lecture
twenty five

eccentric loading:
beam-columns
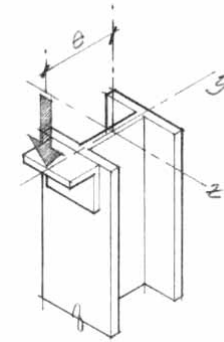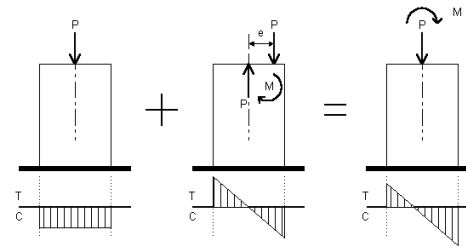
Centric & Eccentric Loading

- *centric*
  - *allowable stress from strength or buckling*
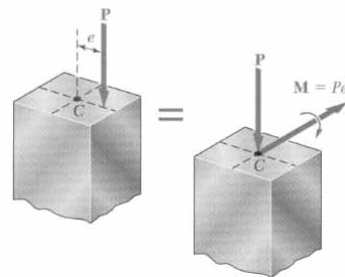- *eccentric*
  - *combined stresses*

Eccentric Loading

- *axial + bending*
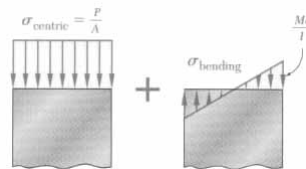
$$f_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$
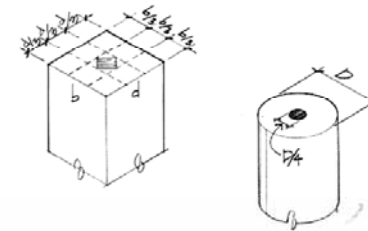
- *design*

$$f_{max} \le F_{cr} = \frac{f_{cr}}{F.S.}$$

Eccentric Loading

- *find e such that the minimum stress = 0*

$$f_{min} = \frac{P}{A} - \frac{(Pe)c}{I} = 0$$

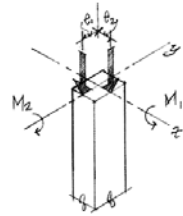- *area defined by e from centroid is the* <u>*kern*</u>

1

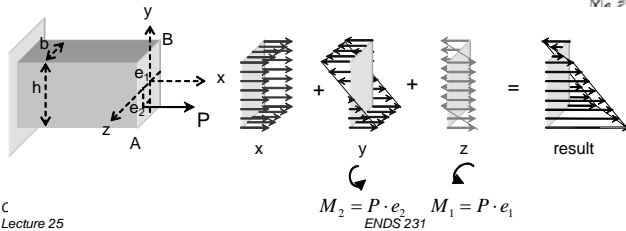## Eccentric Loading

– *when there is eccentricity in two directions*

$$M_1 = P \cdot e_1 \qquad M_2 = P \cdot e_2$$

$$f_{max} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I}$$

– *biaxial bending*

$M_1 = P_1 \times e_1$ (ABOUT THE x-AXIS)
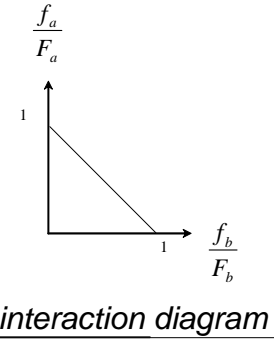$M_2 = P_2 \times e_2$ (ABOUT THE y-AXIS)

$M_2 = P \cdot e_2 \quad M_1 = P \cdot e_1$

Stress Limit Conditions

– *ASD interaction formula*

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.0$$

$\frac{f_a}{F_a}$

1

1 $\frac{f_b}{F_b}$

– *with biaxial bending*

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$

interaction diagram

Stress Limit Conditions

– *in reality, as the column flexes, the moment increases*

– *P-Δ effect*

Δ = DEFLECTION DUE TO SIDE LOAD

LATERAL (SIDE) LOAD

$$\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ factor)}{F_{bx}} \le 1.0$$
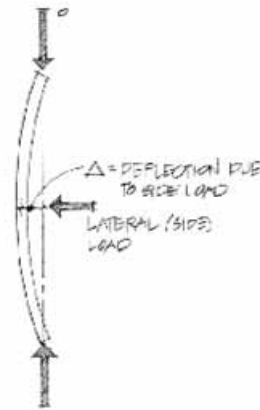
Design

• *satisfy*
  – *strength*
  – *stability*
• *pick*
  – *section*
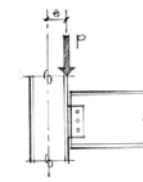
(a) Framed beam (shear) connection.
e = Eccentricity; M = P × e

(b) Moment connection (rigid frame).
M = Moment due to beam bending

(c) Timber beam-column connection.
e = d/2 = eccentricity; M = P × e

(d) Upper chord of a truss—compression plus bending.
$M = \frac{\omega \ell^2}{8}$

2

Design

• *ASD Steel*

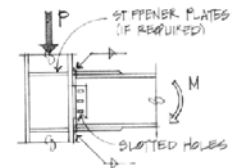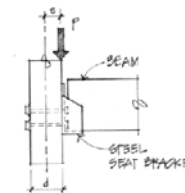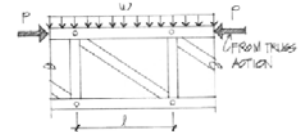$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \dfrac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \dfrac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

C_m – *modification factor for end conditions*
     $= 0.6 - 0.4(M_1/M_2)$ *or 0.85 restrained*
F'_e – *allowable buckling strength*
() term – magnification factor for P-Δ

Design

• *Wood*

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx}\left[1 - \dfrac{f_c}{F_{cEx}}\right]} \leq 1.0$$

[] term – magnification factor for P-Δ
F'_{bx} – *allowable bending strength*

Design

• *LRFD Steel*

– *for* $\dfrac{P_u}{\phi_c P_n} \geq 0.2$: $\quad \dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$

– *for* $\dfrac{P_u}{\phi_c P_n} < 0.2$: $\quad \dfrac{P_u}{2\phi_c P_n} + \left(\dfrac{M_{ux}}{\phi_b M_{nx}} + \dfrac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$

ϕ_c - *resistance factor for compression = 0.85*
ϕ_b - *resistance factor for bending = 0.9*

Design Steps Knowing Loads

1. *assume limiting stress*
   • *buckling, axial stress, combined stress*
2. *solve for r, A or S*
3. *pick trial section*
4. *analyze stresses*
5. *section ok?*
6. *stop when section is ok*

Wind Load Left
Tension Left

Seismic Load Left
Tension Left

Eccentric Load Right
Tension Left

Wind Load Left
Braced at Top