# ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

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SPRING 2008

lecture
twenty four

# column design

# Design Methods

- *know*
  - *loads or lengths*
- *select*
  - *section or load*
  - *adequate for strength and no buckling*

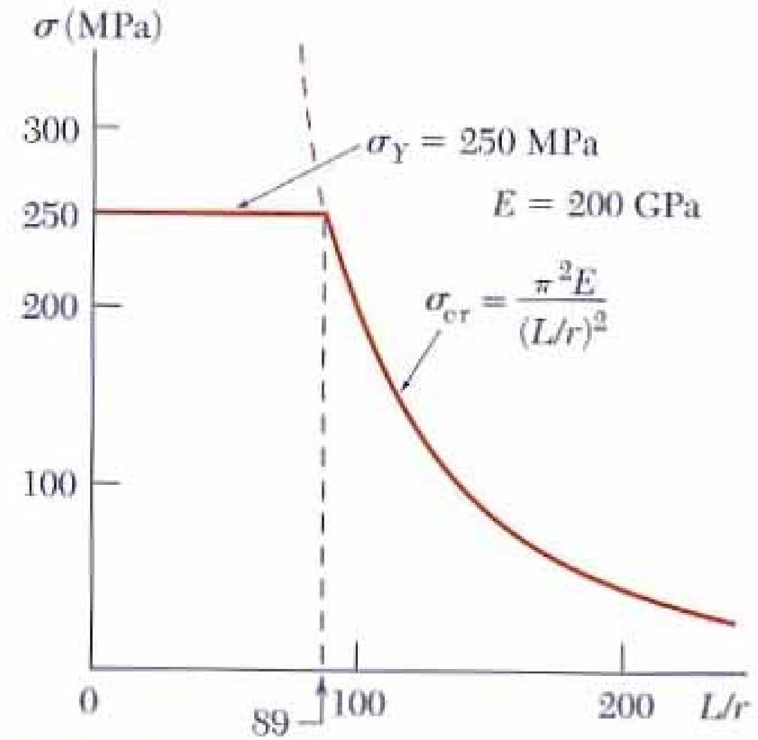


Fig. 10.9

# Allowable Stress Design (ASD)

- AICS 9th ed

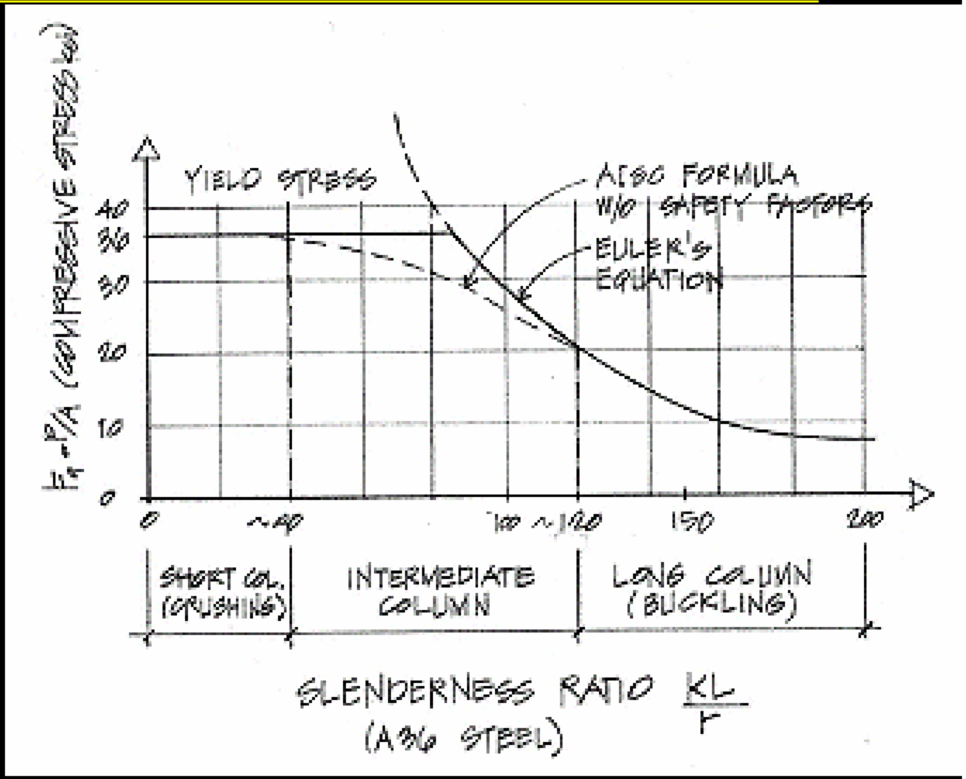
$$F_a = \frac{f_{critical}}{F.S.} = \frac{12\pi^2 E}{23\left(\frac{Kl}{r}\right)^2}$$

