ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231

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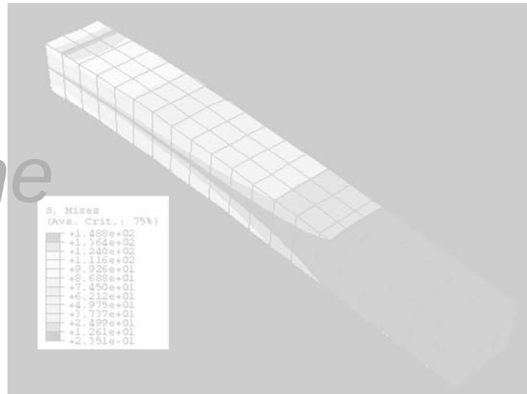
FALL 2007

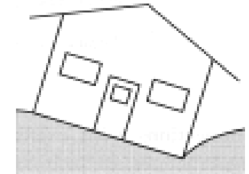
lecture
twenty one

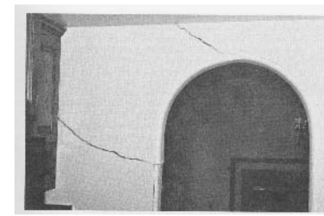


beams:
deflection & design

Design for Strength +...

- strength design
  - forces & material
- serviceability
  - limit deflection and cracking
  - control noise & vibration
  - no excessive settlement of foundations
  - durability
  - appearance
  - component damage
  - ponding

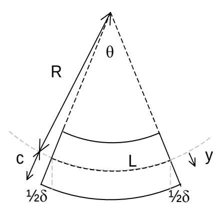


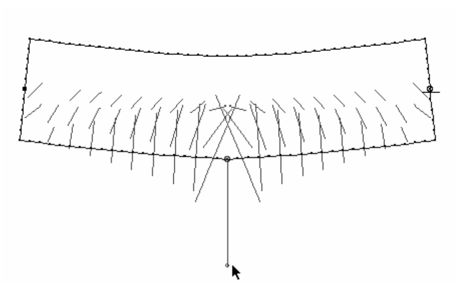


Beam Deformations

- curvature relates to
  - bending moment
  - modulus of elasticity
  - moment of inertia

$$\frac{1}{R} = \frac{M}{EI}$$





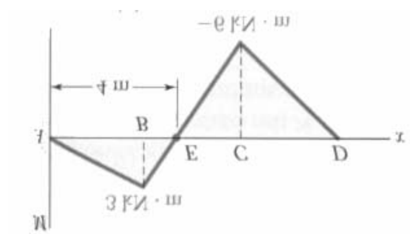
$$curvature = \frac{M(x)}{EI}$$

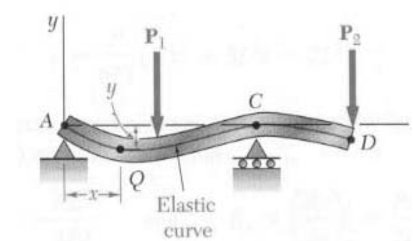
$$\theta = slope = \int \frac{M(x)}{EI} dx$$

$$\Delta = deflection = \int\int \frac{M(x)}{EI} dx$$

Deflected Shape & M(x)

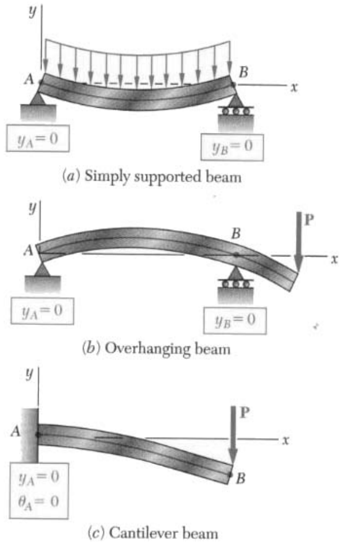
- -M(x) gives shape indication
- boundary conditions must be met





