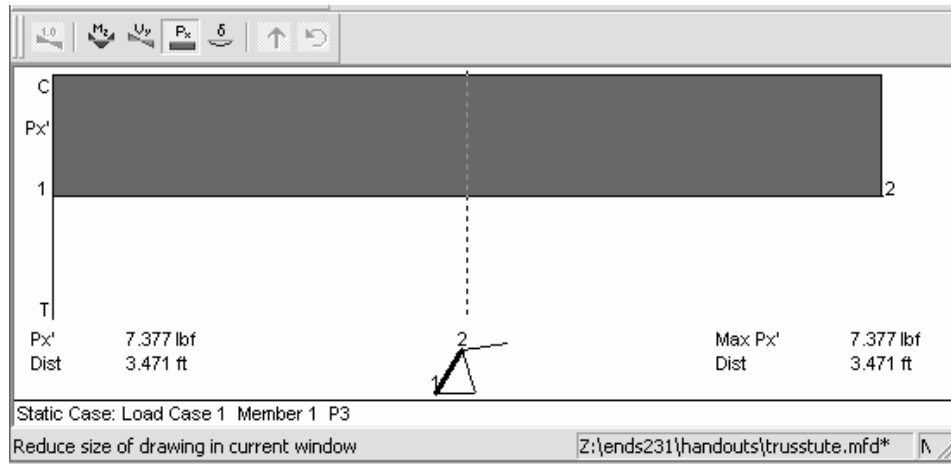
12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):



- To show the axial force diagram, Choose the purple Axial Force button. Tensile members will have “T” by the value (if turned on), while compression members will have “C” by the value
- To show the deflection diagram, Choose the blue Deflection button
- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.



- To see exact values of axial load and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.



13. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.



14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.



Static Case: Load Case 1

	Joint	Label	Rx' lbf	Ry' lbf	Mz' lbf-ft
1	1		5.000	6.246	0.000
2	2		0.000	-0.000	0.000
3	3		-0.000	-0.000	0.000
4	4		0.000	0.000	0.000
5	5		0.000	-1.246	0.000
6	6		0.000	-0.000	0.000
7	Total (Global)		Rx=5.000	Ry=5.000	

Reactions Member Actio

Static Case: Load Case 1

	Memb	Label	Joint	Px' lbf	Vy' lbf	Mz' lbf-ft
1	1		1	7.377	0.000	0.000
2	1		2	-7.377	0.000	0.000
3	2		2	-0.681	0.000	0.000
4	2		3	0.681	0.000	0.000
5	3		1	1.075	0.000	0.000
6	3		3	-1.075	0.000	0.000
7	4		2	4.157	0.000	0.000
8	4		4	-4.157	0.000	0.000

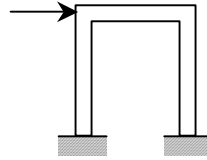
Member Actions Max Ad

NOTE: Px' refers to the axial load (P) in the local axis x direction (x').

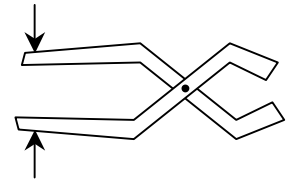
15. To save the file Choose Save from the File menu.  
 16. To load an existing file Choose Open... from the File menu.

### Pinned Frames and Arches

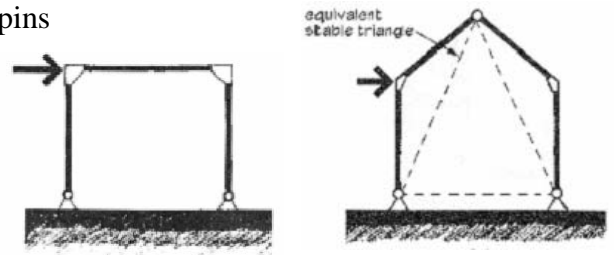
- A FRAME is made up of members where at least one member has more than 3 forces on it
  - Usually stationary and fully constrained



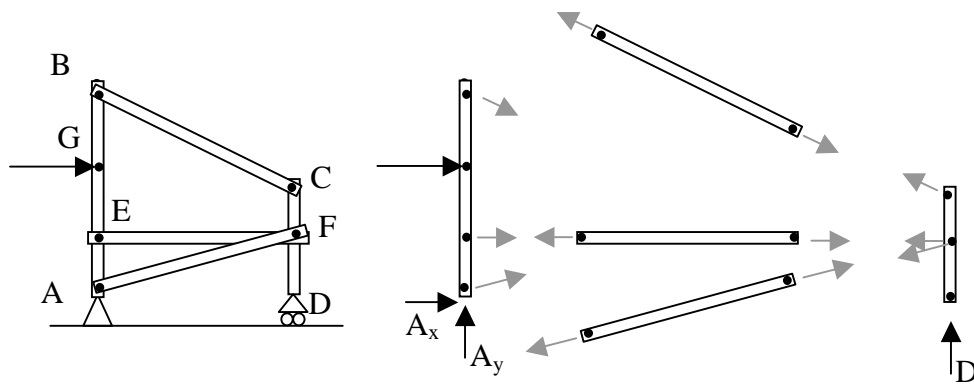
- A PINNED FRAME has member connected by pins
  - Considered *non-rigid* if it would collapse when the supports are removed
  - Considered *rigid* if it retains it's original shape when the supports are removed



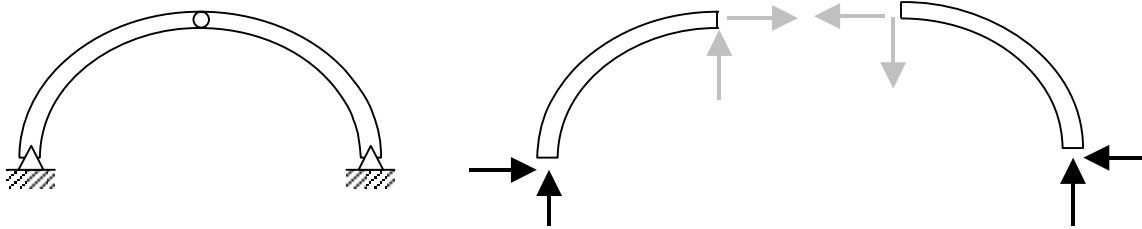
- A RIGID FRAME is all one member with no internal pins
  - Typically *statically indeterminate*
  - **Portal** frames look like door frames
  - Gable frames have a peak.



- INTERNAL PIN CONNECTIONS:
  - Pin connection forces are **equal** and **opposite** between the bodies they connect.
  - There are 2 unknown forces at a pin, but if we know a body is a **two-force** body, the direction of the *resultant* force is known.



- AN ARCH is a structural shape that can span large distances and sees compression along its slope. It may have no hinges (or pins), two hinges at the supports, or two hinges at the supports with a hinge at the apex. The three-hinged arch types are statically determinate with 2 bodies and **6** unknown forces.

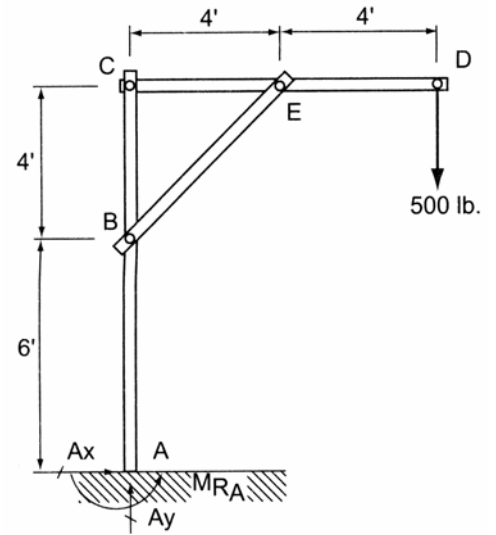


### Solution Procedure

1. Solve for the support forces on the entire frame (FBD) if possible.
2. Draw a FBD of each member:
  - Consider all two-force bodies first.
  - Pins are integral with members
  - Pins with applied forces should belong to members with greater than two forces [Same if pins connect 3 or more members]
  - Draw forces on either side of a pin equal and opposite with arbitrary direction chosen for the first side
  - Consider all multi-force bodies
  - Represent connection forces not known by x & y components
  - There are still three equilibrium equations available, but the moment equations may be more helpful when the number of unknowns is greater than two.

Example 1 (pg 114)**Example Problem 4.12**

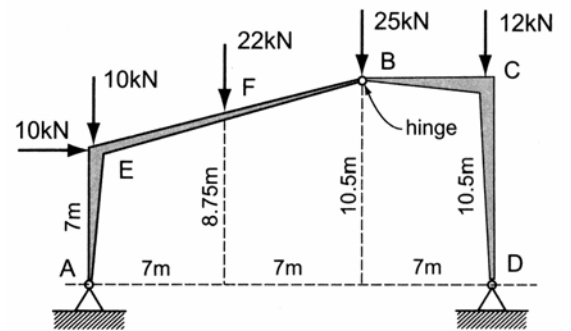
A pinned frame with a fixed base at  $A$  supports a load at the overhang equal to 500 pounds, as shown in Figure 4.68. Draw free body diagrams and solve for the support reactions and the pin reactions at  $B$ ,  $C$ , and  $E$ .





Example 2 (pg 115)**Example 4.13 (Three-Hinged Arch)**

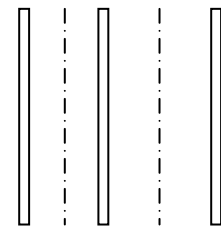
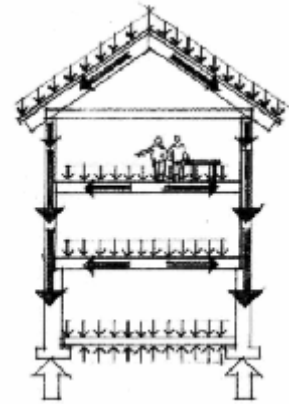
An industrial building is framed using tapered steel sections (haunches) and connected with three hinges (Figure 4.70). Assuming that the loads shown are from gravity loads and wind, determine the support reactions at *A* and *D* and the pin reactions at *B*.



## Loads – Tracing, Concentrated and Distributed

### Load Tracing

- LOAD TRACING is the term used to describe how the loads on and in the structure are transferred through the members (*load paths*) to the foundation, and ultimately supported by the ground.
- It is a sequence of **actions**, NOT reactions. Reactions in statically determinate members (using FBD's) can be solved for to determine the actions on the next member in the hierarchy.
- The *tributary area* is a loaded area that contributes to the load on the member supporting that area, *ex.* the area from the center between two beams to the center of the next two beams for the full span is the load on the center beam
- The *tributary load* on the member is found by **concentrating (or consolidating)** the load into the center.



plan

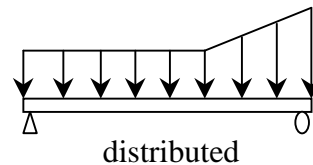
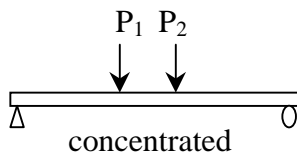
$$w = \left(\frac{\text{load}}{\text{area}}\right) \times (\text{tributary width})$$

where:

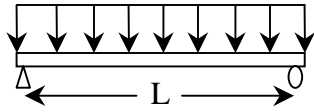
$w$  = distributed load in units of load/length

### Support Conditions & Loading

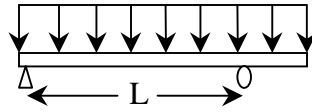
- Types of loads:
  - Concentrated – single load at one point
  - Distributed – loading spread over a distance or area



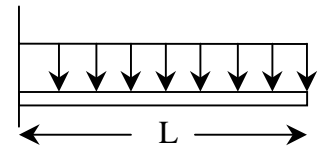
- Types of supports:
  - statically determinate  
(number of unknowns  $\leq$  number of equilibrium equations)



simply supported  
(most common)

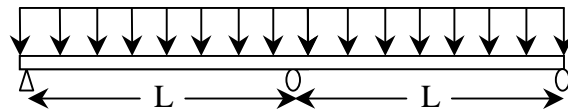


overhang



cantilever

- Statically indeterminate:

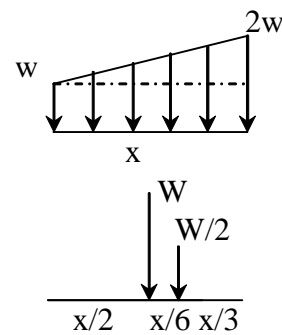
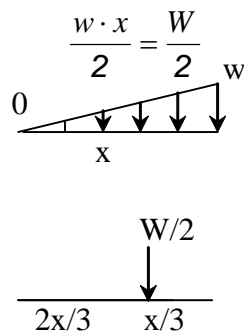
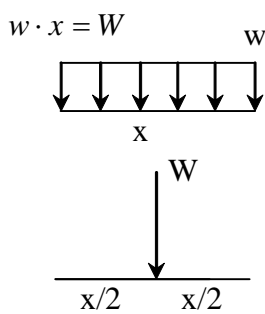


continuous  
(most common case when  $L_1=L_2$ )

Distributed Loads

Distributed loads may be replaced by concentrated loads acting through the balance/center of the distribution or *load area*: **THIS IS AN EQUIVALENT FORCE SYSTEM.**

- $w$  is the symbol used to describe the *load* per unit **length**.
- $W$  is the symbol used to describe the *total load*.

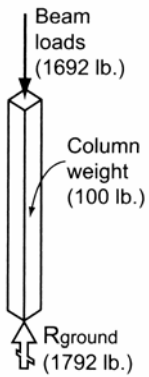
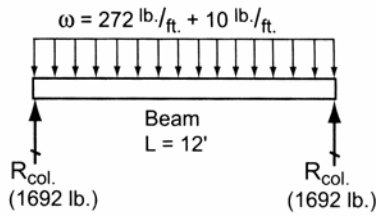
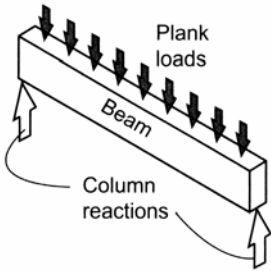
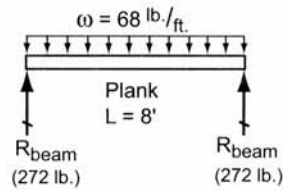
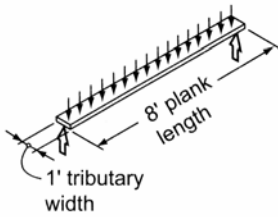
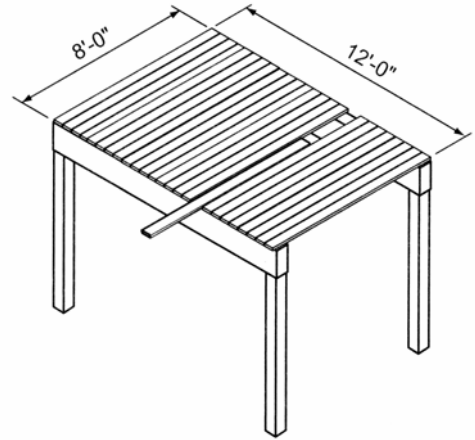


Example 1 (pg 168)

Example Problem 5.2

In the single-bay, post-and-beam deck illustrated, planks typically are available in nominal widths of 4" or 6", but for the purposes of analysis it is permissible to assume a unit width equal to one foot. Determine the plank, beam, and column reactions.

The loads are: 60 lb/ft<sup>2</sup> live load, 8 lb/ft<sup>2</sup> dead load, 10 lb/ft self weight of 12' beams, and 100 lb self weight of columns.



Example 2

**EXAMPLE**

Assume that the average dead plus live load on the structure shown in Figure 3.15 is 60 lbs/ft<sup>2</sup>. Determine the reactions for Beam D. This is the same structure as shown in Figure 3.1.

**Solution:**

Note carefully the directions of the decking span. Beam D carries floor loads from the decking to the left (see the contributory area and load strip), but not to the right, since the

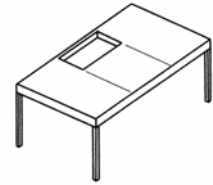
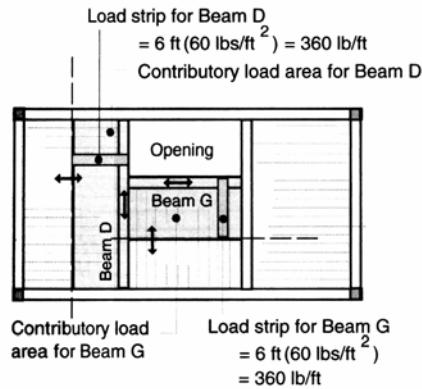
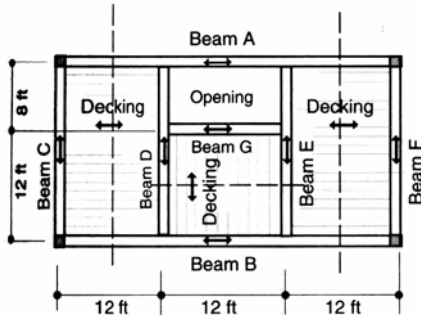


Figure 3.1



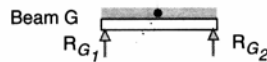
Live and dead load

Assume  $w_{DL+LL} = 60 \text{ lbs/ft}^2$

Beam G carries distributed loads only

Find reactions for Beam G

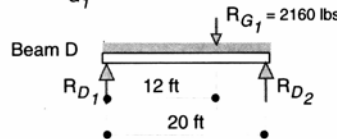
$$W = 6 \text{ ft} (60 \text{ lbs/ft}^2) = 360 \text{ lb/ft}$$



$$R_{G1} = wL/2 = (360 \text{ lb/ft})(12 \text{ ft})/2 = 2160 \text{ lbs}$$

$$R_{G2} = wL/2 = (360 \text{ lb/ft})(12 \text{ ft})/2 = 2160 \text{ lbs}$$

Beam D carries both distributed loads and the reaction  $R_{G1}$  from Beam G



$$\begin{aligned} \sum M_{D_1} = 0 \\ - (12 \text{ ft})(2160 \text{ lb}) - (360 \text{ lb/ft})(20 \text{ ft})(20 \text{ ft}/2) + 20 R_{D_2} = 0 \\ R_{D_2} = 4896 \text{ lb} \end{aligned}$$

$$\begin{aligned} \sum F_y = 0 \\ R_{D_1} + R_{D_2} = (360 \text{ lb/ft})(20 \text{ ft}) + 2160 \text{ lb} \\ R_{D_1} = 4464 \text{ lb} \end{aligned}$$

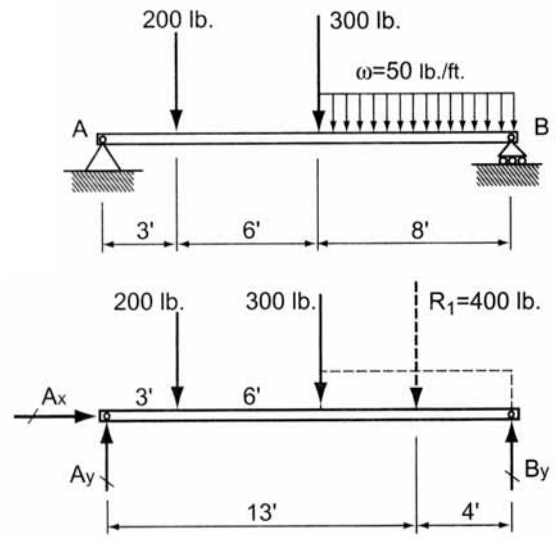
FIGURE 3.15 Load modeling and reaction determination.

center decking runs parallel to Beam D and is not carried by it. Beam D also picks up the end of Beam G and thus also “carries” the reactive force from Beam G. It is therefore necessary to analyze Beam G first to determine the magnitude of this force. The analysis appears in Figure 3.15. The reactive force from Beam G of 2160 lbs is then treated as a downward force acting on Beam D. The load model for Beam D thus consists of distributed forces from the decking plus the 2160-lb force. It is then analyzed by means of the equations of statics to obtain reactive forces of 4896 lbs and 4464 lbs at its ends.

Example 3 (pg 70)

**Example Problem 3.12—Simple Beam**

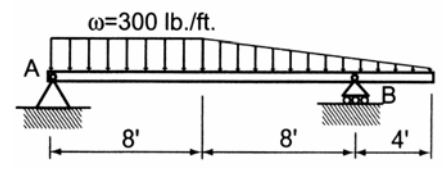
A simple beam supports two concentrated loads and a uniformly distributed load over 8 ft. of the span. See Figure 3.40. Construct a FBD of the beam and solve for the support reactions at *A* and *B*.



Example 4 (pg 71)

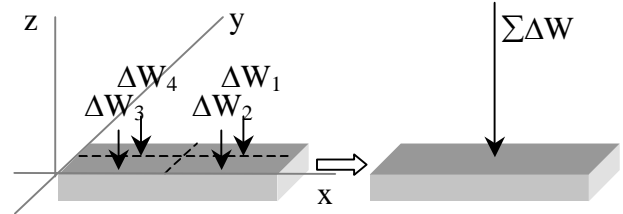
**Example Problem 3.13—Overhang Beam (Figure 3.41)**

A single overhang beam supports a uniformly distributed load over an 8-ft. section and a triangular distribution over the remaining 12 ft. Draw the appropriate FBD and solve for the support reactions at *A* and *B*.



### Centers of Gravity - Centroids

- The *center of gravity* is the location of the equivalent force representing the total weight of a body comprised of particles that each have a mass gravity acts upon.



Resultant force: Over a body of constant thickness in x and y

$$\sum F_z = \sum_{i=1}^n \Delta W_i = W \qquad W = \int dW$$

Location:  $\bar{x}$ ,  $\bar{y}$  is the equivalent location of the force W from all  $\Delta W_i$ 's over all x & y locations (with respect to the moment from each force) from:

$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \qquad \bar{x} W = \int x dW \Rightarrow \bar{x} = \frac{\int x dW}{W} \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x\Delta W)}{W}}$$

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \qquad \bar{y} W = \int y dW \Rightarrow \bar{y} = \frac{\int y dW}{W} \quad \text{OR} \quad \boxed{\bar{y} = \frac{\sum(y\Delta W)}{W}}$$

- The *centroid of an area* is the average x and y locations of the area particles

For a discrete shape ( $\Delta A_i$ ) of a uniform thickness and material, the weight can be defined as:

$\Delta W_i = \gamma t \Delta A_i$  where:  
 $\gamma$  is weight per unit **volume** (= specific weight) with units of  $\text{N/m}^3$  or  $\text{lb/ft}^3$   
 $t \Delta A_i$  is the volume

So if  $W = \gamma A$  :

$$\bar{x} \gamma A = \int x \gamma t dA \Rightarrow \bar{x} A = \int x dA \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x\Delta A)}{A}} \quad \text{and similarly} \quad \boxed{\bar{y} = \frac{\sum(y\Delta A)}{A}}$$

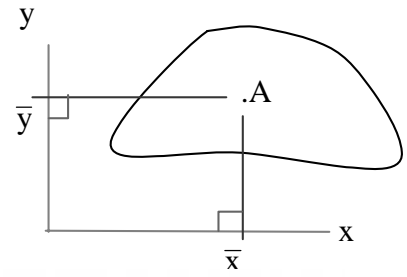
Similarly, for a line with constant cross section,  $a$  ( $\Delta W_i = \gamma a \Delta L_i$ ):

$$\bar{x} L = \int x dL \quad \text{OR} \quad \boxed{\bar{x} = \frac{\sum(x\Delta L)}{L}} \quad \text{and} \quad \bar{y} L = \int y dL \quad \text{OR} \quad \boxed{\bar{y} = \frac{\sum(y\Delta L)}{L}}$$

- $\bar{x}$ ,  $\bar{y}$  **with respect to an x, y coordinate system** is the centroid of an area AND the center of **gravity** for a body of uniform material and thickness.

- The *first moment of the area* is like a force moment: and is the **area** multiplied by the perpendicular distance to an axis.

$$Q_x = \int ydA = \bar{y}A \quad Q_y = \int x dA = \bar{x}A$$



- Centroids of Common Shapes
- Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	$\alpha r^2$
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	$\pi r$
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$



- Symmetric Areas

- An area is symmetric with respect to a line when every point on one side is mirrored on the other. The line divides the area into equal parts and the centroid will be on that axis.
- An area can be symmetric to a *center point* when every (x,y) point is matched by a (-x,-y) point. It does not necessarily have an axis of symmetry. The center point is the *centroid*.
- If the symmetry line is on an axis, the centroid location is on that axis (value of 0). With double symmetry, the centroid is at the intersection.
- Symmetry can also be defined by areas that match across a line, but are 180° to each other.

### Basic Steps

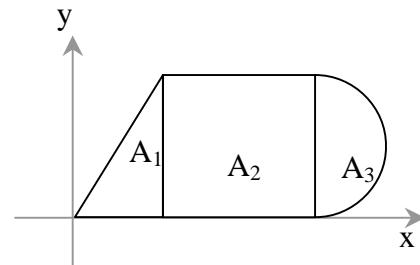
1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of *Component*, *Area*,  $\bar{x}$ ,  $\bar{x}A$ ,  $\bar{y}$ ,  $\bar{y}A$
5. Fill in the table value
6. Draw a summation line. Sum all the areas, all the  $\bar{x}A$  terms, and all the  $\bar{y}A$  terms
7. Calculate  $\hat{x}$  and  $\hat{y}$

- Composite Shapes

If we have a shape made up of basic shapes that we know centroid locations for, we can find an “average” centroid of the areas.

$$\hat{x}A = \hat{x} \sum_{i=1}^n A_i = \sum_{i=1}^n \bar{x}_i A_i \qquad \hat{y}A = \hat{y} \sum_{i=1}^n A_i = \sum_{i=1}^n \bar{y}_i A_i$$

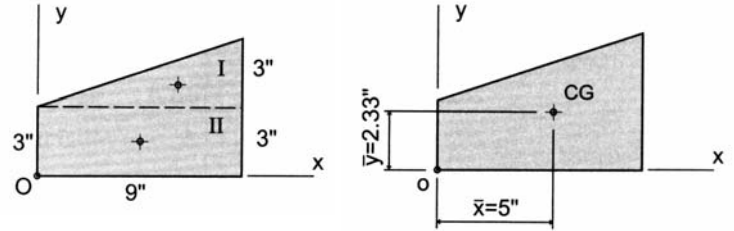
Centroid values can be negative.  
Area values can be negative (holes)



Example 1 (pg 243)

**Example Problem 7.1: Centroids (Figures 7.5 and 7.6)**

Determine the centroidal  $x$  and  $y$  distances for the composite area shown. Use the lower left corner of the trapezoid as the reference origin.



Component	Area ( $\Delta A$ ) (in. <sup>2</sup> )	$\bar{x}$ (in.)	$\bar{x}\Delta A$ (in. <sup>3</sup> )	$\bar{y}$ (in.)	$\bar{y}\Delta A$ (in. <sup>3</sup> )
<p>(a)</p>	$\frac{9(3)}{2} = 13.5 \text{ in.}^2$	6"	81 in. <sup>3</sup>	4"	54 in. <sup>3</sup>
<p>(b)</p>	9" (3") = 27 in. <sup>2</sup>	4.5"	121.5 in. <sup>3</sup>	1.5"	40.5 in. <sup>3</sup>
	$A = \sum \Delta A = 40.5 \text{ in.}^2$		$\sum \bar{x}\Delta A = 202.5 \text{ in.}^3$		$\sum \bar{y}\Delta A = 94.5 \text{ in.}^3$

$$\hat{x} = \frac{202.5 \text{ in.}^3}{40.5 \text{ in.}^2} = 5 \text{ in}$$

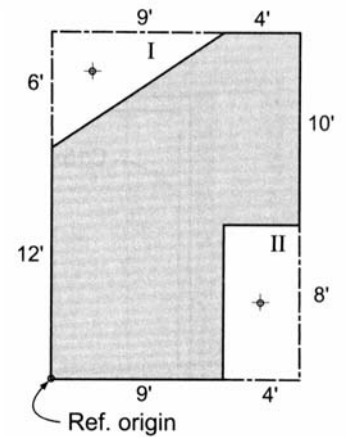
$$\hat{y} = \frac{94.5 \text{ in.}^3}{40.5 \text{ in.}^2} = 2.33 \text{ in}$$

Example 2 (pg 245)

**Example Problem 7.3b (Figure 7.13)**

An alternate method that can be employed in solving this problem is referred to as the *negative area method*.

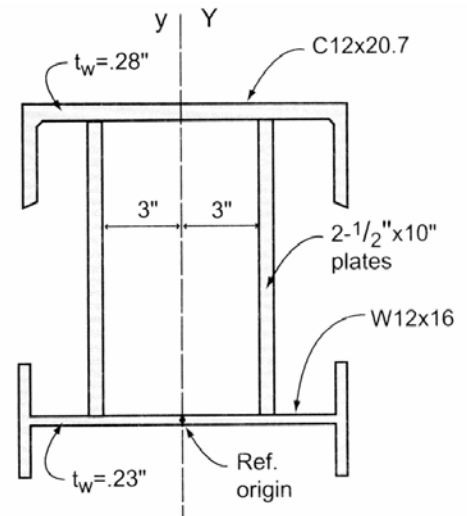
A 6" thick concrete wall panel is precast to the dimensions as shown. Using the lower left corner as the reference origin, determine the center of gravity (centroid) of the panel.



Example 3 (pg 249)

## Example Problem 7.5 (Figures 7.16 and 7.17)

A composite or built-up cross-section for a beam is fabricated using two  $\frac{1}{2} \times 10$ " vertical plates with a C12  $\times$  20.7 channel section welded to the top and a W12  $\times$  16 section welded to the bottom as shown. Determine the location of the major  $x$ -axis using the center of the W12  $\times$  16's web as the reference origin.



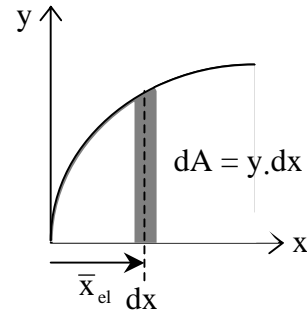
### Moments of Inertia

- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.
- Definition: Moment of Inertia;** the second area moment

$$I_y = \int x^2 dA \qquad I_x = \int y^2 dA$$

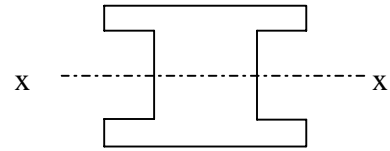
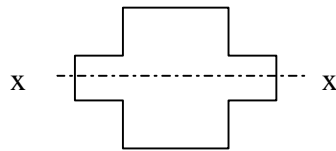
We can define a single integral using a narrow strip:

for  $I_x$ , strip is parallel to  $x$       for  $I_y$ , strip is parallel to  $y$



*\*I can be negative if the area is negative (a hole or subtraction).*

- A shape that has area at a greater distance away from an axis *through its centroid* will have a **larger** value of  $I$ .

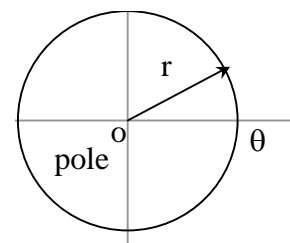


- Just like for center of gravity of an area, the moment of inertia can be determined with respect to *any* reference **axis**.

- Definition: Polar Moment of Inertia;** the second area moment using polar coordinate axes

$$J_o = \int r^2 dA = \int x^2 dA + \int y^2 dA$$

$$J_o = I_x + I_y$$



- Definition: Radius of Gyration;** the distance from the moment of inertia axis for an area at which the entire area could be considered as being concentrated at.

$$I_x = r_x^2 A \Rightarrow r_x = \sqrt{\frac{I_x}{A}} \text{ radius of gyration in } x$$

$$r_y = \sqrt{\frac{I_y}{A}} \text{ radius of gyration in } y$$

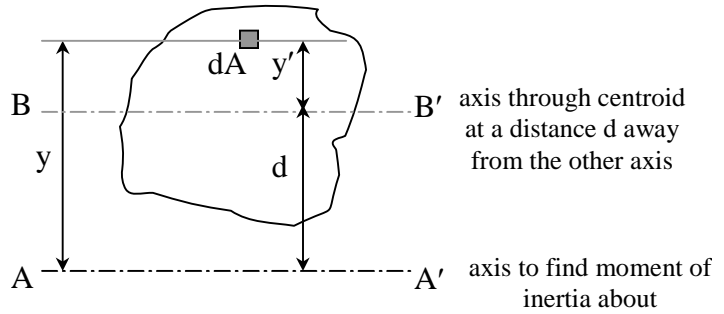
$$r_o = \sqrt{\frac{J_o}{A}} \text{ polar radius of gyration, and } r_o^2 = r_x^2 + r_y^2$$

The Parallel-Axis Theorem

- The moment of inertia of an area with respect to any axis not through its centroid is equal to the moment of inertia of that area with respect to its own parallel centroidal axis plus the product of the area and the square of the distance between the two axes.

$$I = \int y^2 dA = \int (y'-d)^2 dA$$

$$= \int y'^2 dA + 2d \int y' dA + d^2 \int dA$$



but  $\int y' dA = 0$ , because the centroid is on this axis, resulting in:

$$I_x = I_{cx} + Ad_y^2 \quad (\text{text notation}) \quad \text{or} \quad I_x = \bar{I}_x + Ad_y^2$$

where  $I_{cx}$  (or  $\bar{I}_x$ ) is the moment of inertia about the centroid of the area about an x axis and  $d_y$  is the y distance between the parallel axes

Similarly	$I_y = \bar{I}_y + Ad_x^2$	Moment of inertia about a y axis
	$J_o = \bar{J}_c + Ad^2$	Polar moment of Inertia
	$r_o^2 = \bar{r}_c^2 + d^2$	Polar radius of gyration
	$r^2 = \bar{r}^2 + d^2$	Radius of gyration

\* I can be negative again if the area is negative (a hole or subtraction).

\*\* If  $\bar{I}$  is not given in a chart, but  $\bar{x}$  &  $\bar{y}$  are: YOU MUST CALCULATE  $\bar{I}$  WITH  $\bar{I} = I - Ad^2$

Composite Areas:

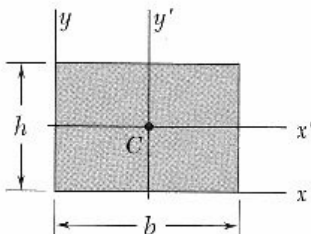
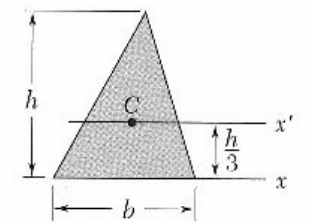
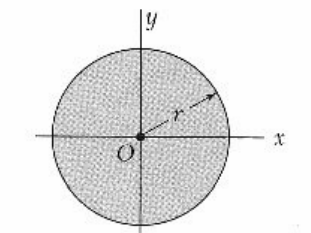
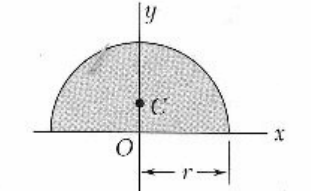
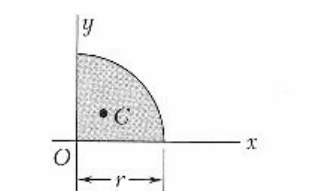
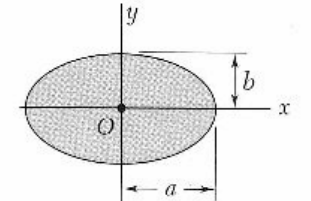
$$I = \sum \bar{I} + \sum Ad^2 \quad \text{where} \quad \bar{I} \text{ is the moment of inertia about the centroid of the component area}$$

$d$  is the distance from the centroid of the component area to the centroid of the composite area (ie.  $d_y = \hat{y} - \bar{y}$ )

**Basic Steps**

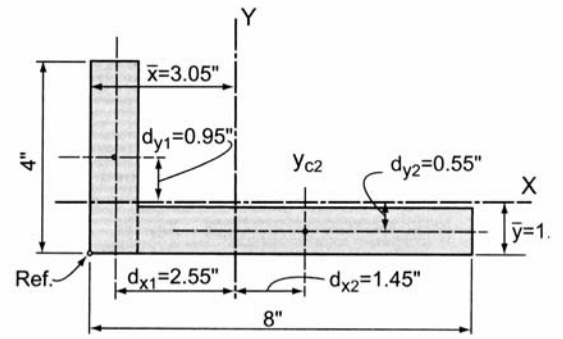
1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of  
 $Component, Area, \bar{x}, \bar{x}A, \bar{y}, \bar{y}A, \bar{I}_x, d_y, Ad_y^2, \bar{I}_y, d_x, Ad_x^2$
5. Fill in the table values needed to calculate  $\hat{x}$  and  $\hat{y}$  for the composite
6. Fill in the rest of the table values.
7. Sum the moment of inertia ( $\bar{I}$ 's) and  $Ad^2$  columns and add together.

Moments of Inertia of Common Shapes

<p>Rectangle</p>		<div style="border: 1px dashed black; padding: 5px; display: inline-block;"> <math>\bar{I}_{x'} = \frac{1}{12}bh^3</math>  <math>\bar{I}_{y'} = \frac{1}{12}b^3h</math> </div> <p>about centroid</p> $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
<p>Triangle</p>		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
<p>Circle</p>		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
<p>Semicircle</p>		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
<p>Quarter circle</p>		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
<p>Ellipse</p>		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Example 1 (pg 257)

Find the moments of inertia ( $\hat{x} = 3.05''$ ,  $\hat{y} = 1.05''$ ).

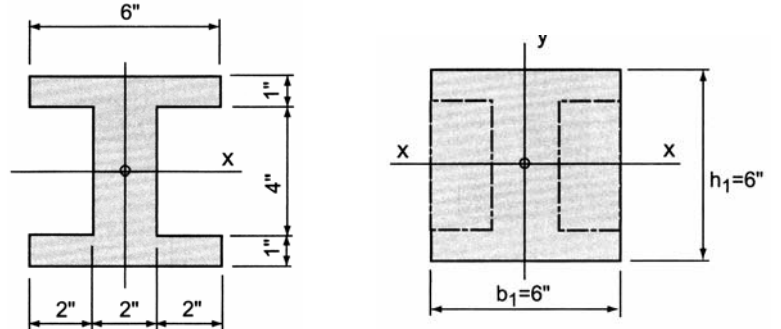


Component	$I_{xc}$ (in. <sup>4</sup> )	$d_y$ (in.)	$Ad_y^2$ (in. <sup>4</sup> )	$I_{yc}$ (in. <sup>4</sup> )	$d_x$ (in.)	$Ad_x^2$ (in. <sup>4</sup> )
	$\frac{(1)(4)^3}{12} = 5.33$	0.95	3.61	$\frac{(4)(1)^3}{12} = 0.33$	2.55	26.01
	$\frac{(7)(1)^3}{12} = 0.58$	0.55	2.12	$\frac{(1)(7)^3}{12} = 28.58$	1.45	14.72
	$\sum I_{xc} = 5.91$		$\sum Ad_y^2 = 5.73$	$\sum I_{yc} = 28.91$		$\sum Ad_x^2 = 40.73$

Example 2 (pg 253)

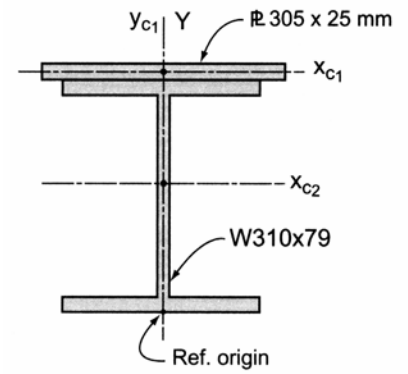
Example Problem 7.6 (Figures 7.24 to 7.26)

Determine the  $I$  about the centroidal  $x$ -axis.



Example 3 (pg 258)**Example Problem 7.10 (Figures 7.35 and 7.36)**

Locate the centroidal  $x$  and  $y$  axes for the cross-section shown. Use the reference origin indicated and assume that the steel plate is centered over the flange of the wide-flange section. Compute the  $I_x$  and  $I_y$  about the major centroidal axes.

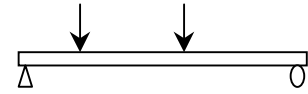




## Beam Structures and Internal Forces

- BEAMS

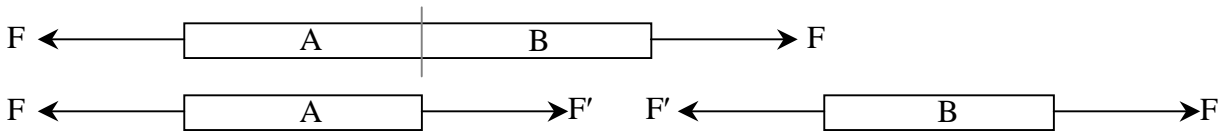
- Important type of structural members (floors, bridges, roofs)
- Usually long, straight and rectangular
- Have loads that are usually perpendicular applied at points along the length



### Internal Forces 2

- *Internal forces* are those that hold the parts of the member together for equilibrium

- Truss members:

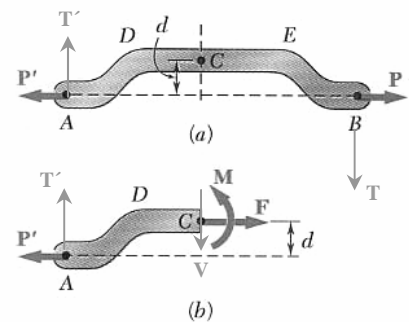


- For any member:

$F$  = internal *axial force*  
(perpendicular to cut across section)

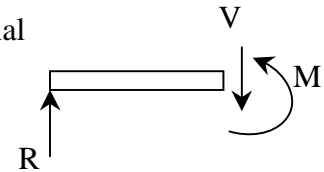
$V$  = internal *shear force*  
(parallel to cut across section)

$M$  = internal *bending moment*



### Support Conditions & Loading

- Most often loads are perpendicular to the beam and cause only internal shear forces and bending moments
- Knowing the internal forces and moments is *necessary* when designing beam size & shape to resist those loads

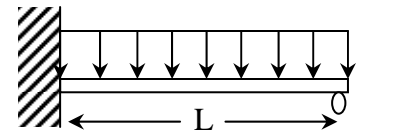


- Types of loads

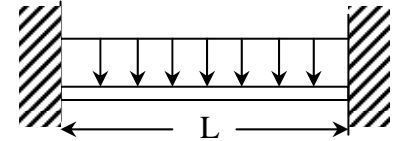
- Concentrated – single load, single moment
- Distributed – loading spread over a distance, uniform or **non-uniform**.

• Types of supports

- *Statically determinate*: simply supported, cantilever, overhang (number of unknowns < number of equilibrium equations)
- *Statically indeterminate*: continuous, fixed-roller, fixed-fixed (number of unknowns > number of equilibrium equations)



Propped

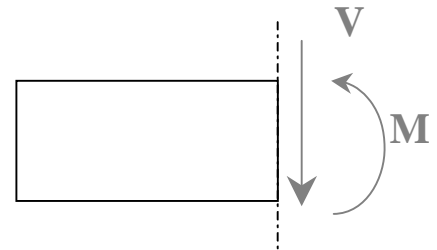



Restrained

**Sign Conventions for Internal Shear and Bending Moment**

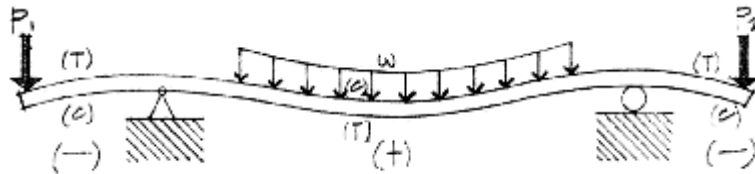
(different from statics and truss members!)

