ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

DR. ANNE NICHOLS

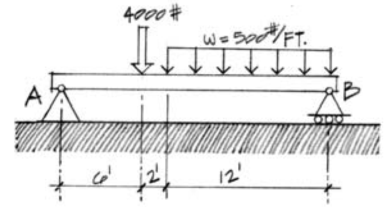
FALL 2007

lecture thirteen

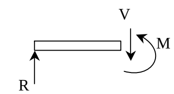
beam forces – internal

Beams

- span horizontally
  - floors
  - bridges
  - roofs

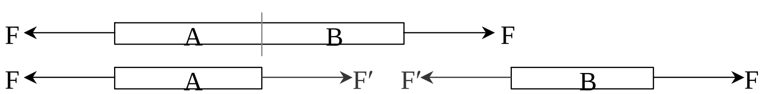


- loaded transversely by gravity loads
- may have internal axial force
- <u>will have</u> internal shear force
- <u>will have</u> internal moment (bending)

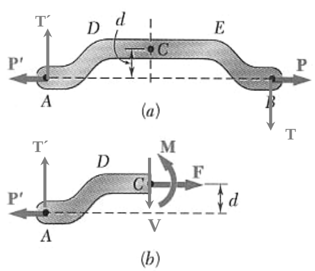


Internal Forces

- trusses
  - axial only, (compression & tension)

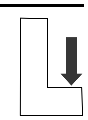


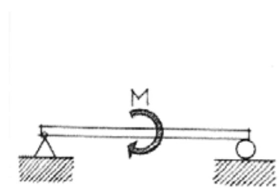
- in general
  - axial force
  - shear force, V
  - bending moment, M



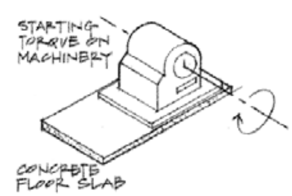
Beam Loading

- concentrated force
- concentrated <u>moment</u>
  - spandrel beams



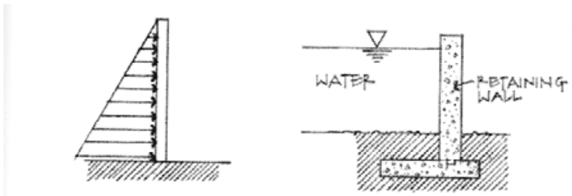


(d) Pure moment.



Beam Loading

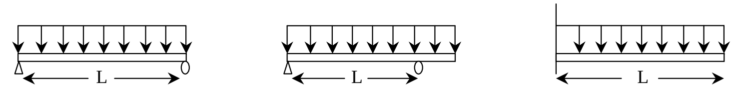
- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure
  - wind loads



(c) Nonuniformly distributed load.

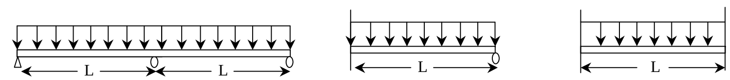
Beam Supports

- statically determinate



simply supported (most common) | overhang | cantilever

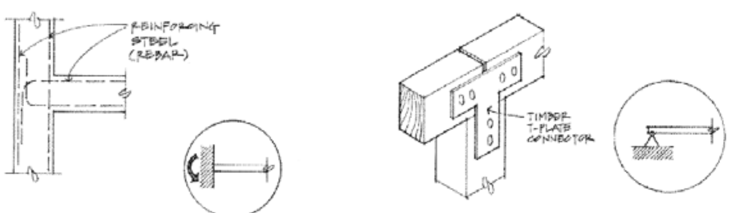
- statically indeterminate



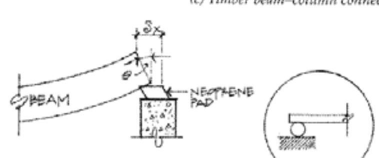
continuous (most common case when $L_1=L_2$) | Propped | Restrained

Beam Supports

- in the real world, modeled type



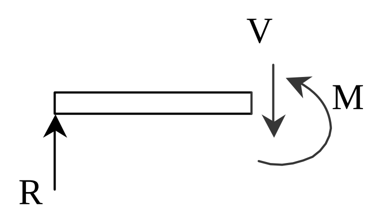
(c) Timber beam–column connection with T-plate.



