ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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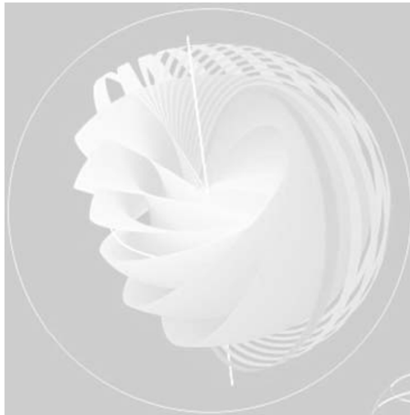
FALL 2007

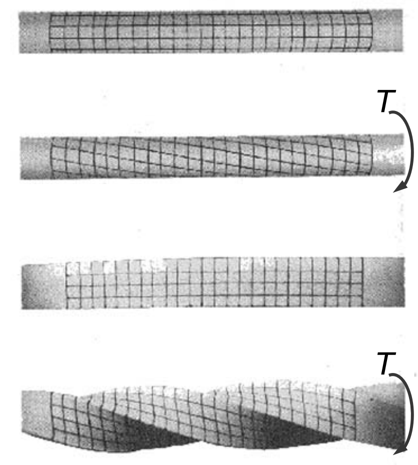
lecture
seventeen



torsion
& thermal effects

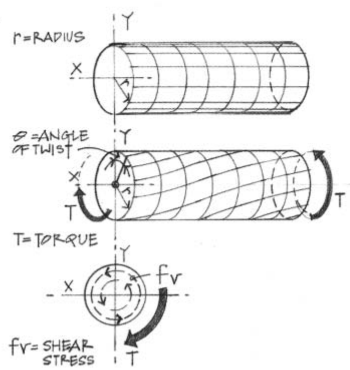
Torsional Stress & Strain

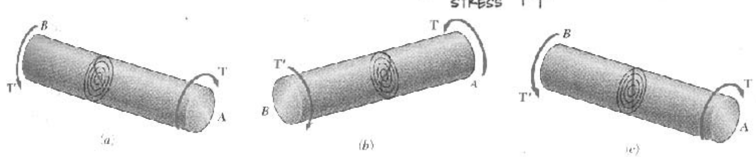
- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
- not true for square sections....



Shear Stress Distribution

- depend on the deformation
- ϕ = angle of twist
  - measure
- can prove planar section doesn't distort



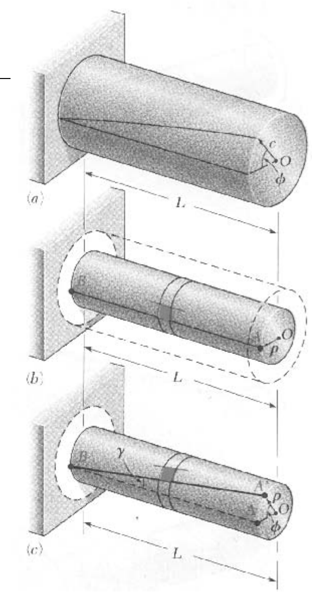


Shearing Strain

- related to ϕ

$$\gamma = \frac{\rho\phi}{L}$$

- ρ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: γ_{max} is at outer diameter

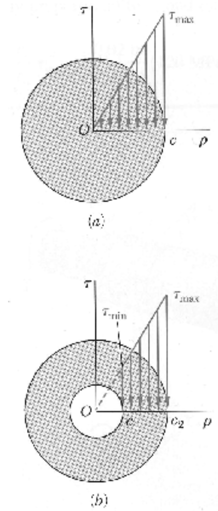


Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \dfrac{\rho\phi}{L}$
- so $\tau = G \cdot \dfrac{\rho\phi}{L}$
- where G is the <u>Shear Modulus</u>

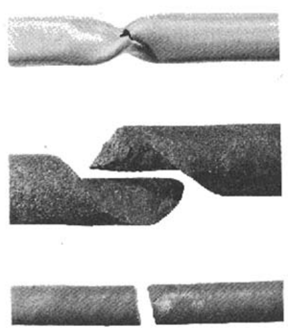
Torsional Stress - Strain

- from $T = \Sigma\tau(\rho)\Delta A$
- can derive $T = \dfrac{\tau J}{\rho}$
  - where J is the polar moment of inertia
  - elastic range $\tau = \dfrac{T\rho}{J}$



Shear Stress

- τ_{max} happens at <u>outer diameter</u>
- combined shear and axial stresses
  - maximum shear stress at 45° "twisted" plane

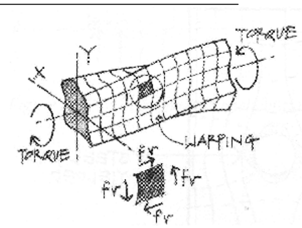


Shear strain

- knowing $\tau = G \cdot \dfrac{\rho\phi}{L}$ and $\tau = \dfrac{T\rho}{J}$
- solve: $\phi = \dfrac{TL}{JG}$
- composite shafts: $\phi = \sum_i \dfrac{T_i L_i}{J_i G_i}$

Noncircular Shapes

- *torsion depends on J*
- *plane sections don't remain plane*
- τ_{max} *is still at outer diameter*



$$\tau_{\max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

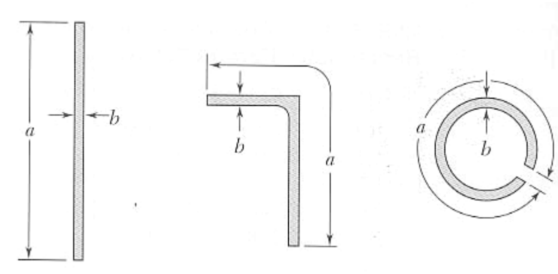
– *where a is longer side (> b)*

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Open Thin-Walled Sections

- *with very large a/b ratios:*



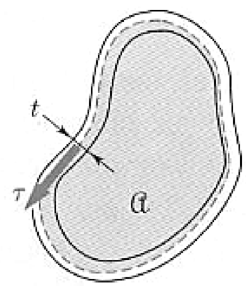
$$\tau_{\max} = \frac{T}{\tfrac{1}{3} ab^2} \qquad \phi = \frac{TL}{\tfrac{1}{3} ab^3 G}$$

Shear Flow in Closed Sections

- *q is the internal shear force/unit length*

$$\tau = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$

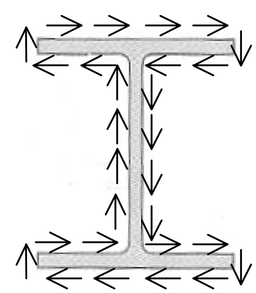


- $\mathcal{A}$ *is the area bounded by the centerline*
- s_i *is the length segment,* t_i *is the thickness*

Shear Flow in Open Sections

- *each segment has proportion of T with respect to torsional rigidity,*

$$\tau_{\max} = \frac{Tt_{\max}}{\tfrac{1}{3} \Sigma b_i t_i^3}$$



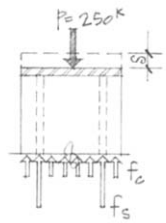
- *total angle of twist:*

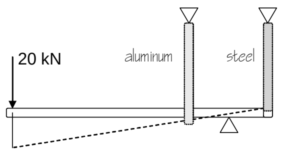
$$\phi = \frac{TL}{\tfrac{1}{3} G \Sigma b_i t_i^3}$$

- *I beams - web is thicker, so* τ_{max} *is in* *<u>web</u>*

Deformation Relationships

- *physical movement*
  - *axially (same or zero)*
  - *rotations from axial changes*

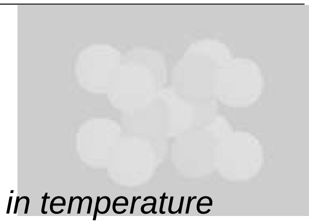


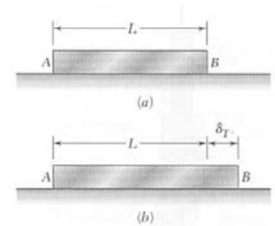


- $\delta = \dfrac{PL}{AE}$ *relates* δ *to P*

Deformations from Temperature

- *atomic chemistry reacts to changes in energy*
- *solid materials*
  - *can contract with decrease in temperature*
  - *can expand with increase in temperature*
- *linear change can be measured per degree*



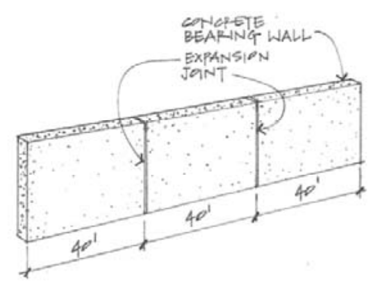


Thermal Deformation

- α *- the rate of strain per degree*
- *UNITS :* $/{}^{\circ}F$, $/{}^{\circ}C$
- *length change:* $\delta_T = \alpha(\Delta T)L$
- *thermal strain:* $\varepsilon_T = \alpha(\Delta T)$
  - *no stress when movement allowed*

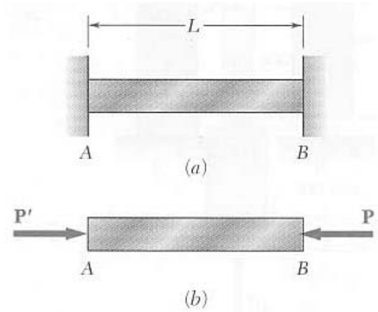
Coefficients of Thermal Expansion

Material	*Coefficients (α) [in./in./°F]*
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

- *if thermal movement is restrained* *<u>stresses</u> are induced*



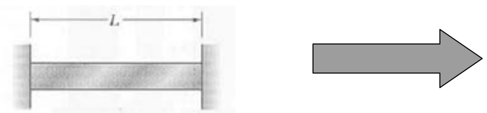
1. *bar pushes on supports*
2. *support pushes back*
3. *reaction causes internal stress* $f = \frac{P}{A} = \frac{\delta}{L}E$

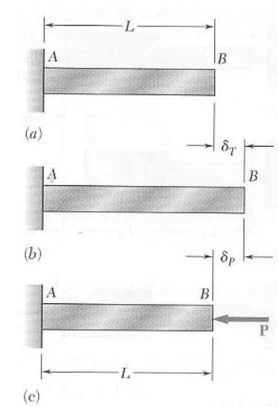
Superposition Method

– *can remove a support to make it look determinant*

– *replace the support with a reaction*

– *enforce the geometry constraint*





Superposition Method

– *total length change restrained to <u>zero</u>*

constraint: $\delta_P + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$