ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

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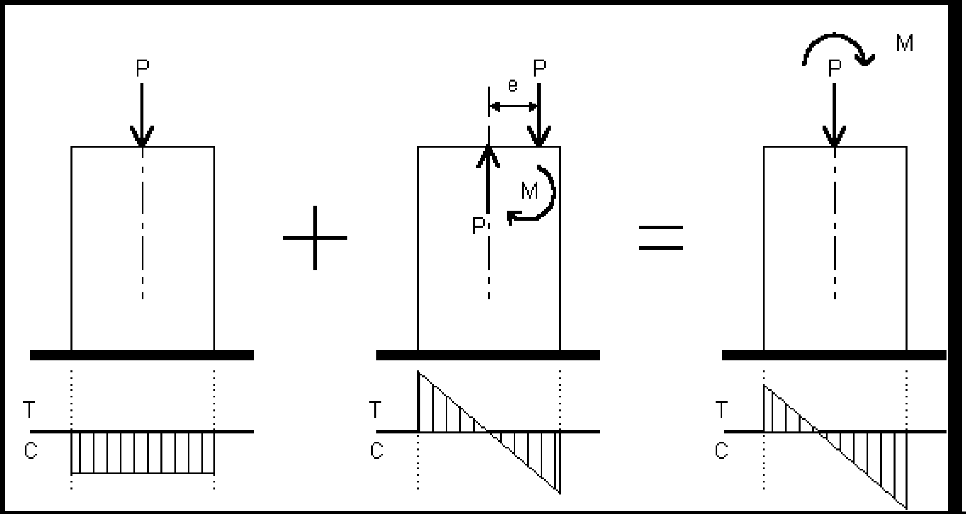
SPRING 2008

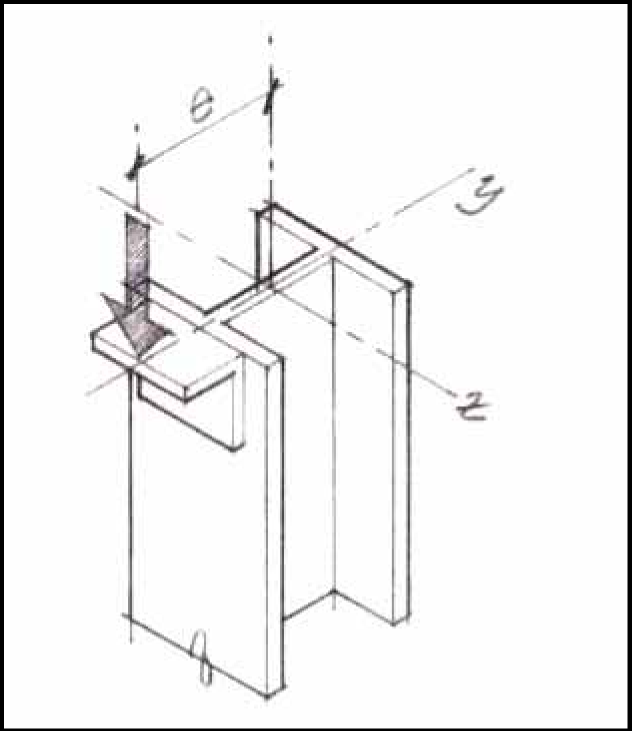
lecture twenty five

eccentric loading: beam-columns

Centric & Eccentric Loading

- centric
  - allowable stress from strength or buckling
- eccentric
  - combined stresses