(a) Beam supported by a neoprene pad.

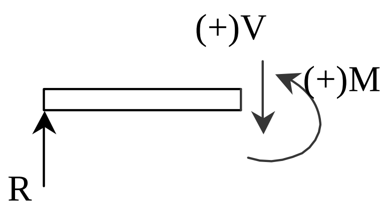
Internal Forces in Beams

- like method of sections / joints
  - no axial forces
- section <u>must</u> be in equilibrium
- want to know where biggest internal forces and moments are for designing



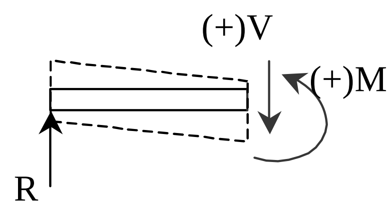
V & M Diagrams

- tool to locate V_{max} and M_{max}
- *necessary* for designing
- have a *different sign convention* than external forces, moments, and reactions

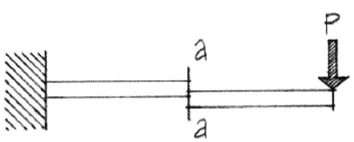


Sign Convention

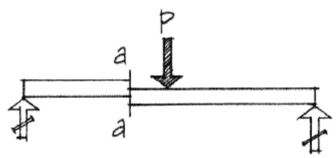
- shear force, V:
  - cut section to LEFT
  - if $\sum F_y$ is positive by statics, V acts down and is POSITIVE
  - beam has to resist shearing apart by V



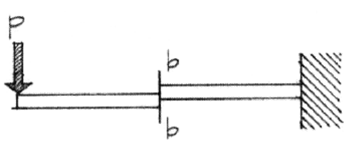
Shear Sign Convention



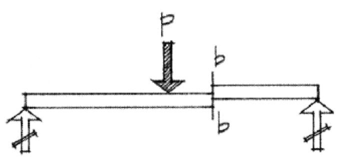
(+) Shear.



(+) Shear.



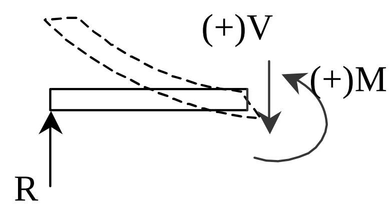
(–) Shear.



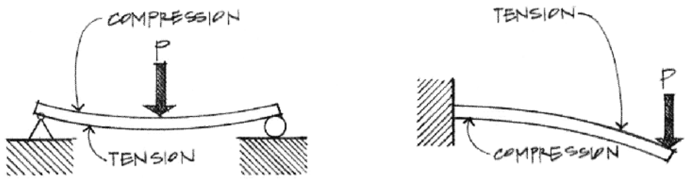
(–) Shear.

Sign Convention

- bending moment, M:
  - cut section to LEFT
  - if $\sum M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a "smiley" beam has to resist bending apart by M



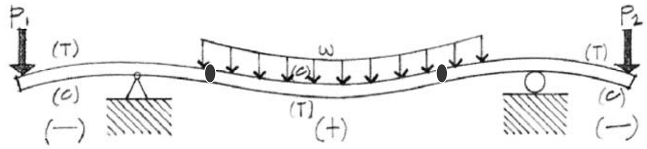
Bending Moment Sign Convention



(+) Moment. (–) Moment.

(+) Moment. (–) Moment.