When  $\sum F_y$  \*\*excluding V\*\* on the left hand side (LHS) section is positive, V will direct down and is considered POSITIVE.



When  $\sum M$  \*\*excluding M\*\* about the cut on the left hand side (LHS) section causes a smile which could hold water (curl upward), M will be counter clockwise (+) and is considered POSITIVE. 

On the deflected shape of a beam, the point where the shape changes from smile up to frown is called the **inflection point**. The bending moment value at this point is **zero**.



**Shear And Bending Moment Diagrams**

The plot of shear and bending moment as they vary across a beam length are *extremely important design tools*: V(x) is plotted on the y axis of the shear diagram, M(x) is plotted on the y axis of the moment diagram.

The *load diagram* is essentially the free body diagram of the beam *with the actual loading (not the equivalent of distributed loads.)*

Maximum Shear and Bending – The maximum *value*, regardless of sign, is important for design.

Method 1: The Equilibrium Method

Isolate FBD sections at significant points along the beam and determine V and M at the cut section. The values for V and M can also be written in equation format as functions of the distance to the cut section.

Important Places for FBD cuts

- at supports
- at concentrated loads
- at start and end of distributed loads
- at concentrated moments

Method 2: The Semigraphical Method

Relationships exist between the loading and shear diagrams, and between the shear and bending diagrams.

Knowing the *area* of the loading gives the *change in shear (V)*.

Knowing the *area* of the shear gives the *change in bending moment (M)*.

Concentrated loads and moments cause a vertical *jump* in the diagram.

$$\frac{\Delta V}{\lim_{\Delta x \rightarrow 0} \Delta x} = \frac{dV}{dx} = -w \quad (\text{the negative shows it is down because we give } w \text{ a positive value})$$

$$V_D - V_C = - \int_{x_C}^{x_D} w dx = \text{the **area** under the load curve between C \& D}$$

\*These shear formulas are NOT VALID at discontinuities like concentrated loads

$$\frac{\Delta M}{\lim_{\Delta x \rightarrow 0} \Delta x} = \frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V dx = \text{the **area** under the shear curve between C \& D}$$

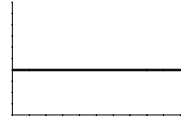
\* These moment formulas ARE VALID even with concentrated loads.

\*These moment formulas are NOT VALID at discontinuities like applied moments.

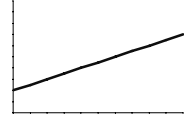
The MAXIMUM BENDING MOMENT from a curve that is continuous can be found when the slope is zero  $\left(\frac{dM}{dx} = 0\right)$ , which is when the value of the shear is 0.

**Basic Curve Relationships (from calculus) for y(x)**

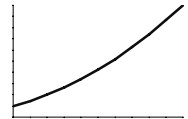
Horizontal Line:  $y = b$  (constant) and the area (change in shear) =  $b \cdot x$ , resulting in a:



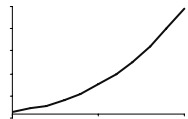
Sloped Line:  $y = mx + b$  and the area (change in shear) =  $\frac{\Delta y \cdot \Delta x}{2}$ , resulting in a:



Parabolic Curve:  $y = ax^2 + b$  and the area (change in shear) =  $\frac{\Delta y \cdot \Delta x}{3}$ , resulting in a:



3<sup>rd</sup> Degree Curve:  $y = ax^3 + bx^2 + cx + d$



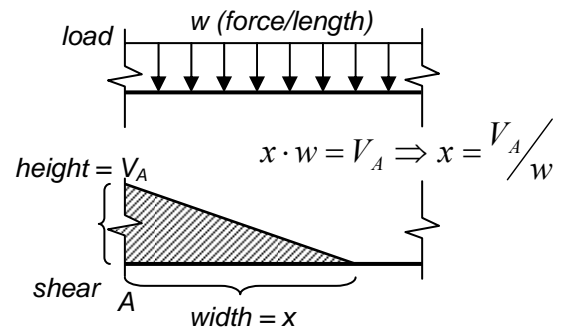
**Free Software Site:** <http://www.rekenwonder.com/atlas.htm>

**BASIC PROCEDURE:**

1. Find all support forces.

*V diagram:*

2. At free ends and at simply supported ends, the shear will have a zero value.
3. At the left support, the shear will equal the reaction force.
4. The shear will not change in x until there is another load, where the shear is reduced if the load is negative. If there is a distributed load, the change in shear is the area under the loading.
5. At the right support, the reaction is treated just like the loads of step 4.
6. At the free end, the shear should go to zero.



*M diagram:*

7. At free ends and at simply supported ends, the moment will have a zero value.
8. At the left support, the moment will equal the reaction moment (if there is one).
9. The moment will not change in x until there is another load or applied moment, where the moment is reduced if the applied moment is negative. If there is a value for shear on the V diagram, the change in moment is the area under the shear diagram.

For a triangle in the shear diagram, the width will equal the height ÷ w!

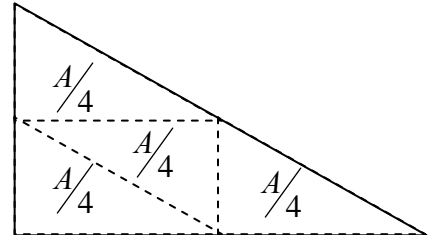
10. At the right support, the moment reaction is treated just like the moments of step 9.
11. At the free end, the moment should go to zero.

**Parabolic Curve Shapes Based on Triangle Orientation**

In order to tell if a parabola curves “up” or “down” from a triangular area in the preceding diagram, the orientation of the triangle is used as a reference.

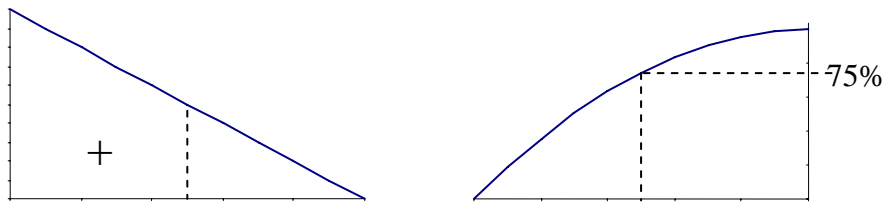
Geometry of Right Triangles

Similar triangles show that four triangles, each with  $\frac{1}{4}$  the area of the large triangle, fit within the large triangle. This means that  $\frac{3}{4}$  of the area is on one side of the triangle, if a line is drawn through the middle of the base, and  $\frac{1}{4}$  of the area is on the other side.

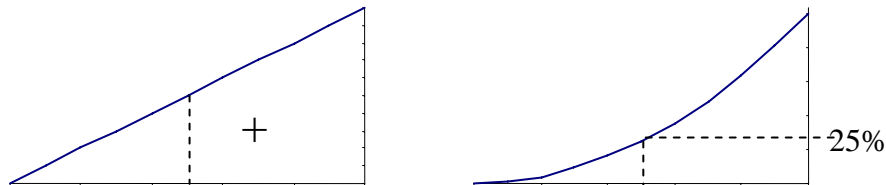


By how a triangle is oriented, we can determine the curve shape in the next diagram.

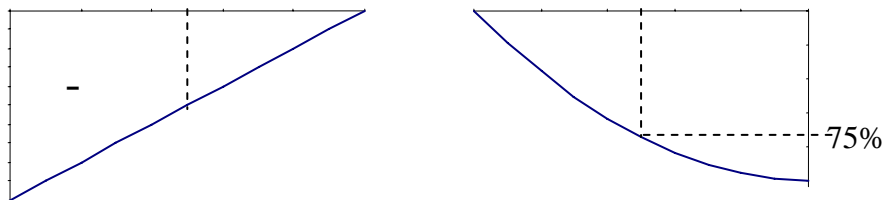
CASE 1: *Positive triangle with fat side to the left.*



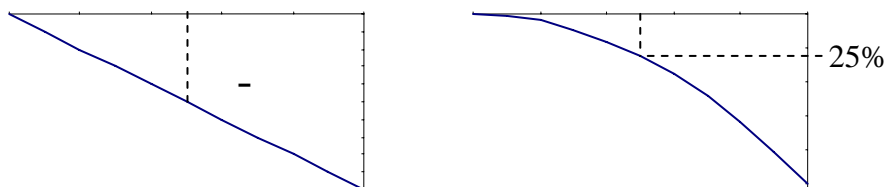
CASE 2: *Positive triangle with fat side to the right.*



CASE 3: *Negative triangle with fat side to the left.*



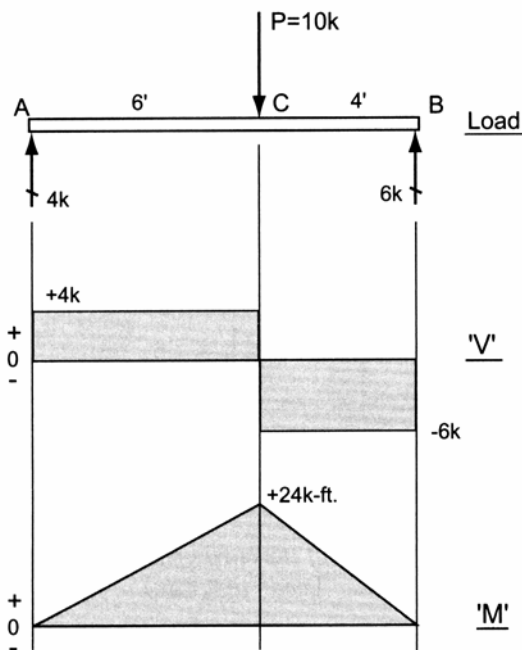
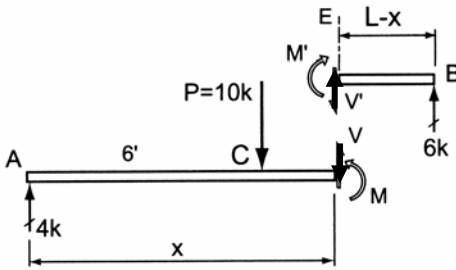
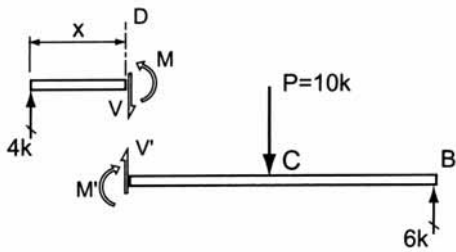
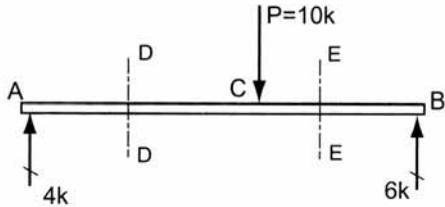
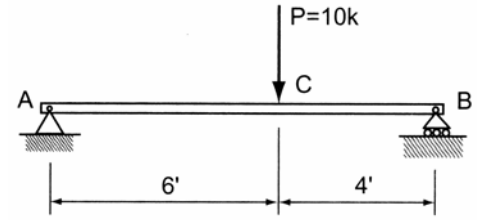
CASE 4: *Negative triangle with fat side to the right.*



Example 1 (pg 273)

Example Problem 8.1 (Equilibrium Method)

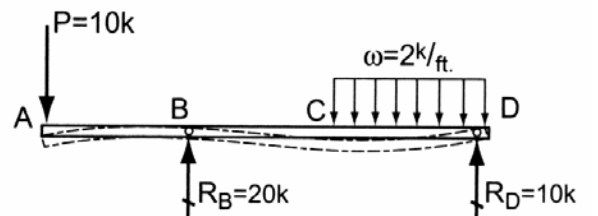
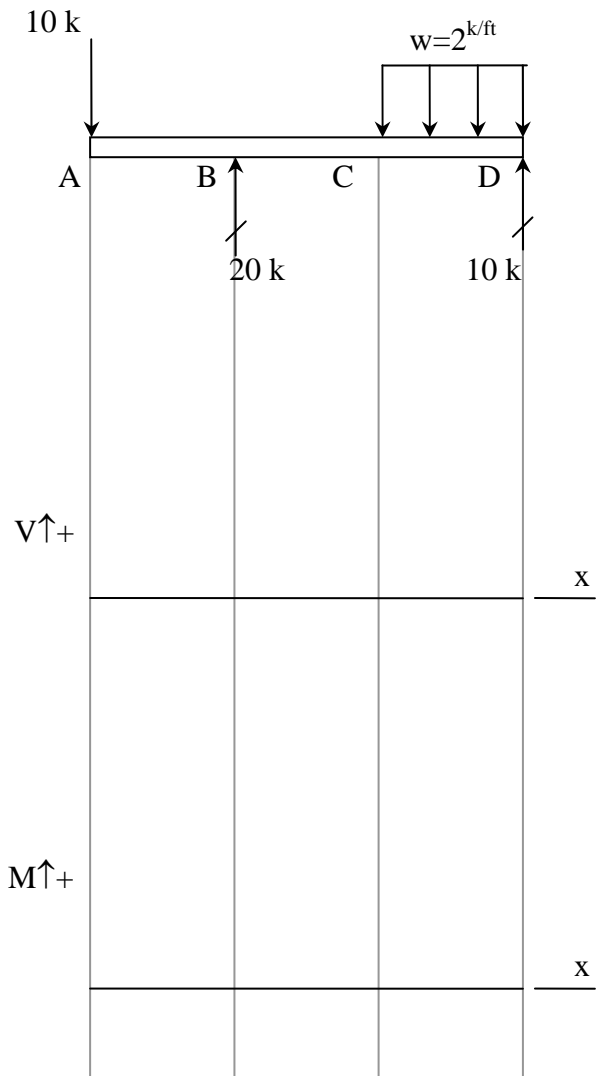
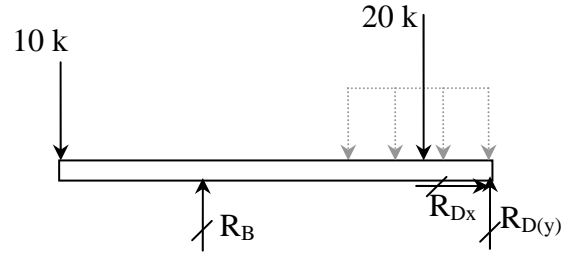
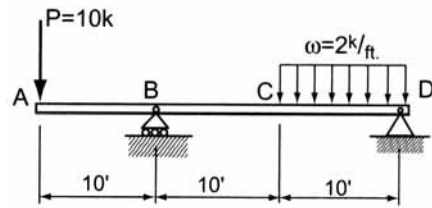
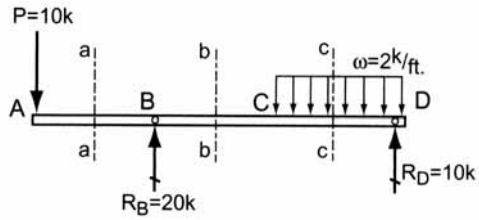
Draw the shear and moment diagram for a simply supported beam with a single concentrated load (Figure 8.8), using the equilibrium method.



Example 2 (pg 275)

Example Problem 8.2(Equilibrium Method)

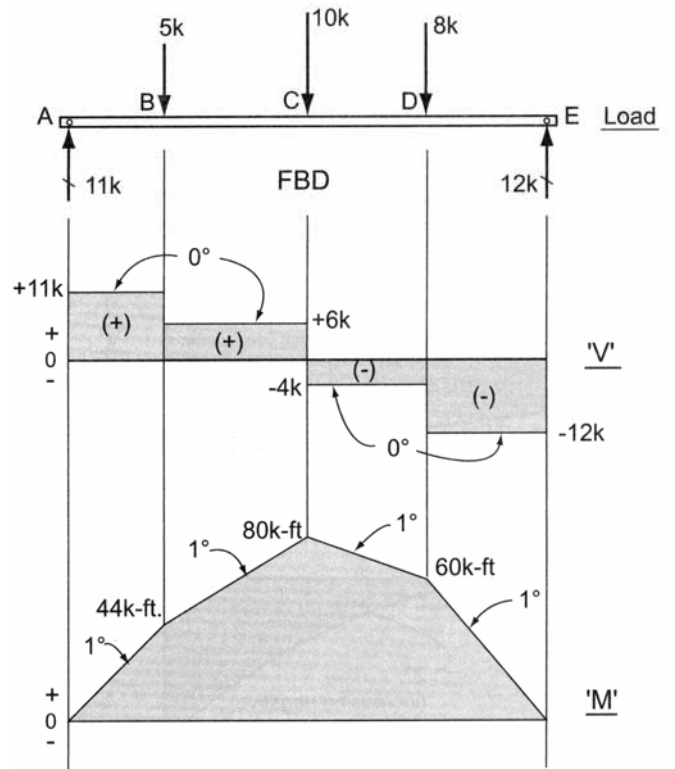
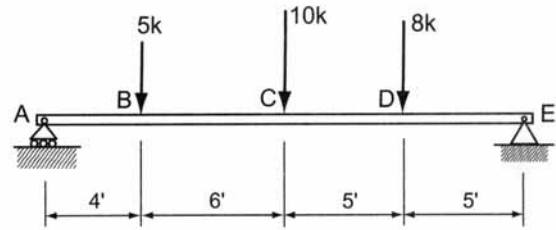
Draw  $V$  and  $M$  diagrams for an overhang beam (Figure 8.12) loaded as shown. Determine the critical  $V_{max}$  and  $M_{max}$  locations and magnitudes.



Example 3 (pg 283)

**Example Problem 8.4**

Construct the  $V$  and  $M$  diagrams for the girder that supports three concentrated loads as shown in Figure 8.28.

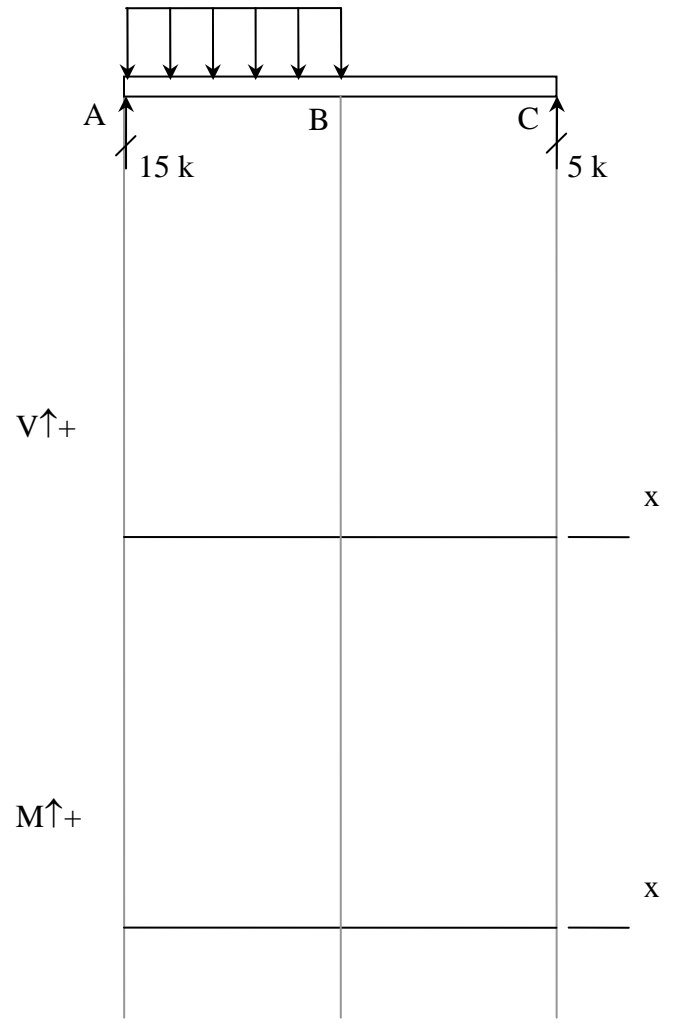
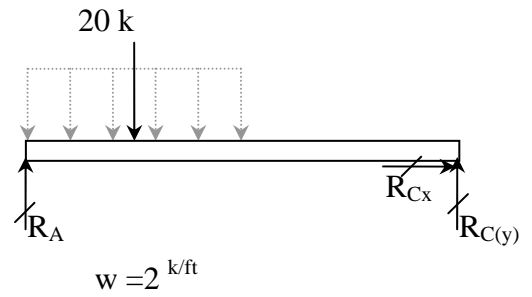
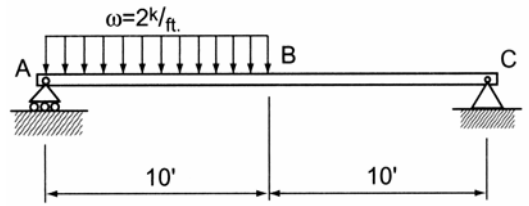




Example 4 (pg 285)

**Example Problem 8.6 (Semi-Graphical Method)**

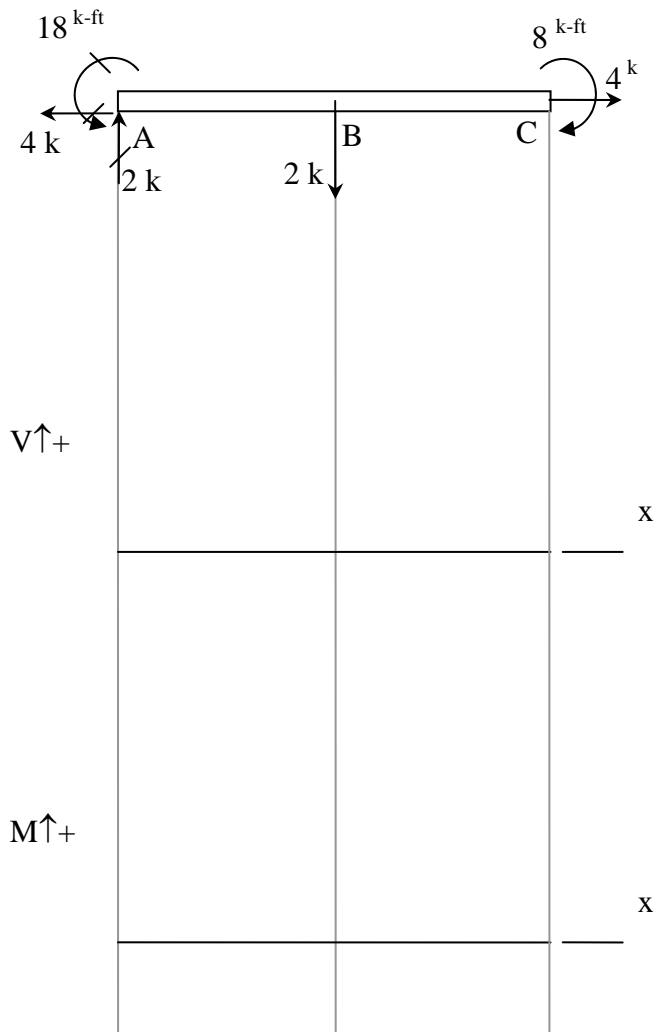
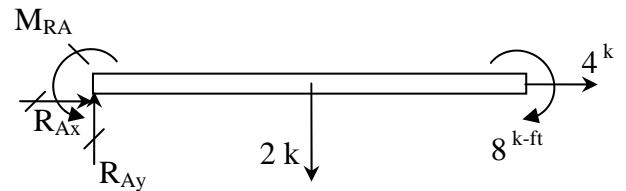
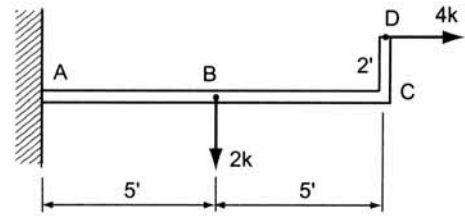
Construct  $V$  and  $M$  diagrams for the simply supported beam  $ABC$ , which is subjected to a partial uniform load (Figure 8.30).



Example 5 (pg 286)

**Example Problem 8.7 (Figure 8.31)**

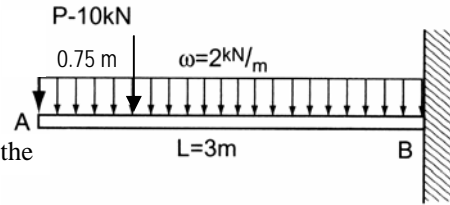
For a cantilever beam with an upturned end, draw the load, shear, and moment diagrams.



**Example 6 (changed from pg 284)**

**Example Problem 8.5 (Semi-Graphical Method)**

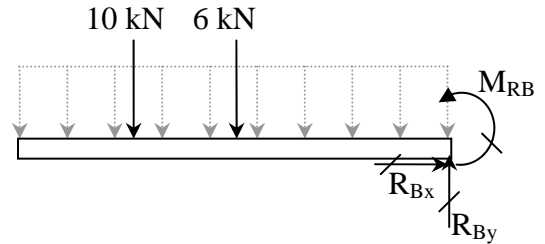
A cantilever beam supports a uniform load of  $\omega = 2 \text{ kN/m}$  over its entire span, plus a concentrated load of 10 kN at 0.75 m from the free end. Construct the  $V$  and  $M$  diagrams (Figure 8.29).



**SOLUTION:**  
Determine the reactions:

$$\begin{aligned} \sum F_x = R_{Bx} &= 0 & R_{Bx} &= 0 \text{ kN} \\ \sum F_y = -10\text{kN} - (2 \text{ kN/m})(3\text{m}) + R_{By} &= 0 & R_{By} &= 16 \text{ kN} \\ \sum M_B = (10\text{kN})(2.25\text{m}) + (6\text{kN})(1.5\text{m}) + M_{RB} &= 0 & M_{RB} &= -31.5\text{kN}\cdot\text{m} \end{aligned}$$

Draw the load diagram *with the distributed load as given* with the reactions.



Shear Diagram:

Label the load areas and calculate:

$$\begin{aligned} \text{Area I} &= (-2 \text{ kN/m})(0.75 \text{ m}) = -1.5 \text{ kN} \\ \text{Area II} &= (-2 \text{ kN/m})(2.25 \text{ m}) = -4.5 \text{ kN} \end{aligned}$$

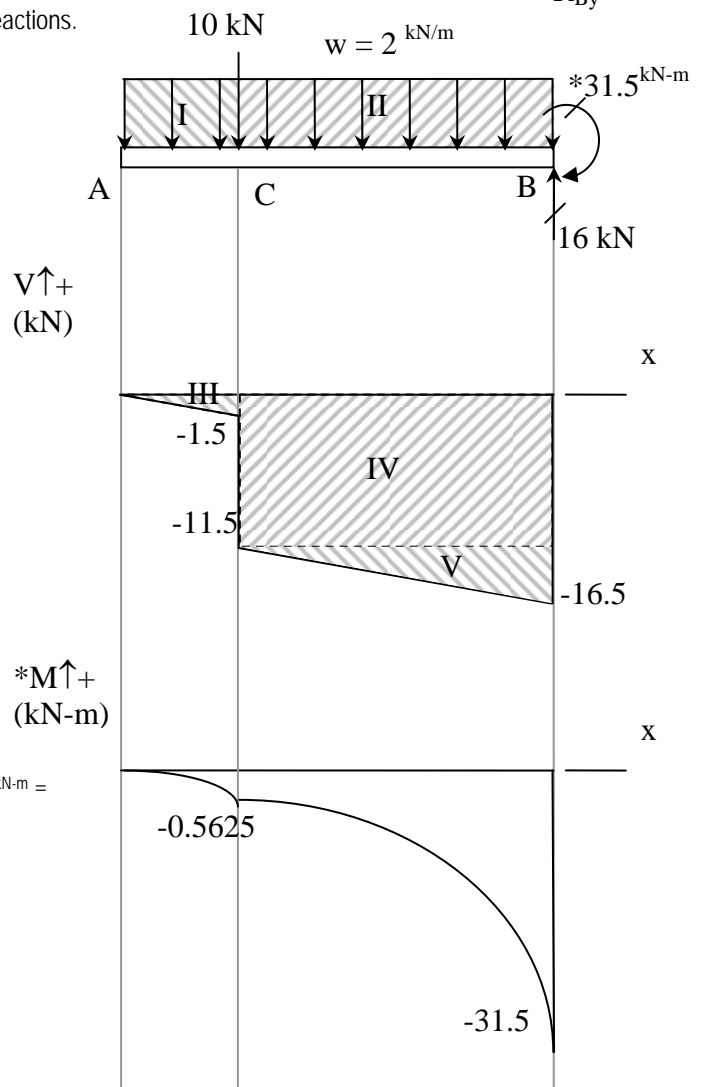
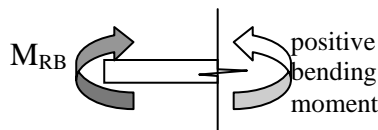
$$\begin{aligned} V_A &= 0 \\ V_C &= V_A + \text{Area I} = 0 - 1.5 \text{ kN} = -1.5 \text{ kN} \text{ and} \\ V_C &= V_C + \text{force at C} = -1.5 \text{ kN} - 10 \text{ kN} = -11.5 \text{ kN} \\ V_B &= V_C + \text{Area II} = -11.5 \text{ kN} - 4.5 \text{ kN} = -16 \text{ kN} \text{ and} \\ V_B &= V_B + \text{force at B} = -16 \text{ kN} + 16 \text{ kN} = 0 \text{ kN} \end{aligned}$$

Bending Moment Diagram:

Label the load areas and calculate:

$$\begin{aligned} \text{Area III} &= (-1.5 \text{ kN})(0.75 \text{ m})/2 = -0.5625 \text{ kN}\cdot\text{m} \\ \text{Area IV} &= (-11.5 \text{ kN})(2.25\text{m}) = -25.875 \text{ kN}\cdot\text{m} \\ \text{Area V} &= (-16 - 11.5 \text{ kN})(2.25\text{m})/2 = -5.0625 \text{ kN}\cdot\text{m} \end{aligned}$$

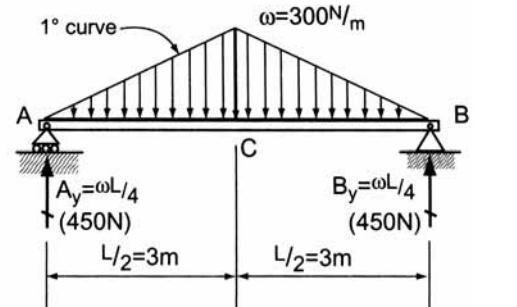
$$\begin{aligned} M_A &= 0 \\ M_C &= M_A + \text{Area III} = 0 - 0.5625 \text{ kN}\cdot\text{m} = -0.5625 \text{ kN}\cdot\text{m} \\ M_B &= M_C + \text{Area IV} + \text{Area V} = -0.5625 \text{ kN}\cdot\text{m} - 25.875 \text{ kN}\cdot\text{m} - 5.0625 \text{ kN}\cdot\text{m} = \\ &= -31.5 \text{ kN}\cdot\text{m} \text{ and} \\ M_B &= M_B + \text{moment at B} = -31.5 \text{ kN}\cdot\text{m} + 31.5 \text{ kN}\cdot\text{m} = 0 \text{ kN}\cdot\text{m} \end{aligned}$$



Example 7 (pg 287)

**Example Problem 8.9 (Figure 8.33)**

A header beam spanning a large opening in an industrial building supports a triangular load as shown. Construct the V and M diagrams and label the peak values.



**SOLUTION:**

Determine the reactions:

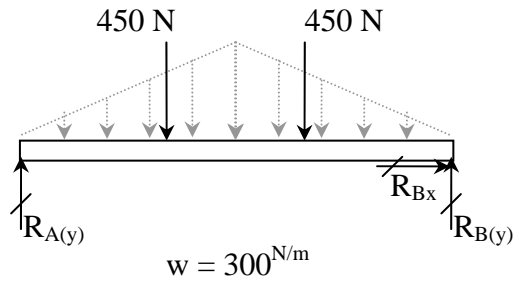
$$\sum F_x = R_{Bx} = 0 \quad R_{Bx} = 0 \text{ kN}$$

$$\sum F_y = R_{Ay} - (300 \text{ N/m})(3\text{m}) \frac{1}{2} - (300 \text{ N/m})(3\text{m}) \frac{1}{2} + R_{By} = 0$$

or by load tracing  $R_{Ay}$  &  $R_{By} = (wL/2)/2 = (300 \text{ N/m})(6 \text{ m})/4 = 450 \text{ N}$

$$\sum M_A = -(450\text{N})(\frac{2}{3} \times 3\text{m}) - (450\text{N})(3 + \frac{1}{3} \times 3\text{m}) + R_{By}(6\text{m}) = 0$$

$$R_{By} = 450 \text{ N}$$



Draw the load diagram *with the distributed load* as given with the reactions.

Shear Diagram:

Label the load areas and calculate:

$$\text{Area I} = (-300 \text{ N/m})(3 \text{ m})/2 = -450 \text{ N}$$

$$\text{Area II} = -300 \text{ N/m})(3 \text{ m})/2 = -450 \text{ N}$$

$$V_A = 0 \text{ and } V_A = V_A + \text{force at A} = 0 + 450 \text{ N} = 450 \text{ N}$$

$$V_C = V_A + \text{Area I} = 450 \text{ N} - 450 \text{ N} = 0 \text{ N}$$

$$V_B = V_C + \text{Area II} = 0 \text{ N} - 450 \text{ N} = -450 \text{ N} \text{ and}$$

$$V_B = V_B + \text{force at B} = -450 \text{ N} + 450 \text{ N} = 0 \text{ N}$$

Bending Moment Diagram:

Label the load areas and calculate:

Areas III & IV happen to be parabolic segments with an area of  $2bh/3$ :

$$\text{Area III} = 2(3 \text{ m})(450 \text{ N})/3 = 900 \text{ N}\cdot\text{m}$$

$$\text{Area IV} = -2(3 \text{ m})(450 \text{ N})/3 = -900 \text{ N}\cdot\text{m}$$

$$M_A = 0$$

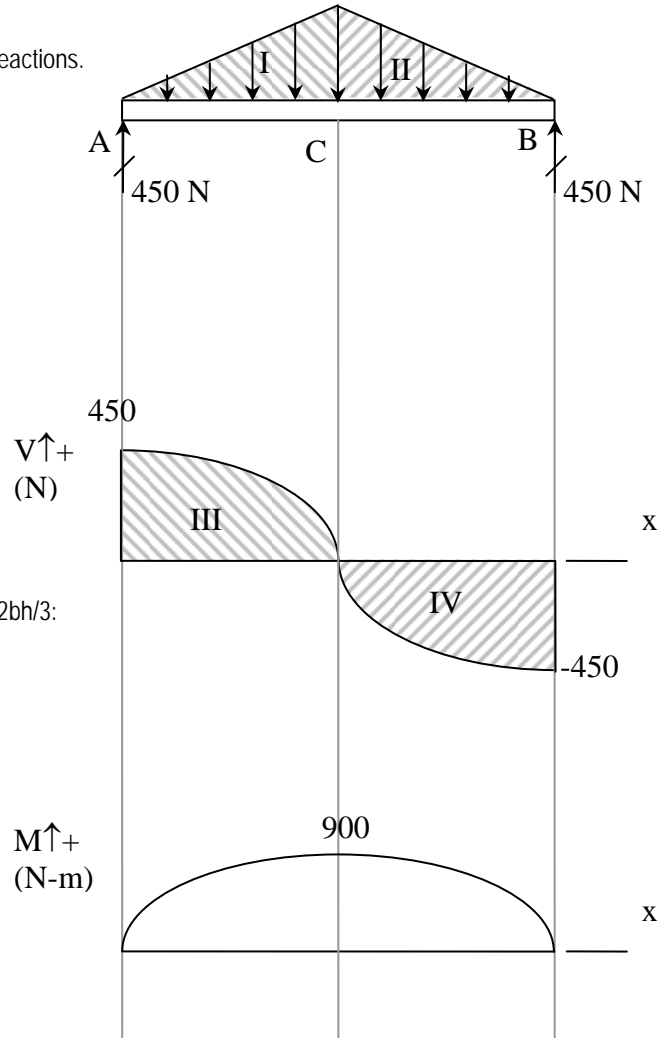
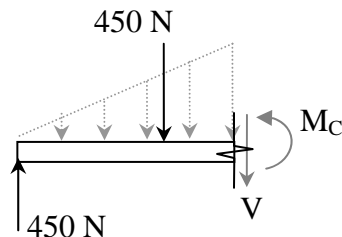
$$M_C = M_A + \text{Area III} = 0 + 900 \text{ N}\cdot\text{m} = 900 \text{ N}\cdot\text{m}$$

$$M_B = M_C + \text{Area IV} = 900 \text{ N}\cdot\text{m} - 900 \text{ N}\cdot\text{m} = 0$$

We can prove that the area is a parabolic segment by using the equilibrium method at C:

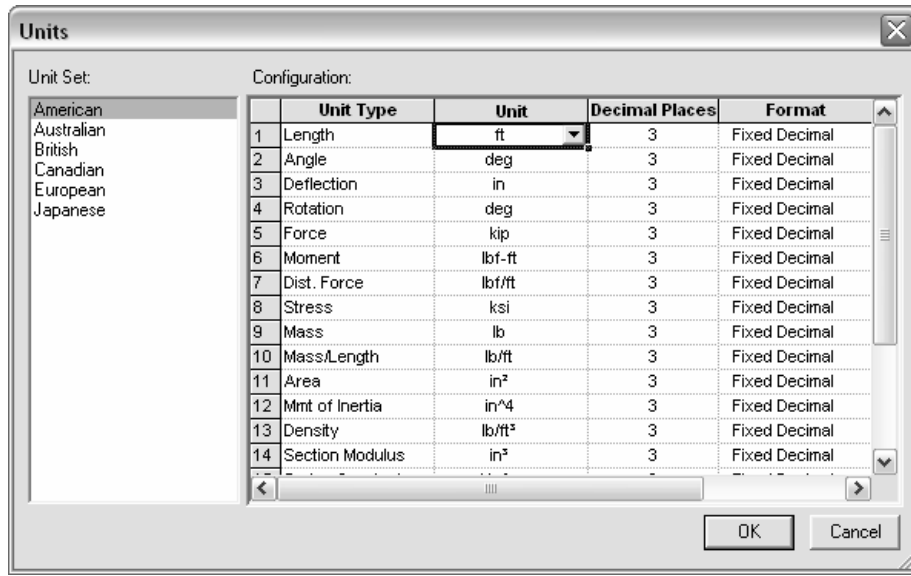
$$\sum M_{\text{section cut}} = M_C - (450\text{N})(3\text{m}) + (450\text{N})(\frac{1}{3} \times 3\text{m}) = 0$$

so  $M_C = 900 \text{ N}\cdot\text{m}$

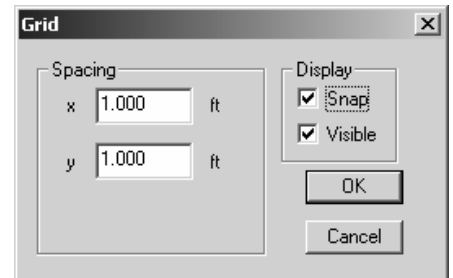


### Beam Analysis Using Multiframe4D

1. The software is on the computers in the college computing lab (<http://thelab.tamu.edu>) in Programs under the Windows Start menu. **Multiframe4D is under the COSC menu.**
2. There is a tutorial in the Help menu (Chapter 1 – 2D Tutorial) that lists the tasks and order in greater detail. The first task is to define the unit system:
  - Choose Units... from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.



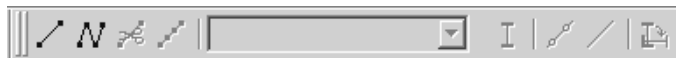
3. To see the scale of the geometry, a grid option is available:
  - Choose Grid... from the View menu



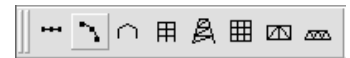
4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:



The Member toolbar shows ways to create members:



The Generate toolbar has convenient tools to create typical structural shapes.



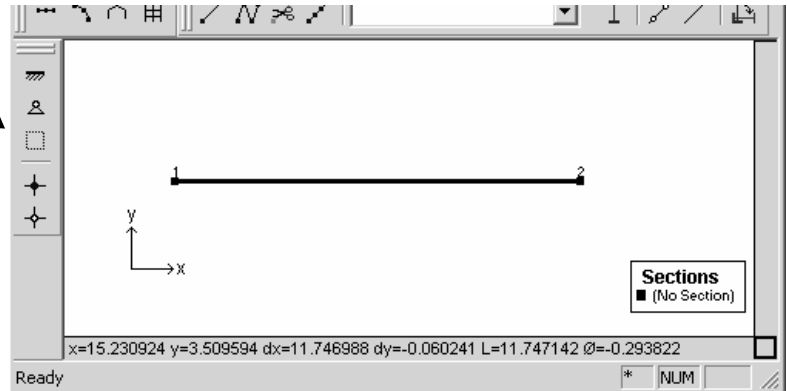
- To create a beam with supports at one or both ends, use the add member button:



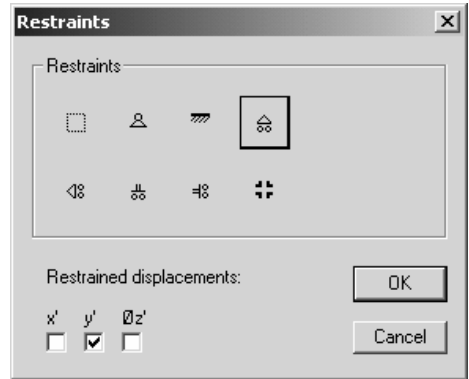
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window.
- To create a beam with supports NOT at the ends, use the add connected members button to create segments between supports and ends
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.



- The geometry can be set precisely by selecting the beam member, bringing up the specific menu (right click), choosing Member Properties to set the length.



- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint ... menu (right click).  
NOTE: If the support appears at both ends of the beam, you had the beam selected rather than the joint. Select the joint to change the support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.