Eccentric Loading

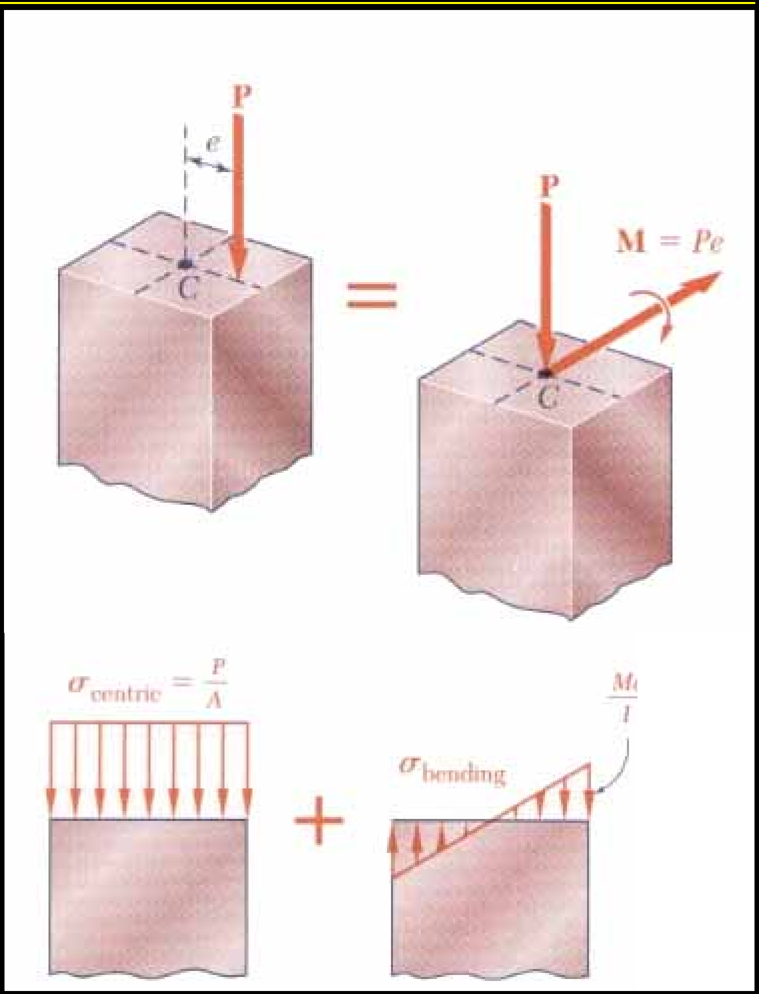
– *axial + bending*

$$f_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$

– *design*

$$f_{max} \le F_{cr} = \frac{f_{cr}}{F.S.}$$

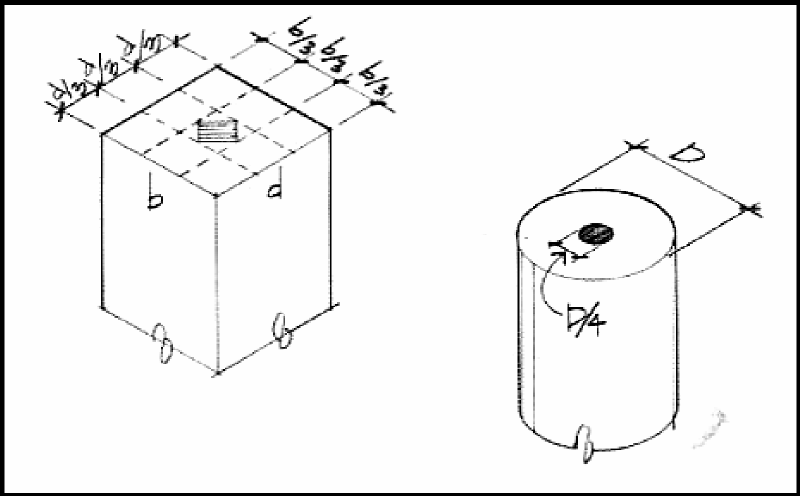


Eccentric Loading

– *find e such that the minimum stress = 0*

$$f_{\min} = \frac{P}{A} - \frac{(Pe)c}{I} = 0$$

– *area defined by e from centroid is the* ***kern***

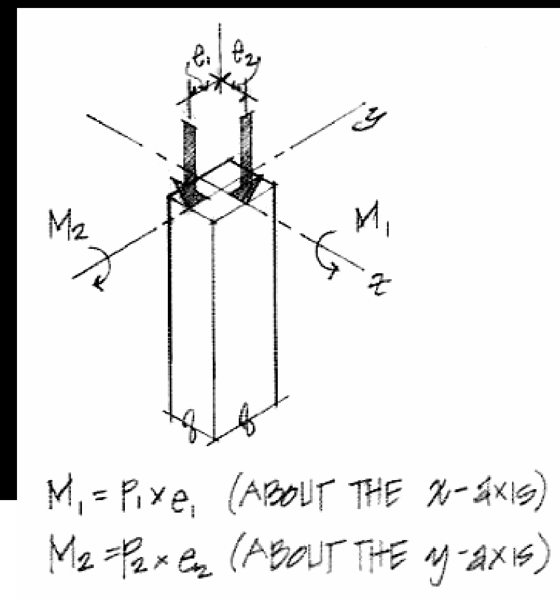


Eccentric Loading

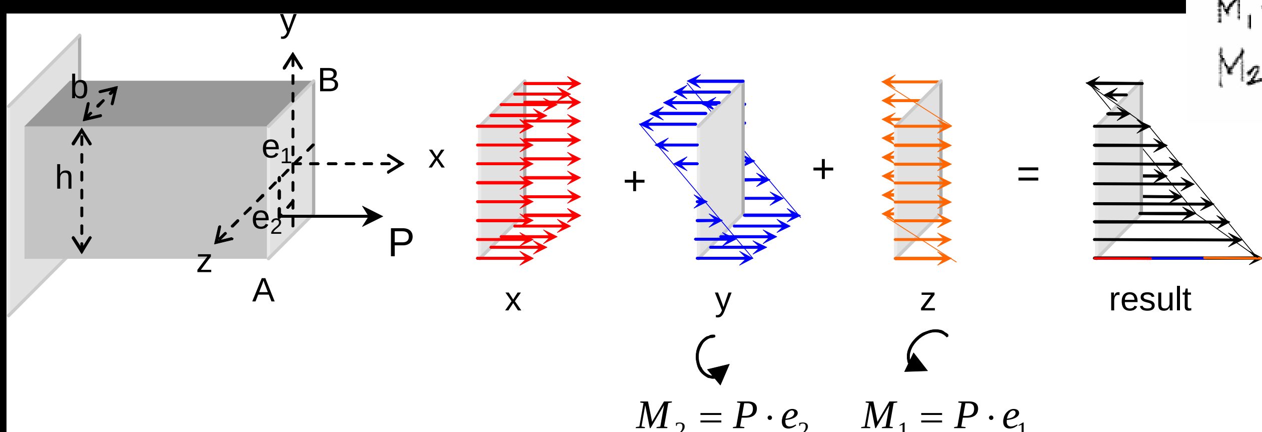
– *when there is eccentricity in two directions*