- slenderness ratio $\frac{Kl}{r}$

  – for $kl/r \geq C_c$ = 126.1 with $F_y$ = 36 ksi
  = 107.0 with $F_y$ = 50 ksi

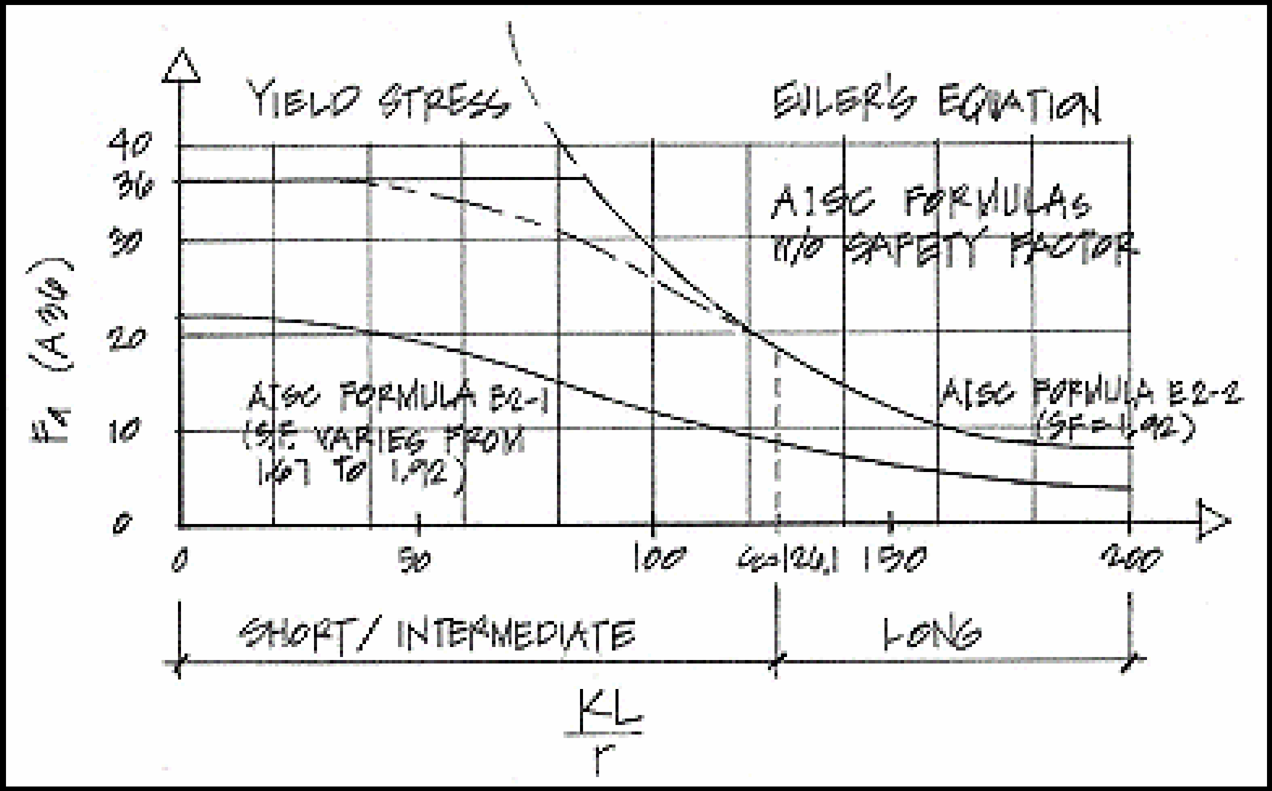
# $C_c$ and Euler's Formula

- *Kl/r < $C_c$*
  - *short and stubby*
  - *parabolic transition*
- *Kl/r > $C_c$*
  - *Euler's relationship*
  - *< 200 preferred*