Boundary Conditions

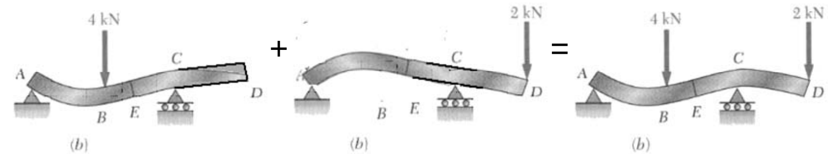
- *at pins, rollers, fixed supports: y = 0*
- *at fixed supports: $\theta = 0$*
- *at inflection points from symmetry: $\theta = 0$*
- y_{max} *at* $\frac{dy}{dx} = 0$



(a) Simply supported beam
(b) Overhanging beam
(c) Cantilever beam

Superpositioning

- *if w can be __superpositioned__*
  - *θ & y can*
  - *elastic range only!*



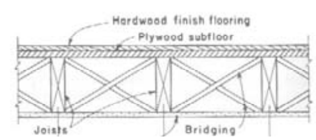
Deflection Limits

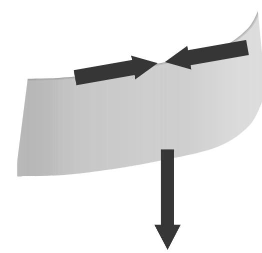
- *based on service condition, severity*

Use	*LL only*	*DL+LL*
Roof beams:		
Industrial	*L/180*	*L/120*
Commercial		
plaster ceiling	*L/240*	*L/180*
no plaster	*L/360*	*L/240*
Floor beams:		
Ordinary Usage	*L/360*	*L/240*
Roof or floor (damageable elements)		*L/480*

Lateral Buckling

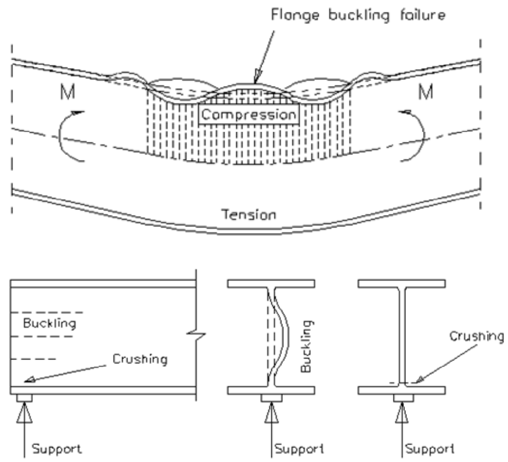
- *lateral buckling caused by compressive forces at top coupled with insufficient rigidity*
- *can occur at low stress levels*
- *stiffen, brace or bigger I_y*





Local Buckling

- steel I beams
- flange
  - buckle in direction of smaller radius of gyration
- web
  - force
  - "crippling"



Local Buckling

- flange

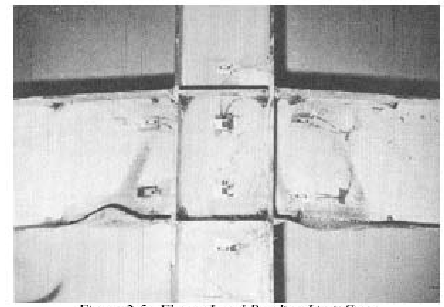


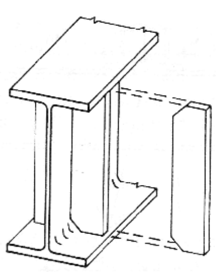
Figure 2-5. Flange Local Bending Limit State (Beedle, L.S., Christopher, R., 1964)

- web

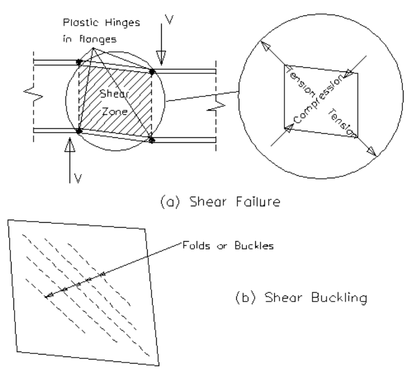
Figure 2-7. Web Local Buckling Limit State (SAC Project)

Shear in Web

- panels in plate girders or webs with large shear
- buckling in compression direction
- add stiffeners



stiffeners to prevent lateral buckling

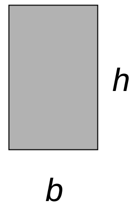


Shear in Web

- plate girders and stiffeners

Beam Design

1. Know F_{all} for the material or F_U for LRFD
2. Draw V & M, finding M_{max}
3. Calculate $S_{req'd}$ $\left(f_b \leq F_b\right)$
4. Determine section size



$$S = \frac{bh^2}{6}$$

Beam Design

4*. Include self weight for M_{max}
   – and repeat 3 & 4 if necessary

5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper

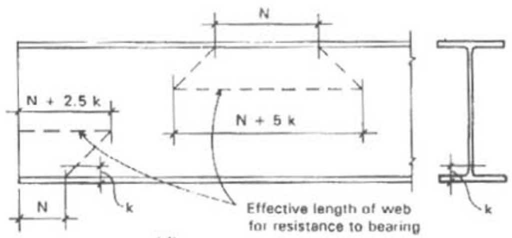
Beam Design

6. Evaluate shear stresses - horizontal $\left(f_v \leq F_v\right)$
   - W and rectangles $f_{v-\max} = \dfrac{3V}{2A} \approx \dfrac{V}{A_{web}}$
   - thin walled sections $f_{v-\max} = \dfrac{VQ}{Ib}$

Beam Design

7. Provide adequate bearing area at supports

$$f_p = \frac{P}{A} \leq F_p$$

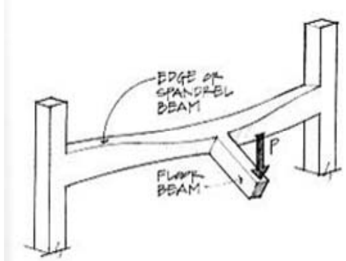


(d)

Beam Design

8. Evaluate torsion

$$\left(f_v \leq F_v\right)$$



- circular cross section

$$f_v = \frac{T\rho}{J}$$

- rectangular

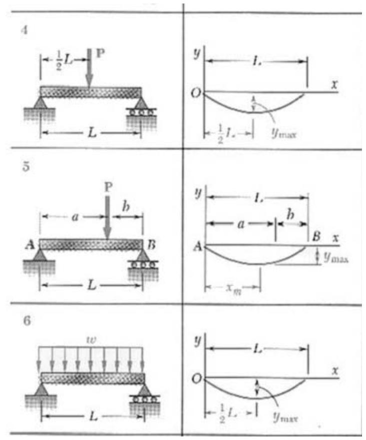
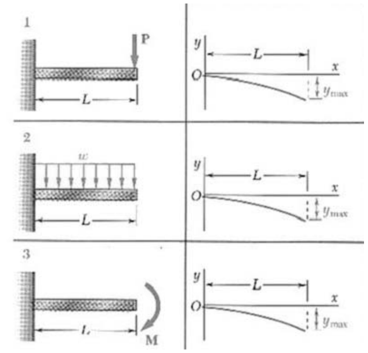
$$f_v = \frac{T}{c_1 ab^2}$$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Beam Design

9. Evaluate deflections

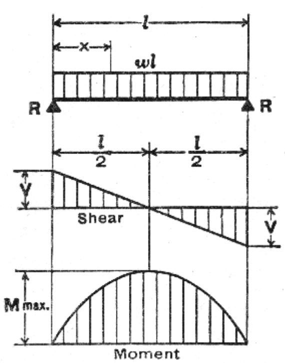



$$y_{max}(x) = \Delta_{actual} \leq \Delta_{allowable}$$

Beam Design

9. – how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$