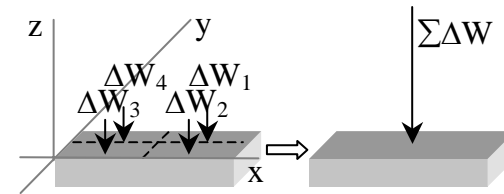




centers of gravity- centroids

Center of Gravity

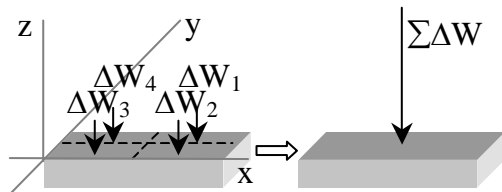
- location of equivalent weight
- determined with calculus



- sum element weights $W = \int dW$

Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- “average” x & y of an area
- for a volume of constant thickness
 - $\Delta W = \gamma \Delta A$ where γ is weight/volume
 - center of gravity = centroid of area

$$\bar{x} = \frac{\sum (x \Delta A)}{A}$$

$$\bar{y} = \frac{\sum (y \Delta A)}{A}$$



Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$



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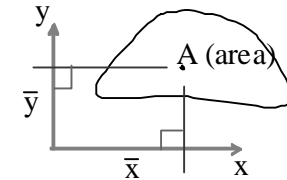
S2004abn

1st Moment Area

- math concept
- the moment of an area about an axis

$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



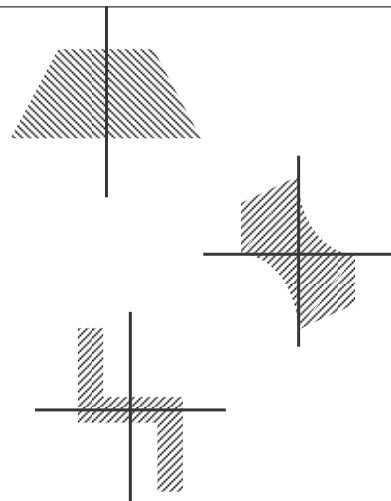
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Symmetric Areas

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry



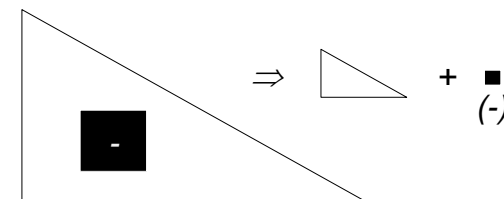
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Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



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Basic Procedure

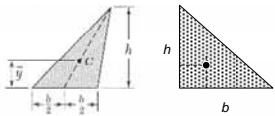
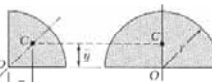

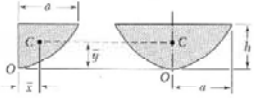

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \hat{x} and \hat{y}

Component	Area	\bar{x}	$\bar{x}A$	\bar{y}	$\bar{y}A$
Σ					

Area Centroids

- Table 7.1 – pg. 242

Centroids of Common Shapes of Areas and Lines

Shape		\bar{x}	\bar{y}
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$