ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

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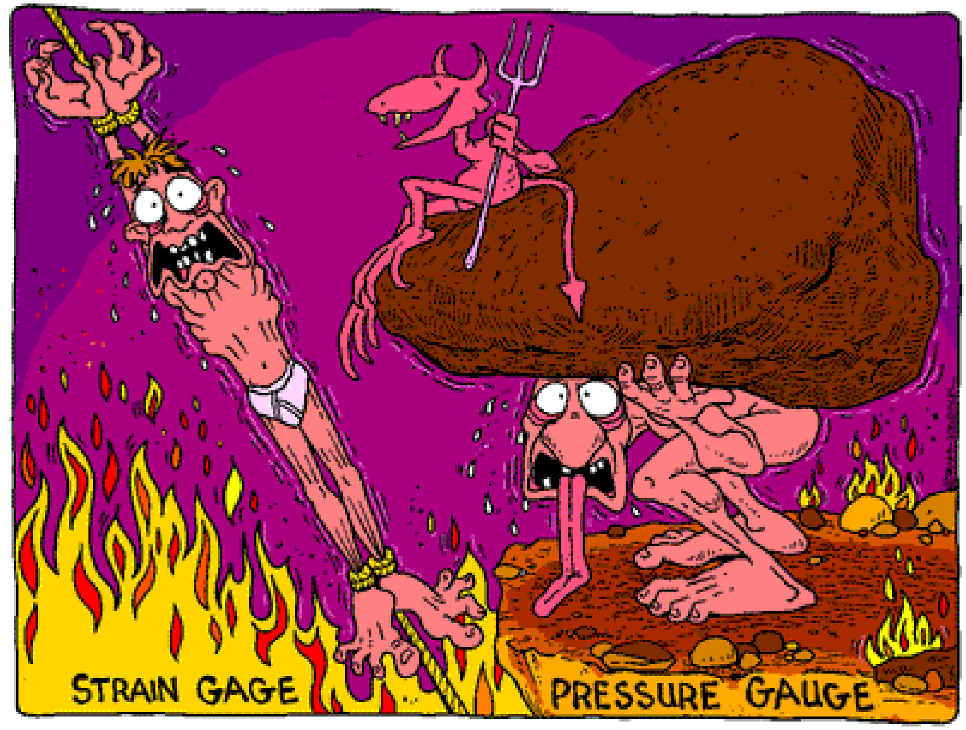
SPRING 2008

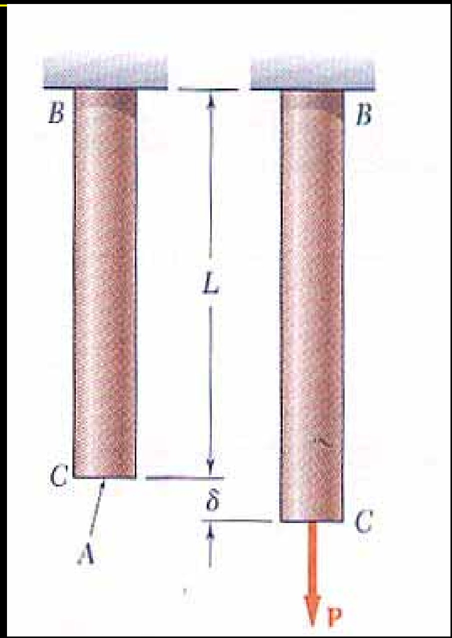
lecture
sixteen



elasticity & strain

Deformations

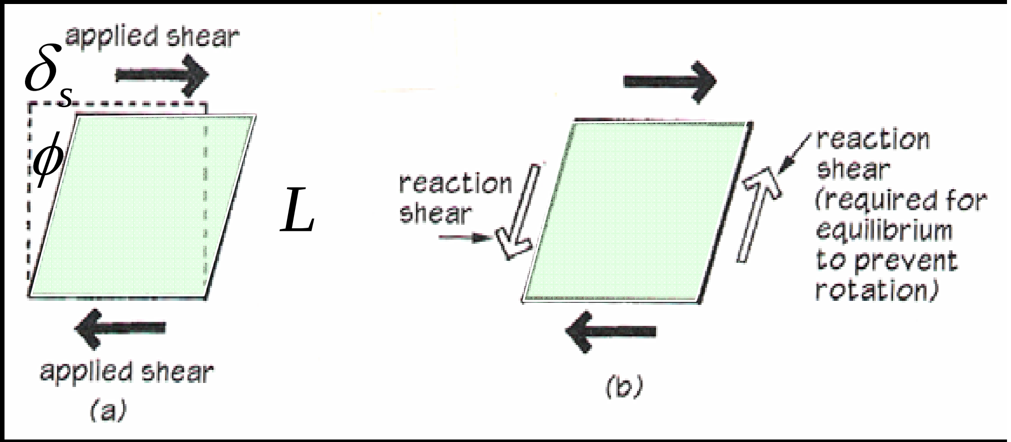
- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS



$$\varepsilon = \frac{\delta}{L}$$

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*



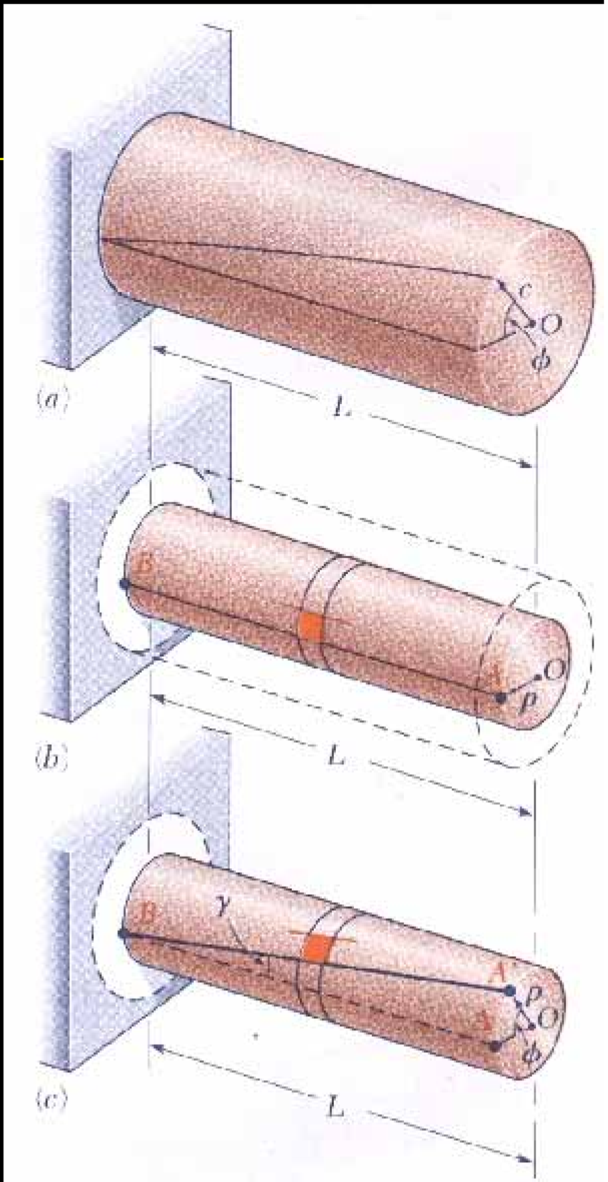
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

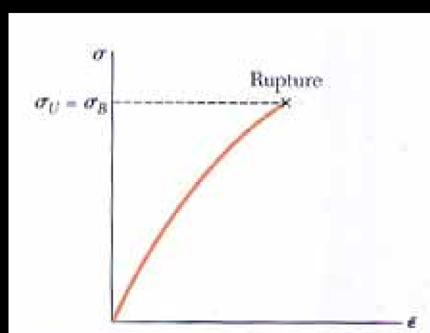
- deformations with torsion
- twist
- change in angle of line
- stress: τ
- strain: γ
  - unitless (radians)

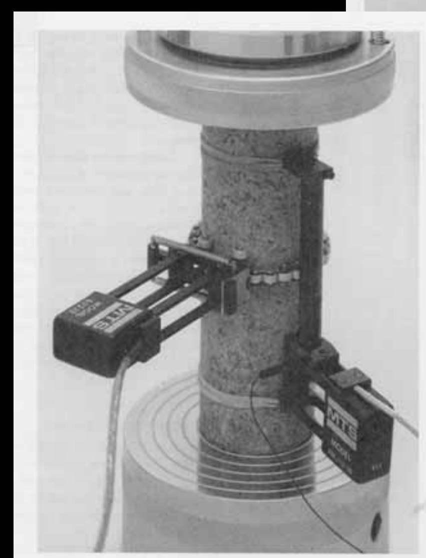
$$\gamma = \frac{\rho\phi}{L}$$

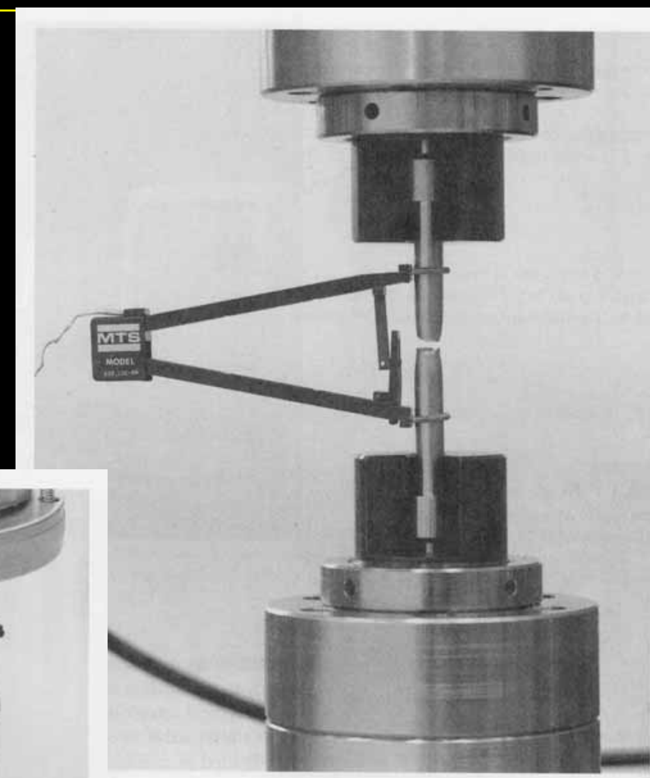


Load and Deformation

- for stress, need P & A
- for strain, need δ & L
  - how?
  - TEST with load and measure
  - plot P/A vs. ε







Material Behavior

- every material has its own response
  - 10,000 psi
  - L = 10 in
  - Douglas Fir vs. steel?