The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

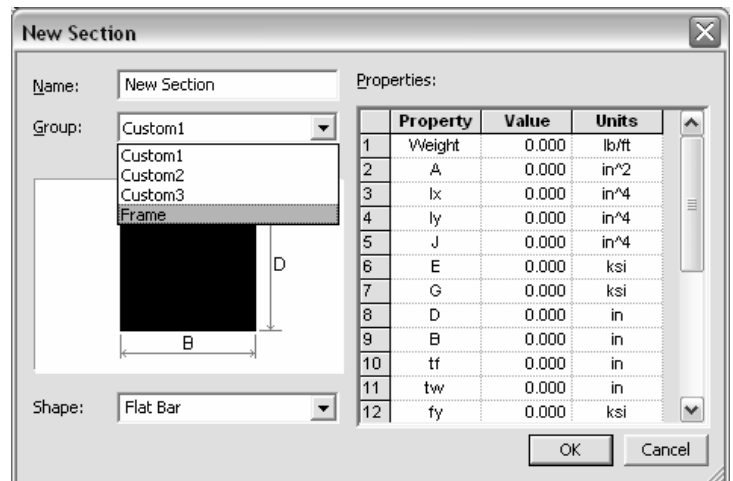


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like  $t_f$ ,  $t_w$ , etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

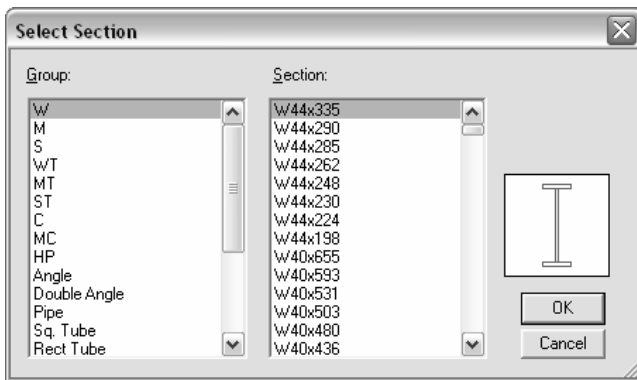
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

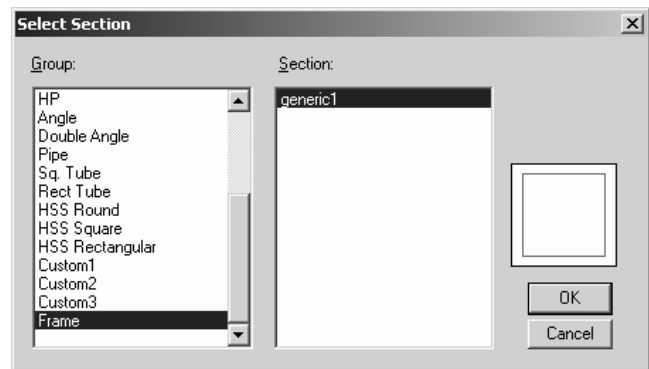


- Choose the group name and section name:

(STANDARD SHAPES)



(CUSTOM)



7. The beam geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:



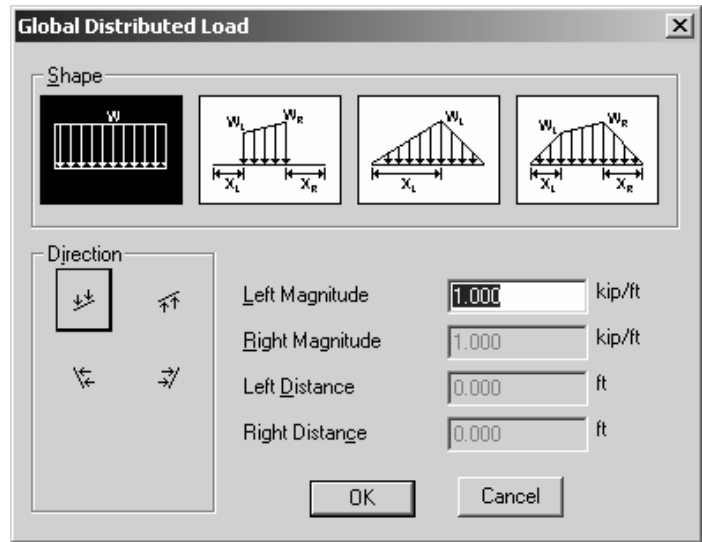
8. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force in the local coordinate system (last two buttons).



- Choose the member to be loaded (drag) and select the load type (here shown for distributed loading):

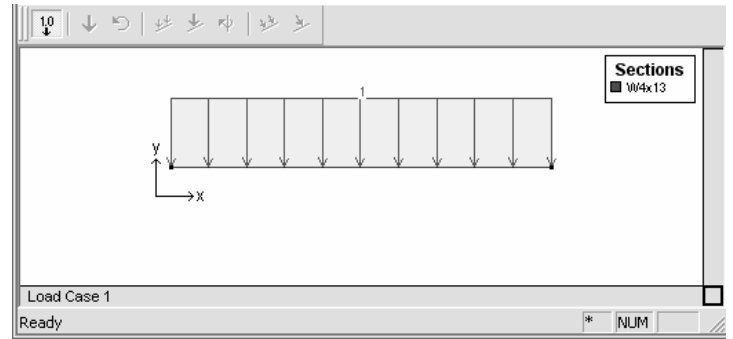


- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.
- Enter the values of the load and distances (if any). Distances can be entered as a function of the length, i.e.  $L/2$ ,  $L/4$ ...



NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.

Multiframe4D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).



9. In order to run the analysis after the geometry, member properties and loading has been defined:

- Choose Analyze Linear from the Case menu

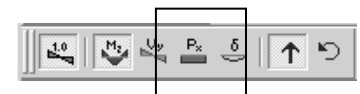
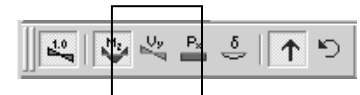
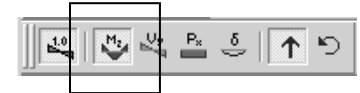
10. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:



11. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):



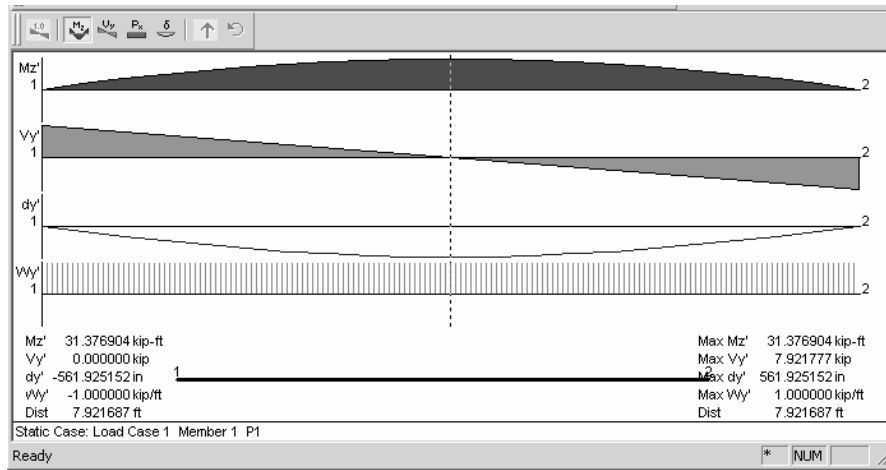
- To show the moment diagram, Choose the red Moment button
- To show the shear diagram, Choose the green Shear button
- To show the axial force diagram, Choose the purple Axial Force button
- To show the deflection diagram, Choose the blue Deflection button



- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.



- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.



12. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.



13. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.



	Joint	Label	Rx' kip	Ry' kip	Mz' lbf-ft
1	1		0.000	7.922	0.000
2	2		0.000	7.922	0.000
3	Total (Global)		Rx=0.000	Ry=15.844	

Reactions

	Membr	Label	Joint	Px' kip	Vy' kip	Mz' lbf-ft
1	1		1	0.000	7.922	0.000
2	1		2	0.000	7.922	0.000

Member Actions

NOTE: Px' refers to the axial load (P) in the local axis x direction (x'). Vy' refers to the shear perpendicular to the local x axis, and Mz' refers to the bending moment.

14. To save the file Choose Save from the File menu.

15. To load an existing file Choose Open... from the File menu.

## Mechanics of Materials

*Mechanics of Materials* is a basic engineering science that deals with the relation between externally applied load and its effect on deformable bodies. The main purpose of Mechanics of Materials is to answer the question of which requirements have to be met to assure **STRENGTH, RIGIDITY, AND STABILITY** of engineering structures.

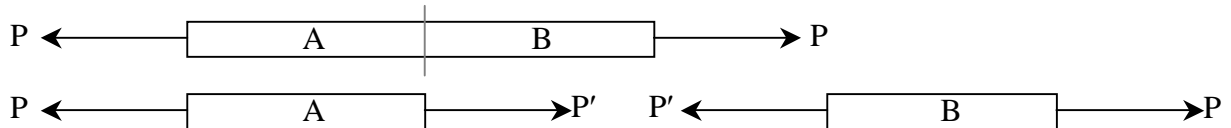
To solve a problem in Mechanics of Materials, one has to consider **THREE ASPECTS OF THE PROBLEM**:

1. **STATICS**: equilibrium of external forces, internal forces, stresses
2. **GEOMETRY**: deformations and conditions of geometric fit, strains
3. **MATERIAL PROPERTIES**: stress-strain relationship for each material, obtained from material testing.

- **STRESS** – The intensity of a force acting over an **area**.

### Normal Stress

Stress that acts along an *axis* of a member; can be internal or external; can be compressive or tensile.

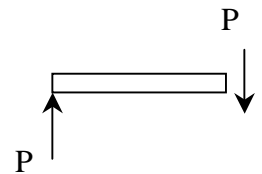


$$f = \sigma = \frac{P}{A_{net}} \quad \text{Strength condition: } f = \frac{P}{A_{net}} < f_{allowable} \text{ or } F_{allowed}$$

### Shear Stress

Stress that acts perpendicular to an *axis or length* of a member, or **parallel** to the cross section is called shear stress.

Shear stress cannot be assumed to be uniform, so we refer to *average shearing stress*.



$$f_v = \tau = \frac{P}{A_{net}} \quad \text{Strength condition: } f_v = \frac{P}{A_{net}} < \tau_{allowable} \text{ or } F_{allowed}$$

Bearing Stress

A compressive normal stress acting *between two bodies*.

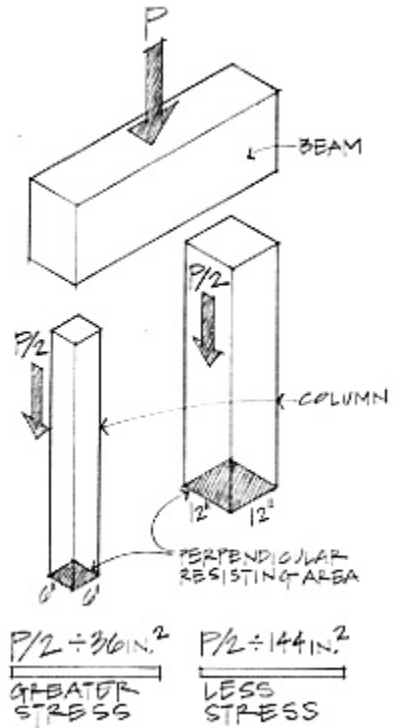
$$f_p = \frac{P}{A_{bearing}}$$

Bending Stress

A normal stress caused by bending; can be compressive or tensile. (Discussed in Note Set on Beam Bending.)

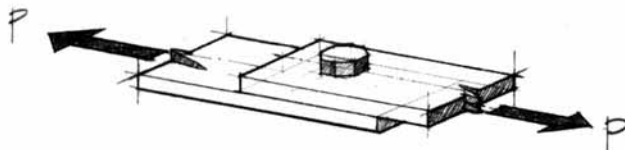
Torsional Stress

A shear stress caused by torsion (moment around the axis). (Discussed in Note Set on Torsion.)

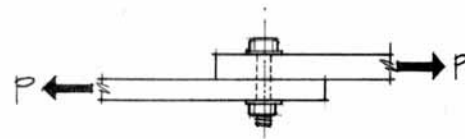


**Bolts in Shear and Bearing**

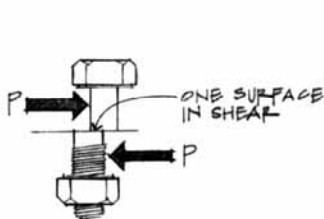
Single shear - forces cause only one shear “drop” across the bolt.



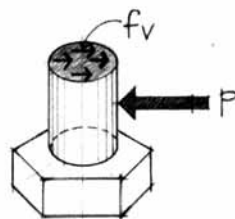
(a) Two steel plates bolted using one bolt.



(b) Elevation showing the bolt in shear.



(c)



(d)

$f_v$  = Average shear stress through bolt cross section

$A$  = Bolt cross-sectional area

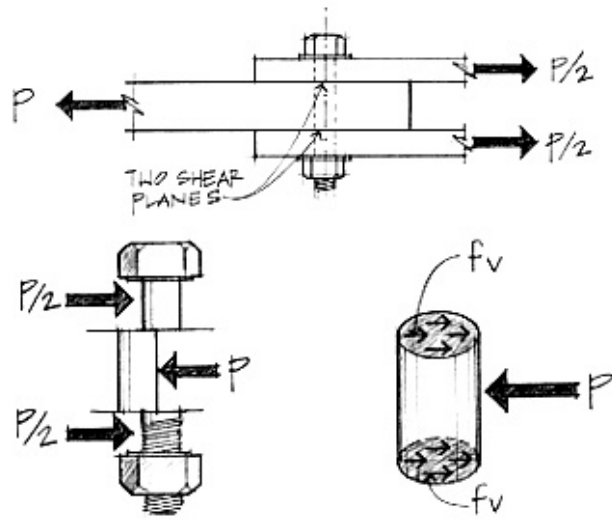
$$f_v = \frac{P}{A}$$

Figure 5.11 A bolted connection—single shear.

Double shear - forces cause two shear changes across the bolt.

$$f_v = \frac{P}{2A}$$

(two shear planes)

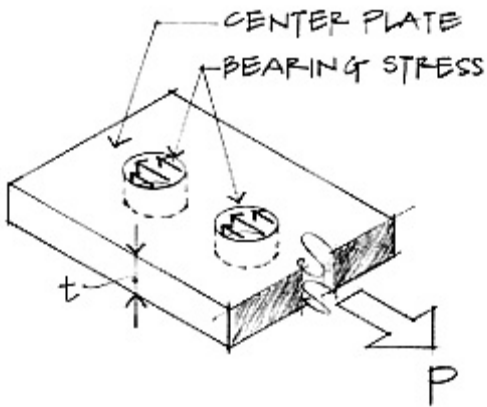


Free-body diagram of middle section of the bolt in shear.

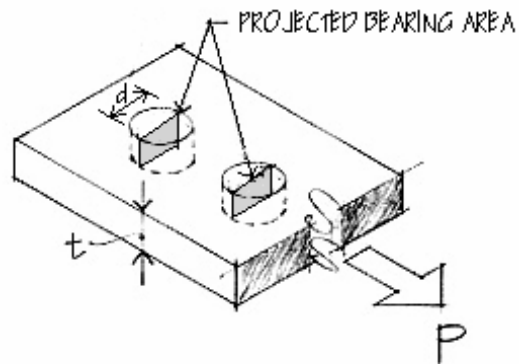
Figure 5.12 A bolted connection in double shear.

Bearing of a bolt on a bolt hole – The bearing surface can be represented by *projecting* the cross section of the bolt hole on a plane (into a rectangle).

$$f_p = \frac{P}{A} = \frac{P}{td}$$



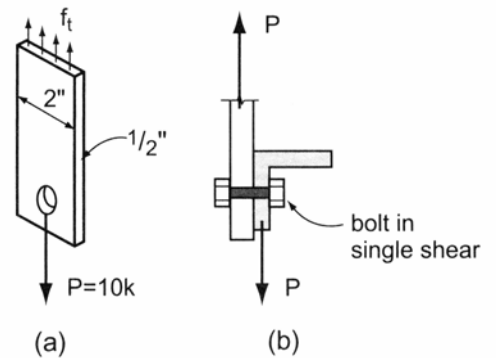
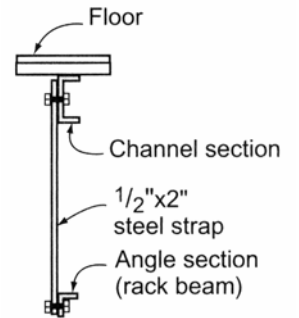
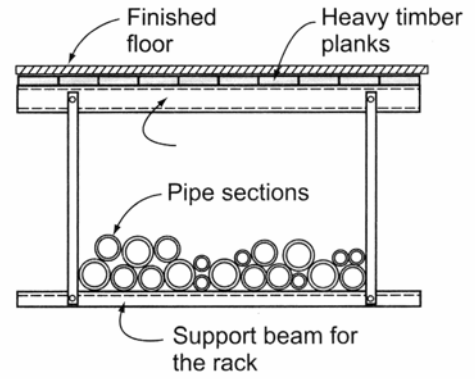
Bearing stress on plate.



Example 1 (pg 201)

Example Problem 6.8 (Figures 6.18 to 6.20)

A pipe storage rack is used for storing pipe in a shop. The support rack beam is fastened to the main floor beam using steel straps  $\frac{1}{2} \times 2$  in dimension. Round bolts are used to fasten the strap to the floor beam in single shear. (a) If the weight of the pipes impose a maximum tension load of 10,000 pounds in each strap, determine the tension stress developed in the steel strap. (b) Also, what diameter bolt is necessary to fasten the strap to the floor beam if the allowable shear stress for the bolts equals  $F_v = 15,000 \text{ lb./in.}^2$ ? Determine the bearing stress in the strap from the bolt diameter chosen.

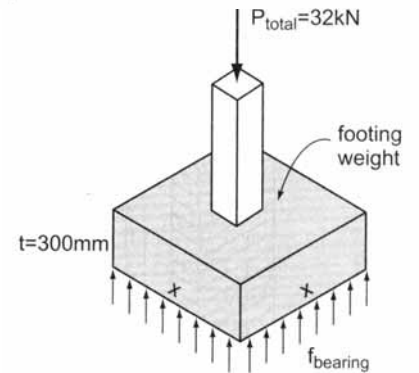
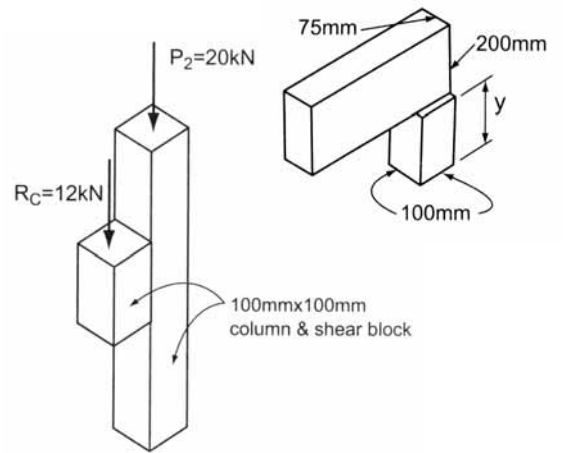
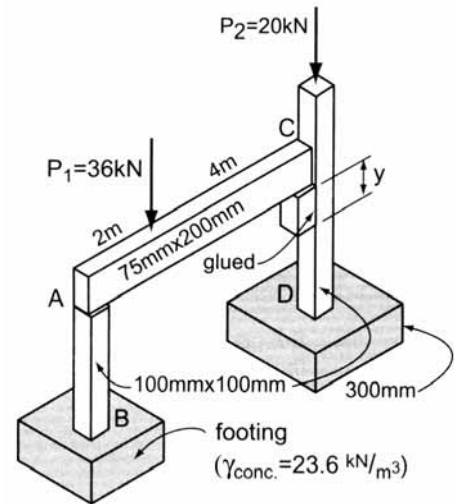


Example 2 (pg 202)

**Example Problem 6.9 (Figures 6.21 to 6.26)**

A 75 mm × 200 mm “rough cut” beam is supported by columns at both ends. Column AB supports the beam in bearing while column CD utilizes a shear block at C. Both columns bear on concrete footings on the ground.

- What is the compressive stress developed in column AB?
- What is the bearing stress that develops at C between the beam and shear block made from a 100 mm × 100 mm block cut from a post?
- What is the required depth  $y$  necessary to resist the shear force developed at the glued joint between the shear block and post? Assume that the glue is capable of safely resisting 500 kPa (72.5 psi) in shear.
- Determine the size of square footing required to take the maximum column load if the allowable soil pressure  $q = 73 \text{ kN/m}^2 = 73 \text{ kPa}$  (1525 psf).



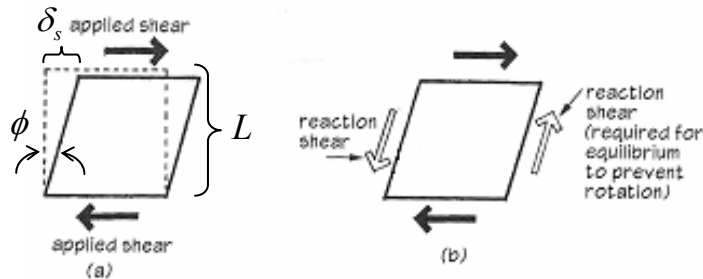
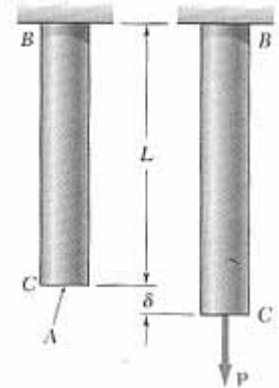
## Stress and Strain – Elasticity

### Normal Strain

In an axially loaded member, normal strain,  $\epsilon$  is the change in the length,  $\delta$  with respect to the original length,  $L$ .

$$\epsilon = \frac{\delta}{L}$$

It is UNITLESS, but may be called strain or microstrain ( $\mu$ ).



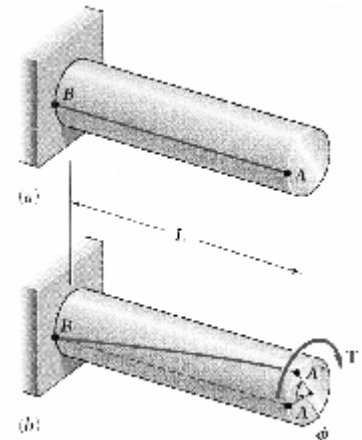
### Shearing Strain

In a member loaded with shear forces, shear strain,  $\gamma$  is the change in the sheared side,  $\delta_s$  with respect to the original height,  $L$ . For small angles:  $\tan \phi \cong \phi$ .

$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

In a member subjected to twisting, the shearing strain is a measure of the angle of twist with respect to the length and distance from the center,  $\rho$ :

$$\gamma = \frac{\rho\phi}{L}$$



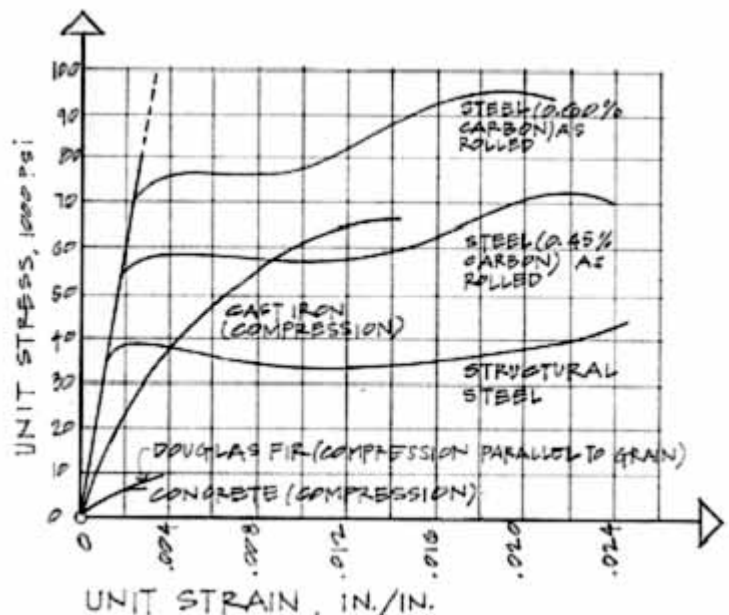
### Testing of Load vs. Strain

Behavior of materials can be measured by recording deformation with respect to the size of the load. For members with constant cross section area, we can plot stress vs. strain.

**BRITTLE MATERIALS** - ceramics, glass, stone, cast iron; show abrupt fracture at small strains.

**DUCTILE MATERIALS** – plastics, steel; show a yield point and large strains (considered *plastic*) and “necking” (give warning of failure)

**SEMI-BRITTLE MATERIALS** – concrete; show no real yield point, small strains, but have some “strain-hardening”.



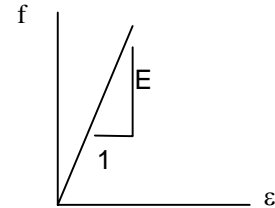
## Linear-Elastic Behavior

In the straight portion of the stress-strain diagram, the materials are *elastic*, which means if they are loaded and unloaded no permanent **deformation** occurs.

### **True Stress & Engineering Stress**

True stress takes into account that the area of the cross section changes with loading.

Engineering stress uses the original area of the cross section.



### **Hooke's Law – Modulus of Elasticity**

In the linear-elastic range, the slope of the stress-strain diagram is *constant*, and has a value of E, called Modulus of Elasticity or Young's Modulus.

$$f = E \cdot \varepsilon$$

Isotropic Materials – have the **same** E with any direction of loading.

Anisotropic Materials – have **different** E's with the direction of loading.

Orthotropic Materials – have **directionally based** E's

### **Plastic Behavior & Fatigue**

Permanent deformations happen outside the linear-elastic range and are called *plastic* deformations. Fatigue is damage caused by reversal of loading.

- The proportional limit (at the end of the **elastic** range) is the greatest stress valid using Hooke's law.
- The elastic limit is the maximum stress that can be applied before permanent deformation would appear upon unloading.
- The yield point (at the **yield stress**) is where a ductile material continues to elongate without an increase of load. (May not be well defined on the stress-strain plot.)
- The ultimate strength is the largest stress a material will see before rupturing, also called the *tensile strength*.
- The rupture strength is the stress at the point of rupture or failure. It may not coincide with the ultimate strength in ductile materials. In brittle materials, it will be the same as the ultimate strength.



- The fatigue strength is the stress at failure when a member is subjected to reverse cycles of stress (up & down or compression & tension). This can happen at much lower values than the ultimate strength of a material.
- Toughness of a material is how much work (a combination of stress and strain) is used for fracture. It is the area under the stress-strain curve.

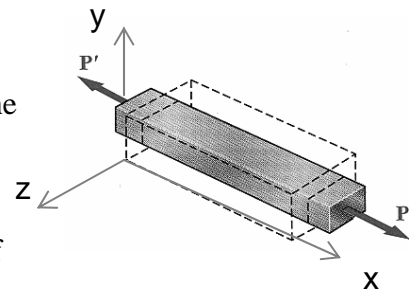
Concrete does not respond well to tension and is tested in compression. The strength at crushing is called the *compression strength*.

Materials that have time dependent elongations when loaded are said to have *creep*. Concrete and wood creep. Concrete also has the property of shrinking over time.

**Poisson’s Ratio**

For an isotropic material that is homogeneous, the properties are the same for the cross section:

$$\epsilon_y = \epsilon_z$$



There exists a linear relationship while in the linear-elastic range of the material between *longitudinal strain* and *lateral strain*:

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x} \quad \epsilon_y = \epsilon_z = -\frac{\mu f_x}{E}$$

Positive strain results from an increase in length with respect to overall length.

Negative strain results from a decrease in length with respect to overall length.

$\mu$  is the Poisson’s ratio and has a value between 0 and 1/2, depending on the material

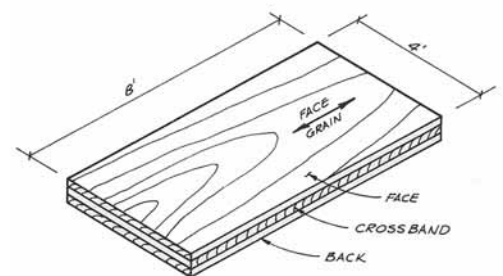
**Relation of Stress to Strain**

$$f = \frac{P}{A}; \quad \epsilon = \frac{\delta}{L} \quad \text{and} \quad E = \frac{f}{\epsilon} \quad \text{so} \quad E = \frac{P/A}{\delta/L} \quad \text{which rearranges to:} \quad \delta = \frac{PL}{AE}$$

**Orthotropic Materials**

One class of non-isotropic materials is *orthotropic materials* that have directionally based values of modulus of elasticity and Poisson’s ratio (E,  $\mu$ ).

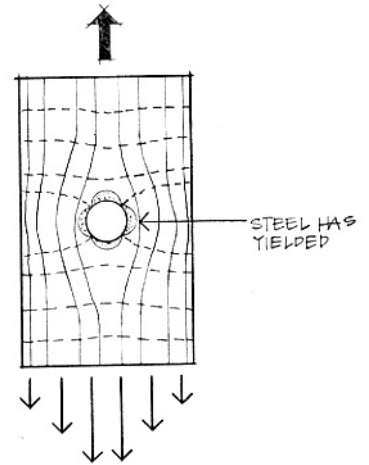
Ex: plywood, laminates, fiber reinforced polymers with direction fibers



**Stress Concentrations**

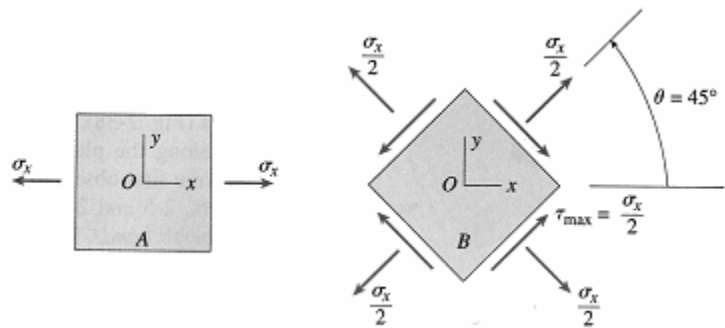
In some sudden changes of cross section, the stress concentration changes (and is why we used *average* normal stress). Examples are sharp notches, or holes or corners.

(Think about airplane window shapes...)



**Maximum Stress**

When both normal stress and shear stress occur in a structural member, the *maximum stresses can occur at some other planes* (angle of  $\theta$ ).



Maximum Normal Stress happens at  $\theta = 0^\circ$  AND

Maximum Shearing Stress happens at  $\theta = 45^\circ$  with only normal stress in the x direction.

**Allowable Stress Design (ASD) and Factor of Safety (F.S.)**

There are uncertainties in material strengths: 
$$F.S. = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Allowable stress design determines the allowable stress by: 
$$\text{allowable stress} = \frac{\text{ultimate stress}}{F.S.}$$

**Load and Resistance Factor Design – LRFD**

There are uncertainties in material strengths *and* in structural loadings.

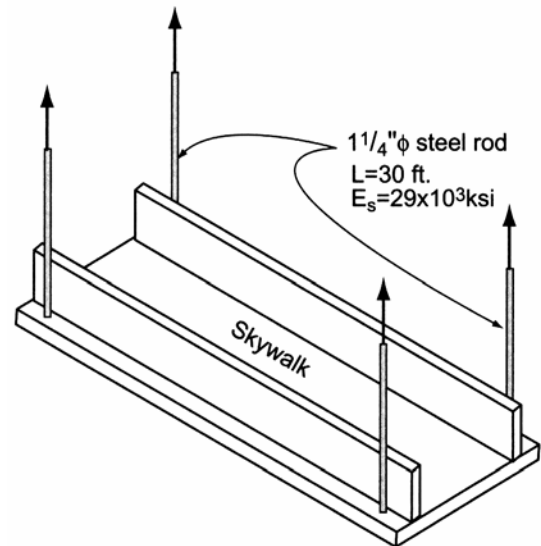
$$\gamma_D R_D + \gamma_L R_L \leq \phi R_u$$

- where  $\gamma$  = load factor for Dead and Live loads
- R = load (dead or live)
- $\phi$  = resistance factor
- $R_n$  = nominal load (capacity)

Example 1 (pg 222)**Example Problem 6.19 (Figure 6.50)**

A skywalk in the atrium area of a major hotel uses a system of  $1\frac{1}{4}$ "  $\phi$  steel rods to support the dead and occupancy loads. Assuming  $F_t = 22$  ksi and  $E = 29 \times 10^3$  ksi, determine the following:

- the allowable load capacity in each rod;
- if the rods supporting the third-floor walkway are 30 ft. in length, how much elongation would occur if the rod was loaded to its maximum allowable stress limit;
- the ultimate load capacity of the rod at failure assuming A36 steel (see Table 6.1a)



Example 2 (metric, pg 223)**Example Problem 6.20 (Figures 6.51 and 6.52)**

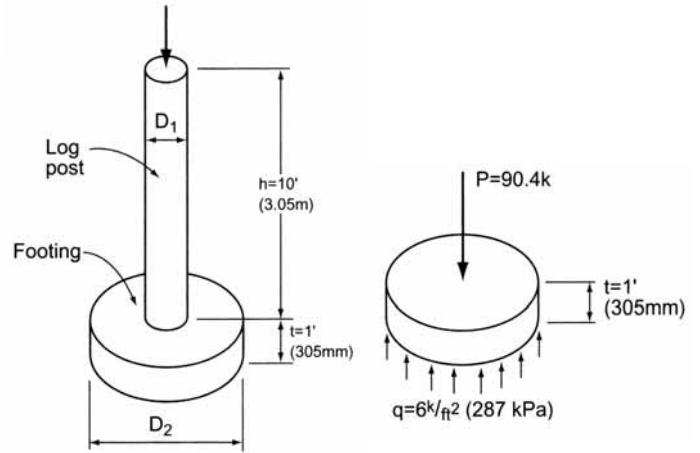
A log post supports a floor load  $P$ . Assuming that the post has a diameter  $D_1 = 12$  in. (0.305 m), (a) determine the allowable load  $P$  if the compression stress in the post is limited to  $F_{\text{allow.}} = 800 \text{ lb./in.}^2$  (5.52 MPa). Once  $P$  has been determined, (b) find the deformation (shortening) that results. Also, (c) determine the size  $D_2$  of the circular footing necessary if the allowable soil pressure is:  $q = 6000$  psf (287 kPa).

$$E_{\text{timber}} = 1.5 \times 10^3 \text{ ksi (10.34 MPa);}$$

$$\gamma_{\text{concrete}} = 150 \text{ lb./ft.}^3 \text{ (23.6 kN/m}^3\text{)}$$

Using the following design values, determine the minimum size of  $D_1$  for bearing on the concrete:

$$\begin{aligned} \phi &= 0.70 & \gamma_D &= 1.4 \\ f'_c &= 24 \text{ MPa} & F_b &= 0.85 f'_c \end{aligned}$$

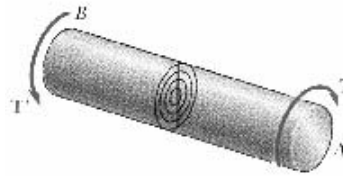
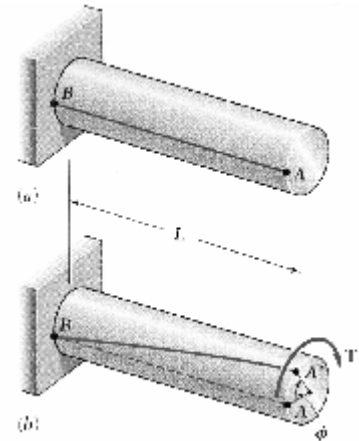


## Torsion, Thermal Effects and Indeterminacy

### Deformation in Torsionally Loaded Members

Axi-symmetric cross sections subjected to axial moment or **torque** will remain plane and undistorted.

At a section, internal torque (resisting applied torque) is made up of shear forces parallel to the area and in the direction of the torque. The distribution of the shearing stresses depends on the angle of twist,  $\phi$ . The cross section remains plane and undistorted.



### Shearing Strain

Shearing strain is the angle change of a straight line segment along the axis.

$$\gamma = \frac{\rho\phi}{L}$$

where

$\rho$  is the radial distance from the centroid to the point under strain.

The maximum strain is at the surface, a distance  $c$  from the centroid:  $\gamma_{max} = \frac{c\phi}{L}$

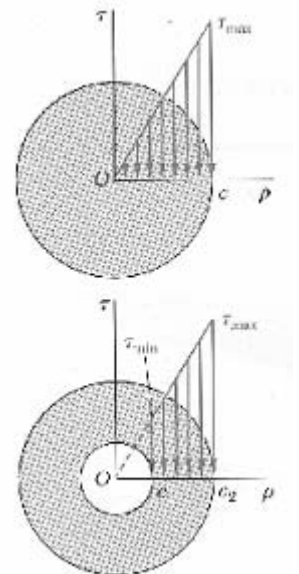
$G$  is the Shear Modulus or Modulus of Rigidity:  $\tau = G \cdot \gamma$

### Shearing Strain and Stress

In the linear elastic range: the torque is the summation of torsion stresses over the area:

$$T = \frac{\tau J}{\rho} \quad \text{gives:} \quad \tau = \frac{T\rho}{J}$$

Maximum torsional stress,  $\tau_{max}$ , occurs at the **outer diameter** (or **perimeter**).



### Polar Moment of Inertia

For axi-symmetric shapes, there is only one value for polar moment of inertia,  $J$ , determined by the radius,  $c$ :

solid section:  $J = \frac{\pi C^4}{2}$       hollow section:  $J = \frac{\pi(c_o^4 - c_i^4)}{2}$

### Combined Torsion and Axial Loading

Just as with combined axial load and shear, combined torsion and axial loading result in maximum shear stress at a 45° oblique “plane” of twist.



### Shearing Strain

In the linear elastic range:  $\phi = \frac{TL}{JG}$  and for composite shafts:  $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

### Torsion in Noncircular Shapes

J is no longer the same along the lateral axes. Plane sections do not remain plane, but distort.  $\tau_{max}$  is still at the furthest distance away from the centroid. For rectangular shapes:

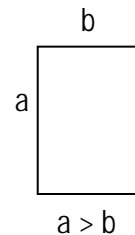
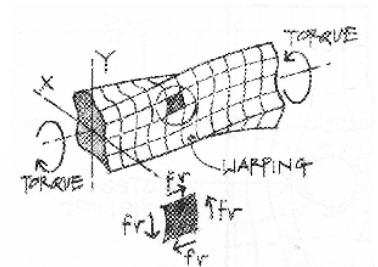
$$\tau_{max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

For  $a/b > 5$ :

$$c_1 = c_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right)$$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

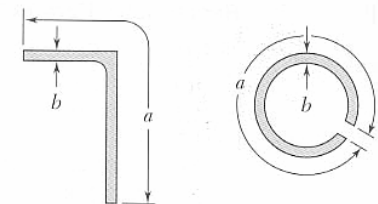
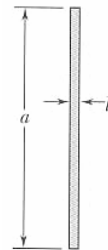
a/b	c <sub>1</sub>	c <sub>2</sub>
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333



### Open Sections

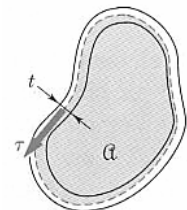
For long narrow shapes where  $a/b$  is very large ( $a/b \rightarrow \infty$ )  $c_1 = c_2 = 1/3$  and:

$$\tau_{max} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}$$



### Shear Flow of Closed Thin Walled Sections

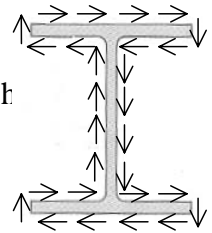
$q$  is the internal shearing force per unit length, and is constant on a cross section even though the thickness of the wall may vary.  $\mathcal{A}$  is the area bounded by the centerline of the wall section;  $s_i$  is a length segment of the wall and  $t_i$  is the corresponding thickness of the length segment.



$$\tau = \frac{T}{2t\mathcal{A}} \quad \phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$

### Shear Flow in Open Sections

The shear flow must wrap around at all edges, and the total torque is distributed among the areas making up the cross section in proportion to the torsional rigidity of each rectangle ( $ab^2/3$ ). The total angle of twist is the sum of the  $\phi$  values from each rectangle.  $t_i$  is the thickness of each rectangle and  $b_i$  is the length of each rectangle.

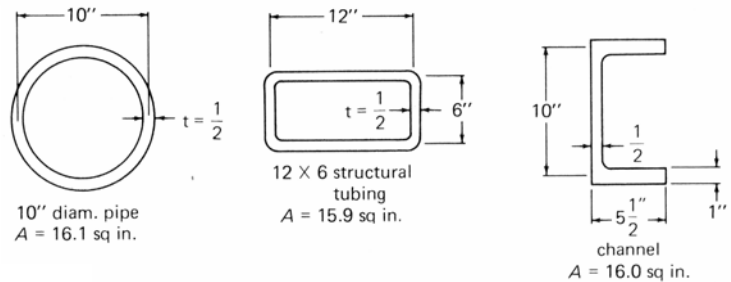


$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3} \sum b_i t_i^3} \quad \phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$

#### Example 1

##### Example 8.9.1

Compare the torsional resisting moment  $T$  and the torsional constant  $J$  for the sections of Fig. 8.9.4 all having about the same cross-sectional area. The maximum shear stress  $\tau$  is 14 ksi.



#### SOLUTION

(a) Circular thin-wall section.

$$T = \frac{\tau J}{\rho} = \frac{(14 \text{ksi})(393.7 \text{in}^4)}{5.25 \text{in}} \cdot \frac{1 \text{ft}}{12 \text{in}} = 87.5 \text{k} - \text{ft}$$

$$J = \frac{\pi(c_o^4 - c_i^4)}{2} = \frac{\pi((5.25 \text{in})^4 - (4.75 \text{in})^4)}{2} = 393.7 \text{in}^4$$

(b) Rectangular box section.  $\tau = \frac{T}{2tA}$

$$T = \tau 2tA = (14 \text{ksi}) 2(0.5 \text{in})(72 \text{in}^2) \cdot \frac{1 \text{ft}}{12 \text{in}} = 84 \text{k} - \text{ft}$$

$$A \approx (12 \text{in})(6 \text{in}) = 72 \text{in}^2$$

(c) Channel section. Since for this open section,

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3} \sum b_i t_i^3} = \frac{Tt}{J} \quad T = \frac{\tau J}{t_{max}} = \frac{(14 \text{ksi})(4.08 \text{in}^4)}{1 \text{in}} \cdot \frac{1 \text{ft}}{12 \text{in}} = 4.8 \text{k} - \text{ft}$$

the maximum shear stress will be in the flange. Also,

$$J = \sum \frac{bt^3}{3} \quad J = \frac{1}{3} [10 \text{in}(0.5 \text{in})^3 + (5.5 \text{in})(1 \text{in})^3 + (5.5 \text{in})(1 \text{in})^3] = 4.08 \text{in}^4$$

### Thermal Strains

Physical restraints limit deformations to be the same, or sum to **zero**, or be proportional with respect to the rotation of a rigid body.

We know axial stress relates to axial strain:  $\delta = \frac{PL}{AE}$  which relates  $\delta$  to  $P$

Deformations can be caused by the *material* reacting to a change in energy with temperature. In general (there are some exceptions):

- Solid materials can **contract** with a decrease in temperature.
- Solid materials can **expand** with an increase in temperature.

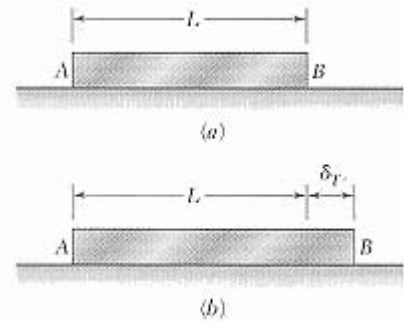
The change in length per unit temperature change is the *coefficient of thermal expansion*,  $\alpha$ . It

has units of  $/^{\circ}F$  or  $/^{\circ}C$  and the deformation is related by:

$$\delta_T = \alpha(\Delta T)L$$

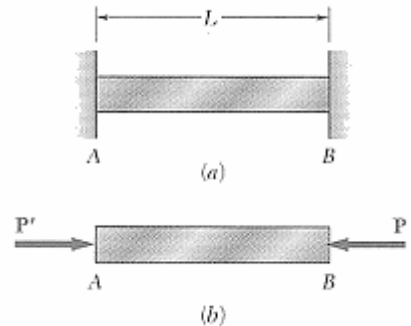
Thermal Strain:  $\epsilon_T = \alpha\Delta T$

There is **no stress** associated with the length change with free movement, BUT if there are restraints, thermal deformations or strains *can cause internal forces and stresses*.