$$M_1 = P \cdot e_1 \qquad M_2 = P \cdot e_2$$

$$f_{\max} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I}$$



– <u>*biaxial bending*</u>



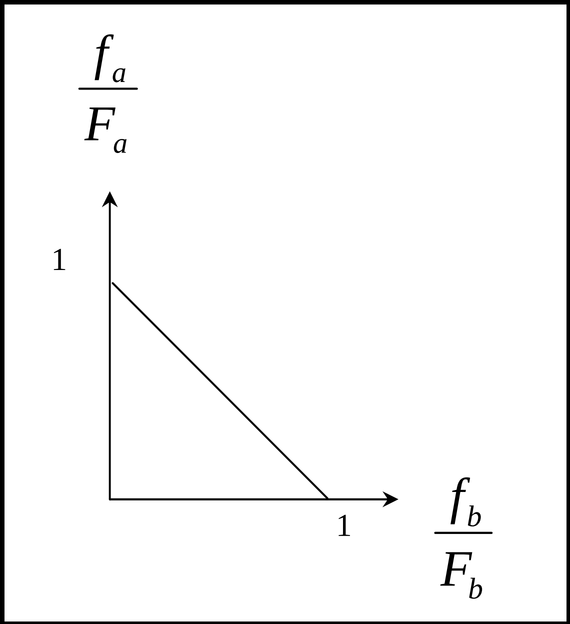
Stress Limit Conditions

– *ASD interaction formula*

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

– *with biaxial bending*

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

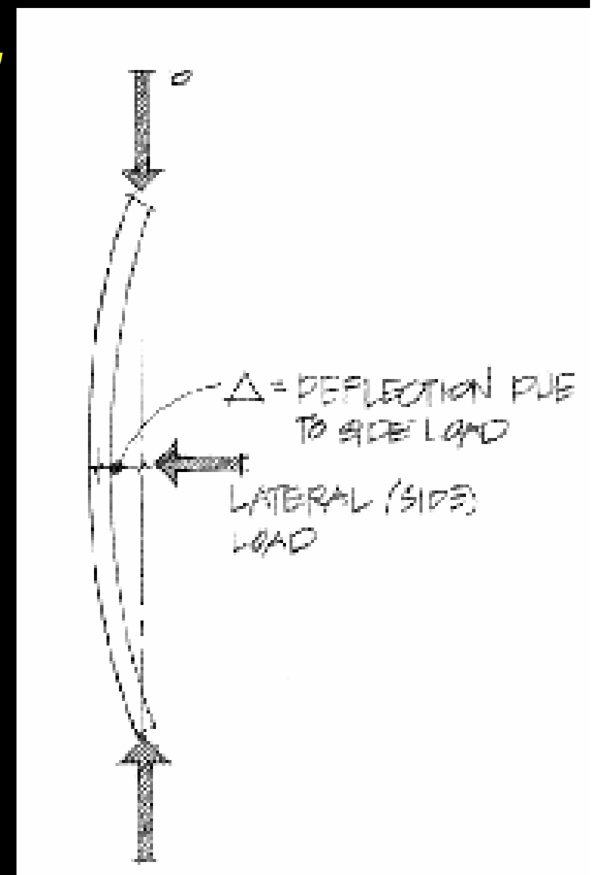


interaction diagram

Stress Limit Conditions

- *in reality, as the column flexes, the moment increases*

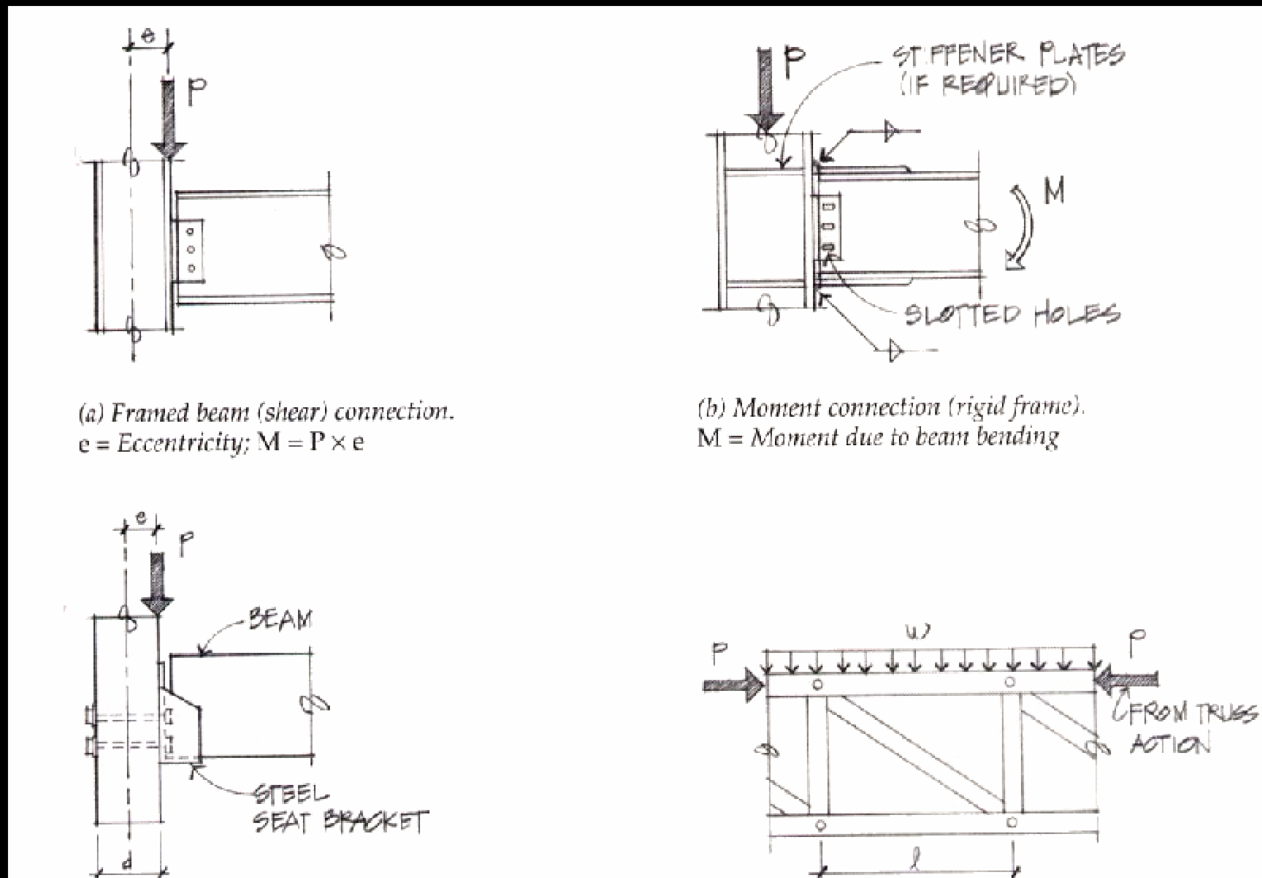
- *<u>P-Δ effect</u>*



$$\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ \ factor)}{F_{bx}} \le 1.0$$

Design

- *satisfy*
  - *strength*
  - *stability*
- *pick*
  - *section*



(a) Framed beam (shear) connection.
e = *Eccentricity;* M = P × e

(b) Moment connection (rigid frame).
M = *Moment due to beam bending*

(c) Timber beam–column connection.
e = d/2 = *eccentricity;* M = P × e

(d) Upper chord of a truss—compression plus bending.
$M = \frac{\omega \ell^2}{8}$

Design

- *ASD Steel*

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

C_m – modification factor for end conditions

= 0.6 – 0.4(M_1/M_2) or 0.85 restrained

F'_e – allowable buckling strength

() term – magnification factor for P-Δ

Design

- *Wood*

$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx}\left[1 - {f_c}/{F_{cEx}}\right]} \le 1.0$$

[] term – magnification factor for P-Δ

F'_{bx} – allowable bending strength

Design

- LRFD Steel
  - for $\frac{P_u}{\phi_c P_n} \geq 0.2:\quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$
  - for $\frac{P_u}{\phi_c P_n} < 0.2:\quad \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0$

ϕ_c - resistance factor for compression = 0.85

ϕ_b - resistance factor for bending = 0.9

Design Steps Knowing Loads

1. assume limiting stress
   - buckling, axial stress, combined stress
2. solve for r, A or S
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok