ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

DR. ANNE NICHOLS

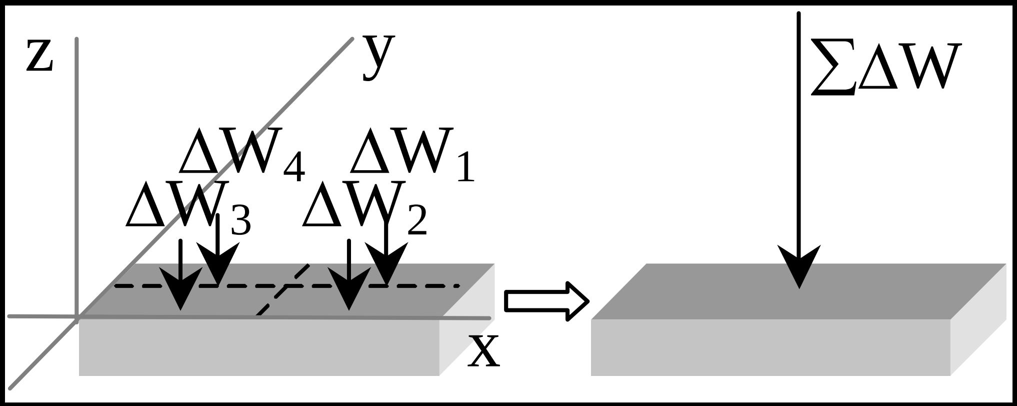
SPRING 2008

lecture
eleven

centers of gravity- centroids

Center of Gravity

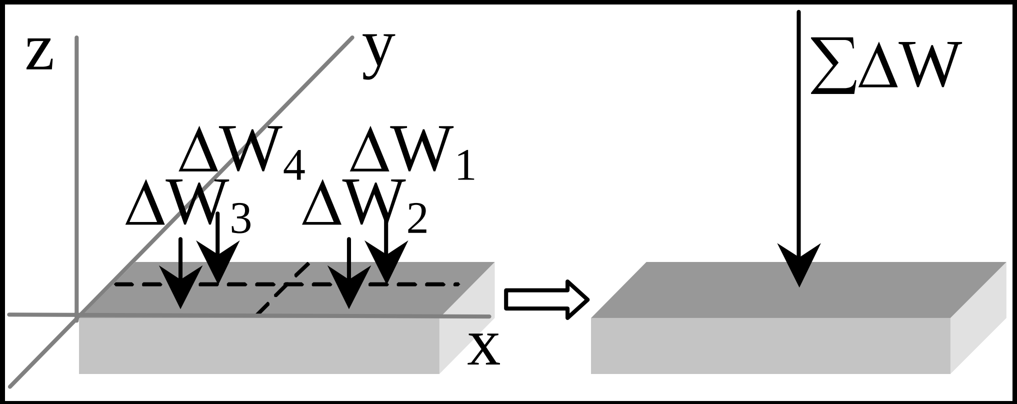
- location of equivalent weight
- determined with calculus



- sum element weights $W = \int dW$

Center of Gravity

- *"average" x & y from moment*



$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}\boldsymbol{W} \Rightarrow \quad \bar{x} = \frac{\sum (x\Delta W)}{\boldsymbol{W}}$$

"bar" means average

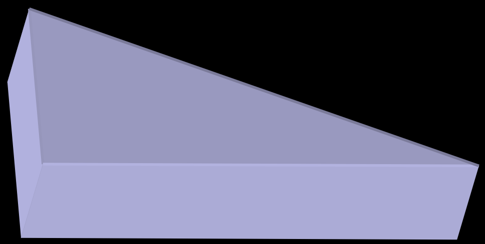
$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}\boldsymbol{W} \Rightarrow \quad \bar{y} = \frac{\sum (y\Delta W)}{\boldsymbol{W}}$$

Centroid

- *"average" x & y of an area*
- *for a volume of constant thickness*
  - $\Delta W = \gamma t \Delta A$ *where γ is weight/volume*
  - *center of gravity = centroid of area*

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$

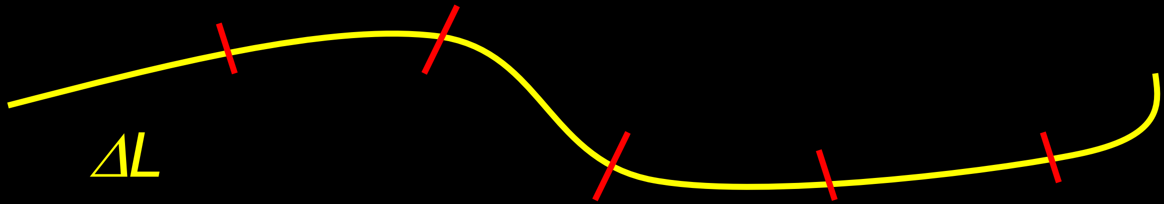


Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$

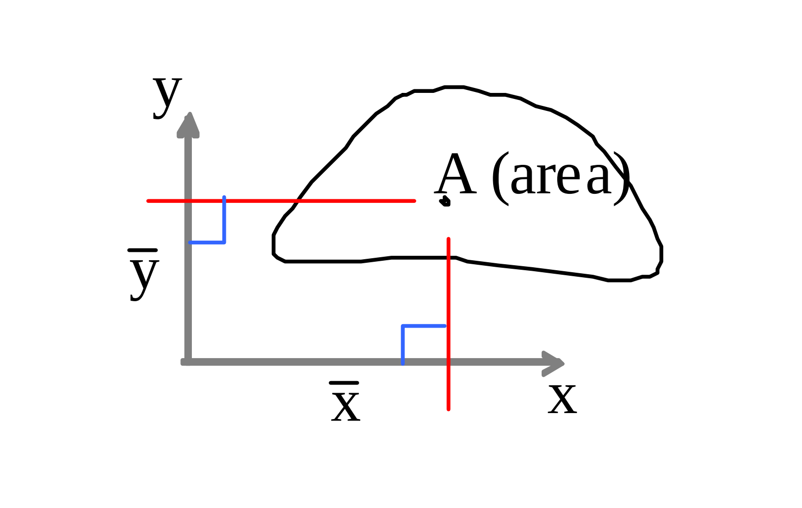


1^{st} Moment Area

- math concept
- the moment of an area about an axis

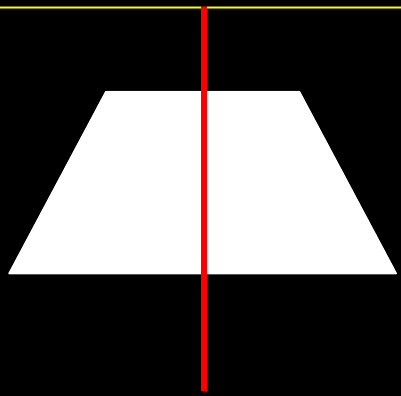
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$

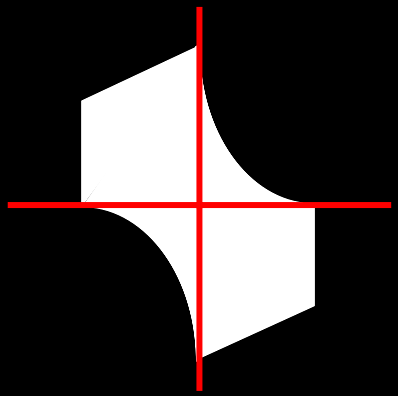


Symmetric Areas

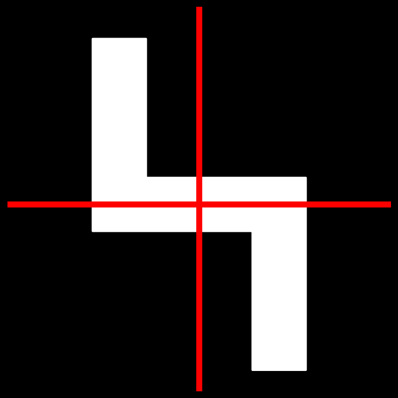
- *symmetric about an axis*



- *symmetric about a center point*

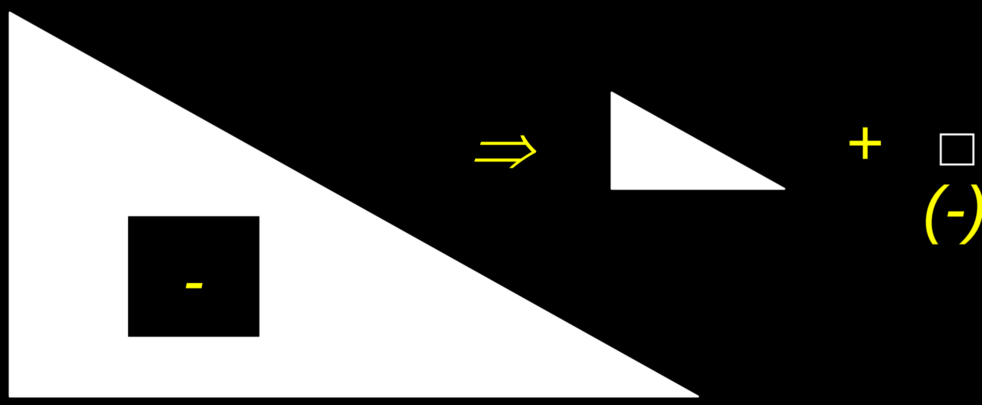


- *mirrored symmetry*



Composite Areas

- *made up of basic shapes*
- *areas can be <u>negative</u>*
- *(centroids can be negative for any area)*



Basic Procedure

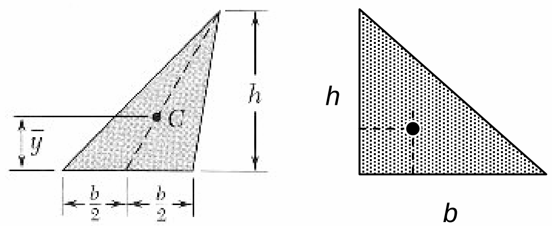
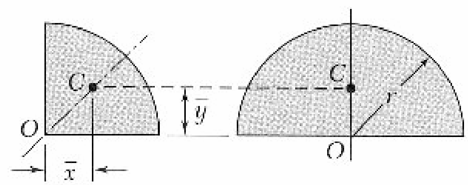
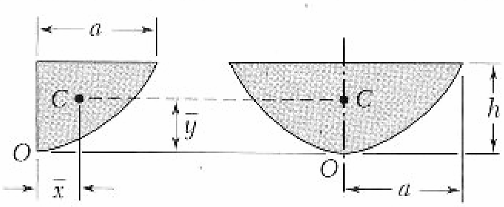
1. *Draw reference origin (if not given)*
2. *Divide into basic shapes (+/-)*
3. *Label shapes*
4. *Draw table*
5. *Fill in table*
6. *Sum necessary columns*
7. *Calculate* $\hat{x}$ *and* $\hat{y}$

Component	*Area*	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
Σ					

Area Centroids

- *Table 7.1 – pg. 242*

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$
Triangular area		$\frac{b}{3}$ *right triangle only*	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$