

lecture  
 twelve



moment of inertia  
 of an area

Moment of Inertia 1  
 Lecture 12

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 ENDS 231

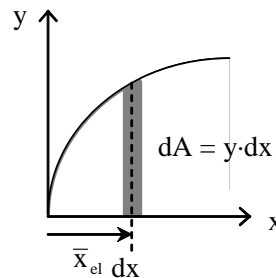
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Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

$$I_x = \int y^2 dA$$



- resistance to bending and buckling

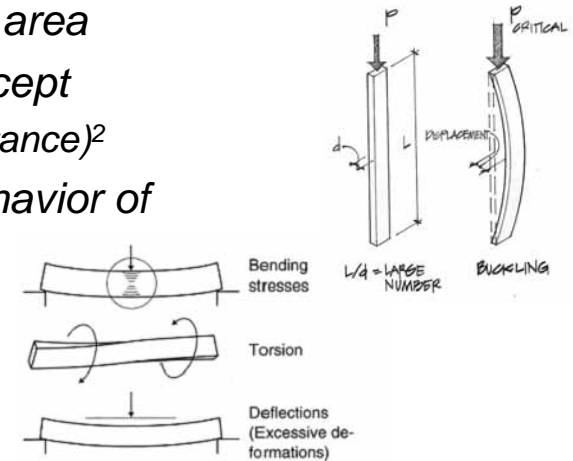
Moment of Inertia 5  
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Moments of Inertia

- 2<sup>nd</sup> moment area
  - math concept
  - area x (distance)<sup>2</sup>
- need for behavior of
  - beams
  - columns



Transverse Loadings

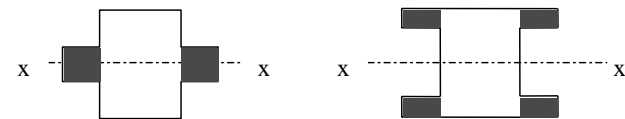
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Moment of Inertia

- same area moved away a distance
  - larger I



Moment of Inertia 6  
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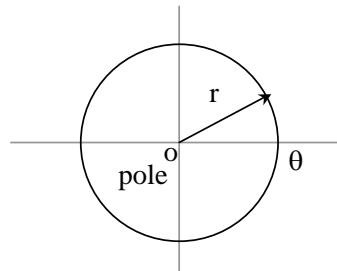
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## Polar Moment of Inertia

- for round-ish shapes
- uses polar coordinates ( $r$  and  $\theta$ )
- resistance to twisting

$$J_o = \int r^2 dA$$



Moment of Inertia 7  
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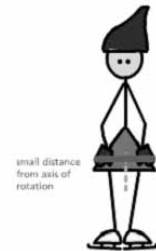
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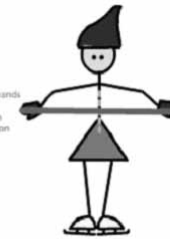
## Radius of Gyration

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$



When a figure skater changes position, he or she is redistributing his or her mass. Thus, every position has its own unique rotational inertia.



The rotational inertia of the figure skater increases when her arms are raised because more of her mass is redistributed further from her axis of rotation.

Moment of Inertia 8  
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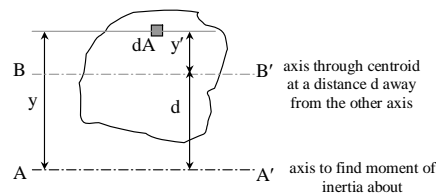
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## Parallel Axis Theorem

- can find composite  $I$  once composite centroid is known (basic shapes)

$$I_x = I_{cx} + Ad_y^2$$

$$= \bar{I}_x + Ad_y^2$$



$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$

Moment of Inertia 9  
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## Basic Procedure

1. Draw reference origin (if not given)
  2. Divide into basic shapes (+/-)
  3. Label shapes
  4. Draw table with  $A$ ,  $\bar{x}$ ,  $\bar{x}A$ ,  $\bar{y}$ ,  $\bar{y}A$ ,  $\bar{I}$ 's,  $d$ 's, and  $Ad^2$ 's
  5. Fill in table and get  $\hat{x}$  and  $\hat{y}$  for composite
  6. Sum necessary columns
  7. Sum  $\bar{I}$ 's and  $Ad^2$ 's
- $$\begin{aligned} (d_x &= \hat{x} - \bar{x}) \\ (d_y &= \hat{y} - \bar{y}) \end{aligned}$$

Moment of Inertia 10  
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# Area Moments of Inertia

• Table 7.2 – pg. 252 (bars refer to centroid)

- $x, y$
- $x', y'$
- $C$

Rectangle		$\bar{I}_x = \frac{1}{12}bh^3$ $\bar{I}_y = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}b^3h$ $I_y = \frac{1}{3}bh^3$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_x = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$