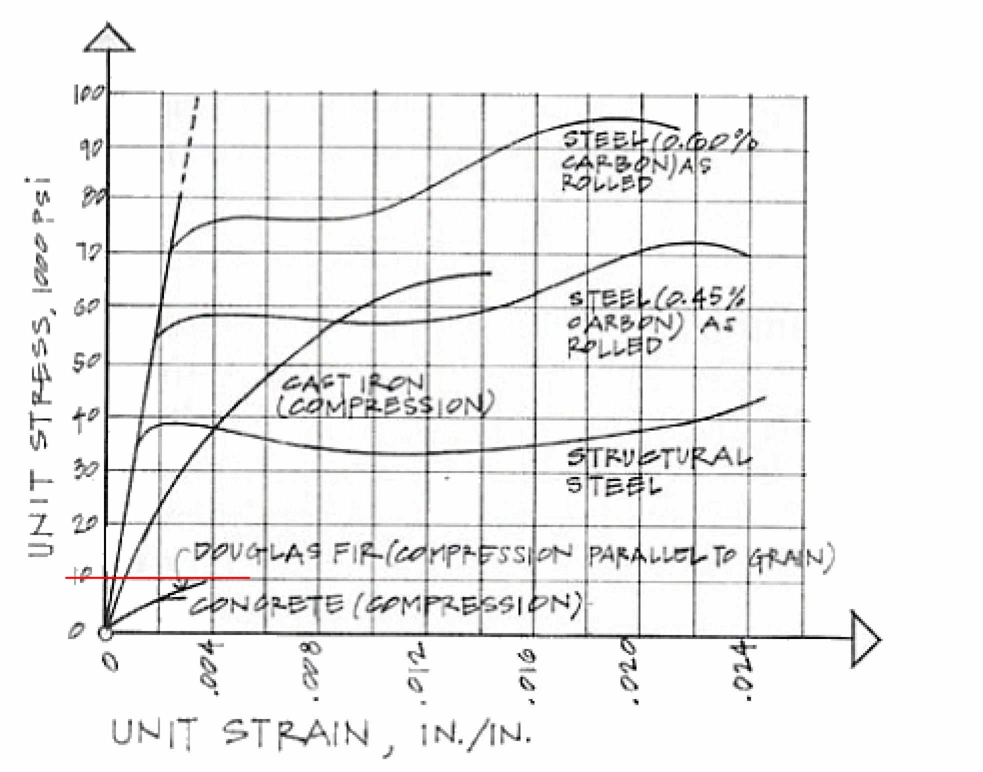


Figure 5.20 Stress-strain diagram for various materials.