$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

# $C_c$ and Euler's Formula



# *Short / Intermediate*

- $L_e/r < C_c$

$$F_a = \left[1 - \frac{\left(\frac{Kl}{r}\right)^2}{2C_c^2}\right]\frac{F_y}{F.S.}$$

  - where

$$F.S. = \frac{5}{3} + \frac{3\left(\frac{Kl}{r}\right)}{8C_c} - \frac{\left(\frac{Kl}{r}\right)^3}{8C_c^3}$$

# *Procedure for Analysis*

1. *calculate KL/r*
   - *biggest of KL/r with respect to x axes and y axis*
2. *find $F_a$ from Table 10.1 or 10.2*
   - *pp. 361 - 364*
3. *compute $P_{allowable} = F_a \cdot A$*
   - *or find $f_{actual} = P/A$*
4. *is $P \le P_{allowable}$? (or is $f_{actual} \le F_a$?)*
   - *yes: ok*
   - *no: overstressed and no good*

# *Procedure for Design*

1. *guess a size (pick a section)*
2. *calculate KL/r*
   - *biggest of KL/r with respect to x axes and y axis*
3. *find $F_a$ from Table 10.1 or 10.2*
   - *pp. 361 - 364*
4. *compute $P_{allowable} = F_a \cdot A$*
   - *or find $f_{actual} = P/A$*

# *Procedure for Design (cont'd)*

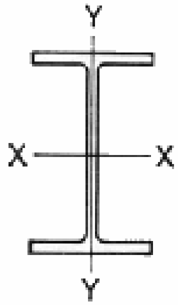
5. *is $P \le P_{allowable}$? (or is $f_{actual} \le F_a$?)*
   - *yes: ok*
   - *no: pick a bigger section and **go back to step 2.***
6. *check design efficiency*
   - *percentage of stress =* $\dfrac{P_{actual}}{P_{allowable}} \cdot 100\%$
   - *if between 90-100%: good*
   - *if < 90%: pick a smaller section and **go back to step 2.***

# Column Charts

Table C-50
Allowable Stress
For Compression Members of 50-ksi Specified Yield Stress Steel[a]

| $\frac{Kl}{r}$ | $F_a$ (ksi) | $\frac{Kl}{r}$ | $F_a$ (ksi) | $\frac{Kl}{r}$ | $F_a$ (ksi) | $\frac{Kl}{r}$ | $F_a$ (ksi) | $\frac{Kl}{r}$ | $F_a$ (ksi) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 29.94 | 41 | 25.69 | 81 | 18.81 | 121 | 10.20 | 161 | 5.76 |
| 2 | 29.87 | 42 | 25.55 | 82 | 18.61 | 122 | 10.03 | 162 | 5.69 |
| 3 | 29.80 | 43 | 25.40 | 83 | 18.41 | 123 | 9.87 | 163 | 5.62 |
| 4 | 29.73 | 44 | 25.26 | 84 | 18.20 | 124 | 9.71 | 164 | 5.55 |
| 5 | 29.66 | 45 | 25.11 | 85 | 17.99 | 125 | 9.56 | 165 | 5.49 |
| 6 | 29.58 | 46 | 24.96 | 86 | 17.79 | 126 | 9.41 | 166 | 5.42 |
| 7 | 29.50 | 47 | 24.81 | 87 | 17.58 | 127 | 9.26 | 167 | 5.35 |
| 8 | 29.42 | 48 | 24.66 | 88 | 17.37 | 128 | 9.11 | 168 | 5.29 |
| 9 | 29.34 | 49 | 24.51 | 89 | 17.15 | 129 | 8.97 | 169 | 5.23 |
| 10 | 29.26 | 50 | 24.35 | 90 | 16.94 | 130 | 8.84 | 170 | 5.17 |
| 11 | 29.17 | 51 | 24.19 | 91 | 16.72 | 131 | 8.70 | 171 | 5.11 |
| 12 | 29.08 | 52 | 24.04 | 92 | 16.50 | 132 | 8.57 | 172 | 5.05 |
| 13 | 28.99 | 53 | 23.88 | 93 | 16.29 | 133 | 8.44 | 173 | 4.99 |
| 14 | 28.90 | 54 | 23.72 | 94 | 16.06 | 134 | 8.32 | 174 | 4.93 |
| 15 | 28.80 | 55 | 23.55 | 95 | 15.84 | 135 | 8.19 | 175 | 4.88 |
| 16 | 28.71 | 56 | 23.39 | 96 | 15.62 | 136 | 8.07 | 176 | 4.82 |
| 17 | 28.61 | 57 | 23.22 | 97 | 15.39 | 137 | 7.96 | 177 | 4.77 |