**How A Restrained Bar Feels with Thermal Strain**

1. Bar pushes on supports because the material needs to expand with an increase in temperature.
2. Supports push *back*.
3. Bar is restrained, can't move and the reaction causes internal *stress*.



**Superposition Method**

If we want to solve a statically indeterminate problem that has extra support forces:

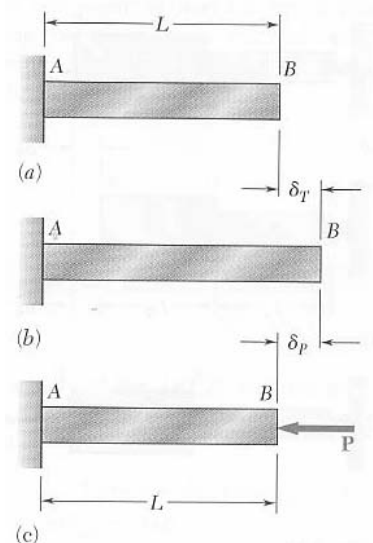
- We can remove a support or supports that *makes the problem look statically determinate*
- Replace it with a reaction and treat it like it is an applied force
- Impose geometry restrictions that the support imposes

For Example:

$$\delta_T = \alpha(\Delta T)L \quad \delta_p = -\frac{PL}{AE}$$

$$\delta_p + \delta_T = 0 \quad -\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$P = \alpha(\Delta T)L \frac{AE}{L} = \alpha(\Delta T)AE \quad f = -\frac{P}{A} = -\alpha(\Delta T)E$$





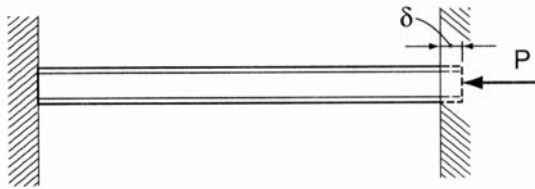
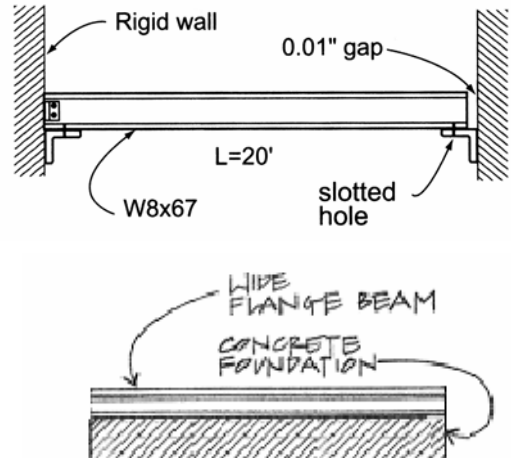
Example 2 (pg 228)

**Example Problem 6.24 (Figures 6.58 and 6.59)**

A W8x67 steel beam, 20 ft. in length, is rigidly attached at one end of a concrete wall. If a gap of 0.010 in. exists at the opposite end when the temperature is 45°F, what results when the temperature rises to 95°F?

ALSO: If the beam is anchored to a concrete slab, and the steel sees a temperature change of 50° F while the concrete only sees a change of 30° F, determine the compressive stress in the beam.

$$\begin{aligned} \alpha_c &= 5.5 \times 10^{-6} / ^\circ \text{F} & E_c &= 3 \times 10^6 \text{ psi} \\ \alpha_s &= 6.5 \times 10^{-6} / ^\circ \text{F} & E_s &= 29 \times 10^6 \text{ psi} \end{aligned}$$



Example 3

5.21 A short concrete column measuring 12 in. square is reinforced with four #8 bars ( $A_s = 4 \times 0.79 \text{ in.}^2 = 3.14 \text{ in.}^2$ ) and supports an axial load of 250k. Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$$E_c = 3 \times 10^6 \text{ psi and}$$

$$E_s = 29 \times 10^6 \text{ psi}$$

**Solution:**

From equilibrium:

$$[\Sigma F_y = 0] - 250 \text{ k} + f_s A_s + f_c A_c = 0$$

$$A_s = 3.14 \text{ in.}^2$$

$$A_c = (12'' \times 12'') - 3.14 \text{ in.}^2 \cong 141 \text{ in.}^2$$

$$3.14 f_s + 141 f_c = 250 \text{ k}$$

From the deformation relationship:

$$\delta_s = \delta_c; L_s = L_c$$

$$\therefore \frac{\delta_s}{L} = \frac{\delta_c}{L}$$

and

$$\epsilon_s = \epsilon_c$$

Since

$$E = \frac{f}{\epsilon}$$

and

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = f_c \frac{E_s}{E_c} = \frac{29 \times 10^3 (f_c)}{3 \times 10^3} = 9.67 f_c$$

Substituting into the equilibrium equation:

$$3.14 (9.67 f_c) + 141 f_c = 250$$

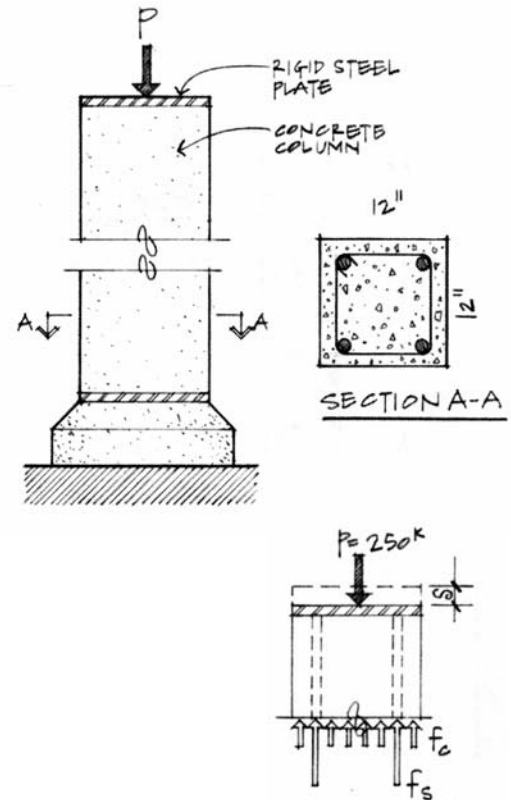
$$30.4 f_c + 141 f_c = 250$$

$$171.4 f_c = 250$$

$$f_c = 1.46 \text{ ksi}$$

$$\therefore f_s = 9.67 (1.46) \text{ ksi}$$

$$f_s = 14.1 \text{ ksi}$$

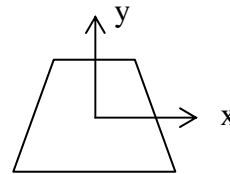
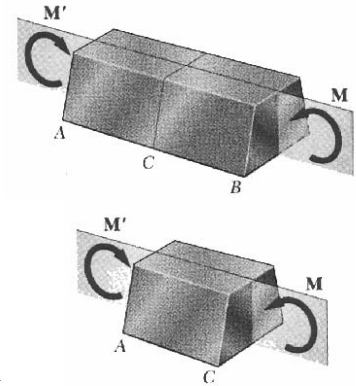


## Beam Bending Stresses and Shear Stress

### Pure Bending in Beams

With bending moments along the axis of the member only, a beam is said to be in pure bending.

Normal stresses due to bending can be found for homogeneous materials having a plane of symmetry in the  $y$  axis that follow Hooke's law.



### Maximum Moment and Stress Distribution

In a member of constant cross section, the maximum bending moment will govern the design of the section size when we know what kind of normal stress is caused by it.

For internal equilibrium to be maintained, the bending moment will be equal to the  $\sum M$  from the normal stresses  $\times$  the areas  $\times$  the moment arms. Geometric fit helps solve this statically indeterminate problem:

1. The normal planes remain normal for pure bending.
2. There is no net internal axial force.
3. Stress varies linearly over cross section.
4. Zero stress exists at the centroid and the line of centroids is the *neutral axis* (n. a)

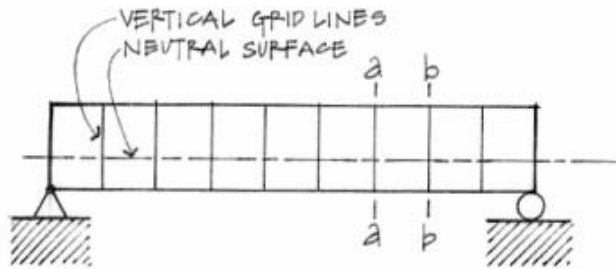
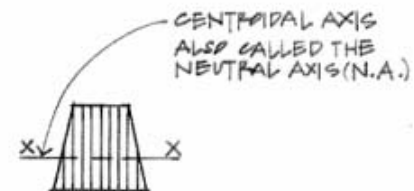


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

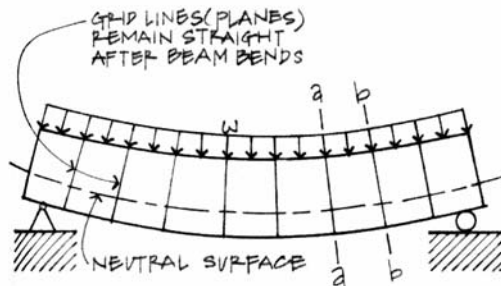


Figure 8.5(b) Beam bending under load.

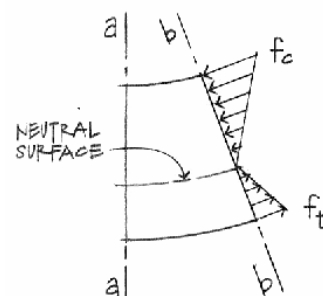
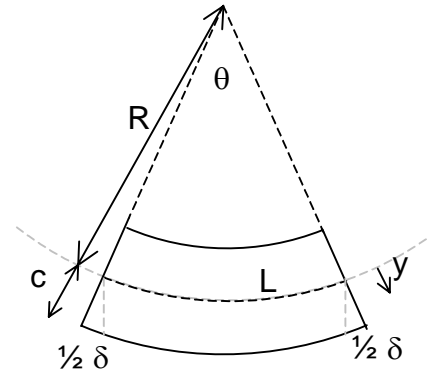


Figure 8.8 Bending stresses on section b-b.

**Relations for Beam Geometry and Stress**

Pure bending results in a circular arc deflection.  $R$  is the distance to the center of the arc;  $\theta$  is the angle of the arc (radians);  $c$  is the distance from the n.a. to the *extreme fiber*;  $f_{max}$  is the maximum normal stress at the *extreme fiber*;  $y$  is a distance in  $y$  from the n.a.;  $M$  is the bending moment;  $I$  is the moment of inertia;  $S$  is the *section modulus*.



$$L = R\theta \quad \epsilon = \frac{\delta}{L} = R \quad f = E\epsilon = \frac{y}{c} f_{max}$$

$$M = \sum f_i A_i \quad M = \frac{f_{max}}{c} \sum y_i^2 A_i \quad I = \sum y^2 A \quad S = \frac{I}{c} \quad f_{max} = \frac{Mc}{I} = \frac{M}{S}$$

Now:  $f_b = \frac{My}{I}$  for a rectangle of height  $h$  and width  $b$ :  $S = \frac{bh^3}{12 \cdot \frac{h}{2}} = \frac{bh^2}{6}$

RELATIONS:

$$\frac{1}{R} = \frac{M}{EI}$$

$$f_b = \frac{My^*}{I}$$

$$S = \frac{I}{c}$$

$$f_{b-max} = \frac{Mc}{I} = \frac{M}{S}$$

$$S_{required} \geq \frac{M}{F_b}$$

\*Note:  $y$  positive goes DOWN. With a positive  $M$  and  $y$  to the bottom fiber as positive, it results in a TENSION stress (we've called positive)

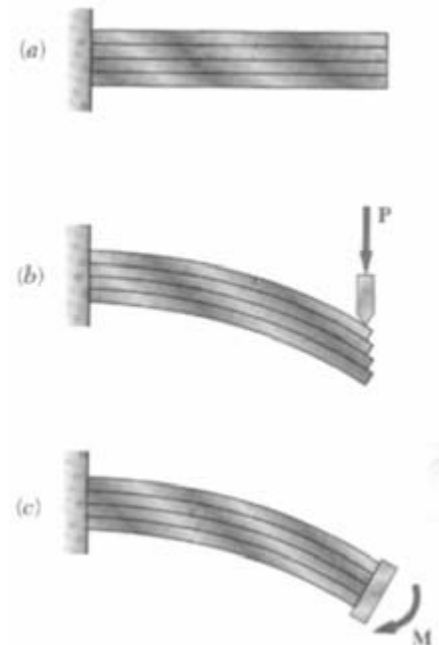
**Transverse Loading in Beams**

We are aware that transverse beam loadings result in internal shear and bending moments.

We designed sections based on bending stresses, since this stress dominates beam behavior.

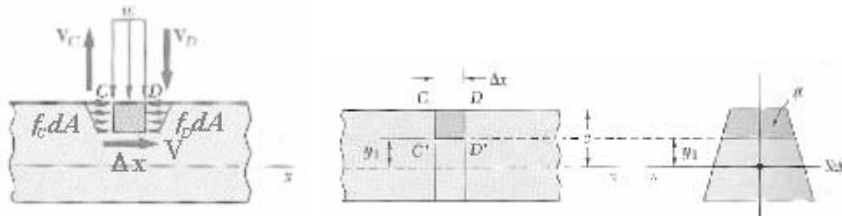
There can be shear stresses *horizontally* within a beam member.

It can be shown that  $f_{horizontal} = f_{vertical}$



**Equilibrium and Derivation**

In order for equilibrium for any element CDD'C', there needs to be a horizontal force ΔH.



$$V = f_D dA - f_C dA$$

Q is a moment area with respect to the neutral axis of the area *above or below* the horizontal where the ΔH occurs.

$$V_{longitudinal} = \frac{V_T Q}{I} \Delta x$$

Q is a maximum when y = 0 (at the **neutral axis**).

q is a horizontal shear per unit length → *shear flow*

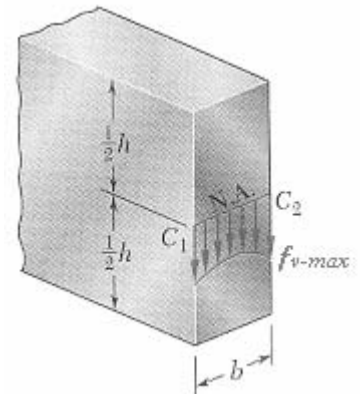
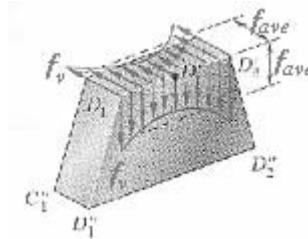
$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

**Shearing Stresses**

$f_{v-ave} = 0$  on the beam's surface. Even if Q is a maximum at y = 0, we don't know that the thickness is a *minimum* there.

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$



**Rectangular Sections**

$f_{v-max}$  occurs at the neutral axis:

$$I = \frac{bh^3}{12} \quad Q = A\bar{y} = b \frac{h}{2} \cdot \frac{1}{2} \frac{h}{2} = \frac{bh^2}{8}$$

then:

$$f_v = \frac{VQ}{Ib} = \frac{V \frac{1}{8} bh^2}{\frac{1}{12} bh^3 b} = \frac{3V}{2bh}$$

$$f_v = \frac{3V}{2A}$$

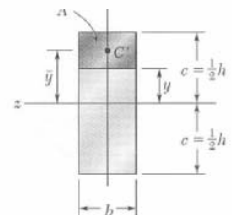
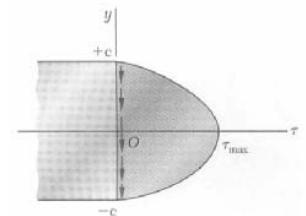


Fig. 6.15

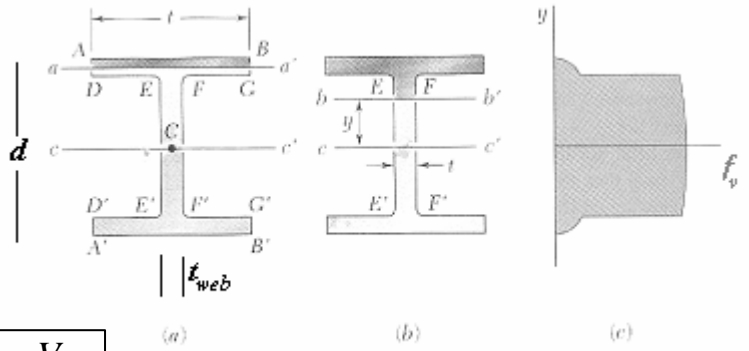


### Webs of Beams

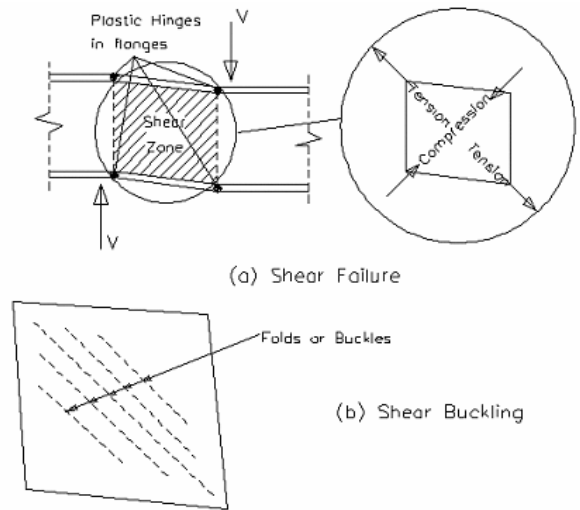
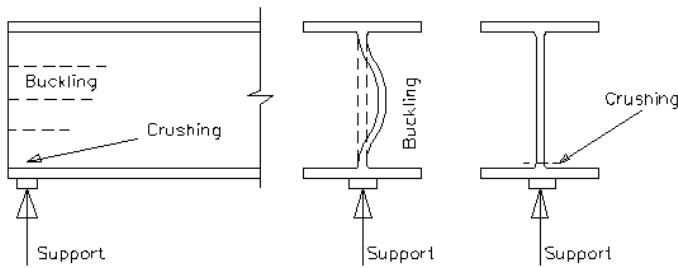
In steel W or S sections the thickness varies from the flange to the web.

We neglect the shear stress in the flanges and consider the shear stress in the web to be constant:

$$f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}} \quad \boxed{f_{v-max} = \frac{V}{t_{web} d}}$$



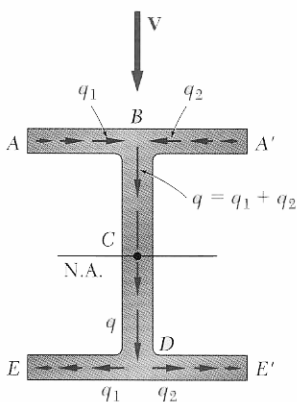
Webs of I beams can fail in tension shear across a panel with stiffeners or the web can buckle.



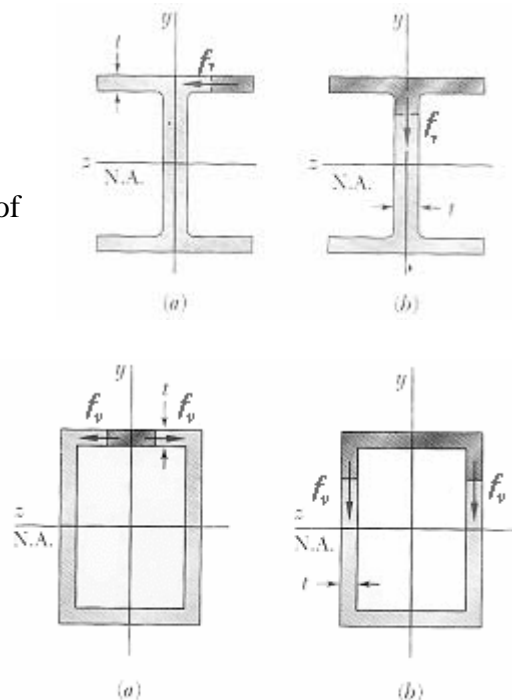
### Shear Flow

Even if the cut we make to find Q is not horizontal, but arbitrary, we can still find the shear flow, q, as long as the loads on thin-walled sections are applied in a plane of symmetry, and the cut is made *perpendicular* to the surface of the member.

$$q = \frac{VQ}{I}$$



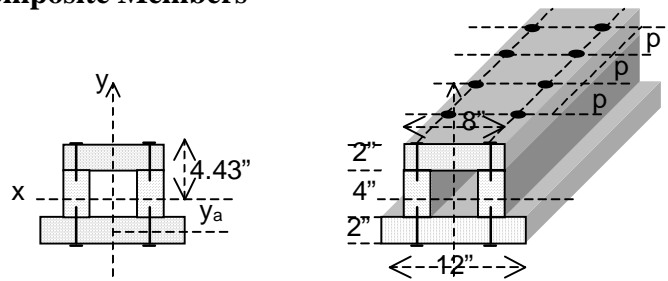
The shear flow magnitudes can be sketched by knowing Q.



**Connectors to Resist Horizontal Shear in Composite Members**

Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices.

The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.



$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$V_{longitudinal} = \frac{VQ}{I} \cdot p$$

where

p = pitch length

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

n = number of connectors connecting the connected area to the rest of the cross section

F = force capacity in one connector

$Q_{connected\ area} = A_{connected\ area} \times y_{connected\ area}$

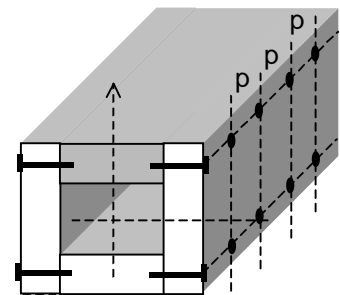
$y_{connected\ area}$  = distance from the centroid of the connected area to the neutral axis

**Connectors to Resist Horizontal Shear in Composite Members**

Even vertical connectors have shear flow across them.

The spacing can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p.

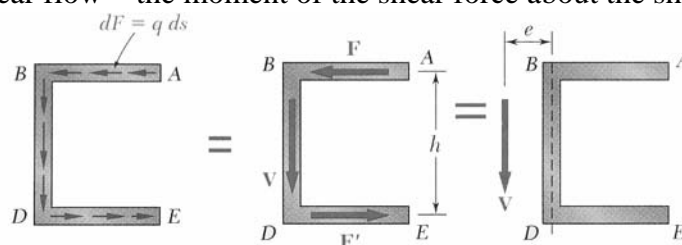
$$p \leq \frac{nF_{connector} I}{VQ_{connected\ area}}$$



**Unsymmetrical Sections or Shear**

If the section is not symmetric, or has a shear not in that plane, the member can bend and twist.

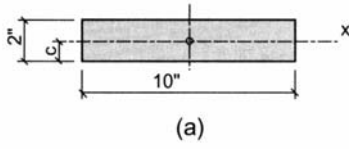
If the load is applied at the *shear center* there will not be twisting. This is the location where the moment caused by shear flow = the moment of the shear force about the shear center.



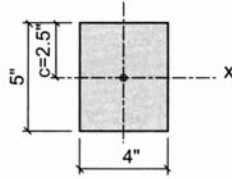
Example 1 (pg 303)

**Example Problem 9.2 (Figures 9.15 to 9.18)**

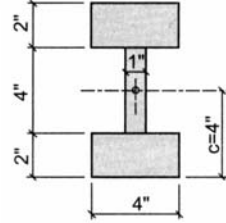
A beam must span a distance of 12' and carry a uniformly distributed load of 120 lb./ft. Determine which cross-section would be the least stressed: *a*, *b*, or *c*.



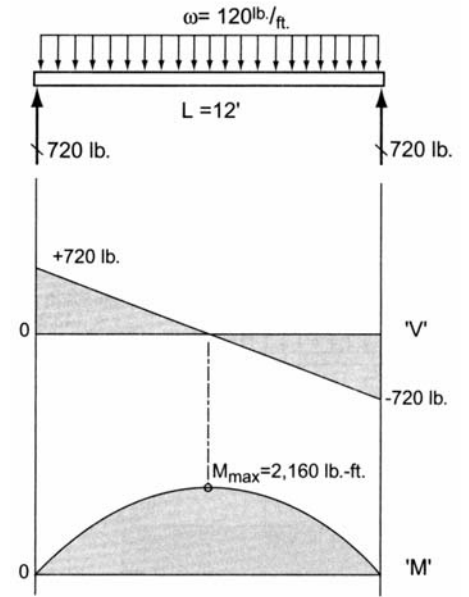
(a)



(b)



(c)





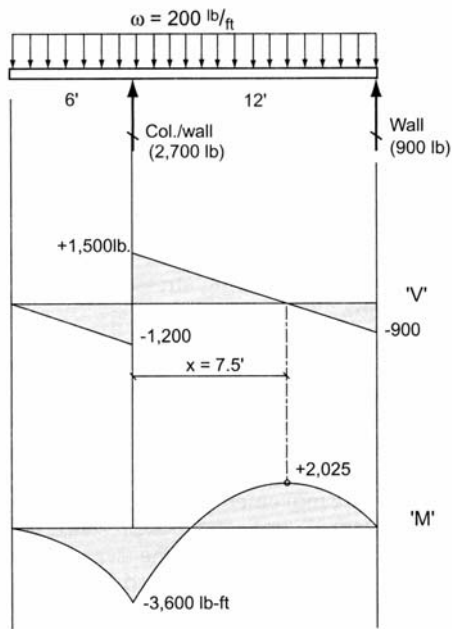
Example 2 (pg 309)

**Example Problem 9.7 (Figures 9.31 to 9.33)**

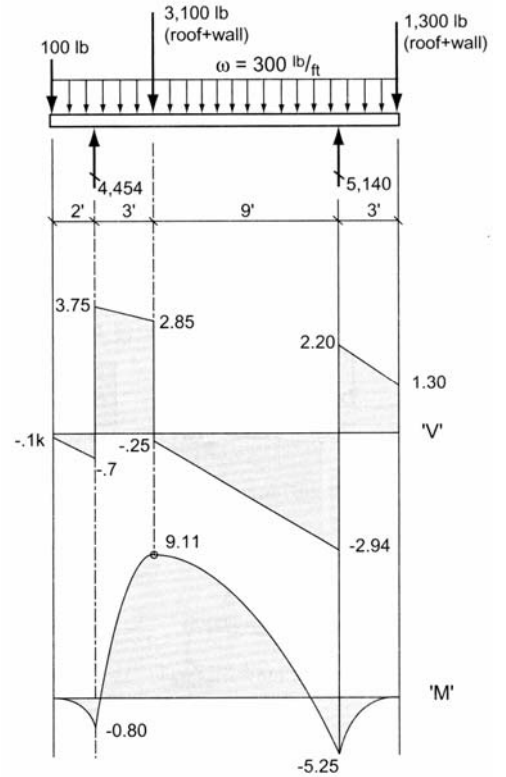
Design the roof and second-floor beams if  $F_b = 1550$  psi (Southern pine No. 1).

- Roof: Snow +DL = 200 lb/ft
- Walls: 400 lb on 2<sup>nd</sup> floor beams
- Railing: 100 lb on beam overhang
- Second Floor: DL + LL = 300 lb/ft (including overhang)

**Roof:**



**Second Floor:**



Example 3 (pg 313)

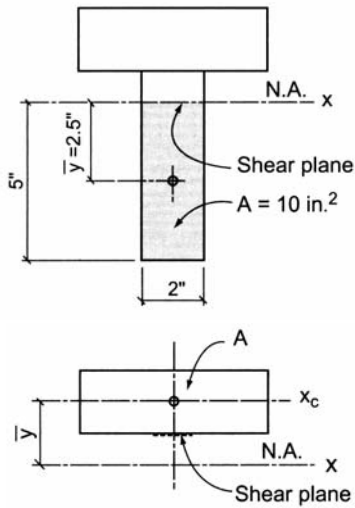
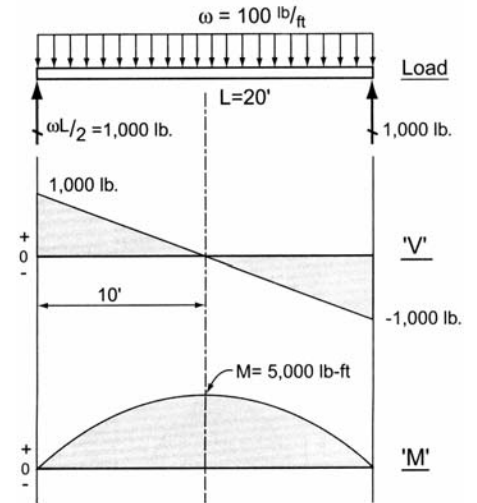
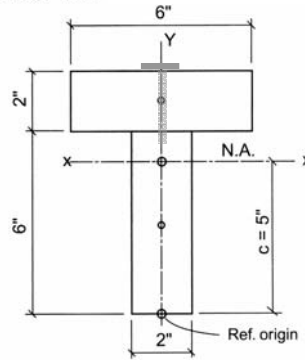
Example Problem 9.8: Shear Stress  
(Figures 9.43 to 9.47)

Calculate the maximum bending and shear stress for the beam shown.

ALSO: Determine the minimum nail spacing required (pitch) if the shear capacity of a nail (F) is 250 lb.

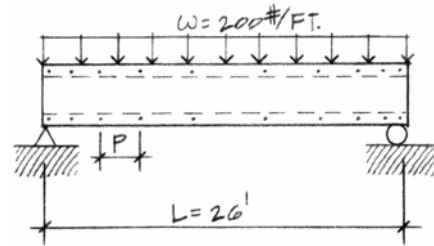
Component	A (in. <sup>2</sup> )	$\bar{y}$ (in.)	$\bar{y}\Delta A$ (in. <sup>3</sup> )
	12	7	84
	12	3	36

Component	$I_{xc}$ (in. <sup>4</sup> )	A (in. <sup>2</sup> )	$d_y$ (in.)	$Ad_y^2$ (in. <sup>4</sup> )
	4	12	2	48
	36	12	2	48



**Example 4**

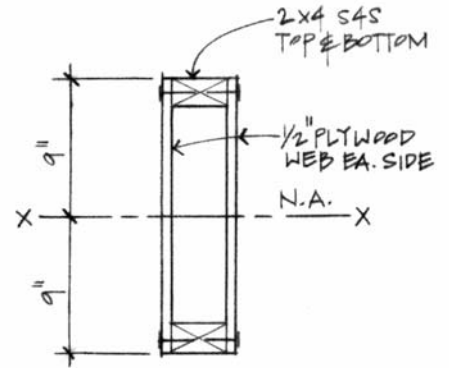
**8.11** A built-up plywood box beam with 2 x 4 S4S top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 #/ft. along the 26-foot span. Assume the nails have a shear capacity of 80# each.



**Solution:**

Construct the shear (V) diagram to obtain the critical shear condition and its location

Note that the condition of shear is critical at the supports, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing P varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.



$$f_v = \frac{VQ}{Ib}$$

$$I_x = \frac{(4.5'')(18'')^3}{12} - \frac{(3.5'')(15'')^3}{12} = 1,202.6 \text{ in.}^4$$

$$Q = A\bar{y} = (5.25 \text{ in.}^2)(8.25'') = 43.3 \text{ in.}^3$$

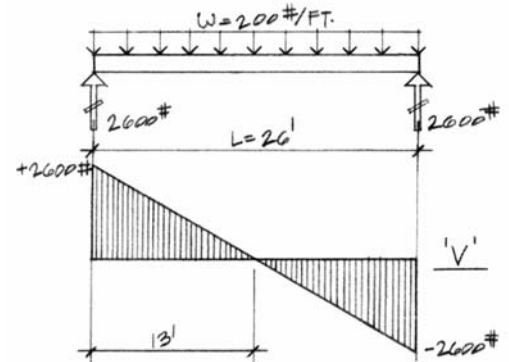
$$\text{Shear force} = f_v \times A_v$$

where:

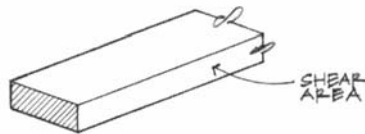
$A_v$  = shear area

Assume:

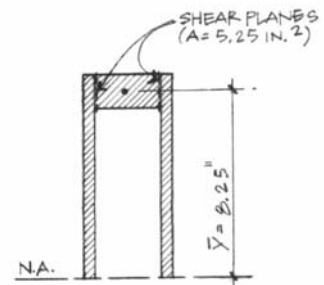
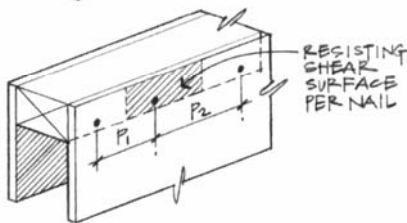
F = Capacity of two nails (one each side) at the flange; representing two shear surfaces



$$(n)F = f_v \times b \times p = \frac{VQ}{Ib} \times bp$$



$$\therefore (n)F = p \times \frac{VQ}{I}; \quad p = \frac{(n)FI_x}{VQ}$$



At the maximum shear location (support) where  $V = 2,600\#$

$$p = \frac{(2 \text{ nails} \times 80 \text{ #/nail})(1,202.6 \text{ in.}^4)}{(2,600\#)(43.3 \text{ in.}^3)} = 1.71''$$

## Beam Design and Deflections

### Criteria for Design

Allowable normal stress or normal stress from LRFD should not be exceeded:

$$F_b \text{ or } \phi F_n \geq f_b = \frac{Mc}{I}$$

Knowing  $M$  and  $F_b$ , the minimum section modulus fitting the limit is:

$$S_{req'd} \geq \frac{M}{F_b}$$

Besides strength, we also need to be concerned about *serviceability*. This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

### Determining Maximum Bending Moment

Drawing  $V$  and  $M$  diagrams will show us the maximum values for design. Remember:

$$\begin{aligned} V &= \Sigma(-w)dx & \frac{dV}{dx} &= -w & \frac{dM}{dx} &= V \\ M &= \Sigma(V)dx \end{aligned}$$

### Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

### Deflections

If the bending moment changes,  $M(x)$  across a beam of constant material and cross section then the curvature will change:

$$\frac{1}{R} = \frac{M(x)}{EI}$$

The slope of the n.a. of a beam,  $\theta$ , will be tangent to the radius of curvature,  $R$ :

$$\theta = slope = \frac{1}{EI} \int M(x) dx$$

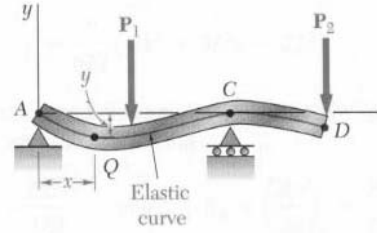
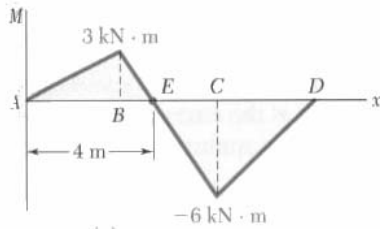
The equation for deflection,  $y$ , along a beam is:

$$y = \frac{1}{EI} \int \theta dx = \frac{1}{EI} \iint M(x) dx$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc... Computer programs can be used as well. (BigBoy Beam freeware: <http://forum.simtel.net/pub/pd/33994.html>)

Elastic curve equations can be **superpositioned** ONLY if the stresses are in the elastic range.

The deflected shape is roughly the same shape as the bending moment diagram flipped but is constrained by supports and geometry.



**Boundary Conditions**

The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.

At Pins, Rollers, Fixed Supports:  $y = 0$

At Fixed Supports:  $\theta = 0$

At Inflection Points From Symmetry:  $\theta = 0$

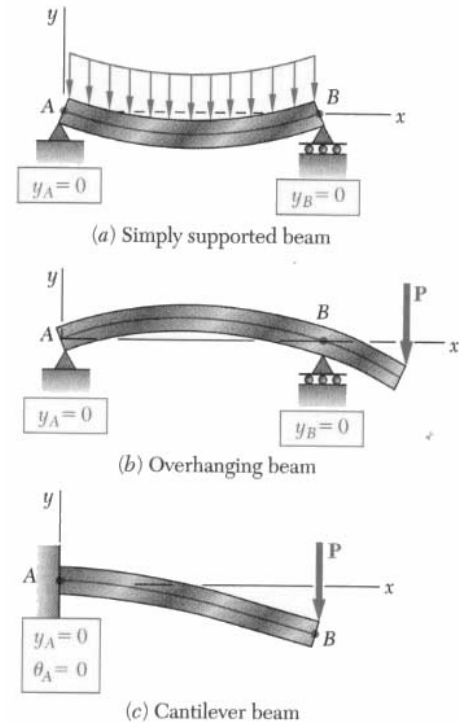
The Slope Is Zero At The Maximum Deflection  $y_{max}$ :

$$\theta = \frac{dy}{dx} = \text{slope} = 0$$

**Allowable Deflection Limits**

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

$$y_{max}(x) = \Delta_{actual} \leq \Delta_{allowable} = L / \text{value}$$



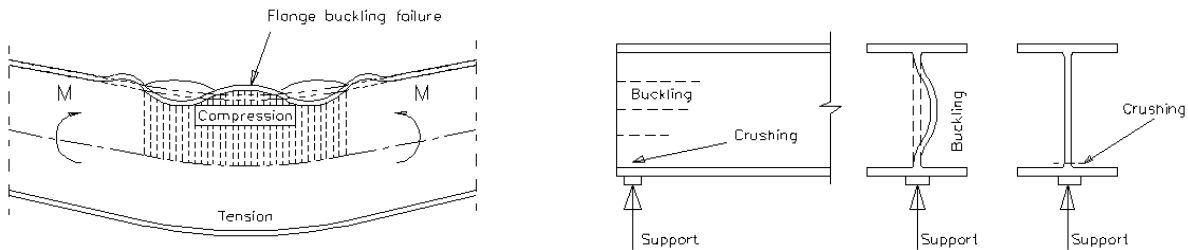
Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

## Lateral Buckling

With compression stresses in the top of a beam, a sudden “popping” or **buckling** can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger  $I_y$ .

## Local Buckling in Steel I Beams– Web Crippling or Flange Buckling

Concentrated forces on a steel beam can cause the web to buckle (called **web crippling**). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.



## Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the tributary area that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

## Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

1. Know  $F_b$  (allowable stress) for the material or  $F_y$  &  $F_u$  for LRFD.
2. Draw V & M, finding  $M_{max}$ .
3. Calculate  $S_{req'd}$ . This step is equivalent to determining  $f_b = \frac{M_{max}}{S} \leq F_b$
4. For rectangular beams  $S = \frac{bh^2}{6}$

- For steel or timber: use the section charts to find S that will work *and remember that the beam self weight will increase  $S_{req'd}$* . And for steel, the design charts show the lightest section within a grouping of similar S's.
- For any thing else, try a nice value for b, and calculate h or the other way around.

\*\*\*\*Determine the “updated”  $V_{max}$  and  $M_{max}$  including the beam self weight, and verify that the updated  $S_{req'd}$  has been met.\*\*\*\*\*

- Consider lateral stability
- Evaluate horizontal shear stresses using  $V_{max}$  to determine if  $f_v \leq F_v$

For I and rectangular beams 
$$f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

- Provide adequate bearing area at supports: 
$$f_p = \frac{P}{A} \leq F_p$$

- Evaluate shear due to torsion 
$$f_v = \frac{T\rho}{J} \text{ or } \frac{T}{c_1 ab^2} \leq F_v$$

(circular section or rectangular)

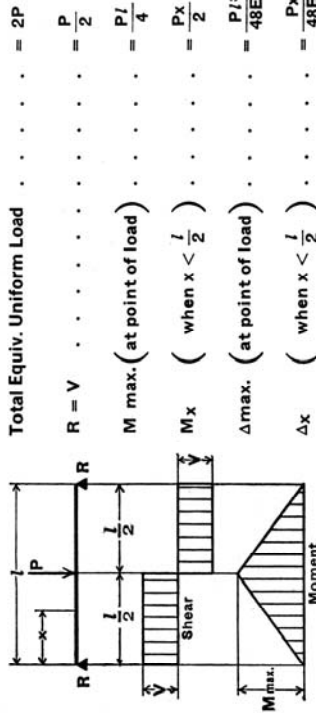
- Evaluate the deflection to determine if  $\Delta_{maxLL} \leq \Delta_{LL-allowed}$  and/or  $\Delta_{maxTotal} \leq \Delta_{T-allowed}$

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

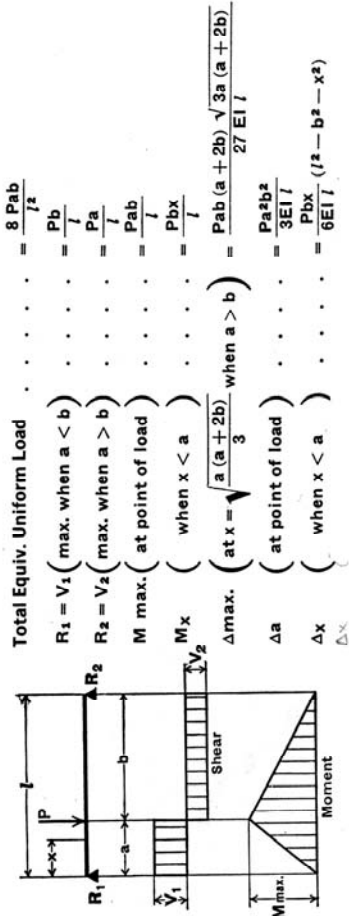
### BEAM DIAGRAMS AND FORMULAS For Various Static Loading Conditions, AISC ASD 8<sup>th</sup> ed.

<p><b>1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD</b></p> <p>                     Total Equiv. Uniform Load . . . . . = <math>wl</math>  <math>R = V</math> . . . . . = <math>\frac{wl}{2}</math>  <math>V_x</math> . . . . . = <math>w \left( \frac{l}{2} - x \right)</math>                      M max. (at center) . . . . . = <math>\frac{wl^2}{8}</math>  <math>M_x</math> . . . . . = <math>\frac{wx}{2} (l-x)</math>  <math>\Delta_{max}</math>. (at center) . . . . . = <math>\frac{5wl^4}{384EI}</math>  <math>\Delta_x</math> . . . . . = <math>\frac{wx}{24EI} (l^3 - 2lx^2 + x^3)</math> </p>	<p><b>2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END</b></p> <p>                     Total Equiv. Uniform Load . . . . . = <math>\frac{16W}{9\sqrt{3}}</math> = 1.0264W  <math>R_1 = V_1</math> . . . . . = <math>\frac{W}{3}</math>  <math>R_2 = V_2</math> max. . . . . = <math>\frac{2W}{3}</math>  <math>V_x</math> . . . . . = <math>\frac{W}{3} - \frac{Wx^2}{l^2}</math>                      M max. (at <math>x = \frac{l}{\sqrt{3}} = .5774l</math>) . . . . . = <math>\frac{2Wl}{9\sqrt{3}} = .1283Wl</math>  <math>M_x</math> . . . . . = <math>\frac{Wx}{3/2} (l^2 - x^2)</math>  <math>\Delta_{max}</math>. (at <math>x = l\sqrt{\frac{5}{15}} = .5193l</math>) . . . . . = <math>\frac{.01304}{EI} \frac{Wl^3}{3/2}</math>  <math>\Delta_x</math> . . . . . = <math>\frac{Wx}{180EI/2} (3x^4 - 10l^2x^2 + 7l^4)</math> </p>	<p><b>3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER</b></p> <p>                     Total Equiv. Uniform Load . . . . . = <math>\frac{4W}{3}</math>  <math>R = V</math> . . . . . = <math>\frac{W}{2}</math>  <math>V_x</math> (when <math>x &lt; \frac{l}{2}</math>) . . . . . = <math>\frac{W}{2} (l^2 - 4x^2)</math>                      M max. (at center) . . . . . = <math>\frac{Wl}{6}</math>  <math>M_x</math> (when <math>x &lt; \frac{l}{2}</math>) . . . . . = <math>Wx \left( \frac{l}{2} - \frac{2x^2}{3} \right)</math>  <math>\Delta_{max}</math>. (at center) . . . . . = <math>\frac{Wl^3}{60EI}</math>  <math>\Delta_x</math> (when <math>x &lt; \frac{l}{2}</math>) . . . . . = <math>\frac{Wx}{480EI/2} (5l^2 - 4x^2)^2</math> </p>
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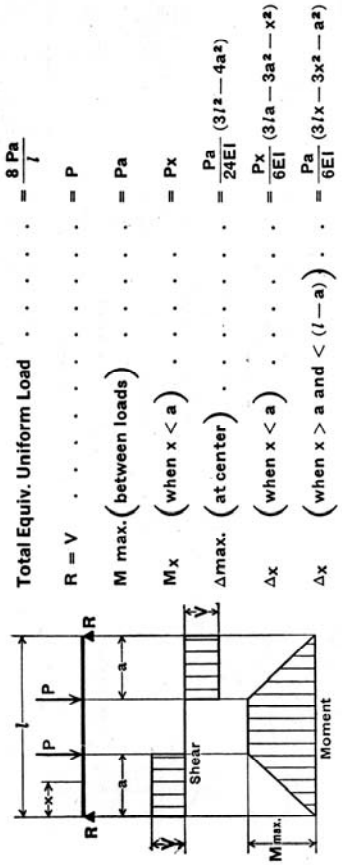
7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



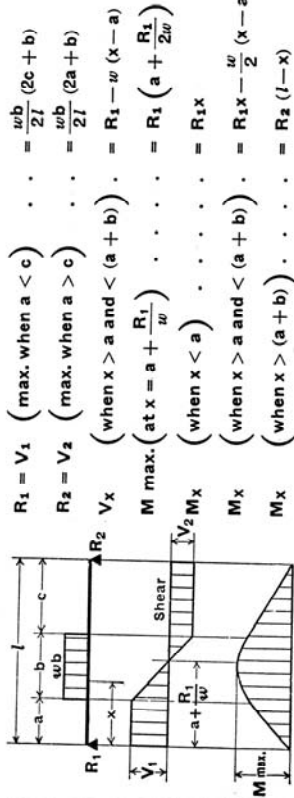
8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



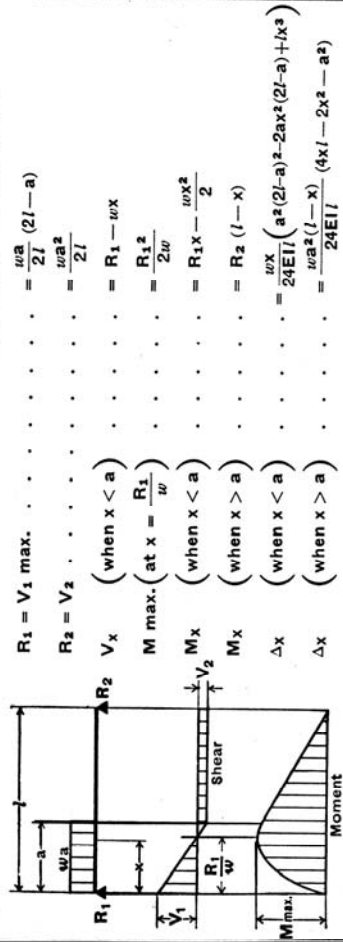
9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



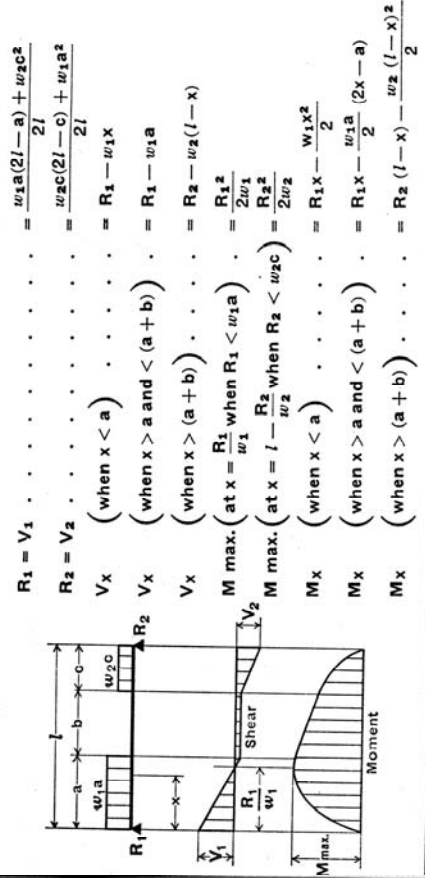
4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END





13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—  
CONCENTRATED LOAD AT CENTER

$\text{Total Equiv. Uniform Load} \dots = \frac{3P}{2}$   
 $R_1 = V_1 \dots = \frac{5P}{16}$   
 $R_2 = V_2 \text{ max.} \dots = \frac{11P}{16}$   
 $M \text{ max. (at fixed end)} \dots = \frac{3Pl}{16}$   
 $M_1 \text{ (at point of load)} \dots = \frac{5Pl}{32}$   
 $M_x \text{ (when } x < \frac{l}{2}) \dots = \frac{5Px}{16}$   
 $M_x \text{ (when } x > \frac{l}{2}) \dots = P \left( \frac{l}{2} - \frac{11x}{16} \right)$   
 $\Delta \text{ max. (at } x = l \sqrt{\frac{1}{5}} = .4472l) \dots = \frac{P l^3}{48EI} \sqrt{5} = .009317 \frac{P l^3}{EI}$   
 $\Delta x \text{ (at point of load)} \dots = \frac{7P l^3}{768EI}$   
 $\Delta x \text{ (when } x < \frac{l}{2}) \dots = \frac{Px}{96EI} (3l^2 - 5x^2)$   
 $\Delta x \text{ (when } x > \frac{l}{2}) \dots = \frac{P}{96EI} (x-l)^2 (11x-2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—  
CONCENTRATED LOAD AT ANY POINT

$R_1 = V_1 \dots = \frac{Pb^2}{2l^2} (a+2l)$   
 $R_2 = V_2 \dots = \frac{Pa}{2l^2} (3l^2 - a^2)$   
 $M_1 \text{ (at point of load)} \dots = R_1 a$   
 $M_2 \text{ (at fixed end)} \dots = \frac{Pab}{2l^2} (a+l)$   
 $M_x \text{ (when } x < a) \dots = R_1 x$   
 $M_x \text{ (when } x > a) \dots = R_1 x - P(x-a)$   
 $\Delta \text{ max. (when } a < .414l \text{ at } x = l \sqrt{\frac{l^2+a^2}{3l^2-a^2}}) = \frac{Pa}{3EI} \sqrt{\frac{l^2+a^2}{3l^2-a^2}}$   
 $\Delta \text{ max. (when } a > .414l \text{ at } x = l \sqrt{\frac{a}{2l+a}}) = \frac{Pab^2}{6EI} \sqrt{\frac{a}{2l+a}}$   
 $\Delta a \text{ (at point of load)} \dots = \frac{Pa^2 b^3}{12EI l^3} (3l+a)$   
 $\Delta x \text{ (when } x < a) \dots = \frac{Pb^2 x}{12EI l^3} (3a l^2 - 2l x^2 - a x^2)$   
 $\Delta x \text{ (when } x > a) \dots = \frac{Pa}{12EI l^3} (l-x)^2 (3l^2 - a^2 x - 2a^2 l)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS  
UNSYMMETRICALLY PLACED

$R_1 = V_1 \text{ (max. when } a < b) \dots = \frac{P}{l} (l-a+b)$   
 $R_2 = V_2 \text{ (max. when } a > b) \dots = \frac{P}{l} (l-b+a)$   
 $V_x \text{ (when } x > a \text{ and } < (l-b)) \dots = \frac{P}{l} (b-a)$   
 $M_1 \text{ (max. when } a > b) \dots = R_1 a$   
 $M_2 \text{ (max. when } a < b) \dots = R_2 b$   
 $M_x \text{ (when } x < a) \dots = R_1 x$   
 $M_x \text{ (when } x > a \text{ and } < (l-b)) \dots = R_1 x - P(x-a)$

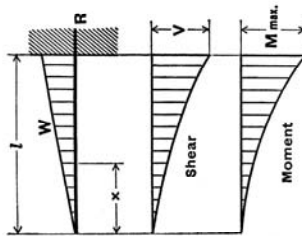
11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS  
UNSYMMETRICALLY PLACED

$R_1 = V_1 \dots = \frac{P_1(l-a) + P_2 b}{l}$   
 $R_2 = V_2 \dots = \frac{P_1 a + P_2(l-b)}{l}$   
 $V_x \text{ (when } x > a \text{ and } < (l-b)) \dots = R_1 - P_1$   
 $M_1 \text{ (max. when } R_1 < P_2) \dots = R_1 a$   
 $M_2 \text{ (max. when } R_2 < P_2) \dots = R_2 b$   
 $M_x \text{ (when } x < a) \dots = R_1 x$   
 $M_x \text{ (when } x > a \text{ and } < (l-b)) \dots = R_1 x - P_1(x-a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—  
UNIFORMLY DISTRIBUTED LOAD

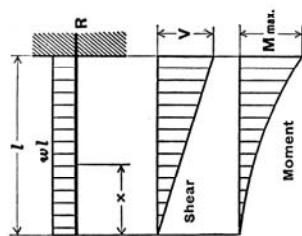
$\text{Total Equiv. Uniform Load} \dots = \frac{wl}{8}$   
 $R_1 = V_1 \dots = \frac{3wl}{8}$   
 $R_2 = V_2 \text{ max.} \dots = \frac{5wl}{8}$   
 $V_x \dots = \frac{R_1 - wx}{8}$   
 $M \text{ max. (at } x = \frac{3}{8}l) \dots = \frac{9}{128} \frac{wl^2}{8}$   
 $M_1 \text{ (at } x = \frac{3}{8}l) \dots = \frac{128}{128} \frac{wl^2}{8}$   
 $M_x \dots = R_1 x - \frac{wlx^2}{2}$   
 $\Delta \text{ max. (at } x = \frac{l}{16} (1 + \sqrt{33}) = .4215l) \dots = \frac{185EI}{185EI} \frac{wl^4}{48E} (l^3 - 3lx^2 + 2x^3)$   
 $\Delta x \dots = \frac{wlx}{48E} (l^3 - 3lx^2 + 2x^3)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



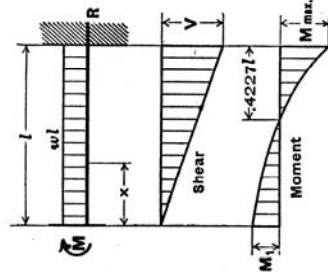
Total Equiv. Uniform Load . . . . . =  $\frac{8}{3} W$   
 $R = V$  . . . . . =  $W$   
 $V_x$  . . . . . =  $W \frac{x^2}{l^2}$   
 $M$  max. (at fixed end) . . . . . =  $\frac{Wl}{3}$   
 $M_x$  . . . . . =  $\frac{Wx^3}{3l^2}$   
 $\Delta$  max. (at free end) . . . . . =  $\frac{Wl^3}{18EI}$   
 $\Delta_x$  . . . . . =  $\frac{W}{60EI l^2} (x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



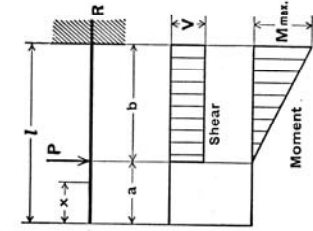
Total Equiv. Uniform Load . . . . . =  $4wl$   
 $R = V$  . . . . . =  $wl$   
 $V_x$  . . . . . =  $wX$   
 $M$  max. (at fixed end) . . . . . =  $\frac{wl^2}{2}$   
 $M_x$  . . . . . =  $\frac{wx^2}{2}$   
 $\Delta$  max. (at free end) . . . . . =  $\frac{wl^4}{8EI}$   
 $\Delta_x$  . . . . . =  $\frac{w}{24EI} (x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



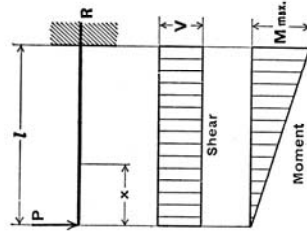
Total Equiv. Uniform Load . . . . . =  $\frac{8}{3} wl$   
 $R = V$  . . . . . =  $wl$   
 $V_x$  . . . . . =  $wX$   
 $M$  max. (at fixed end) . . . . . =  $\frac{wl^2}{3}$   
 $M_1$  (at deflected end) . . . . . =  $\frac{wl^2}{6}$   
 $M_x$  . . . . . =  $\frac{w}{6} (l^3 - 3x^3)$   
 $\Delta$  max. (at deflected end) . . . . . =  $\frac{wl^4}{24EI}$   
 $\Delta_x$  . . . . . =  $\frac{w}{24EI} (l^2 - x^2)^2$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



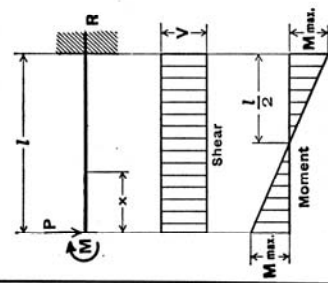
Total Equiv. Uniform Load . . . . . =  $\frac{8Pb}{l}$   
 $R = V$  . . . . . =  $P$   
 $M$  max. (at fixed end) . . . . . =  $Pb$   
 $M_x$  (when  $x > a$ ) . . . . . =  $P(x-a)$   
 $\Delta$  max. (at free end) . . . . . =  $\frac{Pb^2}{6EI} (3l-b)$   
 $\Delta_a$  (at point of load) . . . . . =  $\frac{Pb^2}{3EI}$   
 $\Delta_x$  (when  $x < a$ ) . . . . . =  $\frac{Pb^2}{6EI} (3l-3x-b)$   
 $\Delta_x$  (when  $x > a$ ) . . . . . =  $\frac{P(l-x)^2}{6EI} (3b-l+x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load . . . . . =  $8P$   
 $R = V$  . . . . . =  $P$   
 $M$  max. (at fixed end) . . . . . =  $Pl$   
 $M_x$  . . . . . =  $Px$   
 $\Delta$  max. (at free end) . . . . . =  $\frac{Pl^3}{3EI}$   
 $\Delta_x$  . . . . . =  $\frac{P}{6EI} (2l^3 - 3l^2x + x^3)$

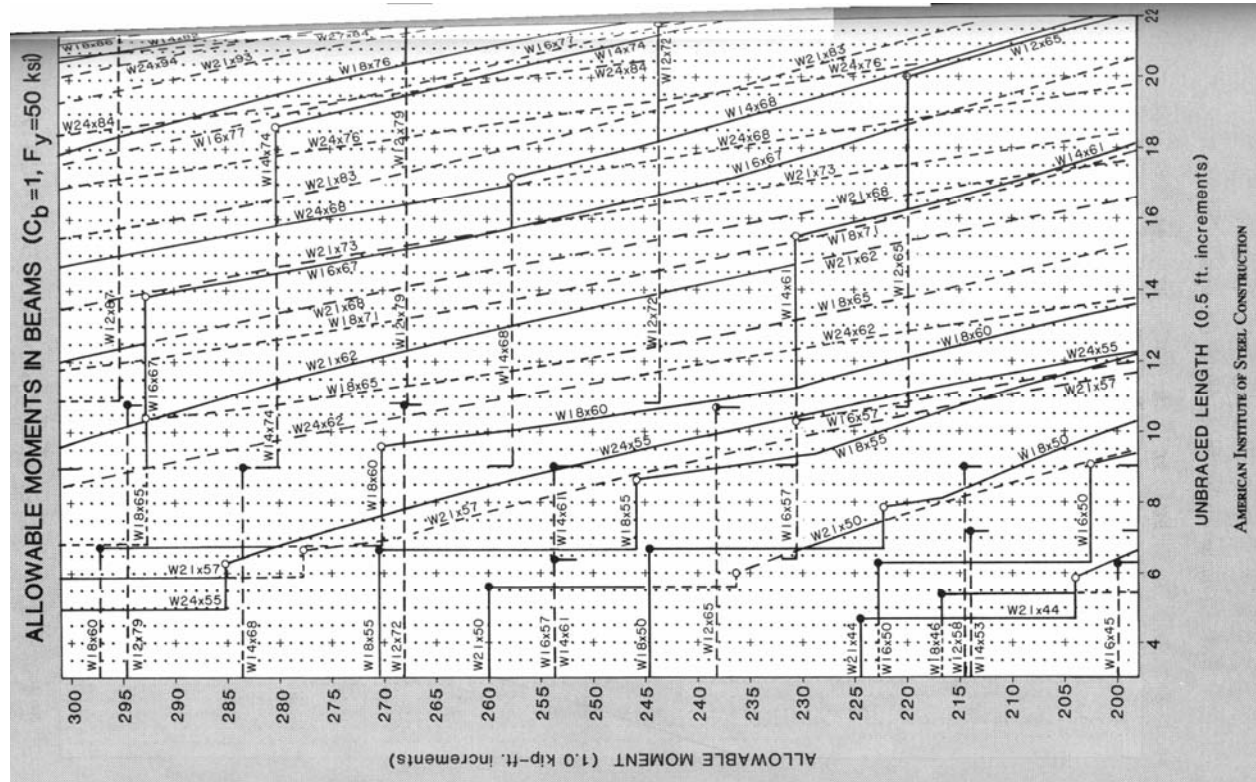
23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load . . . . . =  $4P$   
 $R = V$  . . . . . =  $P$   
 $M$  max. (at both ends) . . . . . =  $\frac{Pl}{2}$   
 $M_x$  . . . . . =  $P(\frac{l}{2}-x)$   
 $\Delta$  max. (at deflected end) . . . . . =  $\frac{Pl^3}{12EI}$   
 $\Delta_x$  . . . . . =  $\frac{P(l-x)^2}{12EI} (l+2x)$

### Allowable Moments in Beams with Unbraced Lengths

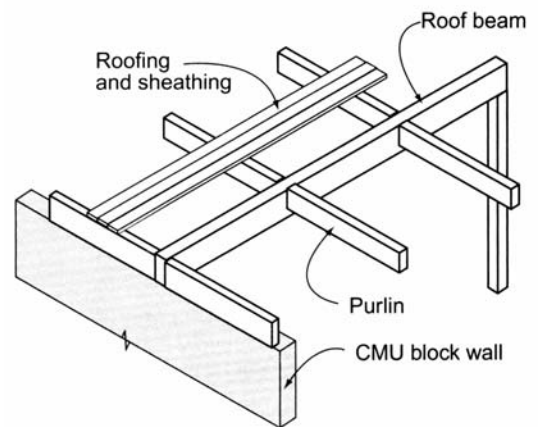
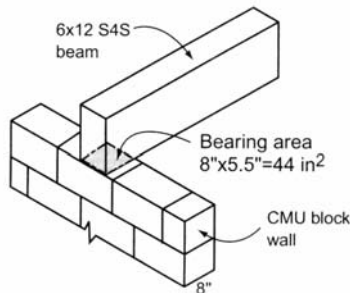
Allowable stresses are reduced when the unbraced length of the compression flange can buckle called  $L_c$ . The limiting unbraced length at the lower stresses is called  $L_u$ . The maximum moment that can be applied (taking self weight into account) can be plotted against the unbraced length. The limit  $L_c$  is indicated by a solid dot (●), while  $L_u$  is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used.  $C_b$ , which is a modification factor for non-zero moments at the ends, is 1 for simply supported beams (0 moments at the ends).

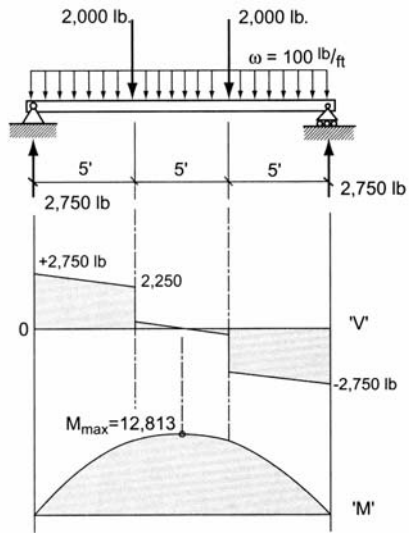


#### Example 1 (pg 328)

#### Example Problem 9.15 (Figures 9.73 to 9.75)

Design a Southern pine No. 1 beam to carry the loads shown (roof beam, no plaster). Assume the beam is supported at each end by an 8" block wall.  $F_b = 1550$  psi;  $F_v = 110$  psi;  $E = 1.6 \times 10^6$  psi.



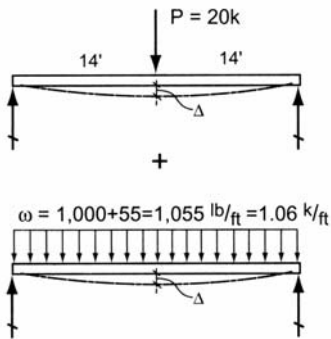
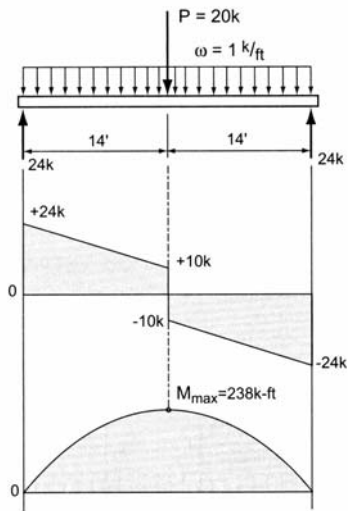
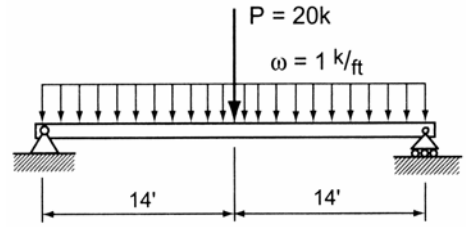


Example 2 (pg 330)

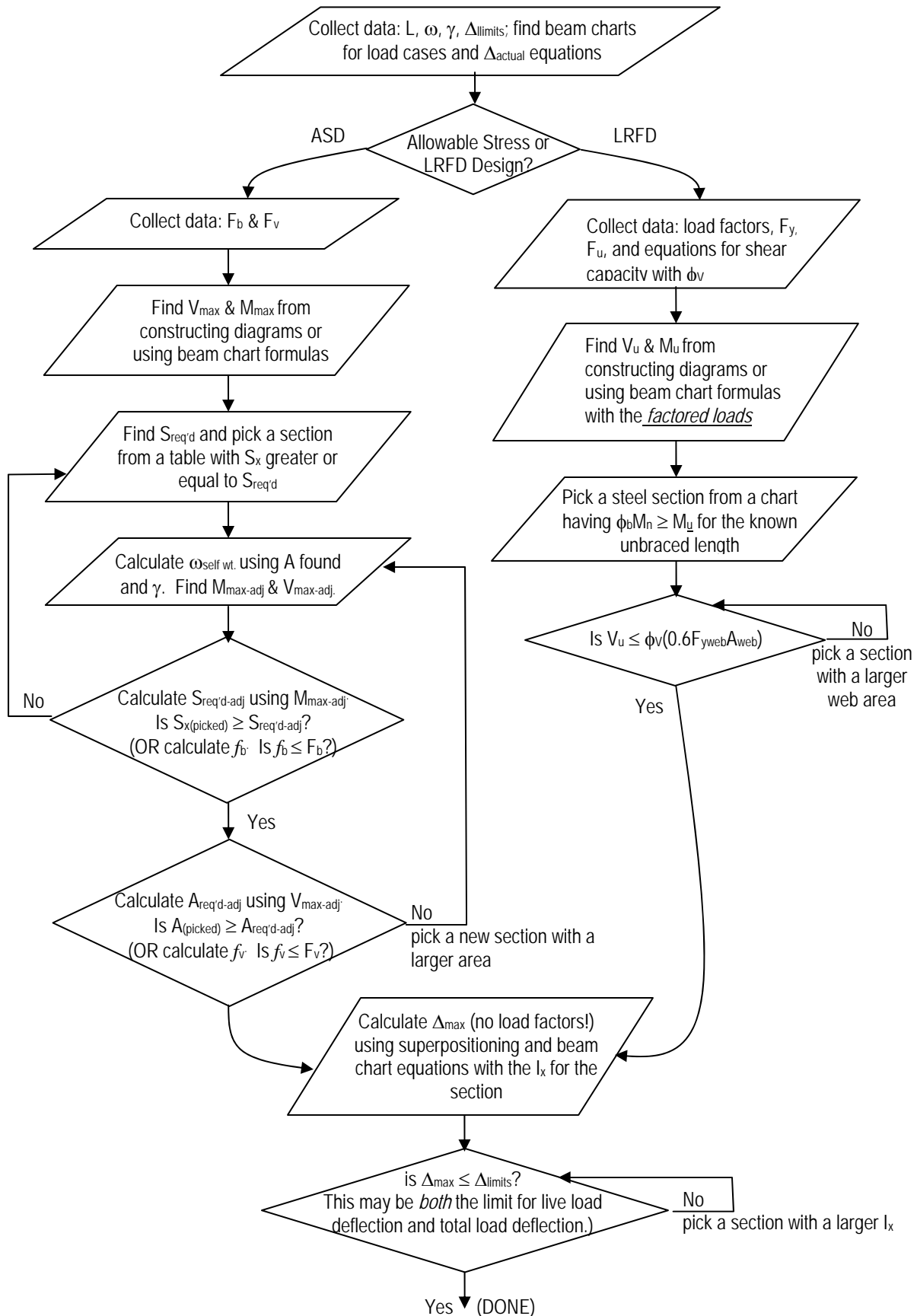
Example Problem 9.16 (Figures 9.76 to 9.78)

A steel beam (A572/50) is loaded as shown. Assuming a deflection requirement of  $\Delta_{total} = L/240$  and a depth restriction of 18" nominal, select the most economical section.

$F_b = 30 \text{ ksi}; F_v = 20 \text{ ksi}; E = 30 \times 10^3 \text{ ksi}$



### Beam Design Flow Chart



## Steel– AISC Load and Resistance Factor Design

### Load and Resistance Factor Design

The Manual of Steel Construction LRFD, 3<sup>rd</sup> ed. by the American Institute of Steel Construction requires that all steel structures and structural elements be proportioned so that no strength limit state is exceeded when subjected to all required **factored** load combinations.

$$\sum \gamma_i R_i \leq \phi R_n$$

where  $\gamma$  = load factor for the type of load  
 $R$  = load (dead or live; force, moment or stress)  
 $\phi$  = resistance factor  
 $R_n$  = nominal load (ultimate capacity; force, moment or stress)

*Nominal strength* is defined as the

capacity of a structure or component to resist the effects of loads, as determined by computations using specified material strengths (such as yield strength,  $F_y$ , or ultimate strength,  $F_u$ ) and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions

### Load Factors and Load Combinations

Nominal loads that must be considered in design include

$D$  = **dead** load due to the weight of the structural elements and other permanent features supported by the structure, such as permanent partitions.  
 $L$  = **live** load due to occupancy and movable equipment  
 $L_r$  = **live roof** load  
 $W$  = wind load  
 $S$  = snow load  
 $E$  = earthquake load  
 $R$  = initial rainwater load or ice water load exclusive of the ponding contribution

The design strength,  $\phi R_n$ , of each structural element or structural assembly must equal or exceed the design strength based on the following combinations of factored nominal loads from ASCE 7 (2005):

$1.4(D + F)$   
 $1.2(D + F) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$   
 $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W)$   
 $1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R)$   
 $1.2D + 1.0E + L + 0.2S$   
 $0.9D + 1.6W + 1.6 H$   
 $0.9D + 1.0E + 1.6 H$

### Steel Materials

W shapes are preferably in steel grade ASTM A992:  $F_y = 50$  ksi,  $F_u = 65$  ksi,  $E = 30,000$  ksi.  
 ASTM A572 can be specified that has  $F_y = 60$  or  $65$  ksi,  $F_u = 75$  or  $80$  ksi,  $E = 30,000$  ksi.  
 ASTM A36 is available for angles and plates with  $F_y = 36$  ksi,  $F_u = 58$  ksi,  $E = 29,000$  ksi.

### Pure Flexure

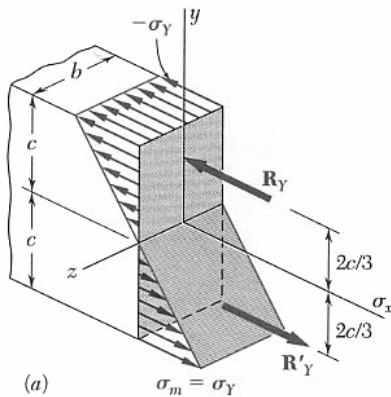
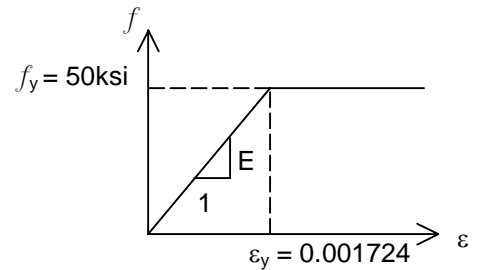
For determining the flexural design strength,  $\phi_b M_n$ , for resistance to pure bending (no axial load) in most flexural members where the following conditions exist, a single calculation will suffice:

$$\sum \gamma_i R_i = M_u \leq \phi_b M_n = 0.9 F_y Z$$

- where  $M_u$  = maximum moment from factored loads
- $\phi_b$  = resistance factor for bending = 0.9
- $M_n$  = nominal moment (ultimate capacity)
- $F_y$  = yield strength of the steel
- $Z$  = plastic section modulus

### Plastic Section Modulus

Plastic behavior is characterized by a yield point and an increase in strain with no increase in stress.



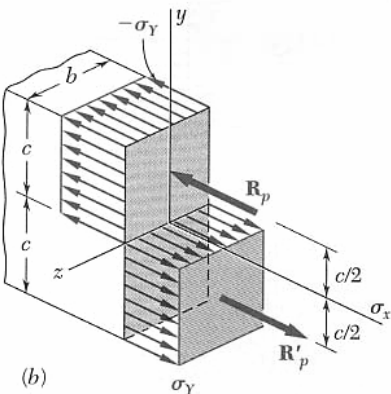
### Internal Moments and Plastic Hinges

Plastic hinges can develop when all of the material in a cross section sees the yield stress. Because all the material at that section can strain without any additional load, the member segments on either side of the hinge can rotate, possibly causing instability.

For a rectangular section:

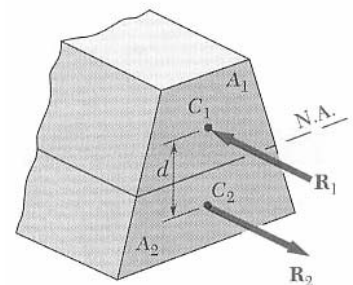
Elastic to  $f_y$ : 
$$M_y = \frac{I}{c} f_y = \frac{bh^2}{6} f_y = \frac{b(2c)^2}{6} f_y = \frac{2bc^2}{3} f_y$$

Fully Plastic: 
$$M_{ult} \text{ or } M_p = bc^2 f_y = \frac{3}{2} M_y$$



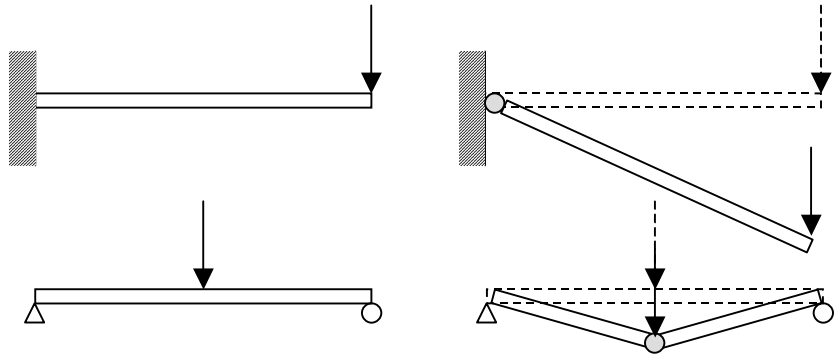
$A_{tension} = A_{compression}$

For a non-rectangular section and internal equilibrium at  $\sigma_y$ , the n.a. will not necessarily be at the centroid. The n.a. occurs where the  $A_{tension} = A_{compression}$ . The reactions occur at the centroids of the tension and compression areas.





Instability from Plastic Hinges



Shape Factor:

The ratio of the plastic moment to the elastic moment at yield:

$$k = \frac{M_p}{M_y}$$

$k = 3/2$  for a rectangle  
 $k \approx 1.1$  for an I beam

Plastic Section Modulus

$$Z = \frac{M_p}{f_y} \quad \text{and} \quad k = \frac{Z}{S}$$

**Shear**

The formulas for the determination of the shear strength on a section are too complex for routine use with the variety of shapes available or possible for steel members. For members that possess an axis of symmetry in the plane of loading, and where web stiffeners are not required, two simplifying assumptions that result in a negligible loss of (theoretical) accuracy are permitted:

1. The contribution of the flanges to shear capacity may be neglected.
2.  $\frac{h}{t_w} \leq \frac{418}{\sqrt{F_y}}$  where  $h$  equals the clear distance between flanges less the fillet or corner radius for rolled shapes.

With these assumptions, the calculated strength becomes simple. Neglecting the flanges, all symmetrical rolled shapes, box shapes, and built-up sections reduce to an equivalent rectangular section with dimensions  $\Sigma t_w d$  and shear strength becomes  $\phi_v V_n$ :

$$\Sigma \gamma_i R_i = V_u \leq \phi_v V_n = 0.9(0.6 F_{yw} A_w)$$

- where
- $V_u$  = maximum shear from factored loads
  - $\phi_v$  = resistance factor for shear = 0.9
  - $V_n$  = nominal shear (ultimate capacity)
  - $F_{yw}$  = yield strength of the steel in the web
  - $A_w = t_w d$  = area of the web

## Design for Flexure

The nominal flexural strength  $M_n$  is the *lowest* value obtained according to the limit states of

1. yielding
2. lateral-torsional buckling
3. flange local buckling
4. web local buckling

For a laterally braced *compact* section (one for which the plastic moment can be reached before local buckling) only the limit state of yielding is applicable. For unbraced compact beams and noncompact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable.

With lateral-torsional buckling the nominal flexural strength is

$$M_n = C_b [\text{constant based on } M\text{'s and } L\text{'s}] \leq M_p \quad (M_u \leq \phi_b M_n)$$

where  $C_b$  is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 2M_A + 4M_B + 3M_C}$$

$M_{\max}$  = absolute value of the maximum moment in the unbraced beam segment

$M_A$  = absolute value of the moment at the quarter point of the unbraced beam segment

$M_B$  = absolute value of the moment at the center point of the unbraced beam segment

$M_C$  = absolute value of the moment at the three quarter point of the unbraced beam segment length.

Beam design charts show  $\phi_b M_n$  for unbraced length ( $L_b$ ) of the compression flange in one-foot increments from 1 to 50 ft. for values of the bending coefficient  $C_b = 1$ . For values of  $1 < C_b \leq 2.3$ , the required flexural strength  $M_u$  can be reduced by dividing it by  $C_b$ .  $L_p$ , the limiting laterally unbraced length for full plastic flexural strength when  $C_b = 1$ , is indicated by a solid dot (●) in the beam design moment charts, while  $L_r$ , the limiting laterally unbraced length for inelastic lateral-torsional buckling, is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used.

**NOTE:** the self weight is not included in determination of  $\phi_b M_n$

Example 1

**EXAMPLE 5.4. W-shape flexural member design (selection using plots of  $\phi_b M_p$  vs.  $L_b$  for strong-axis bending).**

**Given:** Select an ASTM A992 W-shape flexural member ( $F_y = 50$  ksi,  $F_u = 65$  ksi) for a required flexural strength  $M_u$  of 150 kip-ft, a required shear strength  $V_u$  of ~28 kips, and a deflection limit of 1 in. For the deflection calculations, assume the load is a uniformly distributed service load of 2 kips/ft and the length of the simple span is 20 ft. For the strength calculations, assume the beam is braced at the ends and midpoint only ( $L_b = 10$  ft).

**Solution:** From Table 5-5, for an unbraced length  $M_u = 150$  kip-ft and  $L_b = 10$  ft, a W16x31 with  $C_b = 1$  has  $\phi_b M_{nx} \approx 150$  kip-ft. Since  $C_b > 1$ , the actual flexural strength will be higher, so the W16x31 is **o.k.** for flexural design strength.

From Table 5-17, Diagram 1, the maximum deflection  $\Delta_{max}$  occurs at mid-span and can be calculated as:

$$\Delta_{max} = \frac{5wL^4}{384EI} + \Delta_{max-P} = \frac{Pl^3}{48EI} = \frac{3^{kips} (20ft)^3 (12in/ft)^3}{48(30,000ksi)(375in^4)} = 0.077in$$

$$= \frac{5(2^{kips/ft} + 0.031^{kips/ft})(20ft)^4 (12in/ft)^3}{384(30,000ksi)(375in^4)} = 0.650in$$

$$= 0.650 in. + 0.077 in = 0.727 in. < 1. \text{ o.k.}$$

$$\phi_v V_n = 0.9(0.6F_{yw}A_w) = 0.9(0.6)(50ksi)(15.9in)(0.275in) = 118kips > 28kips$$

Thus, the W16x31 flexural member is **o.k.**

**Comments:** Note that end connection limit states, such as block shear rupture and bolt bearing strength must also be checked.

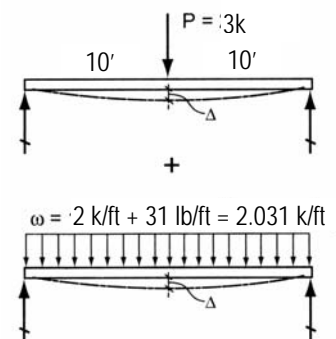
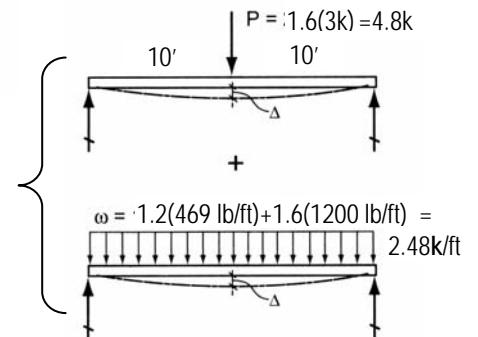
Assume that for the design moment calculation:  
 Dead load = 469 lb/ft  
 Live load = 1200 lb/ft  
 Live point load at midspan = 3 kips

DESIGN LOADS:  
 (before self weight is included as a dead load)

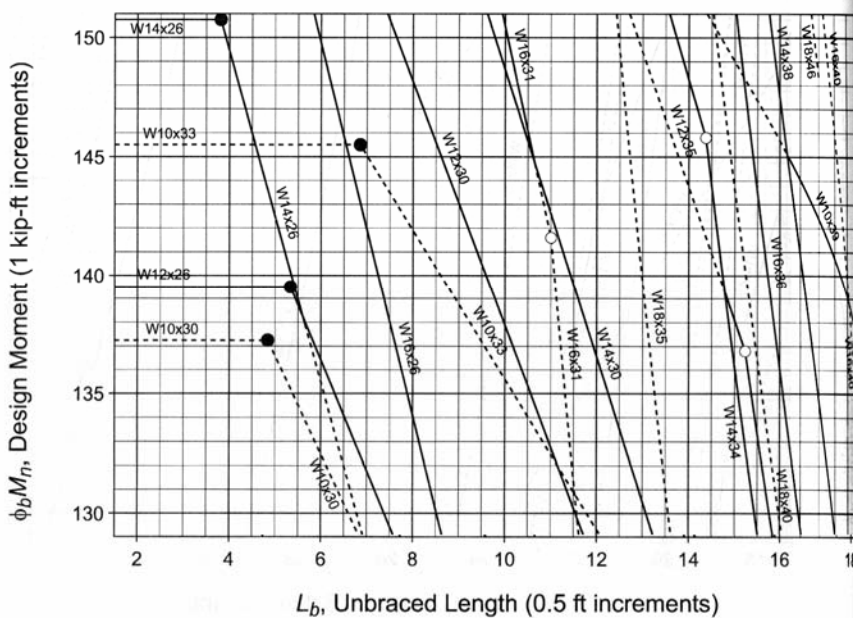
$$M_u = \frac{wl^2}{8} + \frac{Pl}{4} = \frac{2.48k/ft(20ft)^2}{8} + \frac{4.8k(20ft)}{4} = 148k-ft$$

$$V_u = \frac{wl}{2} + \frac{P}{2} = \frac{2.48k/ft(20ft)}{2} + \frac{4.8k}{2} = 27.2k$$

$$(M_{u+self\ weight} = 150\ k\text{-ft}, V_{u+self\ weight} = 27.6\ k)$$



Beam Design Moments ( $\phi_b=0.9, C_b=1.0, F_y=50$  ksi)



## Columns and Stability

### Design Criteria

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be *stable*.

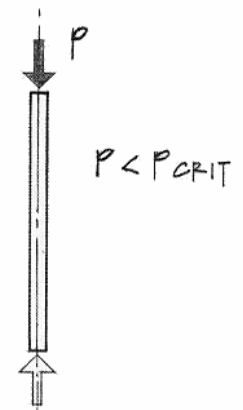
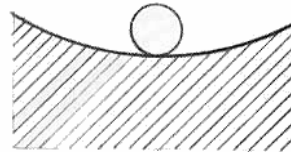
Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

### Physics

Recall that things like to be or *prefer* to be in their lowest energy state (potential energy). Examples include water in a water tank. The energy it took to put the water up there is stored until it is released and can flow due to gravity.

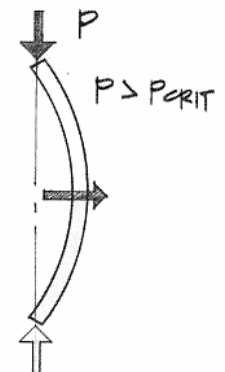
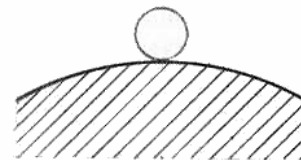
#### Stable Equilibrium

When energy is added to an object in the form of a push or disturbance, the object will return to it's original position. *Things don't change in the end.*



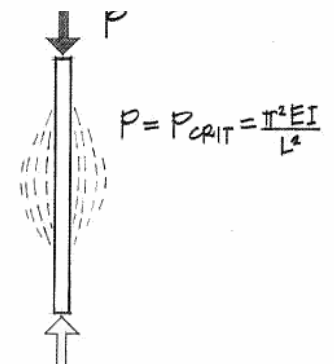
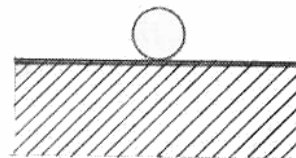
#### Unstable Equilibrium

When energy is added to an object, the object will move and get more "disturbed". *Things change rapidly.*



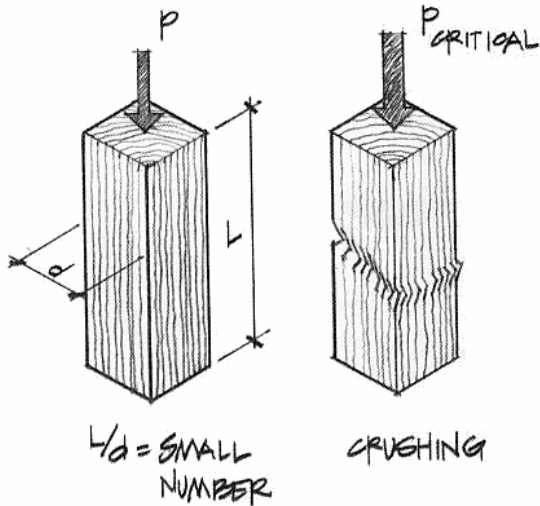
#### Neutral Equilibrium

When energy is added to an object, the object will move some then stop.. *Things change.*



**Column with Axial Loading**

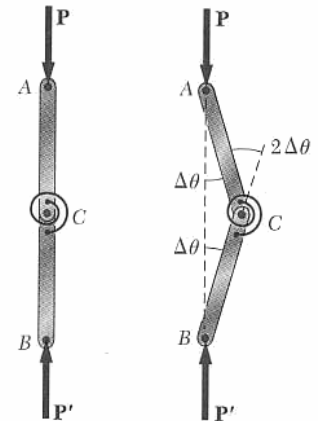
A column loaded centrally can experience unstable equilibrium, called *buckling*, because of how tall and slender they are. This instability is sudden and not good.



Buckling can occur in sheets (like my “memory metal” cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can’t resist the moment in it any longer.

Short (stubby) columns will experience crushing before buckling.



**Critical Buckling Load**

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of  $P \cdot \Delta$ ).

The buckled shape will be in the form of a *sine wave*.

**Euler Formula**

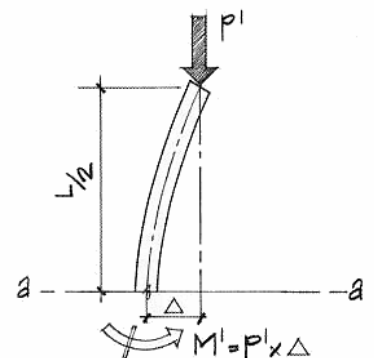
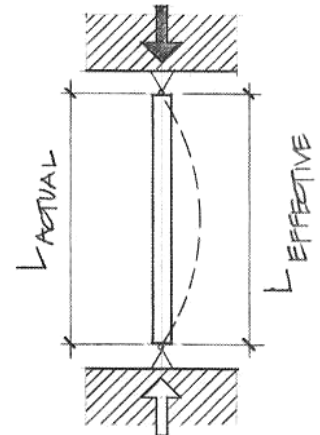
Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

and the critical stress (if less than the normal stress) is:

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EA r^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

where  $I=Ar^2$  and  $L_e/r$  is called the slenderness ratio. The smallest I of the section will govern.



**Yield Stress and Buckling Stress**

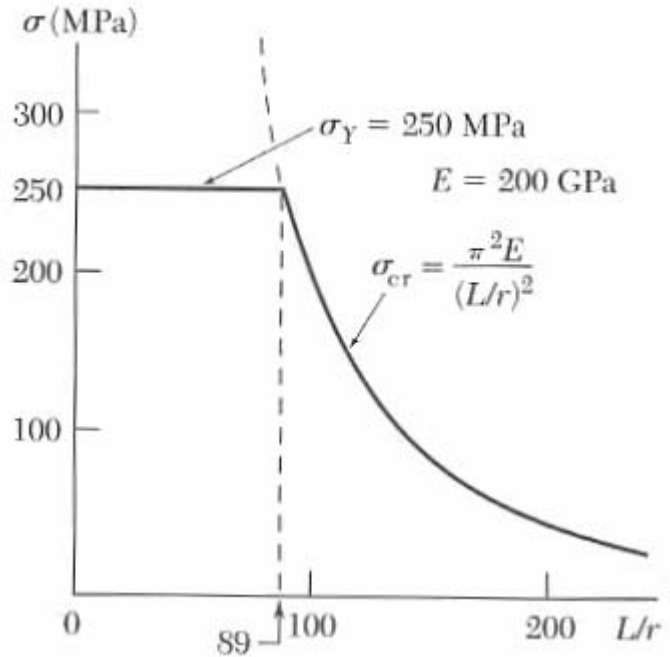
The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler’s formula is used only if the critical stress is less than half of the yield point stress:

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2}; \quad f_{critical} = \frac{P_{critical}}{A} < \frac{F_y}{2}$$

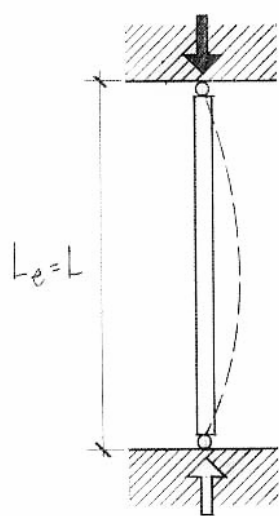
to be used for  $\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$

where  $C_c$  is the column slenderness classification constant and is the slenderness ratio of a column for which the critical stress is equal to half the yield point stress.

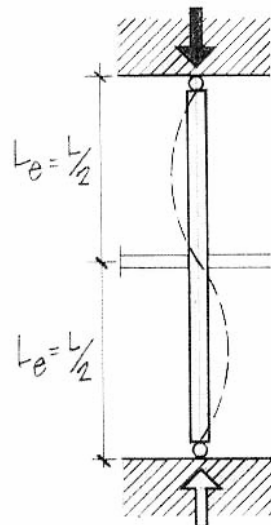


**Effective Length and Bracing**

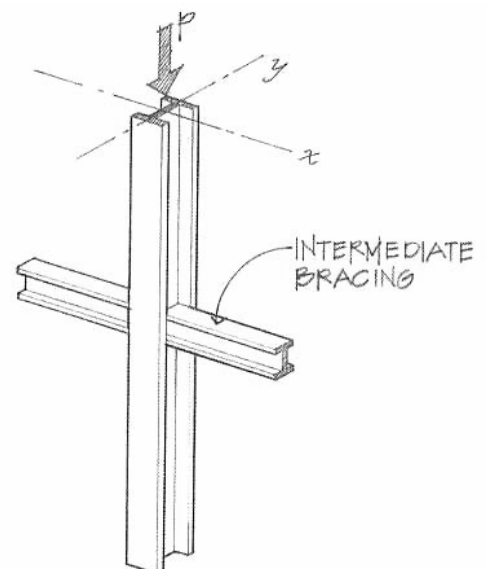
Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K.  $L_e = K \cdot L$

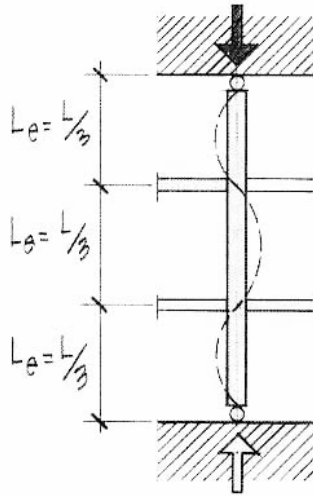


(a) No bracing.

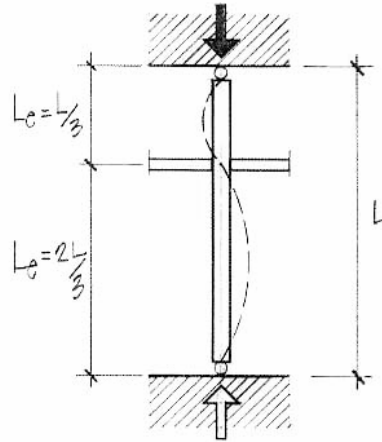


(b) Braced at midpoint.





(c) Third-point bracing.

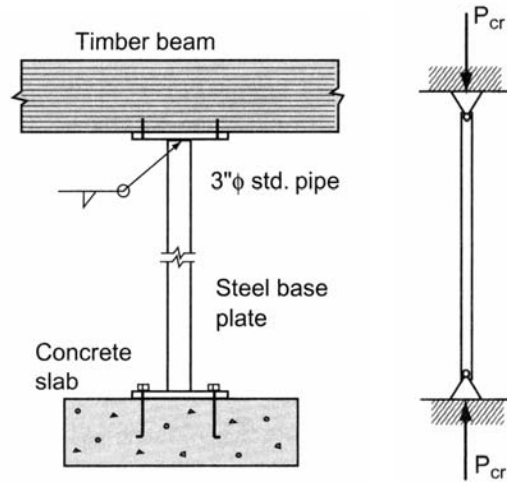


(d) Asymmetric bracing.

Buckled shape of column shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
	Theoretical K value	0.5	0.7	1.0	1.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0
End conditions code	Rotation fixed, Translation fixed Rotation free, Translation fixed Rotation fixed, Translation free Rotation free, Translation free					

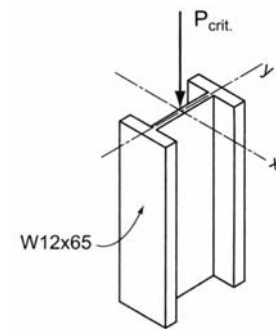
Example 1 (pg 346)**Example Problem 10.1: Short and Long Columns—Modes of Failure (Figures 10.11 and 10.12)**

Determine the critical buckling load for a 3"  $\phi$  standard weight steel pipe column that is 16 ft. tall and pin connected. Assume that  $E = 29 \times 10^6$  psi

Example 2 (pg 346)**Example Problem 10.2 (Figure 10.13)**

Determine the critical buckling stress for a 30-foot-long, W12x65 steel column. Assume simple pin connections at the top and bottom.

$$F_y = 36 \text{ ksi (A36 steel); } E = 29 \times 10^3 \text{ ksi}$$





Example 3 (pg357)

Example Problem 10.8 (Figures 10.33 and 10.34a, b)

Determine the buckling load capacity of a 2x4 stud 12 feet high if blocking is provided at midheight. Assume  $E = 1.2 \times 10^6$  psi.

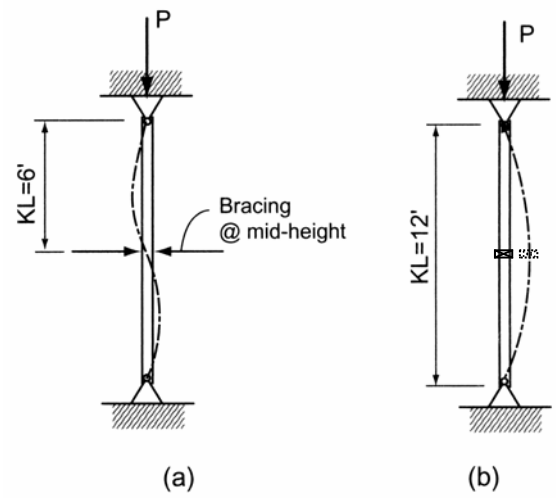
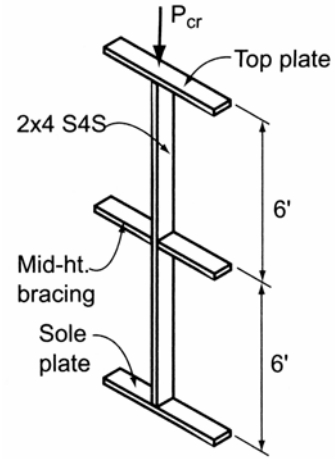


Figure 10.34 (a) Weak axis. (b) Strong axis.

## Steel & Wood Column Design

### Design Aims

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.

### Design Code Methodologies

**Allowable Stress Design (ASD):** the stress in a member must be less than an allowable stress which is equal to the yield stress divided by a factor of safety.

**Load and Resistance Factor Design:** more efficient method that factors loads for importance and compares the summation to a nominal strength that has been adjusted by a reduction factor.

### Allowable Stress Design - Steel

American Institute of Steel Construction (AISC) Manual of ASD, 9<sup>th</sup> ed:

Long and slender: [ $L_c/r \geq C_c$ , preferably < 200]

$$F_{allowable} = \frac{F_{cr}}{F.S.} = \frac{12\pi^2 E}{23(KL/r)^2}$$

The yield limit is idealized into a parabolic curve that blends into the Euler's Formula at  $C_c$ .

With  $F_y = 36$  ksi,  $C_c = 126.1$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

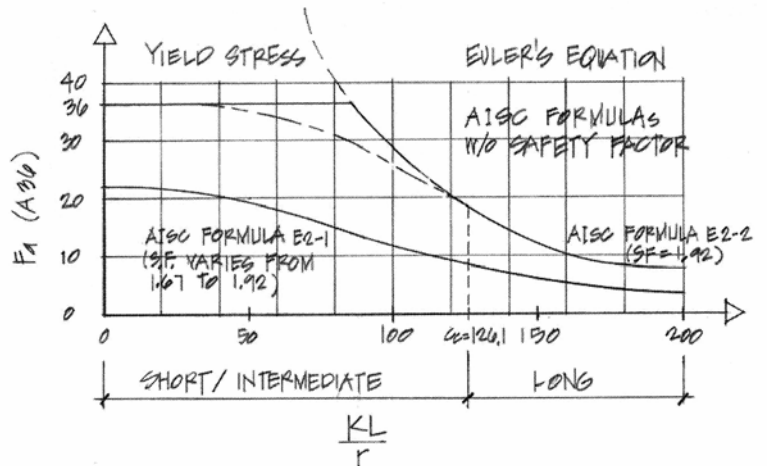
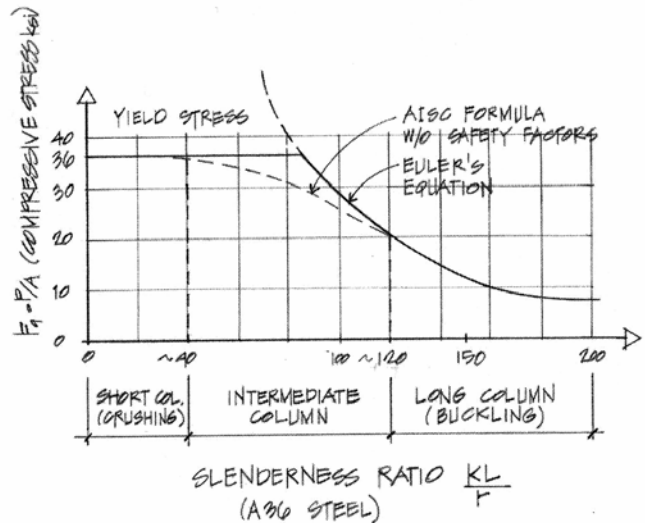
With  $F_y = 50$  ksi,  $C_c = 107.0$

Short and stubby: [ $L_c/r < C_c$ ]

$$F_a = \left[ 1 - \frac{(KL/r)^2}{2C_c^2} \right] \frac{F_y}{F.S.}$$

with:

$$F.S. = \frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3}$$



Procedure for Analysis

1. Calculate  $KL/r$  for each axis (if necessary). The largest will govern the buckling load.
2. Find  $F_a$  as a function of  $KL/r$  from Table 10.1 or 10.2 (pp. 361-364)
3. Compute  $P_{\text{allowable}} = F_a \cdot A$  or alternatively compute  $f_{\text{actual}} = P/A$
4. Is the design satisfactory?

Is  $P \leq P_{\text{allowable}}?$   $\Rightarrow$  yes, it is; no, it is no good

or Is  $f_{\text{actual}} \leq F_a?$   $\Rightarrow$  yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section.
2. Calculate  $KL/r$  for each axis (if necessary). The largest will govern the buckling load.
3. Find  $F_a$  as a function of  $KL/r$  from Table 10.1 or 10.2 (pp. 361-364)
4. Compute  $P_{\text{allowable}} = F_a \cdot A$  or alternatively compute  $f_{\text{actual}} = P/A$
5. Is the design satisfactory?

Is  $P \leq P_{\text{allowable}}?$   $\Rightarrow$  yes, it is; no, pick a bigger section and go back to step 2.

or Is  $f_{\text{actual}} \leq F_a?$   $\Rightarrow$  yes, it is; no, pick a bigger section and go back to step 2.

6. Check design efficiency by calculating percentage of stress used =  $\frac{P_{\text{actual}}}{P_{\text{allowable}}} \cdot 100\%$

If value is between 90-100%, it is efficient.

If values is less than 90%, pick a smaller section and go back to step 2.

The critical load with respect to the slenderness ratio is presented in chart format in ASD, 8<sup>th</sup> ed, as well as the allowable stress charts for compression members.

**Allowable Stress Design - Wood**

National Design Specification for Wood Construction (1992):

Any slenderness ratio,  $L_c/d \leq 50$ :

$$f_c = \frac{P}{A} \leq F'_c$$

$$F'_c = F_c (C_D)(C_M)(C_t)(C_F)(C_p)$$

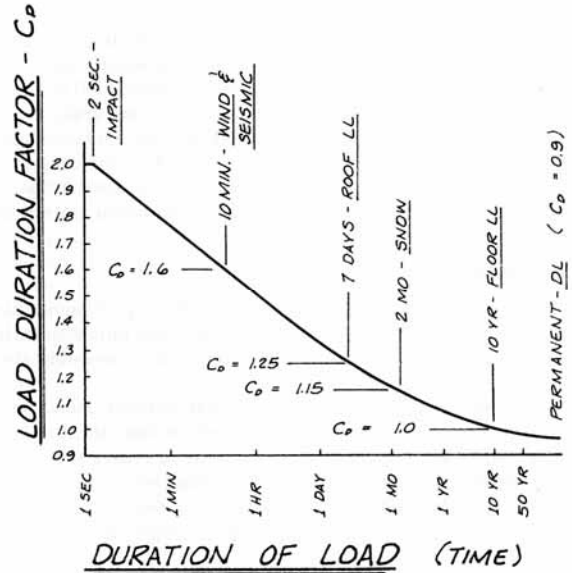
The curve uses factors to replicate the combination curve:

where:

- $F'_c$  = allowable compressive stress parallel to the grain
- $F_c$  = compressive strength parallel to the grain
- $C_D$  = load duration factor
- $C_M$  = wet service factor (1.0 for dry)
- $C_t$  = temperature factor
- $C_F$  = size factor
- $C_p$  = column stability factor off chart

For preliminary column design:

$$F'_c = F_c^* C_p = (F_c C_D) C_p$$



Procedure for Analysis

1. Calculate  $L_e/d_{min}$
2. Obtain  $F'_c$   
 compute  $F_{cE} = \frac{K_{cE} E}{(l_e/d)^2}$  with  $K_{cE} = 0.3$  for sawn, = 0.418 for glu-lam
3. Compute  $F_c^* \cong F_c C_D$  with  $C_D = 1$ , normal,  $C_D = 1.25$  for 7 day roof...
4. Calculate  $F_{cE}/F_c^*$  and get  $C_p$  from Appendix A, Table 14 (pp. 413-414)
5. Calculate  $F'_c = F_c^* C_p$
6. Compute  $P_{allowable} = F'_c \cdot A$  or alternatively compute  $f_{actual} = P/A$
7. Is the design satisfactory?  
 Is  $P \leq P_{allowable}$ ?  $\Rightarrow$  yes, it is; no, it is no good  
 or Is  $f_{actual} \leq F'_c$ ?  $\Rightarrow$  yes, it is; no, it is no good

Procedure for Design

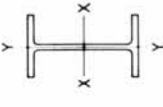
1. Guess a size by picking a section
2. Calculate  $L_e/d_{min}$
3. Obtain  $F'_c$   
 compute  $F_{cE} = \frac{K_{cE} E}{(l_e/d)^2}$  with  $K_{cE} = 0.3$  for sawn, = 0.418 for glu-lam
4. Compute  $F_c^* \cong F_c C_D$  with  $C_D = 1$ , normal,  $C_D = 1.25$  for 7 day roof...
5. Calculate  $F_{cE}/F_c^*$  and get  $C_p$  from Appendix A, Table 14 (pp. 413-414)



Sample AISC Table for Allowable Axial Loads (ASD)

**COLUMNS**  
W shapes  
Allowable axial loads in kips

$F_y = 36 \text{ ksi}$   
 $F_y = 50 \text{ ksi}$



Designation	WT12											
	106		96		87		79		72		65	
Wt./ft.	36	50	36	50	36	50	36	50	36	50	36	50
$F_y$	674	936	609	846	553	768	501	696	456	633	413	573
0	637	872	575	788	522	715	473	647	430	589	389	533
6	7	629	858	568	775	515	703	467	424	579	384	524
8	8	620	844	560	762	508	691	460	418	569	378	514
9	9	611	828	552	748	501	678	453	414	558	373	504
10	10	602	812	544	733	493	665	446	405	547	367	494
11	11	593	795	535	718	485	650	439	399	535	361	483
12	12	583	777	526	701	477	636	431	392	522	354	472
13	13	572	759	516	685	468	620	423	385	509	348	460
14	14	561	740	506	667	459	604	415	377	496	341	448
15	15	550	720	496	649	450	588	407	369	482	334	435
16	16	539	699	486	630	440	570	398	361	468	326	422
17	17	527	678	475	611	430	553	389	353	453	319	408
18	18	514	656	464	591	420	534	379	344	438	311	394
19	19	502	634	452	570	409	515	370	336	422	303	380
20	20	489	611	440	549	398	496	360	326	406	294	365
22	22	462	562	416	505	376	455	339	310	372	277	334
24	24	433	511	390	458	352	412	317	288	336	259	301
26	26	404	457	362	408	327	367	294	267	297	240	266
28	28	372	399	334	356	301	319	270	245	258	220	230
30	30	340	348	304	310	273	278	245	222	225	199	201
32	32	305	306	272	273	244	244	219	197	197	176	176
34	34	271	271	242	242	216	216	194	175	175	156	156
36	36	241	241	215	215	193	193	173	156	156	139	139
38	38	217	217	193	193	173	173	155	140	140	125	125
40	40	196	196	175	175	156	156	140	126	126	113	113

Effective length in ft. KL with respect to least radius of gyration  $r_y$

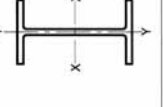
Properties	WT12											
	106		96		87		79		72		65	
$P_n$ (kips)	259	257	161	223	139	193	122	169	106	148	92	128
$P_n$ (kips/in.)	22	31	20	28	19	26	17	24	15	22	14	20
$P_n$ (kips)	588	693	431	508	354	417	269	317	206	243	154	181
$P_n$ (kips)	221	306	182	253	148	205	122	169	101	140	82	114
$L_p$ (ft)	12.9	10.9	12.8	10.9	12.8	10.9	12.8	10.8	12.7	10.8	12.7	10.7
$L_r$ (ft)	43.3	31.2	39.9	28.7	36.2	26.0	33.3	24.0	30.5	21.9	27.7	20.0
$A_g$ (in <sup>2</sup> )	31.2	28.2	25.6	25.6	23.2	21.1	21.1	19.1	17.0	15.6	14.6	13.1
$I_x$ (in <sup>4</sup> )	933	833	740	662	597	533	475	425	391	348	307	266
$I_y$ (in <sup>4</sup> )	301	270	241	216	195	174	156	140	126	113	100	86.6
$r_x$ (in.)	3.11	3.09	3.07	3.05	3.04	3.03	3.02	3.01	2.99	2.98	2.97	2.96
$r_y$ (in.)	1.76	1.76	1.76	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
Ratio $r_x/r_y$	0.215	0.215	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217	0.217
$P_n/(KL)^2 \cdot 10^4$	0.633	0.635	0.645	0.648	0.651	0.651	0.651	0.651	0.651	0.651	0.651	0.651
$P_n/(KL)^2 \cdot 10^4$	139.1	124.3	110.4	98.6	88.6	79.3	71.9	65.0	59.1	53.9	49.1	44.6
$P_n/(KL)^2 \cdot 10^4$	45.0	40.1	36.0	32.2	29.1	26.0	23.2	20.6	18.1	15.6	13.1	11.3
$P_n/(KL)^2 \cdot 10^4$	310	307	300	296	292	289	289	289	289	289	289	289
$P_n/(KL)^2 \cdot 10^4$	100	99.0	97.7	96.5	95.8	95.8	95.8	95.8	95.8	95.8	95.8	95.8

\*Flange is noncompact; see discussion preceding column load tables.

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Sample AISC Table for LRFD Design Strength in Compression

**Table 4-2 (cont.).**  
W-Shapes  
Design Strength in Axial Compression,  $\phi_c P_n$ , kips



$F_y = 50 \text{ ksi}$   
 $\phi_c P_n = 0.85 F_c A_g$

Shape	WT12											
	106		96		87		79		72		65	
Wt./ft.	1330	1200	1090	986	897	812	723	663	621	557	497	
0	1280	1150	1050	947	861	779	690	623	562	504	450	
6	7	1260	1140	1030	933	848	767	696	610	543	486	
8	8	1240	1120	1010	917	834	754	684	594	521	466	
9	9	1210	1100	994	900	818	739	671	577	504	449	
10	10	1190	1070	973	880	800	723	657	561	488	433	
11	11	1160	1050	950	860	781	706	645	545	472	417	
12	12	1130	1020	926	838	761	687	628	524	451	396	
13	13	1100	995	901	814	740	668	611	504	431	376	
14	14	1070	966	874	790	717	647	591	480	407	352	
15	15	1040	935	846	764	694	626	571	456	383	328	
16	16	1000	904	817	738	670	604	551	432	358	303	
17	17	968	871	788	711	645	581	529	406	332	277	
18	18	932	838	758	683	620	558	507	380	306	251	
19	19	895	805	727	655	594	535	484	353	279	224	
20	20	858	771	696	627	568	512	462	327	253	198	
22	22	783	703	634	570	517	464	422	289	215	160	
24	24	708	635	572	514	465	417	379	247	173	118	
26	26	635	569	511	459	415	372	335	210	136	81	
28	28	565	505	453	406	367	328	292	181	106	51	
30	30	497	443	397	355	321	287	254	158	92.3	22.5	
32	32	437	390	349	312	282	252	222	139	81.2	16.7	
34	34	387	345	309	277	250	223	197	123	72.1	11.7	
36	36	345	308	276	247	223	199	172	110	63.7	7.5	
38	38	310	276	248	221	200	179	156	110	56.4	6.5	
40	40	279	249	223	200	181	161	141	99.2	48.9	3.8	

Effective length in ft. KL with respect to least radius of gyration  $r_y$

Properties	WT12											
	106		96		87		79		72		65	
$P_n$ (kips)	242	206	182	156	137	117	101	90.5	80.5	75.2		
$P_n$ (kips/in.)	30.5	27.5	25.8	23.5	21.5	19.5	17.3	16.5	16.8	14.8		
$P_n$ (kips)	609	445	365	278	213	159	125	110	133	98.6		
$P_n$ (kips)	276	228	185	152	126	103	115	93.0	115	93.0		
$L_p$ (ft)	11.0	10.9	10.8	10.8	10.7	11.9	8.87	8.76	6.92	6.89		
$L_r$ (ft)	44.9	41.4	38.4	35.7	33.6	31.7	27.0	25.6	21.5	20.3		
$A_g$ (in <sup>2</sup> )	31.2	28.2	25.6	23.2	21.1	19.1	17.0	15.6	14.6	13.1		
$I_x$ (in <sup>4</sup> )	933	833	740	662	597	533	475	425	391	348		
$I_y$ (in <sup>4</sup> )	301	270	241	216	195	174	156	140	126	113		
$r_x$ (in.)	3.11	3.09	3.07	3.05	3.04	3.03	3.02	3.01	2.99	2.98		
$r_y$ (in.)	1.76	1.76	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75		
Ratio $r_x/r_y$	1.76	1.76	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75		
$P_n/(KL)^2 \cdot 10^4$	25700	23800	21200	18900	17100	15300	13600	12200	11200	9960		
$P_n/(KL)^2 \cdot 10^4$	8620	7730	6900	6180	5580	4980	4360	3740	3120	2500		
$P_n/(KL)^2 \cdot 10^4$	100	99.0	97.7	96.5	95.8	95.8	95.8	95.8	95.8	95.8		

\*Flange is noncompact. Note: Heavy line indicates  $K/r$  equal to or greater than 200.

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Example 1 (pg 367)

**Example Problem 10.10 (Figure 10.41)**

A 24-ft.-tall, A572 grade 50, steel column (W14×82) with an  $F_y = 50$  ksi has pins at both ends. Its weak axis is braced at midheight, but the column is free to buckle the full 24 ft. in the strong direction. Determine the safe load capacity for this column.

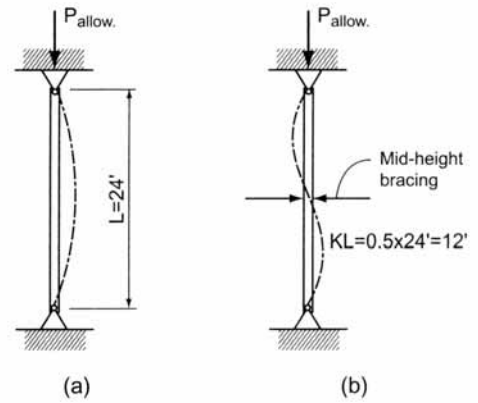


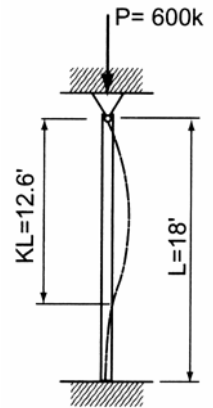
Figure 10.41 (a) Strong axis buckling.  
(b) Weak axis buckling.

Example 2 (pg 371) + chart method

**Example Problem 10.14: Design of Steel Columns (Figure 10.48)**

Select the most economical W12 × column 18' in height to support an axial load of 600 kips using A572 grade 50 steel. Assume that the column is hinged at the top but fixed at the base.

ALSO: Select the column using the ASD design charts, and the LRFD charts assuming that the load is a dead load (factor of 1.4)



Example 3 (pg 379)

**Example Problem 10.18 (Figures 10.60 and 10.61)**

An 18' tall 6x8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

$$F_c = 975 \text{ psi}, E = 1.6 \times 10^6 \text{ psi}$$

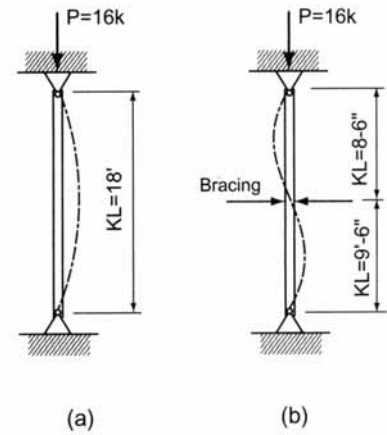
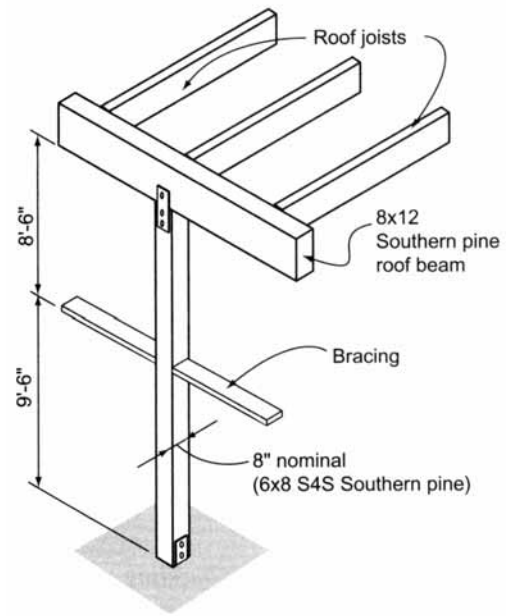


Figure 10.61 (a) Strong axis. (b) Weak axis.



Example 4 (pg 381)**Example Problem 10.20:  
Design of Wood Columns(Figure 10.66)**

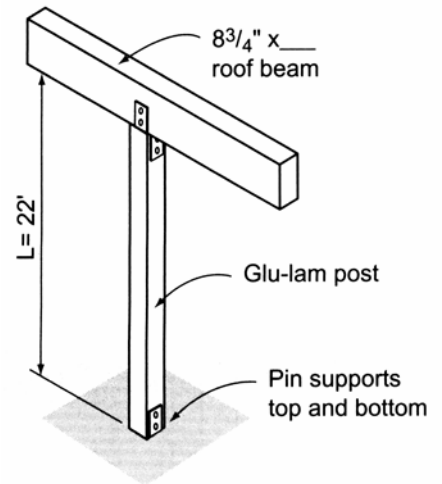
A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming  $8\frac{3}{4}$ " in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

$$8\frac{3}{4}" \times 9" (A = 78.75 \text{ in.}^2)$$

$$8\frac{3}{4}" \times 10\frac{1}{2}" (A = 91.88 \text{ in.}^2)$$

$$8\frac{3}{4}" \times 12" (A = 105.00 \text{ in.}^2)$$

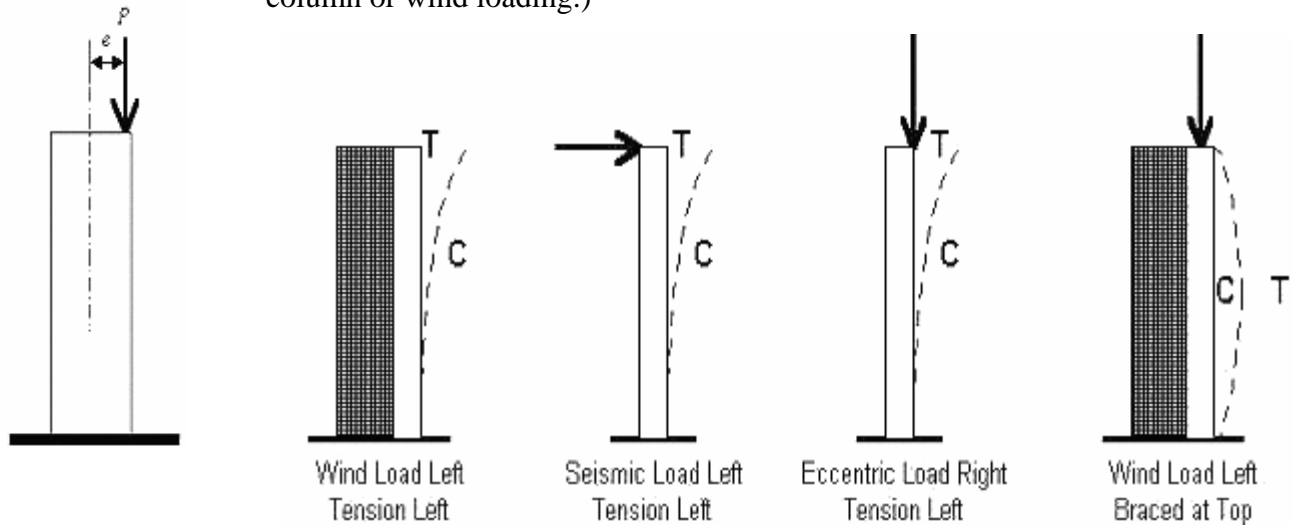


## Column Design – Centric & Eccentric Loading

### Loading Location

*Centric loading:* The load is applied at the centroid of the cross section. The limiting allowable stress is determined from strength ( $P/A$ ) or buckling.

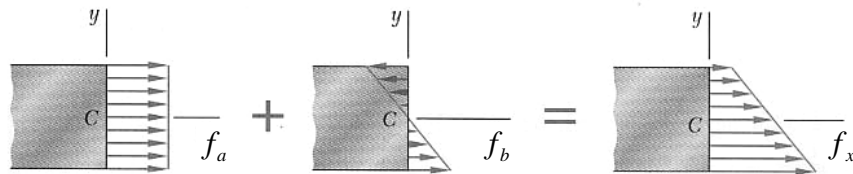
*Eccentric loading:* The load is offset from the centroid of the cross section because of how the beam load comes into the column. This offset introduces bending along with axial stress. (This can also happen with continuous beams across a column or wind loading.)



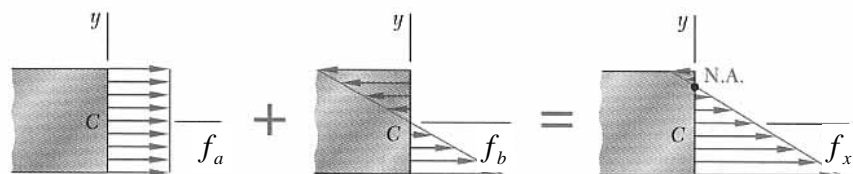
### Eccentric Loading

The eccentricity causes bending stresses by a moment of value  $P \times e$ . Within the elastic range (linear stresses) we can *superposition* or add up the normal and bending stresses:

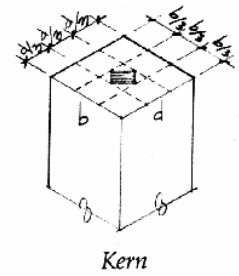
$$f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}$$



The resulting stress distribution is still *linear*. And the n.a. moves (if there is one).



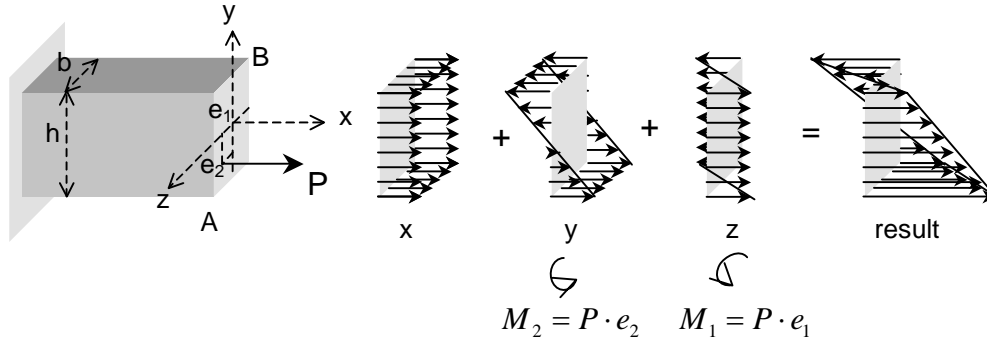
The value of  $e$  (or location of  $P$ ) that causes the stress at an edge to become zero is at the edge of the **kern**. As long as  $P$  stays within the kern, there will *not* be any tension stress.



If there is bending in two directions (**bi-axial bending**), there will be one more bending stress added to the total:

$$f_x = f_a + f_{bx} + f_{by} = \frac{P}{A} + \frac{M_1 y}{I_z} + \frac{M_2 z}{I_y}$$

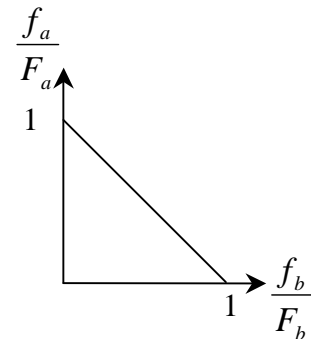
With  $P$ ,  $M_1$ , and  $M_2$ :



### Eccentric Loading Design

Because there are combined stresses, we can't just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the **interaction diagram**. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress = 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).

The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These type of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the **P-Δ (P-delta)** effect.



### Limit Criteria Methods

- 1)  $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$  interaction formula (bending in one direction)
- 2)  $\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$  interaction formula (biaxial bending)
- 3)  $\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_b} \leq 1.0$  interaction formula (P-Δ effect)

Steel - ASD:

The modification factors are included in the form: 
$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

where:

$1 - \frac{f_a}{F'_e}$  = magnification factor accounting for P-Δ

$F'_e = \frac{12\pi^2 E}{23(KL/r)^2}$  = allowable buckling stress

$C_m$  = modification factor accounting for end conditions,  
 =  $0.6 - 0.4 (M_1/M_2)$  where  $M_1$  and  $M_2$  are the end moments and  $M_1 < M_2$ .  
 $M_1/M_2$  is positive when the member is bent in reverse curvature, negative when bent in single curvature  
 = 0.85, members in frames subject to joint translation (sidesway)

Wood: - NDS:

The modification factors are included in the form: 
$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx} \left[1 - f_c / F_{cEx}\right]} \leq 1.0$$

where:

$1 - \frac{f_c}{F_{cEx}}$  = magnification factor accounting for P-Δ

$F'_{bx}$  = allowable bending stress

Steel – LRFD:

The modification factors are included in the form for two conditions.

$$\text{For } \frac{P_u}{\phi_c P_n} \geq 0.2: \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

$$\text{For } \frac{P_u}{\phi_c P_n} < 0.2: \quad \frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

where:

$\phi_c = 0.85$  for compression

$\phi_b = 0.90$  for flexure

## Column Design Methodology

In order to design an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
  - buckling
  - axial stress
  - combined stress
2. See if we can find values for  $r$  or  $A$  or  $S$  ( $=I/c_{\max}$ )
3. Pick a trial section based on if we think  $r$  or  $A$  is going to govern the section size.
4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.
5. Did the section pass the stress test?
  - If not, do you *increase*  $r$  or  $A$  or  $S$ ?
  - If so, is the difference really big so that you could *decrease*  $r$  or  $A$  or  $S$  to make it more efficient (economical)?
6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

### Example 1

#### Example 12.10.1

Investigate the acceptability of a W16×67 used as a column under the loading shown in Fig. 12.10.1. Steel is A992.

$$F_b = 27.8 \text{ ksi.}$$

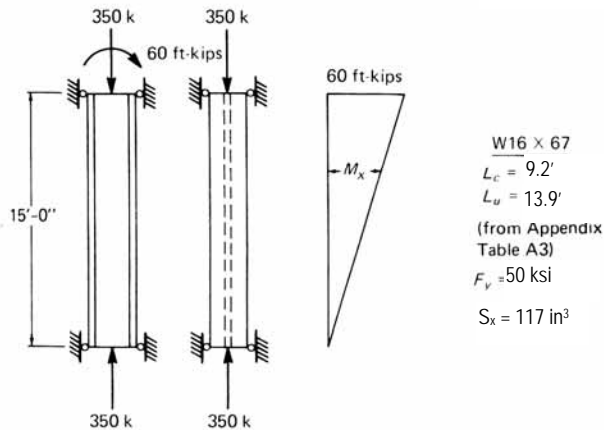


Fig. 12.10.1 Beam-column for Example 12.10.1.