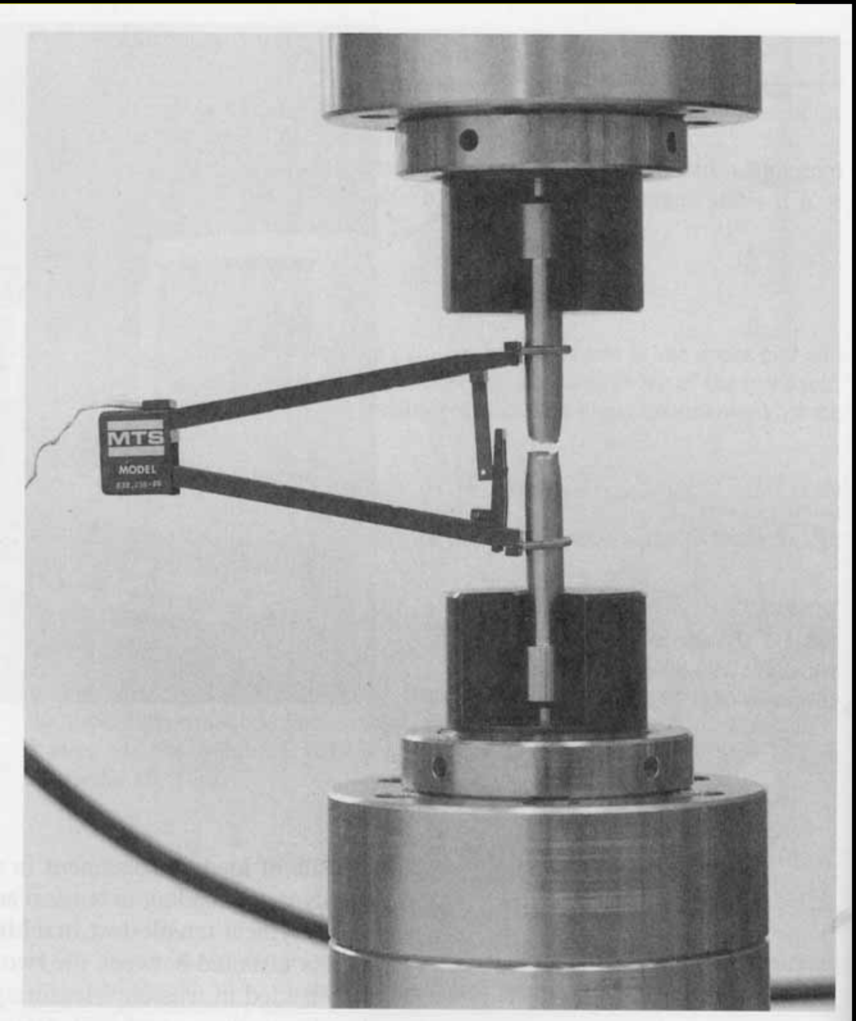
Behavior Types

- *ductile - "necking"*
- *true stress*

$$f = \frac{P}{A}$$

- *engineering stress*
  - *(simplified)*

$$f = \frac{P}{A_o}$$



Behavior Types

- *brittle*

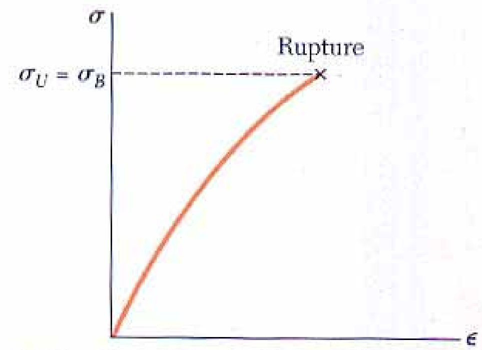


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

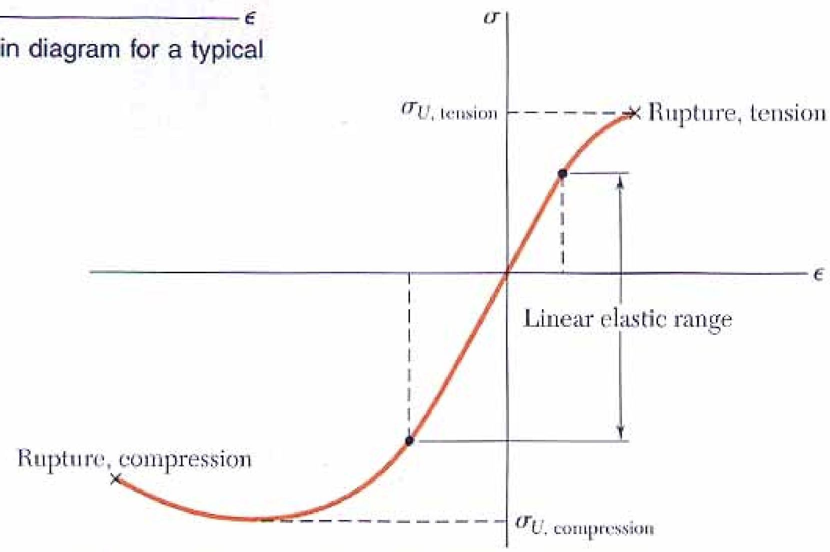


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f-ε diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

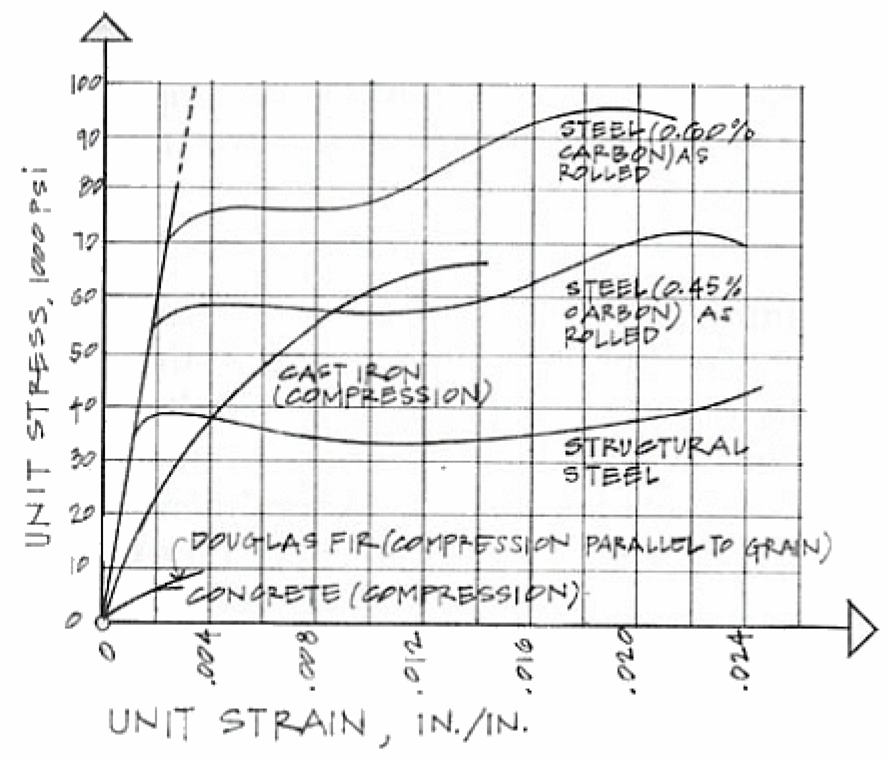
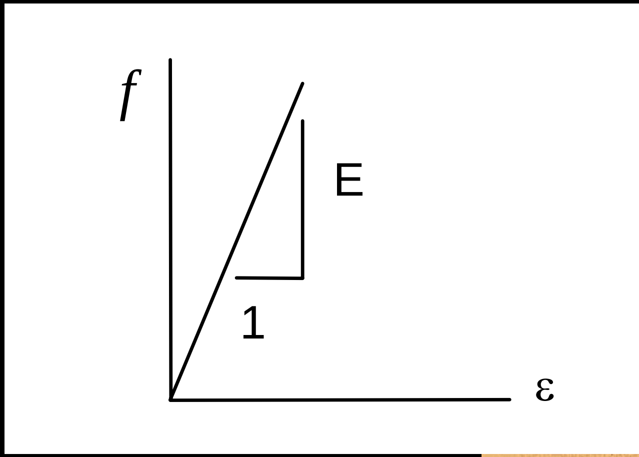


Figure 5.20 Stress-strain diagram for various materials.

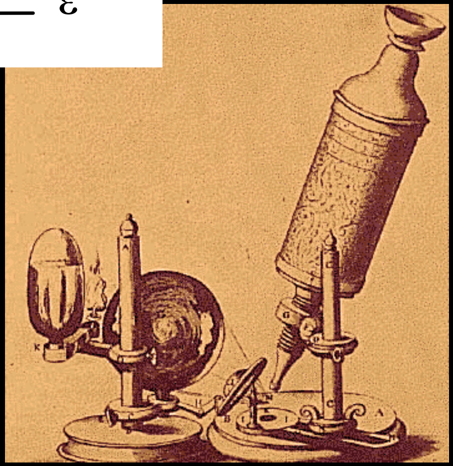
Hooke's Law

- *straight line has constant slope*
- *Hooke's Law*

$$f = E \cdot \varepsilon$$



- *E*
  - *Modulus of elasticity*
  - *Young's modulus*
  - *units just like stress*



Stiffness

- ability to resist strain
- steels
  - same E
  - different yield points
  - different ultimate strength

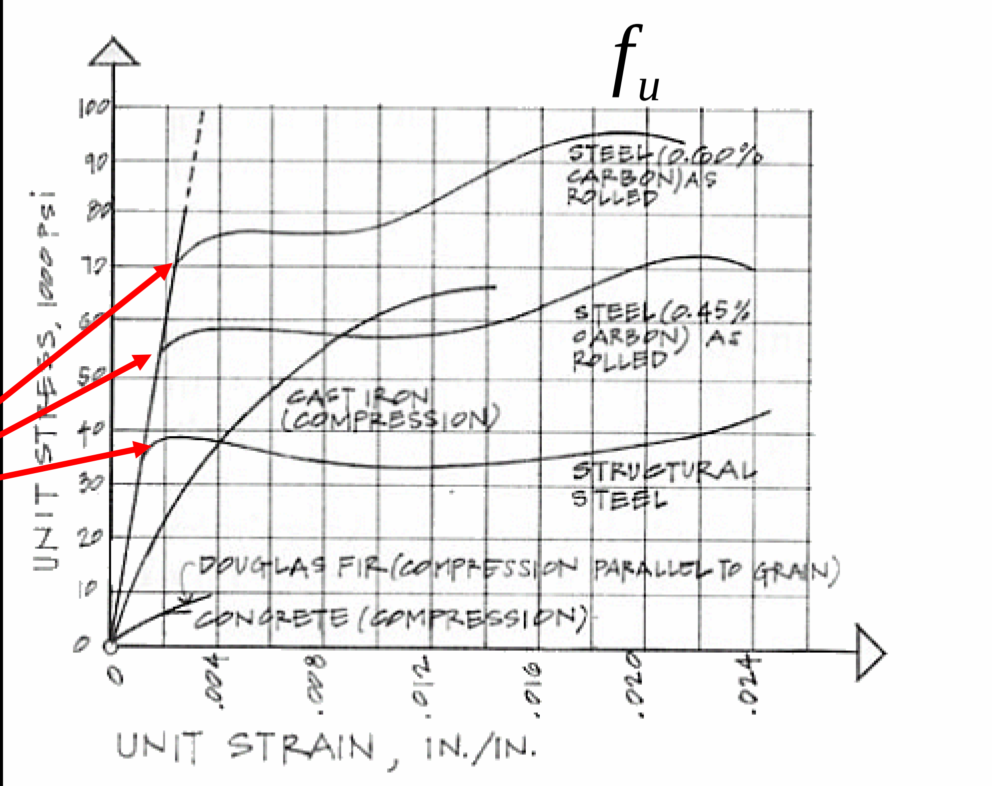
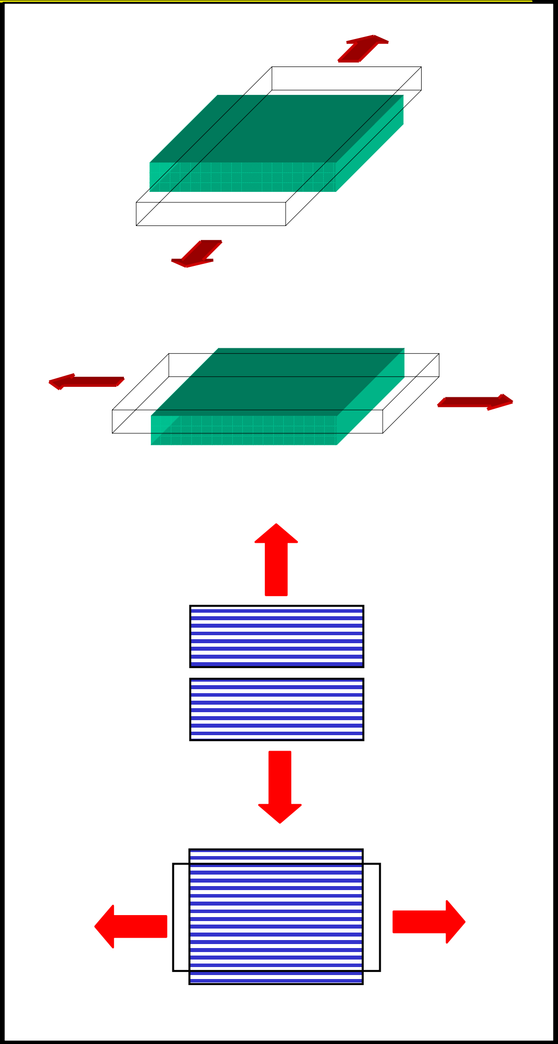


Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

- ISOTROPIC
  - materials with E <u>same</u> at any direction of loading
  - ex. steel
- ANISOTROPIC
  - materials with <u>different</u> E at any direction of loading
  - ex. wood is <u>orthotropic</u>



Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles

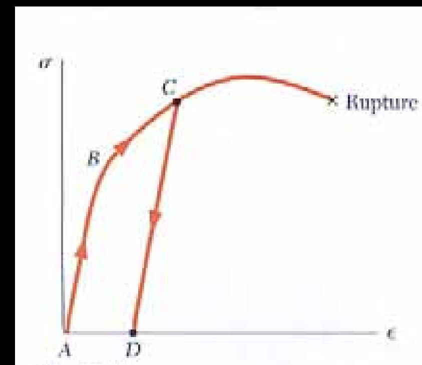


Fig. 2.18

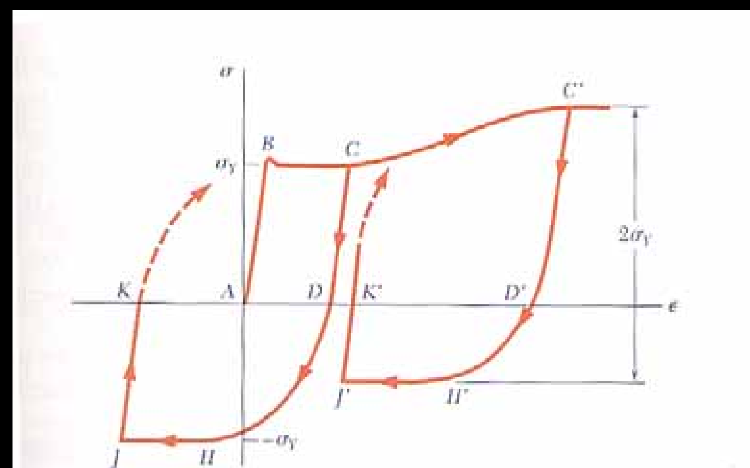


Fig. 2.20

Plastic Behavior

- ductile

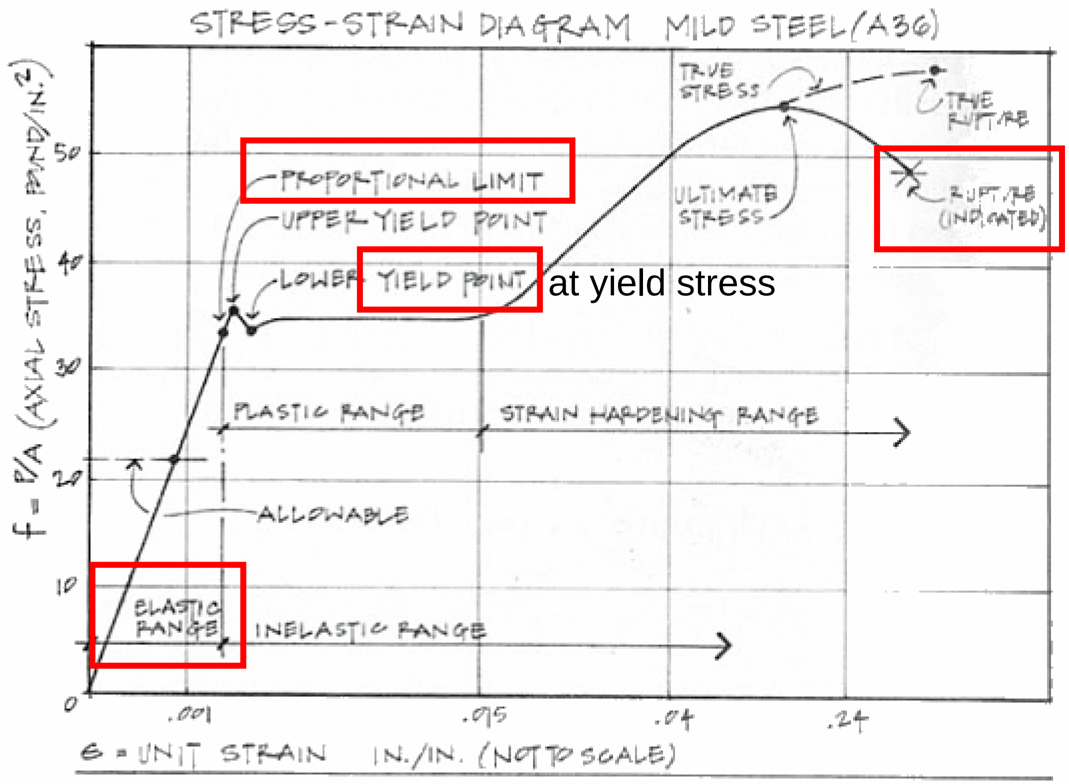


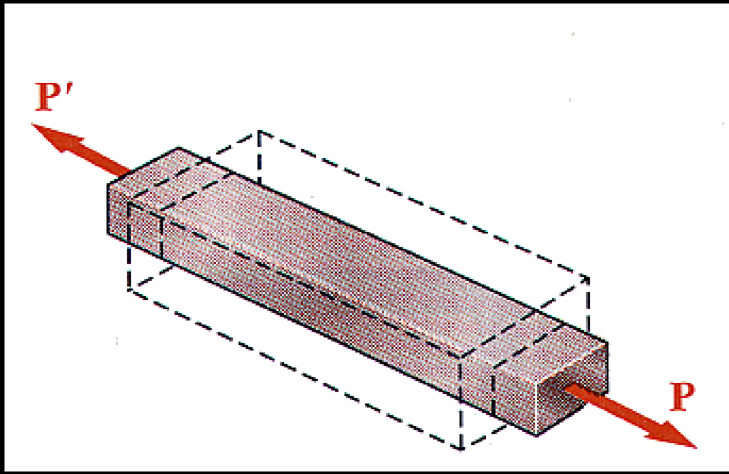
Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- *or "what happens to the cross section with axial stress"*

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$



- *strain in lateral direction*
  - *negative*
  - *equal for isometric materials*

$$\varepsilon_y = \varepsilon_z$$

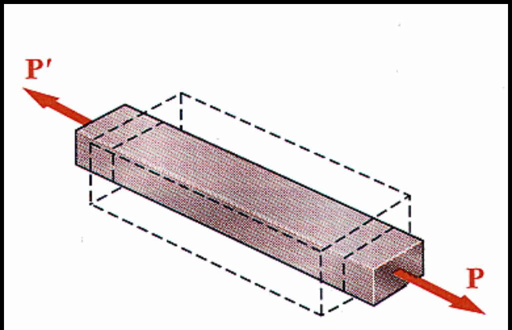
Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- sign! $0 < \mu < 0.5$



Calculating Strain

- *from Hooke's law*

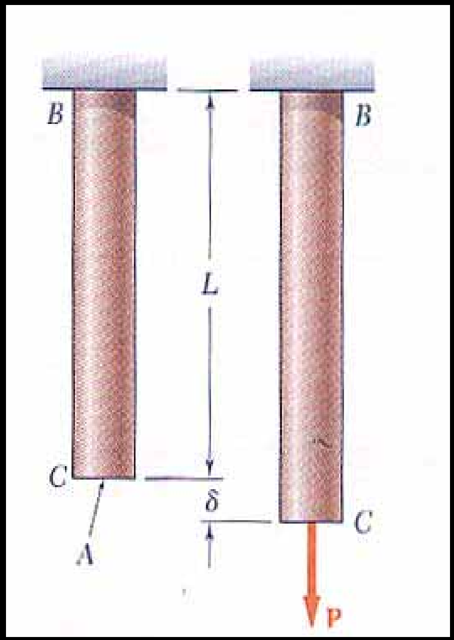
$$f = E \cdot \varepsilon$$

- *substitute*

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

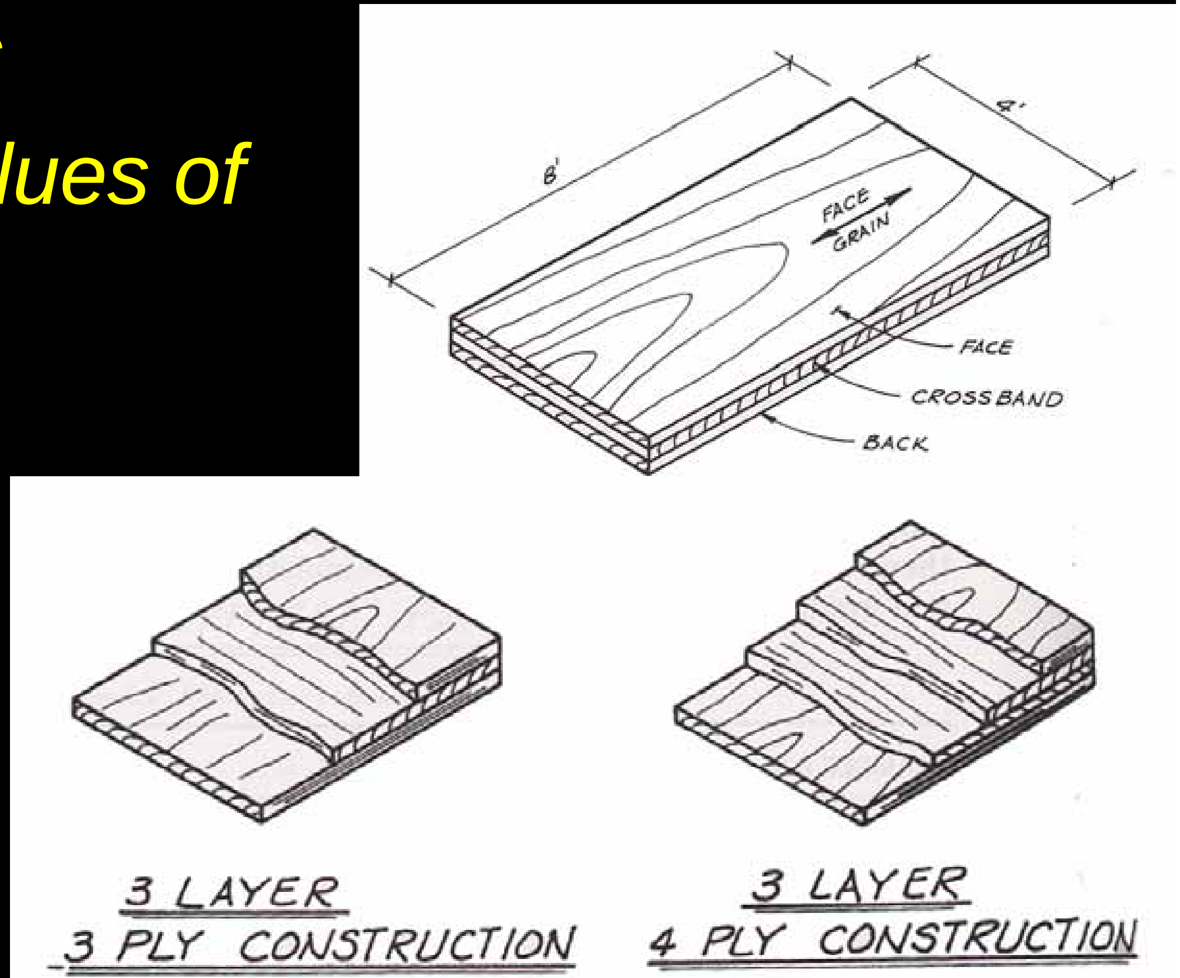
- *get* $\Rightarrow$

$$\delta = \frac{PL}{AE}$$



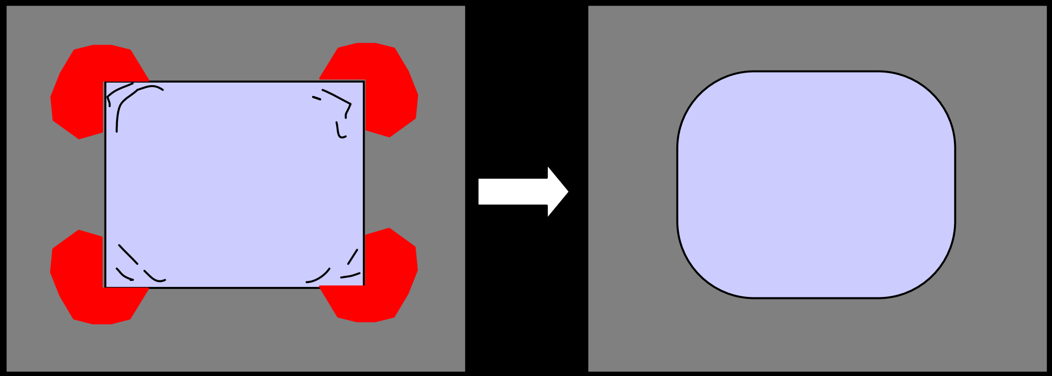
Orthotropic Materials

- *non-isometric*
- *directional values of E and μ*
- *ex:*
  - *plywood*
  - *laminates*
  - *polymer composites*



Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners



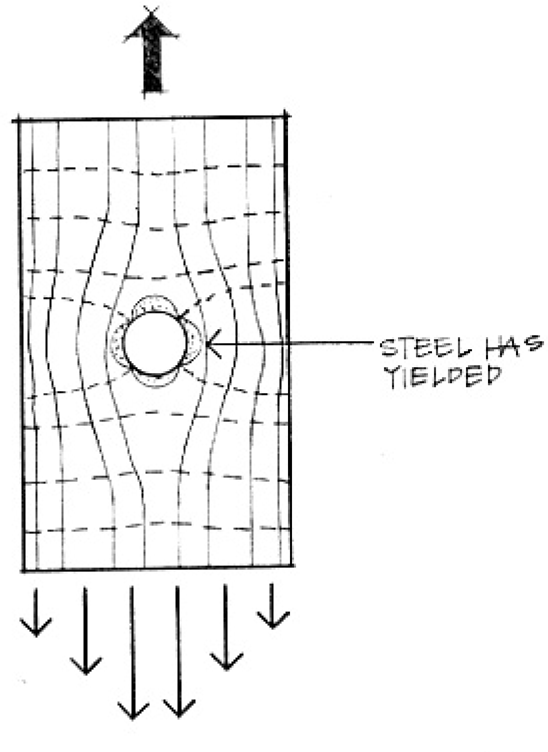
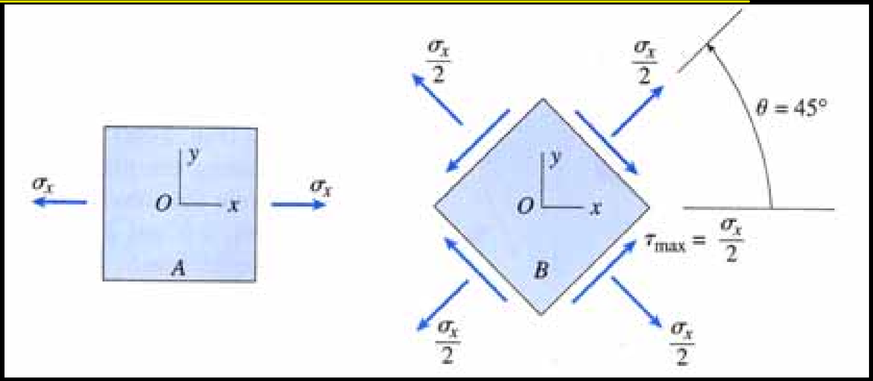


Figure 5.35 Stress trajectories around a hole.

Maximum Stresses