# Column Charts




COLUMNS

W shapes

Allowable axial loads in kips

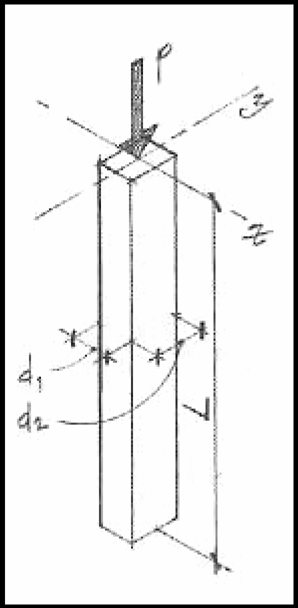
| | $F_y$ = 36 ksi | $F_y$ = 50 ksi |
|---|---|---|

| Designation | W8 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Wt./ft. | 67 | | 58 | | 48 | | 40 | | 35 | | 31 | |
| $F_y$ | 36 | 50 | 36 | 50 | 36 | 50 | 36 | 50 | 36 | 50 | 36 | 50 |
| 0 | 426 | 591 | 369 | 513 | 305 | 423 | 253 | 351 | 222 | 309 | 197 | 274 |
| 6 | 387 | 525 | 336 | 455 | 276 | 375 | 229 | 310 | 201 | 272 | 178 | 241 |
| 7 | 379 | 510 | 328 | 442 | 270 | 363 | 223 | 300 | 197 | 264 | 174 | 234 |
| 8 | 370 | 494 | 320 | 428 | 263 | 352 | 218 | 290 | 191 | 255 | 170 | 226 |
| 9 | 360 | 477 | 312 | 413 | 256 | 339 | 212 | 279 | 186 | 246 | 165 | 217 |
| 10 | 350 | 459 | 303 | 397 | 249 | 326 | 205 | 268 | 180 | 236 | 160 | 208 |
| 11 | 339 | 440 | 293 | 380 | 241 | 312 | 199 | 256 | 174 | 225 | 154 | 199 |
| 12 | 328 | 420 | 283 | 363 | 233 | 297 | 192 | 244 | 168 | 214 | 149 | 189 |
| 13 | 316 | 399 | 273 | 344 | 224 | 282 | 184 | 231 | 162 | 202 | 143 | 179 |
| 14 | 304 | 378 | 263 | 325 | 215 | 266 | 177 | 217 | 155 | 190 | 137 | 168 |
| 15 | 292 | 355 | 251 | 305 | 206 | 249 | 169 | 203 | 148 | 177 | 131 | 156 |
| 16 | 279 | 331 | 240 | 284 | 196 | 232 | 160 | 188 | 141 | 164 | 124 | 145 |
| 17 | 265 | 307 | 228 | 263 | 186 | 214 | 152 | 172 | 133 | 150 | 117 | 132 |

spect to least radius of gyration $r_y$

# Wood Columns

- slenderness ratio = $L/d_{min} = L/d_1$
  - $d_1$ = smaller dimension
  - $l_e/d \leq 50$ (max)




$$f_c = \frac{P}{A} \leq F'_c$$

- where $F'_c$ is the allowable compressive strength parallel to the grain

# Allowable Wood Stress

$$F_c' = F_c\left(C_D\right)\left(C_M\right)\left(C_t\right)\left(C_F\right)\left(C_p\right)$$

- where:

  $F_c$ = compressive strength parallel to grain

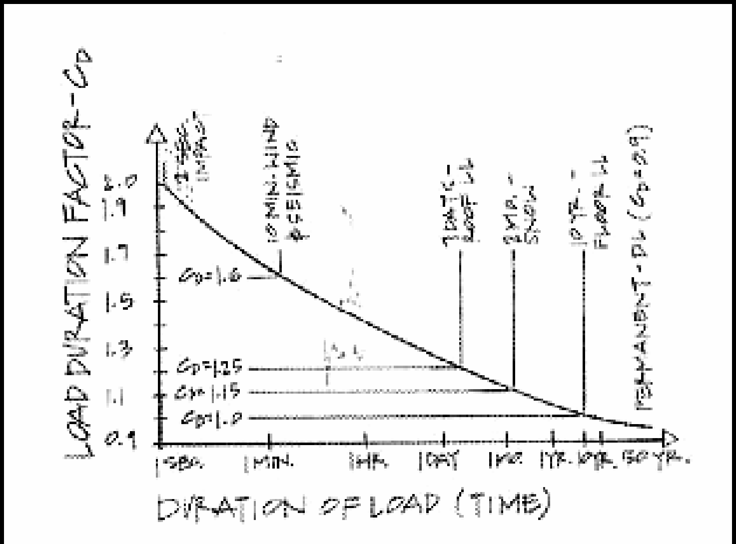
  $C_D$ = load duration factor

  $C_M$ = wet service factor (1.0 dry)

  $C_t$ = temperature factor

  $C_F$ = size factor

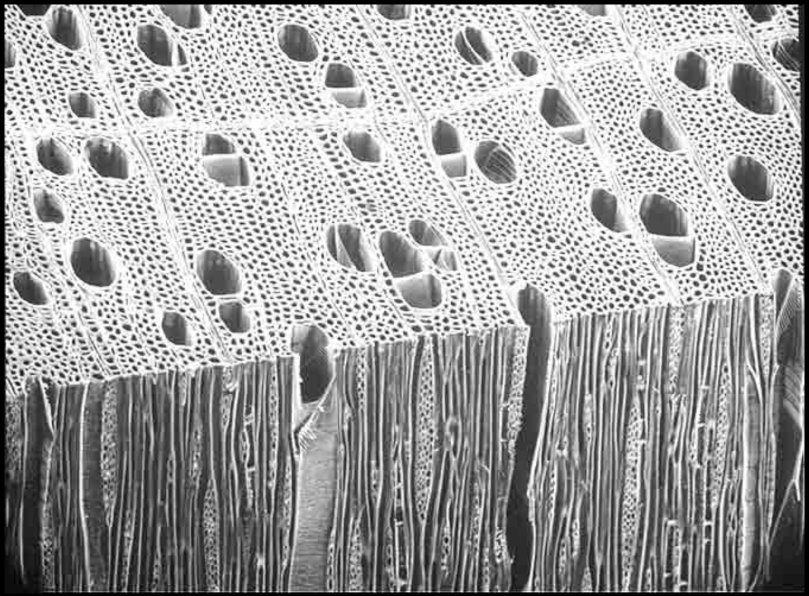
  $C_p$ = column stability factor




(Table 10.3)

# Strength Factors

- wood properties and load duration, $C_D$
  - short duration
    - higher loads
  - normal duration
    - > 10 years



- stability, $C_p$
  - combination curve - tables

$$F_c' = F_c^* C_p = \left(F_c C_D\right) C_p$$

# $C_p$ Charts – Appendix A, Table 14

## Column Stability Factor $C_p$

"$C_p$"

$F_c' = C_p \cdot F_c^*$  $F_{cE} = \frac{.30\,E}{(L/d)^2}$ for sawn posts  $F_{cE} = \frac{.418\,E}{(L/d)^2}$ for Glu-Lam posts