#### SOLUTION

(a) Column effect.

$$\frac{KL}{r_y} = \frac{15(12)}{2.46} = 73$$

$$F_a = 20.38 \text{ ksi}$$

$$f_a = \frac{P}{A} = \frac{350}{19.7} = 17.8 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{17.8}{20.38} = 0.873$$

(b) bending

$$C_m = 0.6 - 0.4(M_1/M_2) = 0.60$$

$$f_b = \frac{60(12)}{117} = 6.15 \text{ ksi}$$

$$\frac{C_m f_b}{F_b} = \frac{0.6(6.15)}{27.8} = 0.13$$

(c) Moment magnification.

$$\frac{KL}{r_x} = \frac{15(12)}{6.96} = 25.9; \quad F'_e = \frac{12\pi^2(30000)}{23(25.9)^2} = 230 \text{ ksi}$$

where the  $x$ -axis is the axis of bending.

$$\frac{1}{1 - f_a/F'_e} = \frac{1.0}{1 - 17.8/230} = 1.084$$

(d) Check of AISC Formulas:

For stability, Formula (1.6-1a),

$$\frac{f_a}{F_a} + \frac{C_m f_b}{F_b} \left( \frac{1.0}{1 - f_a/F'_e} \right) =$$

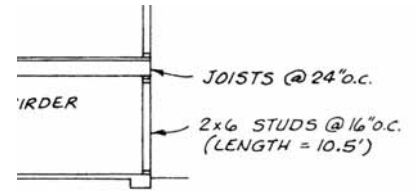
$$0.873 + 0.13(1.084) = 1.01 \approx 1.0$$

The W16×67 is acceptable for the given loading.

Example 2

**EXAMPLE 7.16 Combined Bending and Compression in a Stud Wall**

Check the 2 × 6 stud in the first-floor bearing wall in the building shown in Fig. 7.20a. Consider the given vertical loads and lateral forces. Lumber is No. 2 DF-L.  $MC \leq 19$  percent and normal temperatures apply. Allowable stresses are to be in accordance with the NDS.  $F'_b = 2152$  psi  $F_c = 1350$  psi



$$A = 8.25 \text{ in}^2$$

$$S_x^* = 7.56 \text{ in}^3$$

COLUMN CAPACITY:

Sheathing provides lateral support about the weak axis of the stud. Therefore, check column buckling about the  $x$  axis only ( $L = 10.5$  ft and  $d_x = 5.5$  in.):

$$\left(\frac{l_e}{d}\right)_y = 0 \quad \text{because of sheathing}$$

$$\left(\frac{l_e}{d}\right)_{\max} = \left(\frac{l_e}{d}\right)_x = \frac{10.5 \text{ ft} \times 12 \text{ in./ft}}{5.5 \text{ in.}} = 22.9$$

$$E = 1,600,000 \text{ psi}$$

For visually graded sawn lumber:

$$K_{cE} = 0.3$$

$$c = 0.8$$

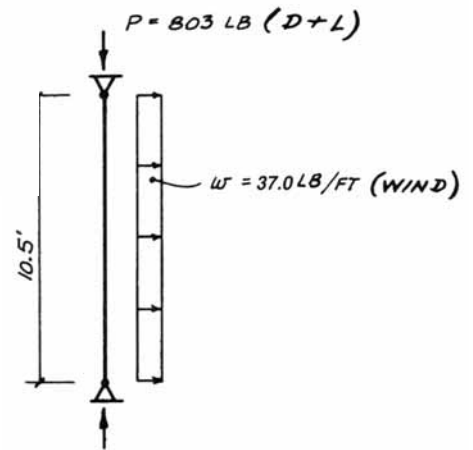
$$F_{cE} = \frac{K_{cE} E'}{(l_e/d)^2} = \frac{0.3(1,600,000)}{(22.9)^2} = 915 \text{ psi}$$

$$F_c^* = F_c(C_D) \quad C_D = 1.6 \text{ from wind loading}$$

$$= 1350(1.6) = 2376 \text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{915}{2376} = 0.385 \quad C_p = 0.36$$

$$F'_c = F_c(C_D)(C_p) = 2376(0.36) = 855 \text{ psi}$$



**Load Case 2: Gravity Loads + Lateral Forces**

BENDING:

Wind governs over seismic. Force to one stud:

$$\text{Wind} = 27.8 \text{ psf}$$

$$w = 27.8 \text{ psf} \times \frac{16 \text{ in}}{12 \text{ in/ft}} = 37.0 \text{ lb/ft}$$

$$M = \frac{wL^2}{8} = \frac{37.0(10.5)^2}{8} = 510 \text{ ft-lb} = 6115 \text{ in.-lb}$$

AXIAL:

$$f_b = \frac{M}{S} = \frac{6115}{7.56} = 809 \text{ psi} \quad F'_b = 2152 \text{ psi}$$

$$D + W: f_c = \frac{P}{A} = \frac{378}{8.25} = 46 \text{ psi}$$

COMBINED STRESS:

The simplified interaction formula from Example 7.13 (Sec. 7.12) applies:

$$\left(\frac{f'_c}{F'_c}\right)^2 + \frac{f_{bx}}{F'_{bx}(1 - f'_c/F_{cEx})} \leq 1.0$$

$$F_{cEx} = F_{cE} = 915 \text{ psi}$$

D + W:

In this load combination, D produces the axial stress  $f_c$  and W results in the bending stress  $f_{bx}$ .

$$\left(\frac{f'_c}{F'_c}\right)^2 + \left(\frac{1}{1 - f'_c/F_{cEx}}\right) \frac{f_{bx}}{F'_{bx}} =$$

$$\left(\frac{46}{855}\right)^2 + \left(\frac{1}{1 - 46/915}\right) \frac{809}{2152} = 0.399 < 1.0$$

2 × 6 No. 2 DF-L exterior bearing wall OK

## Connections and Tension Member Design

### Connections

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Steel construction accomplishes this with bolt and welds. Wood construction uses nails, bolts, shear plates, and split-ring connectors.

#### Bolted and Welded Connections

The limit state for connections depends on the loads:

1. tension yielding
2. shear yielding
3. bearing yielding
4. bending yielding due to eccentric loads
5. rupture

Welds must resist tension AND shear stress. The design strengths depend on the weld materials.

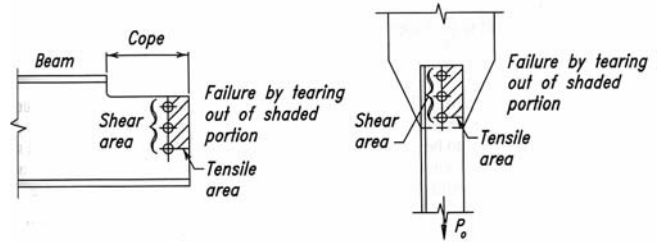


Fig. C-J4.1. Failure for block shear rupture limit state.

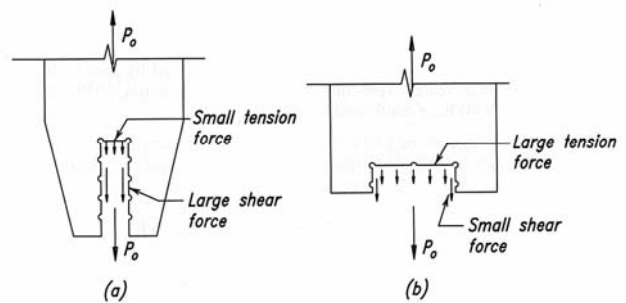


Fig. C-J4.2. Block shear rupture in tension.

### Bolted Connection Design

Bolt designations signify material and type of connection where

SC: slip critical

N: bearing-type connection with bolt threads *included* in shear plane

X: bearing-type connection with bolt threads *excluded* from shear plane

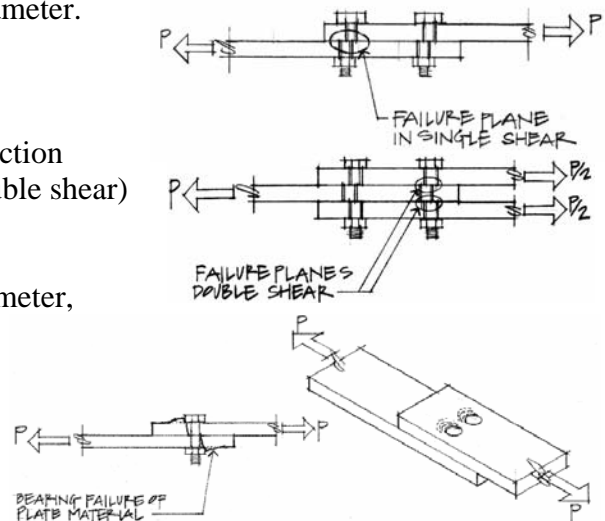
Bolts rarely fail in **bearing**. The material with the hole will more likely yield first.

Standard bolt holes are 1/16" larger than the bolt diameter.

#### ASD

Allowable shear values are given by bolt type, connection type, hole type, diameter, and loading (Single or Double shear) in AISC manual tables.

Allowable bearing force values are given by bolt diameter, ultimate tensile strength,  $F_u$ , of the connected part, and thickness of the connected part in AISC manual tables.



**BOLTS AND THREADED PARTS**  
Bearing  
Allowable loads in kips

**TABLE I-E. BEARING**  
Slip-critical and Bearing-type Connections

Material Thickness	$F_u = 58$ ksi Bolt dia.		$F_u = 65$ ksi Bolt dia.		$F_u = 70$ ksi Bolt dia.		$F_u = 100$ ksi Bolt dia.	
	3/4	7/8	3/4	7/8	3/4	7/8	3/4	7/8
1/8	6.5	7.6	7.3	8.5	9.8	7.9	9.2	10.5
3/16	9.8	11.4	11.0	12.8	14.6	11.8	13.8	15.8
1/4	13.1	15.2	14.6	17.1	19.5	15.8	18.4	21.0
5/16	16.3	19.0	18.3	21.3	24.4	19.7	23.0	26.3
3/8	19.6	22.8	21.9	25.6	29.3	23.6	27.6	31.5
7/16	22.8	26.6	30.5	25.6	29.9	34.1	27.6	32.2
1/2	26.1	30.5	34.8	29.3	34.1	39.0	31.5	36.8
5/8	29.4	34.3	39.2	32.9	38.4	43.9	41.3	47.3
3/4	32.6	38.1	43.5	42.7	48.8	45.9	52.5	57.8
7/8	41.9	47.9	46.9	53.6	58.5	60.9	69.6	84.0
1	52.2	60.9	58.5	68.3	78.0	84.0	90.0	105.0

**Notes:**  
This table is applicable to all mechanical fasteners in both slip-critical and bearing-type connections utilizing standard holes. Standard holes shall have a diameter nominally 1/16-in. larger than the nominal bolt diameter ( $d + 1/16$  in.).  
Tabulated bearing values are based on  $F_u = 1.2 F_u$ .  
 $F_u$  = specified minimum tensile strength of the connected part.  
In connections transmitting axial force whose length between extreme fasteners measured parallel to the line of force exceeds 50 in., tabulated values shall be reduced 20%.  
Connections using high-strength bolts in slotted holes with the load applied in a direction other than approximately normal (between 80 and 100 degrees) to the axis of the hole and connections with bolts in oversize holes shall be designed for resistance against slip at work load in accordance with AISC ASD Specification Sect. J3.8.  
Tabulated values apply when the distance  $l$  parallel to the line of force from the center of the bolt to the edge of the connected part is not less than  $1 1/2 d$  and the distance from the center of a bolt to the center of an adjacent bolt is not less than  $3d$ . See AISC ASD Commentary J3.8.  
Under certain conditions, values greater than the tabulated values may be justified under Specification Sect. J3.7.  
Values are limited to the double-shear bearing capacity of A490-X bolts.  
Values for decimal thicknesses may be obtained by multiplying the decimal value of the unlisted thickness by the value given for a 1-in. thickness.

**BOLTS, THREADED PARTS AND RIVETS**  
Shear  
Allowable load in kips

**TABLE I-D. SHEAR**

ASTM Designation	Connection Type*	Hole Type <sup>b</sup>	$F_u$ ksi	Load <sup>c</sup> in <sup>2</sup>	Nominal Diameter $d$ , in.								
					%	3/4	1	1 1/4	1 1/2				
A307	—	STD	10.0	S	3.068	.4418	.6013	.7854	.9940	1.227	1.485	1.767	
		NSL	10.0	D	3.1	4.4	6.0	7.9	9.9	12.3	14.8	17.7	
	SC <sup>a</sup> Class A	STD	17.0	S	5.22	7.51	10.2	13.4	16.9	20.9	25.2	30.0	
		NSL	17.0	D	10.4	15.0	20.4	26.7	33.8	41.7	50.5	60.1	
	A325	—	OVS, SSL	15.0	S	4.60	6.63	9.02	11.8	14.9	18.4	22.3	26.5
			LSL	15.0	D	9.20	13.3	18.0	23.6	29.8	36.8	44.6	53.0
		N	STD	12.0	S	3.68	5.30	7.22	9.42	11.9	14.7	17.8	21.2
			NSL	12.0	D	7.36	10.6	14.4	18.8	23.9	29.4	35.6	42.4
		X	STD	21.0	S	6.4	9.3	12.6	16.5	20.9	25.8	31.2	37.1
			NSL	21.0	D	12.9	18.6	25.3	33.0	41.7	51.5	62.4	74.2
A490	—	STD	21.0	S	6.44	9.28	12.6	16.5	20.9	25.8	31.2	37.1	
		NSL	21.0	D	12.9	18.6	25.3	33.0	41.7	51.5	62.4	74.2	
	SC <sup>a</sup> Class A	OVS, SSL	18.0	S	5.52	7.95	10.8	14.1	17.9	22.1	26.7	31.8	
		LSL	18.0	D	11.0	15.9	21.6	28.3	35.8	44.2	53.5	63.6	
	N	STD	15.0	S	4.60	6.63	9.02	11.8	14.9	18.4	22.3	26.5	
		NSL	15.0	D	9.20	13.3	18.0	23.6	29.8	36.8	44.6	53.0	
	X	STD	28.0	S	8.6	12.4	16.8	22.0	27.8	34.4	41.6	49.5	
		NSL	28.0	D	17.2	24.7	33.7	44.0	55.7	68.7	83.2	99.0	
	A502-1	—	STD	40.0	S	12.3	17.7	24.1	31.4	39.8	49.1	59.4	70.7
			NSL	40.0	D	24.5	35.3	48.1	62.8	79.5	98.2	119.0	141.0
A502-2	—	STD	17.5	S	5.4	7.7	10.5	13.7	17.4	21.5	26.0	30.9	
		NSL	17.5	D	10.7	15.5	21.0	27.5	34.8	42.9	52.0	61.8	
A502-3	—	STD	22.0	S	6.7	9.7	13.2	17.3	21.9	27.0	32.7	38.9	
		NSL	22.0	D	13.5	19.4	26.5	34.6	43.7	54.0	65.3	77.7	
A36 ( $F_u = 58$ ksi)	N	STD	9.9	S	3.0	4.4	6.0	7.8	9.8	12.1	14.7	17.5	
		NSL	9.9	D	6.1	8.7	11.9	15.6	19.7	24.3	29.4	35.0	
A572, Gr. 50 ( $F_u = 65$ ksi)	N	STD	12.8	S	3.9	5.7	7.7	10.1	12.7	15.7	19.0	22.6	
		NSL	12.8	D	7.9	11.3	15.4	20.1	25.4	31.4	38.0	45.6	
A572, Gr. 50 ( $F_u = 65$ ksi)	X	STD	11.1	S	3.4	4.9	6.7	8.7	11.0	13.6	16.5	19.6	
		NSL	11.1	D	6.8	9.8	13.3	17.4	22.1	27.2	33.0	39.2	
A588 ( $F_u = 70$ ksi)	N	STD	14.3	S	4.4	6.3	8.6	11.2	14.2	17.5	21.2	25.3	
		NSL	14.3	D	8.8	12.6	17.2	22.5	28.4	35.1	42.5	50.5	
A588 ( $F_u = 70$ ksi)	X	STD	11.9	S	3.7	5.3	7.2	9.3	11.8	14.6	17.7	21.0	
		NSL	11.9	D	7.3	10.5	14.3	18.7	23.7	29.2	35.3	42.1	
A588 ( $F_u = 70$ ksi)	X	STD	15.4	S	4.7	6.8	9.3	12.1	15.3	18.9	22.9	27.2	
		NSL	15.4	D	9.4	13.6	18.5	24.2	30.6	37.8	45.7	54.4	

**Notes:**  
\*SC = Slip critical connection.  
N = Bearing-type connection with threads included in shear plane.  
X = Bearing-type connection with threads excluded from shear plane.  
STD = Standard round holes ( $d + 1/16$  in.).  
NSL = Long-or short-slotted holes normal to load direction.  
SSL = Long-or short-slotted hole normal to load direction (required in bearing-type connection).  
% = Single shear.  
D = Double shear.  
For threaded parts of materials not listed, use  $F_u = 0.17 F_u$  when threads are included in a shear plane, and  $F_u = 0.22 F_u$  when threads are excluded from a shear plane.  
<sup>b</sup> If predrilled bolts 1 1/8-in. dia. and greater, special impact wrenches may be required.  
When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in., tabulated values shall be reduced by 20%. See AISC ASD Commentary Sect. J3.4.

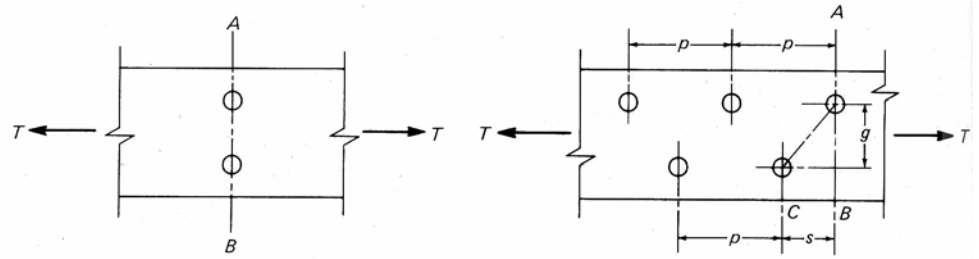


### Tension Member Design

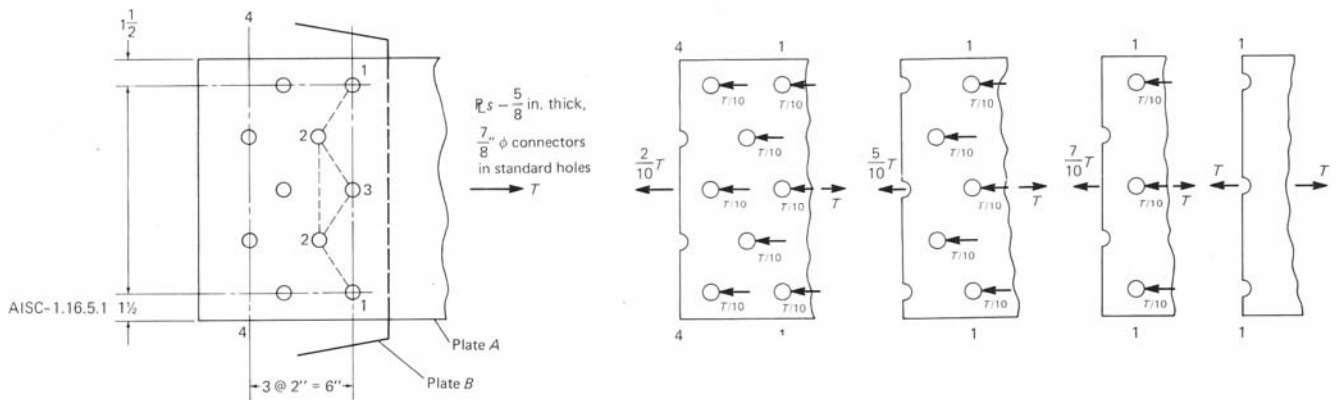
In steel tension members, there may be bolt holes that reduce the size of the cross section.

*Effective Net Area:*

The smallest effective are must be determined by subtracting the bolt hole areas. With staggered holes, the shortest length must be evaluated.



A series of bolts can also transfer a portion of the tensile force, and some of the effective net areas see reduced stress.



### ASD

For other than pin connected members:  $F_t = 0.60F_y$  on gross area

$F_t = 0.50F_u$  on net area

For pin connected members:

$F_t = 0.45F_y$  on net area

For threaded rods of approved steel:

$F_t = 0.33F_u$  on major diameter (static loading only)

### LRFD

The limit state for tension members are:

$$P_u \leq \phi_t P_n$$

1. yielding

$$\phi_t = 0.9 \quad P_n = F_y A_g$$

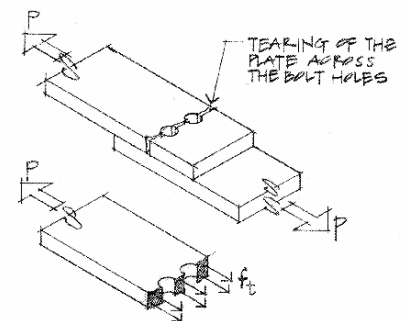
2. rupture

$$\phi_t = 0.75 \quad P_n = F_u A_e$$

where  $A_g$  = the gross area of the member (excluding holes)

$A_e$  = the effective net area (with holes, etc.)

$F_u$  = the tensile strength of the steel (ultimate)



**Welded Connections**

Weld designations include the strength in the name, i.e. E70XX has  $F_y = 70$  ksi.

The throat size,  $T$ , of a fillet weld is determined trigonometry by:  $T = 0.707 \times \text{weld size}$

ASD

Allowable shear stress of a weld is limited to 30% of the nominal strength.

$$F_v = 18 \text{ ksi for E60XX}$$

$$F_v = 21 \text{ ksi for E70XX}$$

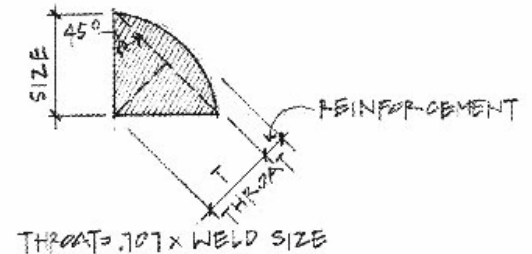
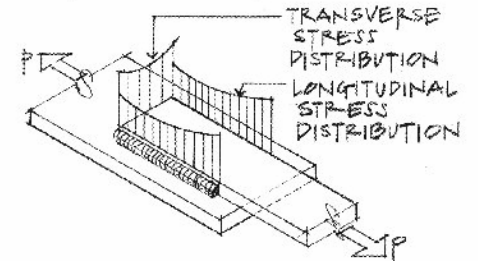
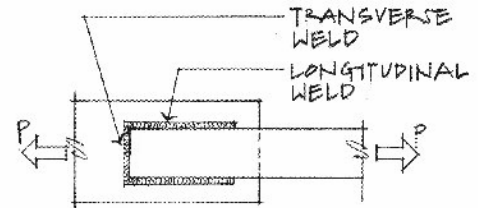
Weld sizes are limited by the size of the parts being put together and are given in AISC manual table J2.4 along with the allowable strength per length of fillet weld, referred to as  $S$ .

The *maximum* size of a fillet weld:

- a) can't be greater than the material thickness if it is  $\frac{1}{4}$ " or less
- b) is permitted to be  $\frac{1}{16}$ " less than the thickness of the material if it is over  $\frac{1}{4}$ "

The *minimum length* of a fillet weld is 4 times the nominal size. If it is not, then the weld size used for design is  $\frac{1}{4}$  the length.

Intermittent fillet welds can not be less that four times the weld size, not to be less than  $1 \frac{1}{2}$ ".



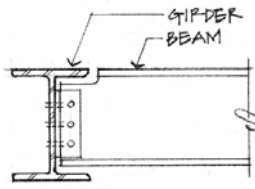
Allowable Strength of Fillet Welds per inch of weld ( $S$ )		
Weld Size (in.)	E60XX (k/in.)	E70XX (k/in.)
$\frac{3}{16}$	2.39	2.78
$\frac{1}{4}$	3.18	3.71
$\frac{5}{16}$	3.98	4.64
$\frac{3}{8}$	4.77	5.57
$\frac{7}{16}$	5.57	6.94
$\frac{1}{2}$	6.36	7.42
$\frac{5}{8}$	7.95	9.27
$\frac{3}{4}$	9.55	11.13

**TABLE J2.4**  
**Minimum Size of Fillet Welds**

Material Thickness of Thicker Part Joined (in.)	Minimum Size of Fillet Weld <sup>a</sup> (in.)
To $\frac{1}{4}$ inclusive	$\frac{1}{8}$
Over $\frac{1}{4}$ to $\frac{1}{2}$	$\frac{3}{16}$
Over $\frac{1}{2}$ to $\frac{3}{4}$	$\frac{1}{4}$
Over $\frac{3}{4}$	$\frac{5}{16}$

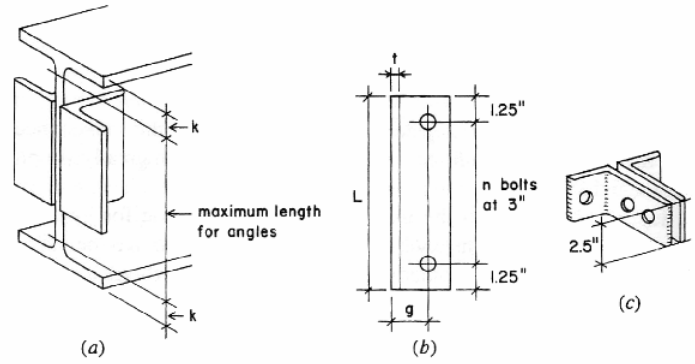
<sup>a</sup>Leg dimension of fillet welds. Single-pass welds must be used.

## Framed Beam Connections



*Coping* is the term for cutting away part of the flange to connect a beam to another beam using welded or bolted angles.

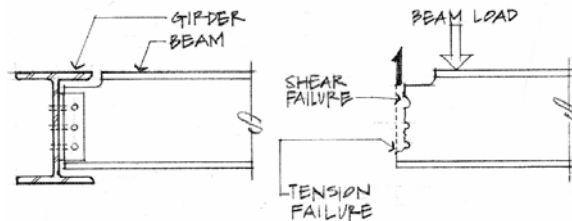
AISC provides tables that give angle sizes knowing bolt type, bolt diameter, angle leg thickness, and number of bolts (determined by *shear capacity*).



## Load and Factor Resistance Design

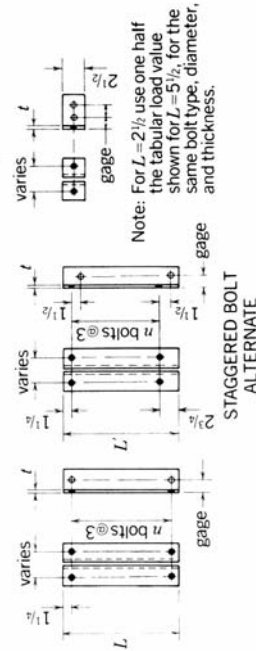
In addition to resisting shear and tension in bolts and shear in welds, the connected materials may be subjected to shear, bearing, tension, flexure and even prying action. Coping can significantly reduce design strengths and may require web reinforcement. All the following must be considered:

- shear yielding
- shear rupture
- block shear rupture -  
failure of a block at a beam as a result of shear and tension
- tension yielding
- tension rupture
- local web buckling
- lateral torsional buckling



**FRAMED BEAM CONNECTIONS**  
Bolted

**TABLE II Allowable loads in kips**



STAGGERED BOLT  
ALTERNATE

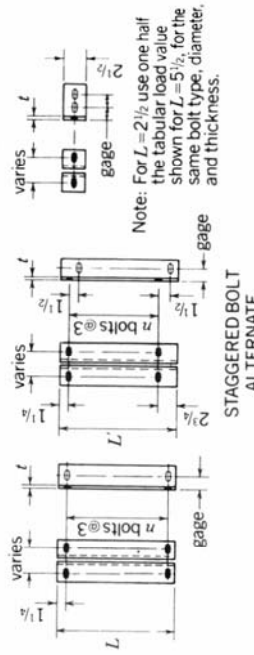
**TABLE II-A Bolt Shear<sup>a</sup>**  
For A307 bolts in standard or slotted holes and for A325 and A490 bolts in **slip-critical** connections with standard holes and Class A, clean mill scale surface condition.

Bolt Type	A307		A325-SC		A490-SC		Note:
	F <sub>v</sub> , Ksi	21.0	17.0	21.0	21.0	21.0	
Bolt Dia., d In.	3/4	7/8	3/4	7/8	3/4	7/8	For slip-critical connections with oversize or slotted holes, see Table II-B.
Angle Thickness t, In.	1/4	5/16	1/4	5/16	1/2	5/8	
L In.	29 1/2	31	10	186	253	330	
L' In.	26 1/2	28	9	167	227	297	
n	23 1/2	25	8	148	202	264	
	20 1/2	22	7	130	177	231	
	17 1/2	19	6	111	152	198	
	14 1/2	16	5	92.8	126	165	
	11 1/2	13	4	74.2	101	132	
	8 1/2	10	3	55.7	75.8	99.0	
	5 1/2	7	2	37.1	50.5	66.0	

**Notes:**  
<sup>a</sup>Tabulated load values are based on double shear of bolts unless noted. See RCSC Specification for other surface conditions.  
<sup>b</sup>Capacity shown is based on double shear of the bolts; however, for length L, net shear on the angle thickness specified is critical. See Table II-C.

**FRAMED BEAM CONNECTIONS**  
Bolted

**TABLE II Allowable loads in kips**



STAGGERED BOLT  
ALTERNATE

**TABLE II-A Bolt Shear**  
For bolts in **bearing-type** connections with standard or slotted holes.

Bolt Type	A325-N		A490-N		A325-X		A490-X	
	F <sub>v</sub> , Ksi	21.0	28.0	30.0	40.0	40.0	40.0	40.0
Bolt Dia., d In.	3/4	7/8	3/4	7/8	3/4	7/8	3/4	7/8
Angle Thickness t, In.	5/16	3/8	1/2	5/8	3/8	5/8	1/2	5/8
L In.	29 1/2	31	10	186	253	330	247	337
L' In.	26 1/2	28	9	167	227	297	223	303
n	23 1/2	25	8	148	202	264	198	269
	20 1/2	22	7	130	177	231	173	236
	17 1/2	19	6	111	152	198	148	202
	14 1/2	16	5	92.8	126	165	124	168
	11 1/2	13	4	74.2	101	132	99.0	135
	8 1/2	10	3	55.7	75.8	99.0	74.2	101
	5 1/2	7	2	37.1	50.5	66.0	49.5	67.3

**Notes:**  
 Tabulated load values are based on double shear of bolts.  
 Shaded values are based on double shear of the bolts; however, for length L, net shear on the angle thickness specified is critical. See Table II-C.  
 For shaded cells without values, shear rupture is critical for lengths L and L' on angle thickness specified. See Table II-C.

### Example 1

**10.2** The butt splice shown in Figure 10.22 uses two  $8 \times \frac{3}{8}$ " plates to "sandwich" in the  $8 \times \frac{1}{2}$ " plates being joined. Four  $\frac{7}{8}$ "  $\phi$  A325-SC bolts are used on both sides of the splice. Assuming A36 steel and standard round holes, determine the allowable capacity of the connection.

**Solution:**

Shear, bearing, and net tension will be checked to determine the critical condition that governs the capacity of the connection.

(Table I-D)

*Shear:* Using the AISC allowable shear in Table 10.1:

$$P_v = 20.4 \text{ k/bolt} \times 4 \text{ bolts} = 81.6 \text{ k (double shear)}$$

(Table I-E)

*Bearing:* Using the AISC bearing in Table 10.2:

The thinner material with the largest proportional load governs, therefore, the  $\frac{1}{2}$ " center plate governs. Assume the bolts are at a  $3d$  spacing, center to center.

$$P_b = 30.5 \text{ k/bolt} \times 4 \text{ bolts} = 122 \text{ k}$$

*Tension:* The center plate is critical since its thickness is less than the combined thickness of the two outer plates.

Hole diameter = (bolt diameter) +  $\frac{1}{16}$ " =  $\frac{7}{8}$ " +  $\frac{1}{16}$ " =  $\frac{15}{16}$ ".

$$A_{net} = (8'' - 2 \times \frac{15}{16}'') \times (\frac{1}{2}'') = 3.06 \text{ in.}^2$$

$$P_t = F_t \times A_{net}$$

where:

$$F_t = 0.5F_u = 0.5(58 \text{ ksi}) = 29 \text{ ksi}$$

$$P_t = 29 \text{ k/in.}^2 \times 3.06 \text{ in.}^2 = 88.7 \text{ k}$$

For yielding in the cross section without holes:

$$A_{gross} = (8'') \times (\frac{1}{2}'') = 4.0 \text{ in.}^2$$

$$P_t = F_t \times A_{gross}$$

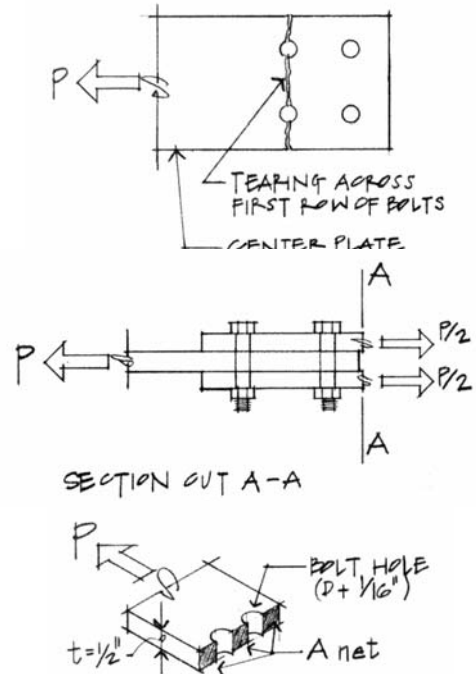
where:

$$F_t = 0.6F_y = 0.6(36 \text{ ksi}) = 21.6 \text{ ksi}$$

$$P_t = 21.6 \text{ k/in.}^2 \times 4.0 \text{ in.}^2 = 86.4 \text{ k}$$

The maximum connection capacity is governed by shear.

$$P_{allow} = 81.6 \text{ k}$$



Example 2

10.7 Determine the capacity of the connection in Figure 10.44 assuming A36 steel with E70XX electrodes.

Solution:

Capacity of weld:

For a 5/16" fillet weld,  $S = 4.64 \text{ k/in}$

Weld length = 22"

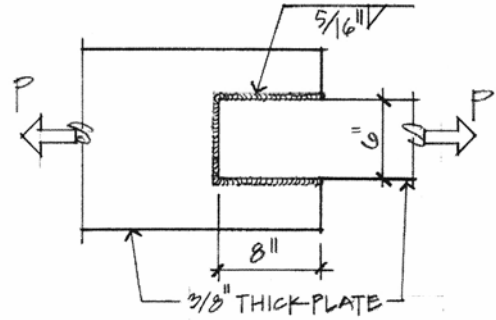
Weld capacity = 22" × 4.64 k/in = 102.1 k

Capacity of plate:

$F_{t, \text{allow}} = 0.6F_y = 22 \text{ ksi}$

Plate capacity = 3/8" × 6" × 22 k/in.<sup>2</sup> = 49.5 k

∴ Plate capacity governs,  $P_{\text{allow}} = 49.5 \text{ k}$



The weld size used is obviously too strong. What size, then, can the weld be reduced to so that the weld strength is more compatible to the plate capacity? To make the weld capacity ≈ plate capacity:

22" × (weld capacity per in.) = 49.5 k

Weld capacity per inch =  $\frac{49.5 \text{ k}}{22 \text{ in.}} = 2.25 \text{ k/in.}$

(page 4)

From Table 10.5, use 3/16" weld ( $S = 2.78 \text{ k/in.}$ ).

Minimum size fillet = 3/16" based on a 3/8" thick plate.

**FRAMED BEAM CONNECTIONS  
BOLTED**  
TABLE II Allowable loads in kips

**TABLE II-C**  
**Allowable Shear in Connection Angles for A36 Material**

Bolt Dia., d In.	3/4						7/8						1								
	1/4		3/8		1/2		1/4		5/16		3/8		1/2		3/8		1/2		3/4		
	L	n	L	n	L	n	L	n	L	n	L	n	L	n	L	n	L	n	L	n	
29 1/2	10	186	232	279	372	175	219	263	350	438	164	205	246	328	411						
26 1/2	9	167	209	256	334	157	196	236	314	393	147	184	221	295	368						
23 1/2	8	148	185	222	296	139	174	209	278	348	131	163	196	261	326						
20 1/2	7	129	161	193	258	121	152	182	243	303	114	142	170	227	284						
17 1/2	6	110	137	165	220	103	129	155	207	258	96.8	121	145	194	242						
14 1/2	5	90.8	114	136	182	85.4	107	128	171	213	79.9	99.9	120	160	200						
11 1/2	4	71.8	89.7	108	144	67.4	84.3	101	135	169	63.1	78.8	94.6	126	158						
8 1/2	3	52.7	65.9	79.1	105	49.5	61.9	74.2	99.0	124	46.2	57.8	69.3	92.4	116						
5 1/2	2	33.7	42.1	50.6	67.4	31.5	39.4	47.3	63.1	78.8	29.4	36.7	44.0	58.7	73.4						
L'	n																				
31	10	199	249	299	398	188	235	282	376	470	177	222	266	355	443						
28	9	180	225	270	360	170	213	255	340	425	160	201	241	321	401						
25	8	161	201	241	322	152	190	228	305	381	144	179	215	287	359						
22	7	142	177	213	284	134	168	201	269	336	127	158	190	253	317						
19	6	123	154	184	246	116	145	175	233	291	110	137	165	220	275						
16	5	104	130	156	208	98.4	123	148	197	246	93.0	116	139	186	232						
13	4	84.8	106	127	170	80.5	101	121	161	201	76.1	95.2	114	152	190						
10	3	65.8	82.2	98.7	132	62.5	78.2	93.8	125	156	59.3	74.1	88.9	119	148						
7	2	46.8	58.5	70.1	93.5	44.6	55.7	66.9	89.2	111	42.4	53.0	63.6	84.8	106						

NOTES: Table based on an allowable shear of 0.3F<sub>u</sub> (17.4 ksi for A36 angles) of the net section of two angles.  
Net section based on diameter of fastener + 1/16 in.

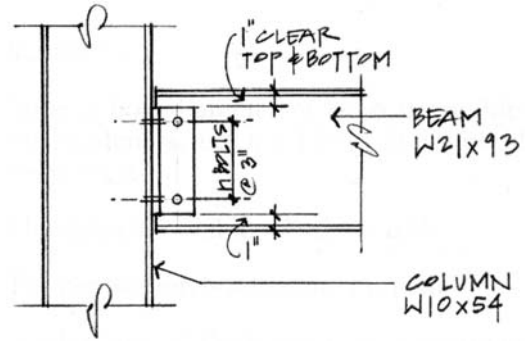
Example 3

The steel used in the connection and beams is A992 with  $F_y = 50$  ksi, and  $F_u = 65$  ksi. Using A490-N bolt material, determine the maximum capacity of the connection based on shear in the bolts, bearing in all materials and pick the number of bolts and angle length (not staggered). Use A36 steel for the angles.

W21x93:  $d = 21.62$  in,  $t_w = 0.58$  in,  $t_f = 0.93$  in

W10x54:  $t_f = 0.615$  in

SOLUTION:



The maximum length the angles can be depends on how it fits between the top and bottom flange with some clearance allowed for the fillet to the flange, and getting an air wrench in to tighten the bolts. This example uses 1" of clearance:

$$\begin{aligned} \text{Available length} &= \text{beam depth} - \text{both flange thicknesses} - 1" \text{ clearance at top} \& \& 1" \text{ at bottom} \\ &= 21.62 \text{ in} - 2(0.93 \text{ in}) - 2(1 \text{ in}) = 17.76 \text{ in.} \end{aligned}$$

The standard lengths for non-staggered holes ( $L$ ) and staggered holes ( $L'$ ) are shown in Table II-A. The closest size within the available length is  $17 \frac{1}{2}$  in. This will fit 6 bolts ( $n$ ) with a standard spacing.

We have a choice of bolt diameters of  $\frac{3}{4}$ ",  $\frac{7}{8}$ " and 1" in Table II-A. These have allowable loads for **shear** (double) of 148 kips, 202 kips, and 264 kips. But the last two values are shaded and the note says that "net shear on the angle thickness specified is critical" and to see Table II-C. The angle thickness ( $t$ ) is listed below the bolt diameter.

Table II-C gives a value of 207 kips for a  $\frac{7}{8}$ " bolt diameter,  $\frac{1}{2}$ " angle thickness, and 17.5" length. It gives a value of 242 kips for a 1" bolt diameter,  $\frac{5}{8}$ " angle thickness, and 17.5" length. Therefore, 242 kips is the maximum value limited by shear in the *angle*.

$$P_p = 264 \text{ kips for double shear of 1" bolts (Table I-D: 6 bolts} \cdot (44 \text{ k/bolt}) = 264 \text{ kips)}$$

$$P_v = 242 \text{ kips for net shear in angle}$$

We also need to evaluate **bearing** of bolts on the angles, beam web, and column flange where there are bolt holes. Table I-E provides allowable bearing load for the material type, bolt diameter and some material thicknesses. The last note states that "Values for decimal thicknesses may be obtained by multiplying the decimal value of the unlisted thickness by the value given for a 1-in. thickness". This comes from the definition for bearing stress:

$$f_p = \frac{P}{td} \leq F_p, \text{ where } P_p = t \cdot d \cdot F_p \text{ at the allowable bearing stress}$$

For a constant diameter and allowable stress, the allowable load depends only on the thickness.

a) Bearing for  $\frac{5}{8}$ " thick angle: There are 12 bolt holes through two angle legs to the column, and 12 bolt holes through two angle legs either side of the beam. The material is A36 ( $F_u = 58$  ksi), with 1" bolt diameters.

$$P_p = 12 \text{ bolts} \cdot (43.5 \text{ k/bolt}) = 522 \text{ kips}$$

b) Bearing for column flange: There are 12 bolt holes through two angle legs to the column. The material is A992 ( $F_u = 65$  ksi), 0.615" thick, with 1" bolt diameters.

$$P_p = 12 \text{ bolts} \cdot (78 \text{ k/bolt}/1") \cdot (0.615 \text{ in}) = 576 \text{ kips.}$$

c) Bearing for beam web: There are 6 bolt holes through two angle legs either side of the beam. The material is A992 ( $F_u = 65$  ksi), 0.58" thick, with 1" bolt diameters

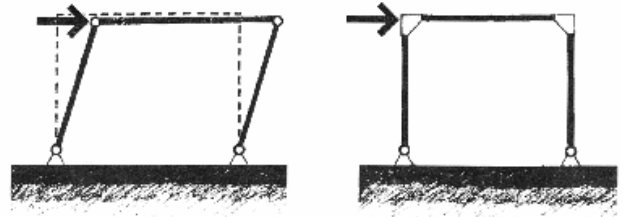
$$P_p = 6 \text{ bolts} \cdot (78 \text{ k/bolt}/1") \cdot (0.58 \text{ in}) = 271 \text{ kips.}$$

Although, the bearing in the beam web is the smallest at 271 kips, with the shear on the bolts even smaller at 264 kips, the maximum capacity for the simple-shear connector is 242 kips limited by net shear in the angles.

## Rigid and Braced Frames

### Rigid Frames

Rigid frames are identified by the lack of pinned joints within the frame. The joints are *rigid* and resist rotation. They may be supported by pins or fixed supports. They are typically statically indeterminate.

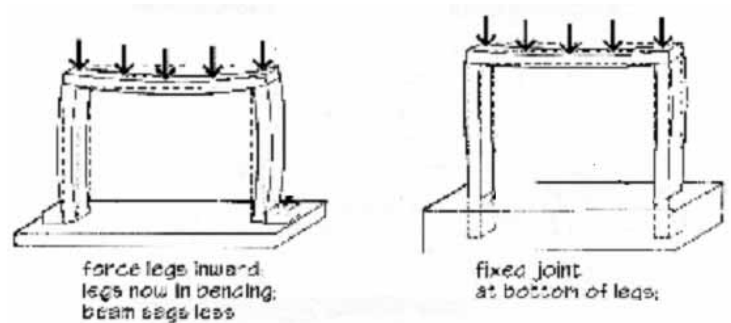


Frames are useful to resist **lateral** loads.

Frame members will see

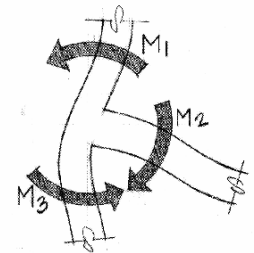
- shear
- bending
- axial forces

and behave like *beam-columns*.



### Behavior

The relation between the joints has to be maintained, but the whole joint can *rotate*. The amount of rotation and distribution of moment depends on the *stiffness* ( $EI/L$ ) of the members in the joint.



End restraints on columns reduce the effective length, allowing columns to be more slender. Because of the rigid joints, deflections and moments in beams are reduced as well.

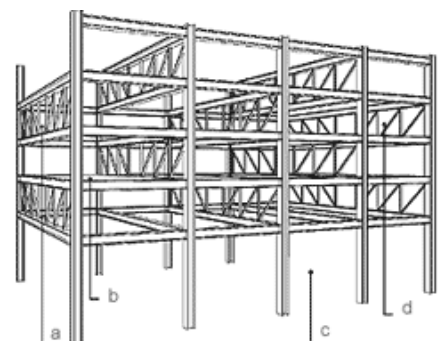
Frames are sensitive to settlement because it induces strains and changes the stress distribution.

### Types

*Gabled* – has a peak

*Portal* – resembles a door. Multi-story, multiple bay portal frames are commonly used for commercial and industrial construction. The floor behavior is similar to that of continuous beams.

*Staggered Truss* – Full story trusses are staggered through the frame bays, allowing larger clear stories.

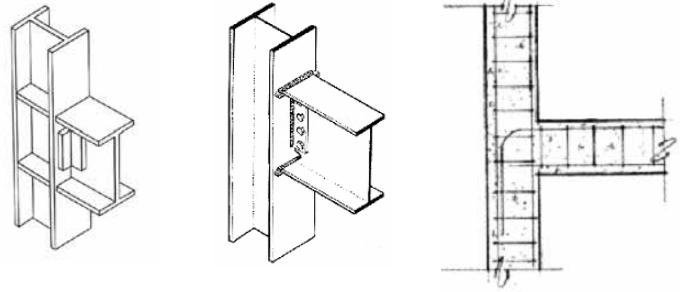


Staggered Truss



Connections

*Steel* – Flanges of members are fully attached to the flanges of the other member. This can be done with welding, or bolted plates.



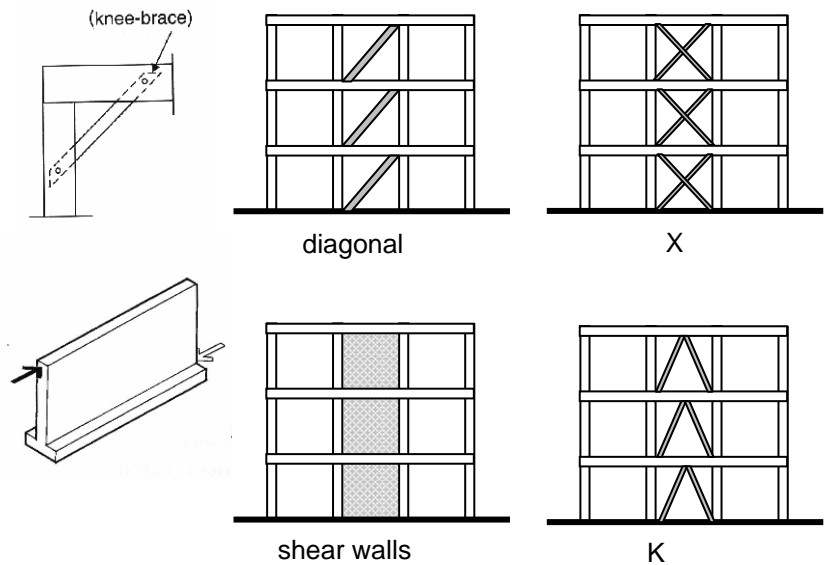
*Reinforced Concrete* – Joints are monolithic with continuous reinforcement for bending. Shear is resisted with stirrups and ties.

**Braced Frames**

Braced frames have beams and columns that are “pin” connected with bracing to resist lateral loads.

Types of Bracing

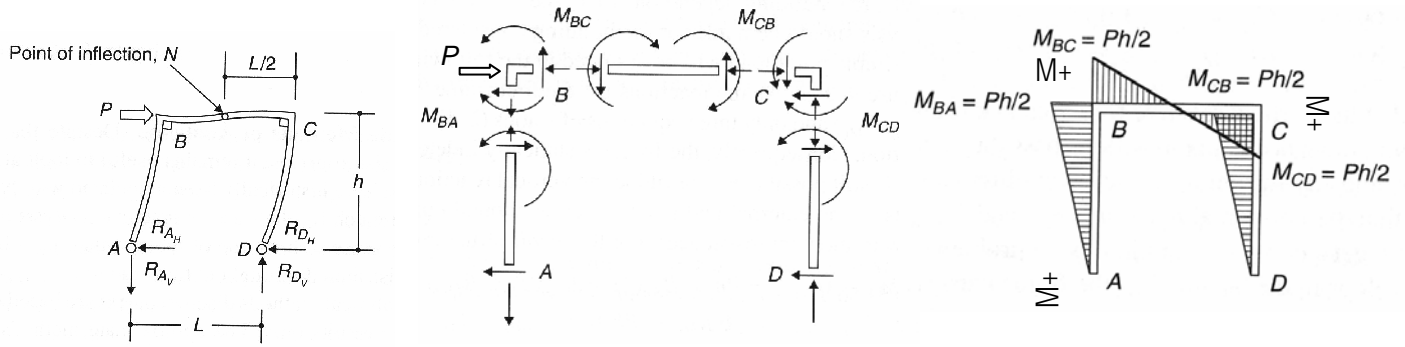
- knee-bracing
- diagonal (including eccentric)
- X
- K or chevron
- shear walls – which resist lateral forces in the plane of the wall



**Rigid Frame Analysis**

Structural analysis methods such as the *portal method* (approximate), the *method of virtual work*, *Castigliano’s theorem*, the *force method*, the *slope-displacement method*, the *stiffness method*, and *matrix analysis*, can be used to solve for internal forces and moments and support reactions.

Shear and bending moment diagrams can be drawn for frame members by isolating the member from a joint and drawing a free body diagram. The internal forces at the end will be equal and opposite, just like for connections in *pinned frames*. Direction of the “beam-like” member is usually drawn by looking from the “inside” of the frame.

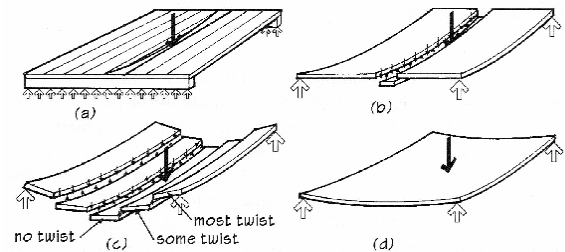


**Frame Design**

The possible load combinations for frames with dead load, live load, wind load, etc. is critical to the design. The maximum moments (positive and negative) may be found from different combinations and at different locations. Lateral wind loads can significantly affect the maximum moments.

Plates and Slabs

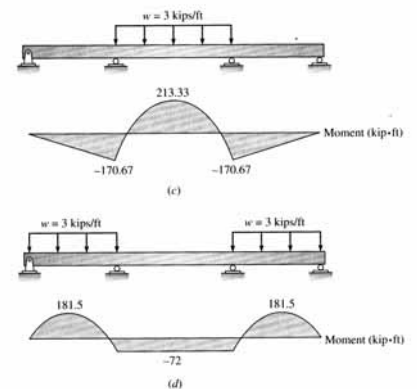
If the frame is rigid or non-rigid, the floors can be a plate or slab (which has drop panels around columns). These elements behave differently depending on their supports and the ratio of the sides.



- one-way behavior: like a “wide” beam, when ratio of sides > 1.5
- two-way behavior: complex, non-determinate, look for handbook solutions

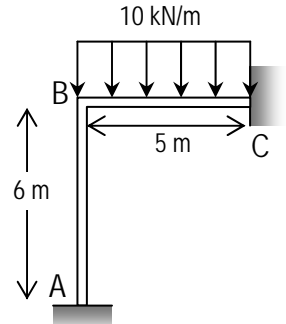
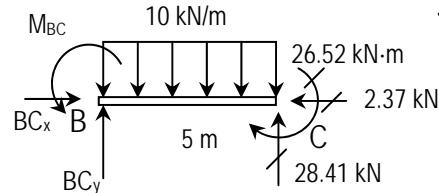
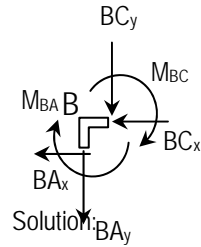
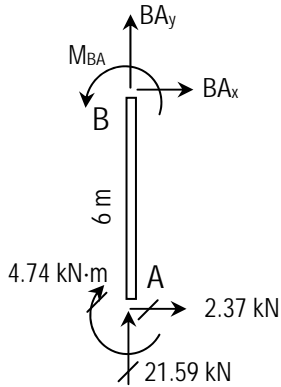
Floor Loading Patterns

With continuous beams or floors, the worst case loading typically occurs when alternate spans are loaded with live load (not every span). The maximum positive and negative moments may not be found for the same loading case! If you are designing with reinforced concrete, you must provide flexure reinforcement on the top and bottom and take into consideration that the maximum may move.



**Example 1**

The rigid frame shown has been analyzed using an advanced structural analysis technique. The reactions at support A are:  $A_x = 2.37 \text{ kN}$ ,  $A_y = 21.59 \text{ kN}$ ,  $M_A = -4.74 \text{ kN}\cdot\text{m}$ . The reactions at support C are:  $C_x = -2.37 \text{ kN}$ ,  $C_y = 28.4 \text{ kN}$ ,  $M_C = -26.52 \text{ kN}\cdot\text{m}$ . Draw the shear and bending moment diagrams, and identify  $V_{\max}$  &  $M_{\max}$ .



**Reactions** These values must be given or found from non-static analysis techniques. The values are given with respect to the global coordinate system we defined for positive and negative forces and moments for equilibrium.

**Member End Forces** The free-body diagrams of all the members and joints of the frame are shown above. The unknowns on the members are drawn positive, and the opposite directions are drawn on the joint. We can begin the computation of internal forces with either member AB or BC, both of which have only three unknowns.

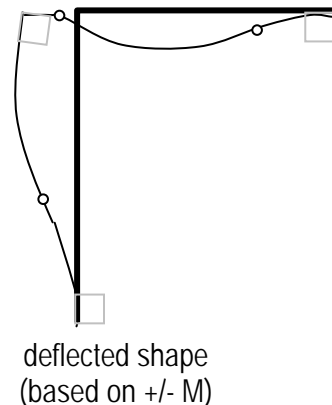
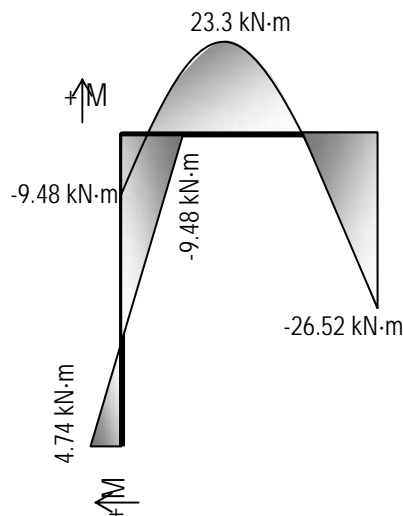
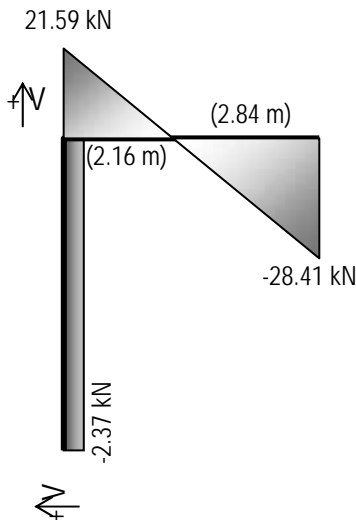
**Member AB** With the magnitudes of reaction forces at A known, the unknowns are at end B of  $BA_x$ ,  $BA_y$ , and  $M_{BA}$ , which can get determined by applying  $\sum F_x = 0$ ,  $\sum F_y = 0$ , and  $\sum M_B = 0$ . Thus,

$$\begin{aligned} \sum F_x = 2.37 \text{ kN} + BA_x = 0 \quad BA_x = -2.37 \text{ kN}, \quad \sum F_y = 21.59 \text{ kN} + BA_y = 0 \quad BA_y = -21.59 \text{ kN} \\ \sum M_B = 2.37 \text{ kN}(6 \text{ m}) - 4.74 \text{ kN}\cdot\text{m} + M_{BA} = 0 \quad M_{BA} = -9.48 \text{ kN}\cdot\text{m} \end{aligned}$$

**Joint B** Because the forces and moments must be equal and opposite,  $BC_x = 2.37 \text{ kN}$ ,  $BC_y = 21.59 \text{ kN}$  and  $M_{BC} = 9.48 \text{ kN}\cdot\text{m}$

**Member BC** All forces are known, so equilibrium can be checked:

$$\begin{aligned} \sum F_x = 2.37 \text{ kN} - 2.37 \text{ kN} = 0 \quad \sum F_y = 21.59 \text{ kN} + 28.49 \text{ kN} - (10 \text{ kN} / \text{m})5 \text{ m} = 0 \\ \sum M_B = 28.41 \text{ kN}(5 \text{ m}) - 10 \text{ kN} / \text{m}(5 \text{ m})(2.5 \text{ m}) - 26.52 \text{ kN}\cdot\text{m} + 9.48 \text{ kN}\cdot\text{m} = 0 \end{aligned}$$

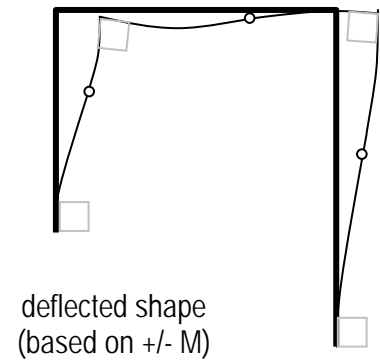
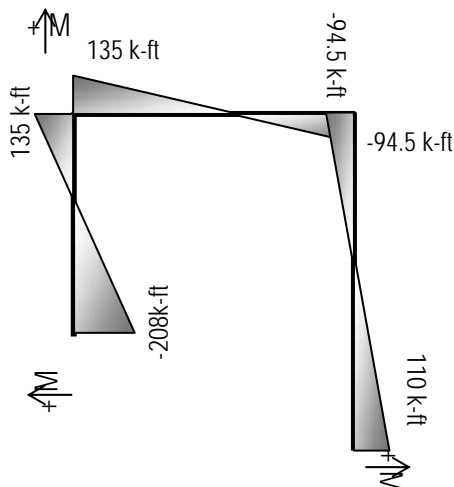
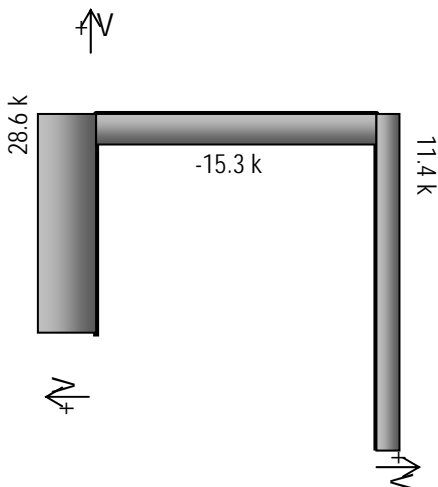
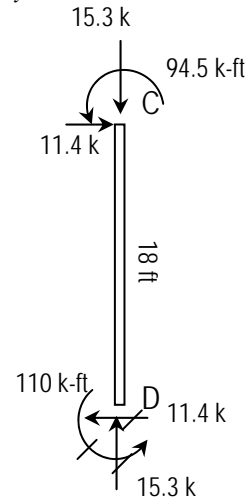
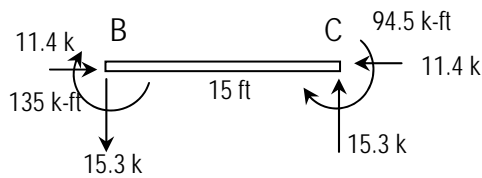
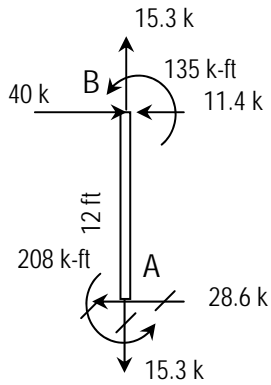
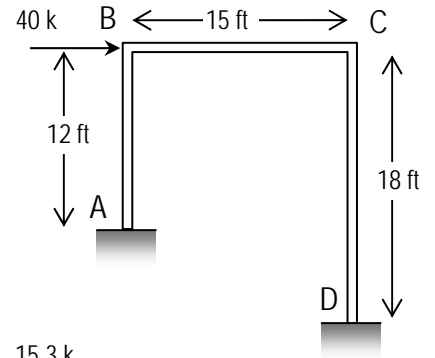


**Example 2**

The rigid frame shown has been analyzed using an advanced structural analysis technique. The reactions at support A are:  $A_x = -28.6$  k,  $A_y = -15.3$  k,  $M_A = 208$  k-ft. The reactions at support D are:  $D_x = -11.4$  k,  $D_y = 15.3$  k,  $M_D = 110$  ft-k. Draw the shear and bending moment diagrams, and identify  $V_{max}$  &  $M_{max}$ .

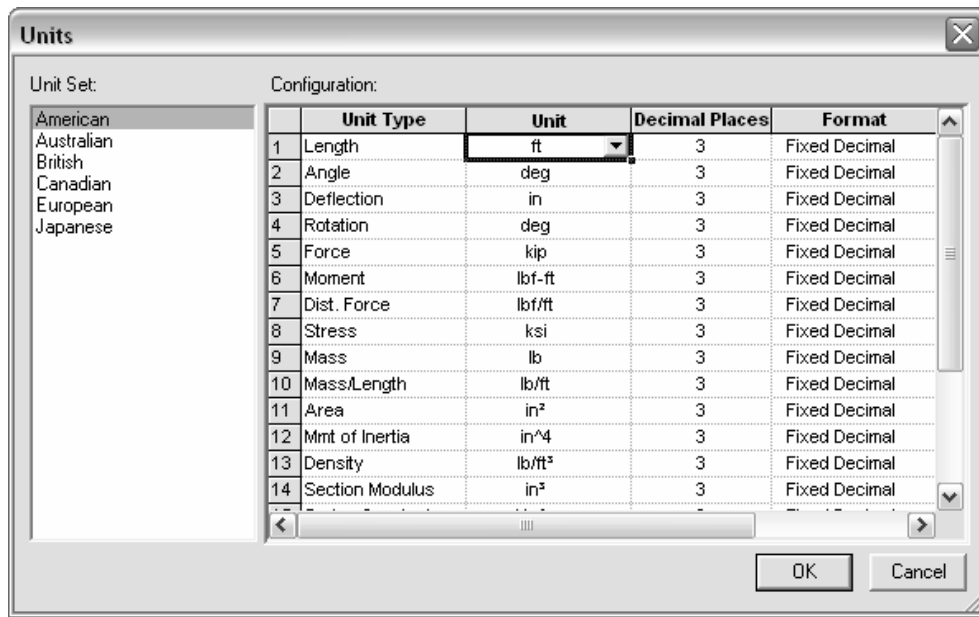
Solution:

*NOTE: The joints are not shown, and the load at joint B is put on only one body.*

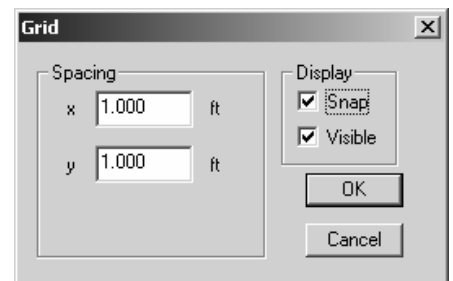


### Frame Analysis Using Multiframe4D

1. The software is on the computers in the college computing lab (<http://thelab.tamu.edu>) in Programs under the Windows Start menu. ~~Multiframe4D is under the COSC menu.~~
2. There is a tutorial in the Help menu (Chapter 1 – 2D Tutorial) that lists the tasks and order in greater detail. The first task is to define the unit system:
  - Choose Units... from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.



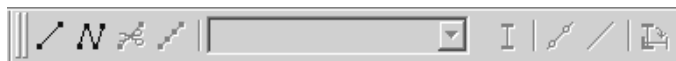
3. To see the scale of the geometry, a grid option is available:
  - Choose Grid... from the View menu



4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:



The Member toolbar shows ways to create members:

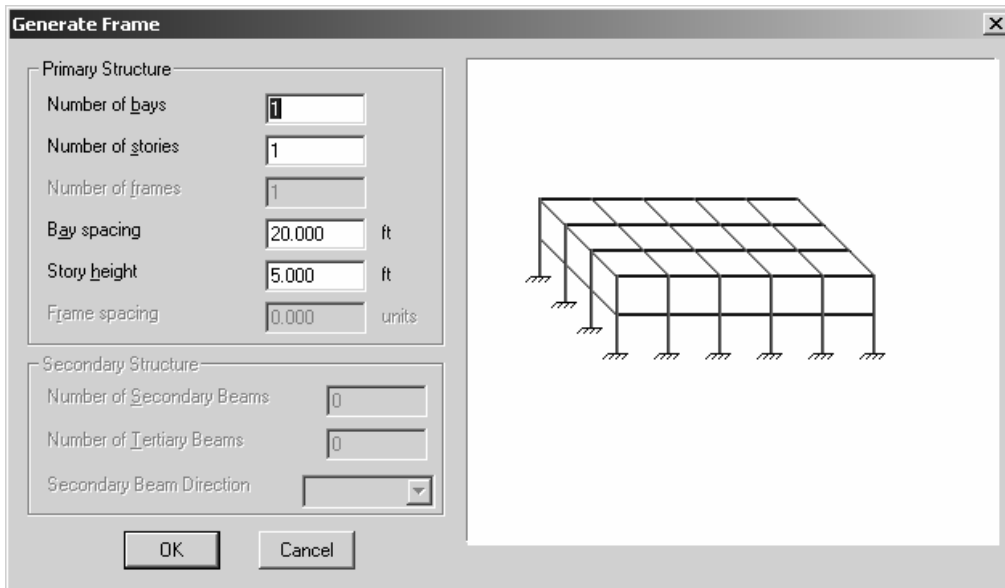


The Generate toolbar has convenient tools to create typical structural shapes.

- To create a frame, use the multi-bay frame button:



- Enter the number of bays (horizontally), number of stories (vertically) and the corresponding spacings:

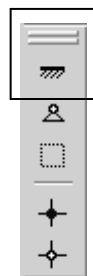


- If the frame does not have regular bays, use the add connected members button to create segments:

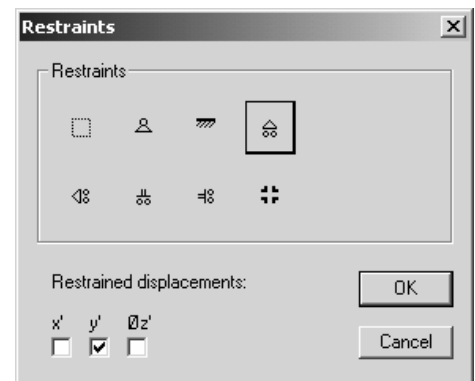


- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.
- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.

- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (fixed shown), or the Frame / Joint Restraint ... menu (right click).



NOTE: If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.



The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

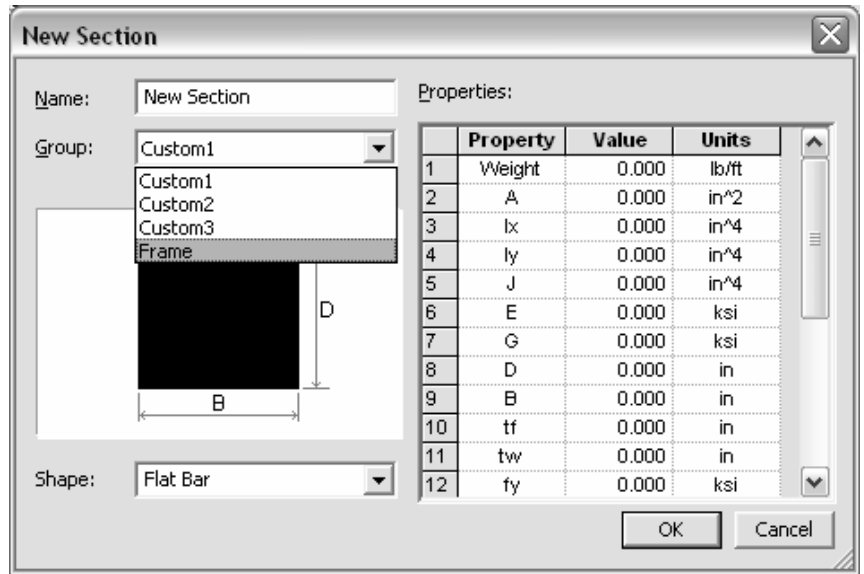


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like t<sub>f</sub>, t<sub>w</sub>, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

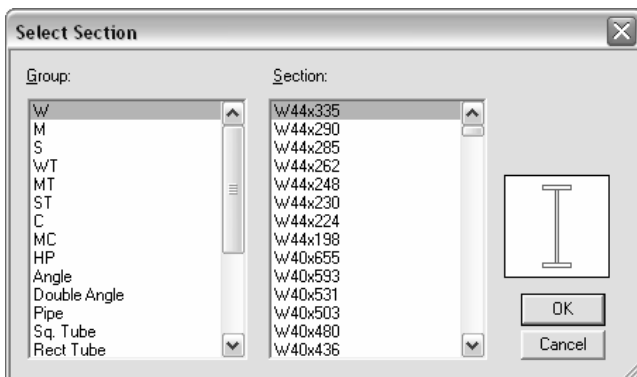
6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

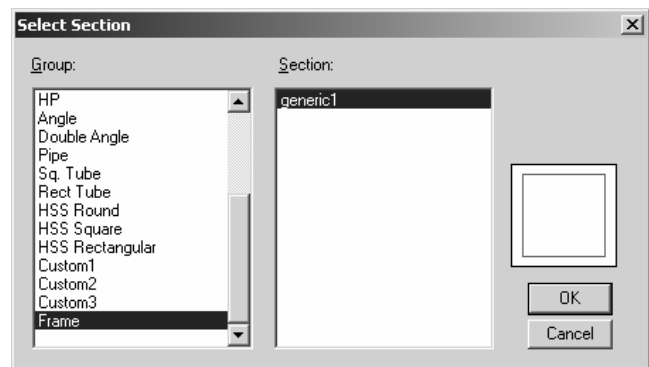


- Choose the group name and section name:

(STANDARD SHAPES)



(CUSTOM)



- The frame geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:



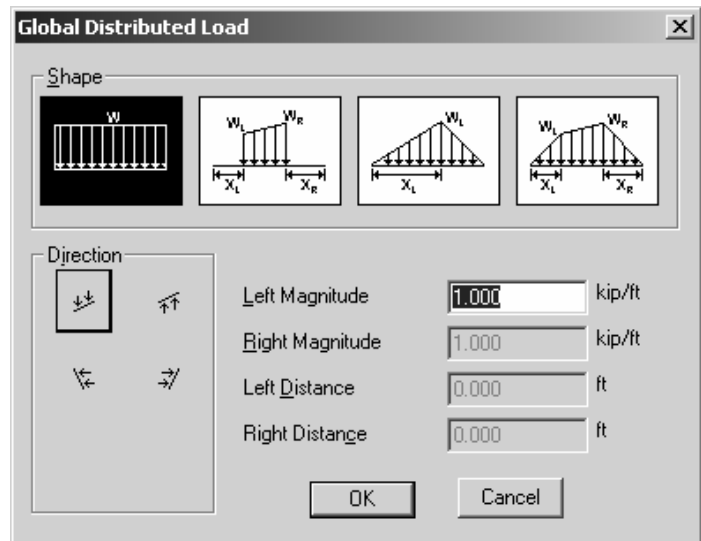
- The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force in the local coordinate system (last two buttons).



- Choose the member to be loaded (drag) and select the load type (here shown for distributed loading):



- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.
- Enter the values of the load and distances (if any). Distances can be entered as a function of the length, i.e.  $L/2$ ,  $L/4$ ...



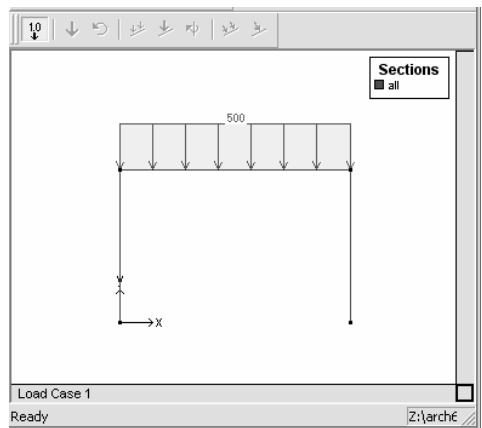
NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.

Multiframe4D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

- In order to run the analysis after the geometry, member properties and loading has been defined:

- Choose Analyze Linear from the Case menu

- If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

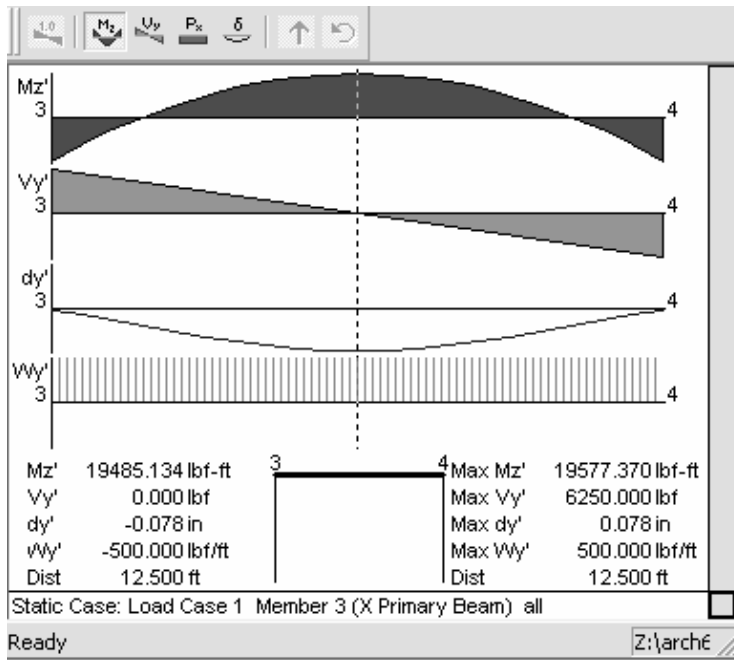
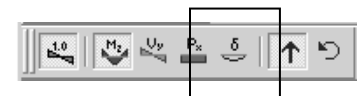
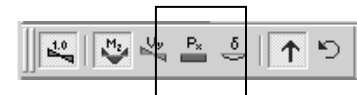
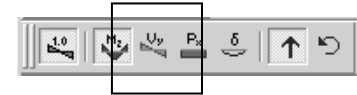
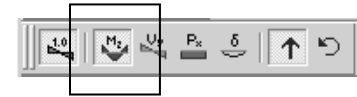




11. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):



- To show the moment diagram, Choose the red Moment button
- To show the shear diagram, Choose the green Shear button
- To show the axial force diagram, Choose the purple Axial Force button
- To show the deflection diagram, Choose the blue Deflection button
- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.



12. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.



13. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

NOTE: Px' refers to the axial load (P) in the local axis x direction (x'). Vy' refers to the shear perpendicular to the local x axis, and Mz' refers to the bending moment.



	Memb	Label	Joint	Px' lbf	Vy' lbf	Mz' lbf-ft
1	1	Column	1	6250.000	-1786.320	-9725.784
2	1	Column	3	-6250.000	1786.320	-19577.371
3	2	Column	2	6250.000	1786.320	9725.784
4	2	Column	4	-6250.000	-1786.320	19577.371
5	3	X Prima	3	1786.320	6250.000	19577.371
6	3	X Prima	4	-1786.320	6250.000	-19577.371

Member Actions Max Acti

14. To save the file Choose Save from the File menu.

15. To load an existing file Choose Open... from the File menu.

Example of Combined Stresses:

for member 3:  $M_{max} = 19.6 \text{ k-ft}$ ,  $P = 1.76 \text{ k}$

knowing  $A = 21.46 \text{ in}^2$ ,  $I = 796.0 \text{ in}^4$ ,  $c = 7.08 \text{ in}$

$$f_{max} = \frac{1.76k}{21.46in^2} + \frac{19.6^{k-ft} \cdot 7.08in}{796in^4} \cdot \frac{12in}{ft} = 0.082ksi + 2.092ksi = 2.174ksi$$

Results window:

	Memb	Label	Joint	Sbz' top ksi	Sbz' bot ksi	Sy' ksi	Sx' ksi	Sx'+Sbz' top ksi	Sx'+Sbz' bot ksi
1	1	Column	1	1.039	-1.039	-1152.461	0.286	1.325	-0.753
2	1	Column	3	-2.092	2.092	-1152.461	0.286	-1.806	2.378
3	2	Column	2	-1.039	1.039	1152.461	0.286	-0.753	1.325
4	2	Column	4	2.092	-2.092	1152.461	0.286	2.378	-1.806
5	3	X Prima	3	-2.092	2.092	4032.245	0.082	-2.011	2.174
6	3	X Prima	4	-2.092	2.092	-4032.245	0.082	-2.011	2.174

Member Stresses Max Stresses Member

where Sx' refers to the axial stress, Sy' refers to the bending stress around the local vertical axis and Sz' refers to the bending stress around the local horizontal axis.

## System Selection and Design

from *Architectural Structures*,  
Wayne Place, Wiley, 2007:

### STRUCTURAL DESIGN PROCESS

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#### 1.1 Nature of the Process

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Architects have a huge array of issues to address in architectural practice. Among these are the following: keeping rain out of a building, getting water off a site, thermal comfort, visual comfort, space planning, fire egress, fire resistance, corrosion and rot resistance, vermin resistance, marketing, client relations, the law, contracts, construction administration, the functional purposes of architecture, the role of the building in the larger cultural context, security, economy, resource management, codes and standards, and how to make a building withstand all the forces to which it will likely be subjected during its lifetime. This last subject area is referred to as *architectural structures*.

Because of the extraordinary range of demands on an architect's time and skills and the extraordinary number of subjects that architecture students must master, architectural structures are typically addressed in only two or three lecture courses in an accredited architectural curriculum in the United States. These two or three lecture courses must be contrasted with the ten or twelve courses that will normally be taken by a graduate of an accredited structural engineering curriculum. This contrast in level of focus makes it clear why a good structural engineering consultant is a very valuable asset to an architect. However, having a good structural consultant does not relieve the architect of serious responsibility in the structural domain. All architects must be well versed in matters related to structures. The architect has the primary responsibility for establishing the structural concept for a building, as part of the overall design concept, and must be able to speak the language of the structural consultant with sufficient skill and understanding to take full advantage of the consultant's capabilities.

#### 1.2 General Comments Regarding Architectural Education

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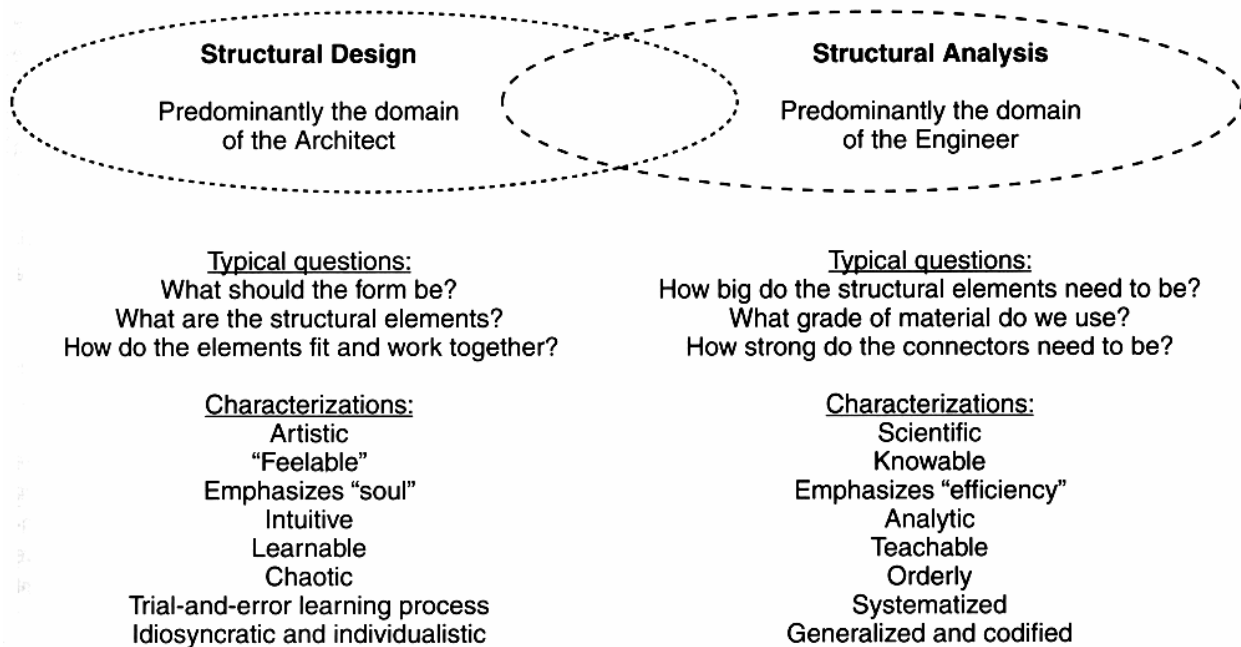
Structural design is one of the more rigorous aspects of architectural design. Much knowledge has been generated and codified over the centuries that human beings have been practicing in and developing this field. This book gives primary attention to those things that are known, quantified, and codified.

However, very few things in the realm of architecture yield a single solution. To any given design problem, there are many possible solutions, and picking the best solution is often the subject of intense debate. Therefore, no one should come to this subject matter assuming that this text, or any text, is going to serve up a single, optimized solution to any design problem, unless that design problem has been so narrowly defined as to be artificial.

In design, there is always a great deal of latitude for personal expression. Design is purposeful action. The designer must have an attitude to act. Architecture students develop an attitude through a chaotic learning process involving a lot of trial and error. In going through this process, an architecture student must remain aware of a fundamental premise: the process is more important than the product; that is, the student's learning and development are more important than the output. The student has a license to make mistakes. It is actually more efficient to plow forward and make mistakes than to spend too much time trying to figure out how to do it perfectly the first time. To paraphrase the immortal words of Thomas Edison: To have good ideas, you should have many ideas and then throw out the bad ones. Of course, throwing out the bad ones requires a lot of rigorous and critical thinking. No one should ever fall in love with any idea that has not been subjected to intense and prolonged critical evaluation and withstood the test with flying colors. Furthermore, important ideas should be subjected to periodic reevaluation. Times and conditions change. Ideas that once seemed unassailable may outlive their usefulness or, at the very least, need updating in the light of new knowledge and insights.

In pursuing this subject matter, it is valuable to have a frame of reference regarding the roles of the architect, as the leader of the design team, and the structural engineer, as a crucial contributor of expertise and hard work needed to execute the project safely and effectively. The diagram in Figure 1.1 will help provide that frame of reference.

In contemplating the diagram in Figure 1.1, keep in mind that design and analysis are two sides of the same coin and that the skills and points of view of architects and engineers, although distinctive, also overlap and sometimes blur together. The most effective design teams consist of individuals with strong foci who can play their respective roles while having enough overlap in understanding and purpose that they can see each other's point of view and cooperate in working toward mutually understood and shared goals. The most harmful poison to a design team is to have such a separation in points of view and understanding that a rift develops between the members of the team. Cooperation is the watchword in this process, as in all other team efforts.

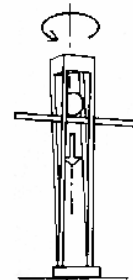


**Figure 1.1** Nature of the design process and roles of the design participants.

## Design Criteria for the Behavior of the Overall System

Components of a system consist of vertical and horizontal elements. Connections of the vertical to horizontal elements are also necessary. For the structural elements to behave and respond as designed, the system must have the following qualities:

- the components stay together
- the system resists overturning, sliding, twisting and excessive distortion
- the system has internal stability
- the system has overall strength and stiffness



**Twisting**

## “Order” of Design

There is no set order to design of a structural system. But there are certain stages that can be recognized. These may be referred to as *preliminary*, *revised* and *final*, or more formally as:

First order: which can include determining structural type and organization, design intent, and contextual or programmatic emphasis. Preliminary member size charts are useful at this stage.

Second order: which can include evaluating structural strategies, choice of construction materials, and structural system options with those materials. System selection design aids are useful at this stage.

Third order: which, after the design has been narrowed down, is where analysis and design (shape and size) of individual structural elements (beams, columns, connections, etc.) is performed. The outcome here may direct further first order or second order investigations!!!

from Understanding Structures, Fuller Moore, McGraw-Hill, 1999:

DESIGN CRITERIA	Light-frame timber	Heavy-frame timber	Masonry bearing wall	Steel frame (hinge connections)	Steel frame (rigid connections)	Steel open-web joists	Steel space frame	Steel decking	Site-cast concrete: one-way slab	Site-cast concrete: two-way plate	Site-cast concrete: two-way slab	Site-cast concrete: one-way joists	Site-cast concrete: waffle slab	Precast concrete: solid slab	Precast concrete: hollow-core slab	Precast concrete: single tee	Precast concrete: double tee	RATIONALE
Exposed, fire-resiant construction																		Inherently fire-resistive construction
Irregular building form																		Simple, site-fabricated systems
Irregular column placement																		Systems without beams in roof or floors
Minimize floor thickness																		Precast-concrete systems without ribs
Allow for future renovations																		Short-span, one-way, easily modified
Permit construction in poor weather																		Quickly erected; avoid site-cast concrete
Minimize off-site fabrication time																		Easily formed or built on site
Minimize on-site erection time																		Highly prefabricated; modular components
Minimize low-rise construction time																		Lightweight, easily formed or prefabricated
Minimize medium-rise construction time																		Precast, site-cast concrete; steel frames
Minimize high-rise construction time																		Strong; prefabricated; lightweight
Minimize shear walls or diagonal bracing																		Capable of forming rigid joints
Minimize dead load on foundations																		Lightweight, short-span systems
Minimize damage due to foundation settlement																		Systems without rigid joints
Minimize the number of separate trades on job																		Multipurpose components
Provide concealed space for mech. services																		Systems that inherently provide voids
Minimize the number of supports																		Two-way, long-span systems
Long spans																		Long-span systems

Figure 18.6: Framing system selection chart.