- *if we need to know where max f and f_v happen:*



$$\theta = 0° \rightarrow \cos\theta = 1 \qquad f_{\max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_o} = \frac{f_{\max}}{2}$$

Maximum Stresses

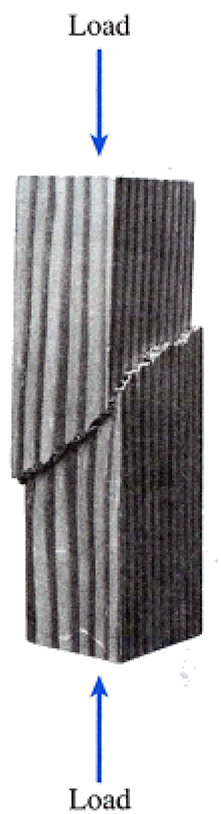
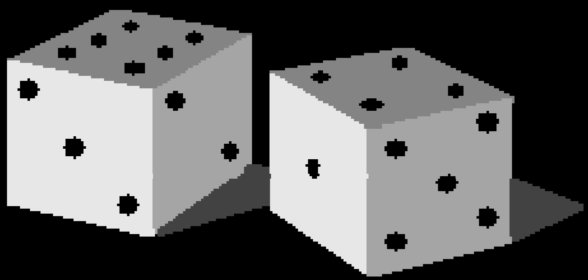


FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- *RISK & UNCERTAINTY*



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

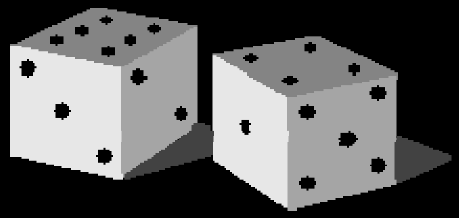
$$allowable\ load = \frac{ultimate\ load}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$$

Load and Resistance Factor Design

- loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ - resistance factor

γ - load factor for (D)ead & (L)ive load