| $F_{cE}/F_c^*$ | Sawn $C_p$ | Glu-Lam $C_p$ | $F_{cE}/F_c^*$ | Sawn $C_p$ | Glu-Lam $C_p$ | $F_{cE}/F_c^*$ | Sawn $C_p$ | Glu-Lam $C_p$ | $F_{cE}/F_c^*$ | Sawn $C_p$ | Glu-Lam $C_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.000 | 0.000 | 0.60 | 0.500 | 0.538 | 1.20 | 0.750 | 0.822 | 2.40 | 0.894 | 0.940 |
| 0.01 | 0.010 | 0.010 | 0.61 | 0.506 | 0.545 | 1.22 | 0.755 | 0.826 | 2.45 | 0.897 | 0.941 |
| 0.02 | 0.020 | 0.020 | 0.62 | 0.512 | 0.552 | 1.24 | 0.760 | 0.831 | 2.50 | 0.899 | 0.943 |
| 0.03 | 0.030 | 0.030 | 0.63 | 0.518 | 0.559 | 1.26 | 0.764 | 0.836 | 2.55 | 0.901 | 0.944 |
| 0.04 | 0.040 | 0.040 | 0.64 | 0.524 | 0.566 | 1.28 | 0.769 | 0.840 | 2.60 | 0.904 | 0.946 |
| 0.05 | 0.049 | 0.050 | 0.65 | 0.530 | 0.573 | 1.30 | 0.773 | 0.844 | 2.65 | 0.906 | 0.947 |
| 0.06 | 0.059 | 0.060 | 0.66 | 0.536 | 0.580 | 1.32 | 0.777 | 0.848 | 2.70 | 0.908 | 0.949 |
| 0.07 | 0.069 | 0.069 | 0.67 | 0.542 | 0.587 | 1.34 | 0.781 | 0.852 | 2.75 | 0.910 | 0.950 |
| 0.08 | 0.079 | 0.079 | 0.68 | 0.548 | 0.593 | 1.36 | 0.785 | 0.855 | 2.80 | 0.912 | 0.951 |
| 0.09 | 0.088 | 0.089 | 0.69 | 0.553 | 0.600 | 1.38 | 0.789 | 0.859 | 2.85 | 0.914 | 0.952 |
| 0.10 | 0.098 | 0.099 | 0.70 | 0.559 | 0.607 | 1.40 | 0.793 | 0.862 | 2.90 | 0.916 | 0.953 |
| 0.11 | 0.107 | 0.109 | 0.71 | 0.564 | 0.613 | 1.42 | 0.796 | 0.865 | 2.95 | 0.917 | 0.954 |
| 0.12 | 0.117 | 0.118 | 0.72 | 0.569 | 0.619 | 1.44 | 0.800 | 0.868 | 3.00 | 0.919 | 0.955 |
| 0.13 | 0.126 | 0.128 | 0.73 | 0.575 | 0.626 | 1.46 | 0.803 | 0.871 | 3.05 | 0.920 | 0.956 |
| 0.14 | 0.136 | 0.138 | 0.74 | 0.580 | 0.632 | 1.48 | 0.807 | 0.874 | 3.10 | 0.922 | 0.957 |
| 0.15 | 0.145 | 0.147 | 0.75 | 0.585 | 0.638 | 1.50 | 0.810 | 0.877 | 3.15 | 0.923 | 0.958 |
| 0.16 | 0.154 | 0.157 | 0.76 | 0.590 | 0.644 | 1.52 | 0.813 | 0.879 | 3.20 | 0.925 | 0.959 |
| 0.17 | 0.164 | 0.167 | 0.77 | 0.595 | 0.650 | 1.54 | 0.816 | 0.882 | 3.25 | 0.926 | 0.960 |
| 0.18 | 0.173 | 0.176 | 0.78 | 0.600 | 0.655 | 1.56 | 0.819 | 0.884 | 3.30 | 0.927 | 0.961 |
| 0.19 | 0.182 | 0.186 | 0.79 | 0.605 | 0.661 | 1.58 | 0.822 | 0.887 | 3.35 | 0.929 | 0.961 |

# Column Charts – Appendix A, 12 & 13

*Table 12 Allowable Column Loads—Selected Species/Sizes. (Continued)*

| Eff. Col. Len(ft) | *l/d* | (*l/d*)sq | *Fce* | *Fce/Fc** Norm | Snow | *Cp* Norm | Snow | *F'c*(psi) Norm | Snow | 8×8 *Pa* (*k*) Norm | *A* = 56.25 Snow | 8×10 *Pa* (*k*) Norm | *A* = 71.25 Snow | 8×12 *Pa* Norm | *A* = 86.25 Snow |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 19.2 | 368.64 | 1302.08 | 1.30 | 1.13 | .7731 | .7315 | 773 | 841 | 43.5 | 47.3 | 55.1 | 59.9 | 66.7 | 72.6 |
| 13 | 20.8 | 432.64 | 1109.47 | 1.11 | 0.96 | .7258 | .6767 | 726 | 778 | 40.8 | 43.8 | 51.7 | 55.4 | 62.6 | 67.1 |
| 14 | 22.4 | 501.76 | 956.63 | 0.96 | 0.83 | .6767 | .6235 | 677 | 717 | 38.1 | 40.3 | 48.2 | 51.1 | 58.4 | 61.8 |
| 15 | 24.00 | 576.00 | 833.33 | 0.83 | 0.72 | .6235 | .5694 | 624 | 655 | 35.1 | 36.8 | 44.4 | 46.7 | 53.8 | 56.5 |
| 16 | 25.60 | 655.36 | 732.42 | 0.73 | 0.64 | .5747 | .5244 | 575 | 603 | 32.3 | 33.9 | 40.9 | 43.0 | 49.6 | 52.0 |
| 17 | 27.20 | 739.84 | 648.79 | 0.65 | 0.56 | .5303 | .4744 | 530 | 546 | 29.8 | 30.7 | 37.8 | 38.9 | 45.7 | 47.1 |
| 18 | 28.80 | 829.44 | 578.70 | 0.58 | 0.50 | .4873 | .4336 | 487 | 499 | 27.4 | 28.0 | 34.7 | 35.5 | 42.0 | 43.0 |
| 19 | 30.40 | 924.16 | 519.39 | 0.52 | 0.45 | .4475 | .3975 | 448 | 457 | 25.2 | 25.7 | 31.9 | 32.6 | 38.6 | 39.4 |
| 20 | 32.00 | 1024.00 | 468.75 | 0.47 | 0.41 | .4122 | .3673 | 412 | 422 | 23.2 | 23.8 | 29.4 | 30.1 | 35.6 | 36.4 |
| 21 | 33.60 | 1128.96 | 425.17 | 0.43 | 0.37 | .3826 | .3360 | 383 | 386 | 21.5 | 21.7 | 27.3 | 27.5 | 33.0 | 33.3 |
| 22 | 35.20 | 1239.04 | 387.40 | 0.39 | 0.34 | .3518 | .3118 | 352 | 359 | 19.8 | 20.2 | 25.1 | 25.5 | 30.3 | 30.9 |
| 23 | 36.80 | 1354.24 | 354.44 | 0.35 | 0.31 | .3199 | .2869 | 320 | 330 | 18.0 | 18.6 | 22.8 | 23.5 | 27.6 | 28.5 |
| 24 | 38.40 | 1474.56 | 325.52 | 0.33 | 0.28 | .3035 | .2615 | 304 | 301 | 17.1 | 16.9 | 21.6 | 21.4 | 26.2 | 25.9 |
| 25 | 40.00 | 1600.00 | 300.00 | 0.30 | 0.26 | .2785 | .2442 | 279 | 281 | 15.7 | 15.8 | 19.8 | 20.0 | 24.0 | 24.2 |
| 26 | 41.60 | 1730.56 | 277.37 | 0.28 | 0.24 | .2615 | .2267 | 262 | 261 | 14.7 | 14.7 | 18.6 | 18.6 | 22.6 | 22.5 |
| 27 | 43.20 | 1866.24 | 257.20 | 0.26 | 0.22 | .2442 | .2090 | 244 | 240 | 13.7 | 13.5 | 17.4 | 17.1 | 21.1 | 20.7 |
| 28 | 44.80 | 2007.04 | 239.16 | 0.24 | 0.21 | .2267 | .2000 | 227 | 230 | 12.8 | 12.9 | 16.2 | 16.4 | 19.6 | 19.8 |
| 29 | 46.40 | 2152.96 | 222.95 | 0.22 | 0.19 | .2090 | .1819 | 209 | 209 | 11.8 | 11.8 | 14.9 | 14.9 | 18.0 | 18.0 |
| 30 | 48.00 | 2304.00 | 208.33 | 0.21 | 0.18 | .2000 | .1728 | 200 | 199 | 11.3 | 11.2 | 14.3 | 14.2 | 17.3 | 17.1 |
| | DF-L No.1 | | (P&T) | | *Fc* = 1000 | | *E* = 1.6 | | | | | | | | |
| | DF-L No.1 & Btr | | Dim.Lum | | *Fc* = 1500 | | *E* = 1.8 | | | | | | | | |

# Procedure for Analysis

1. calculate $L_e/d_{min}$
2. obtain $F'_c$
   – compute $$F_{cE} = \frac{K_{cE}E}{\left(L_e/d\right)^2}$$
     - $K_{cE}$ =0.3 sawn
     - $K_{cE}$ = 0.418 glu-lam
3. compute $F_c^* \approx F_c C_D$
4. calculate $F_{cE}/F_c^*$ and get $C_p$ (table 14)
5. calculate $F'_c = F_c^* C_p$

# *Procedure for Analysis (cont'd)*

6. *compute* $P_{allowable} = F'_c \cdot A$
   - *or find* $f_{actual} = P/A$
7. *is* $P \leq P_{allowable}$*?* *(or* $f_{actual} \leq F'_c$*?)*
   - *yes: OK*
   - *no: overstressed & no good*

# *Procedure for Design*

1. *guess a size (pick a section)*
2. *calculate $L_e/d_{min}$*
3. *obtain $F'_c$* $$F_{cE} = \frac{K_{cE}E}{\left(\frac{L_e}{d}\right)^2}$$
   - *compute*
     - $K_{cE}$ *=0.3 sawn*
     - $K_{cE}$ *= 0.418 glu-lam*
4. *compute* $F_c^* \approx F_c C_D$
5. *calculate $F_{cE}/F_c^*$ and get $C_p$ (table 14)*
6. *calculate* $F'_c = F_c^* C_p$

# *Procedure for Design (cont'd)*

6. *compute* $P_{allowable} = F'_c \cdot A$
   - *or find* $f_{actual} = P/A$
7. *is* $P \leq P_{allowable}$*?* *(or* $f_{actual} \leq F'_c$*?)*
   - *yes: OK*
   - *no: pick a bigger section and **go back to step 2**.*

# LRFD design

- *limit states for failure* $P_u \le \phi_c P_n$

$$\phi_c = 0.85 \quad P_n = F_{cr} A_g$$

1. *yielding* $\lambda_c \le 1.5$
2. *buckling* $\lambda_c > 1.5$

$$\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}} \qquad L_e/r$$

$\lambda_c$ – *column slenderness parameter*

$A_g$ - *gross area*

# *Compact Sections*

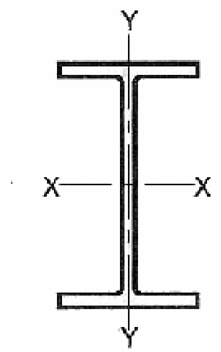
- *flanges continuously connected to the web or webs and width-thickness rations < limiting values*
  - *no local buckling of flange or web*
  - *for* $\lambda_c \leq 1.5$ $$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y$$
  - *for* $\lambda_c > 1.5$ $$F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right]F_y$$

# Column Charts

$F_y = 50$ ksi

$\phi_c P_n = 0.85 F_{cr} A_g$

**Table 4-2 (cont.).**
**W-Shapes**
**Design Strength in Axial Compression, $\phi_c P_n$, kips**




| | Shape | W12× | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 106 | 96 | 87 | 79 | 72 | 65†† | 58 | 53 | 50 | 45 | 40 |
| adius of gyration $r_y$ | 0 | 1330 | 1200 | 1090 | 986 | 897 | 812 | 723 | 663 | 621 | 557 | 497 |
| | 6 | 1280 | 1150 | 1050 | 947 | 861 | 779 | 680 | 623 | 562 | 504 | 450 |
| | 7 | 1260 | 1140 | 1030 | 933 | 848 | 767 | 666 | 610 | 543 | 486 | 434 |
| | 8 | 1240 | 1120 | 1010 | 917 | 834 | 754 | 649 | 594 | 521 | 466 | 416 |
| | 9 | 1210 | 1100 | 994 | 900 | 818 | 739 | 631 | 577 | 497 | 445 | 396 |
| | 10 | 1190 | 1070 | 973 | 880 | 800 | 723 | 611 | 559 | 472 | 422 | 376 |
| | 11 | 1160 | 1050 | 950 | 860 | 781 | 706 | 590 | 539 | 445 | 398 | 354 |
| | 12 | 1130 | 1020 | 926 | 838 | 761 | 687 | 568 | 518 | 418 | 374 | 332 |
| | 13 | 1100 | 995 | 901 | 814 | 740 | 668 | 545 | 496 | 390 | 349 | 310 |
| | 14 | 1070 | 966 | 874 | 790 | 717 | 647 | 521 | 474 | 363 | 324 | 287 |