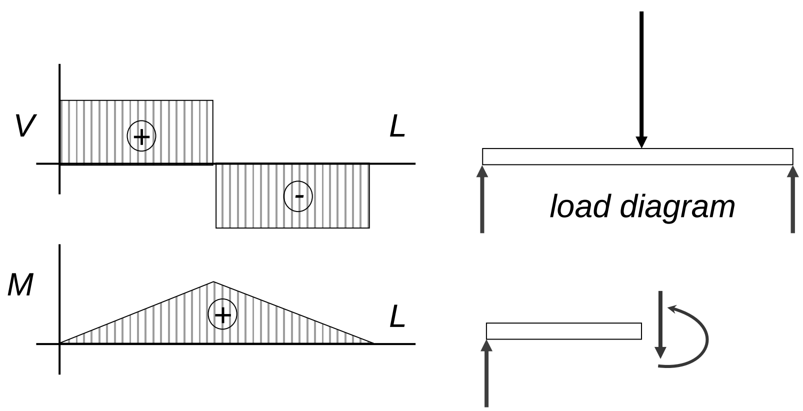
Deflected Shape



- positive bending moment
  - – tension in bottom, compression in top
- negative bending moment
  - – tension in top, compression in bottom
- zero bending moment
  - – <u>inflection point</u>

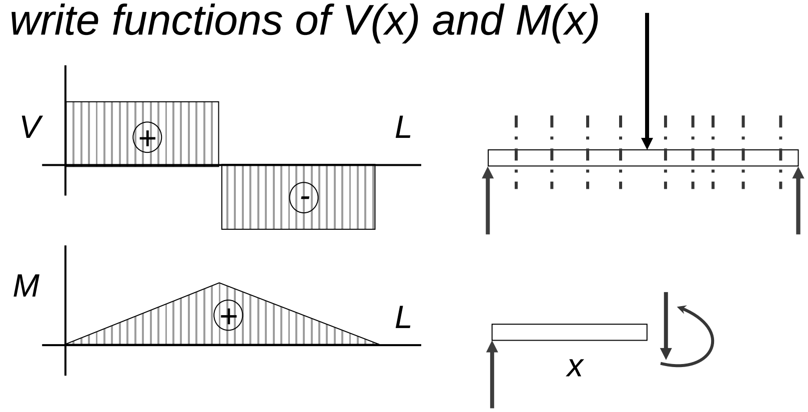
Constructing V & M Diagrams

- along the beam length, plot V, plot M



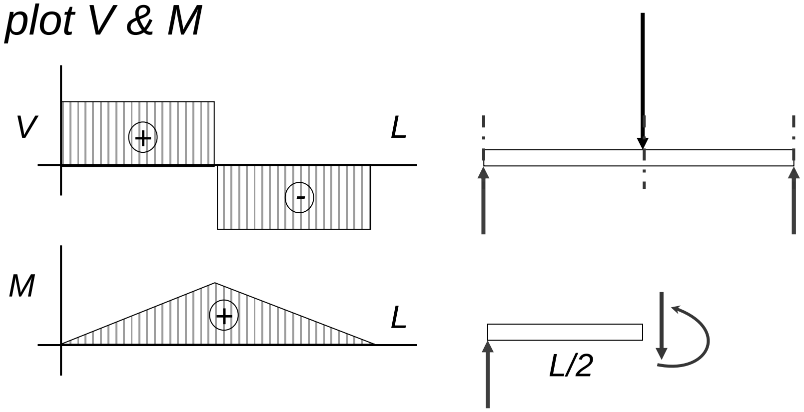
Mathematical Method

- cut sections with x as width
- write functions of V(x) and M(x)



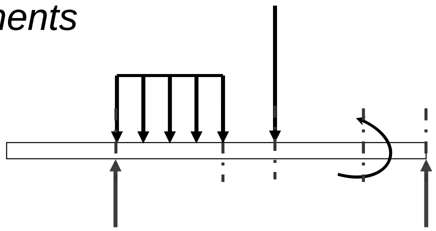
Method 1: Equilibrium

- *cut sections at important places*
- *plot V & M*



Method 1: Equilibrium

- *important places*
  - *supports*
  - *concentrated loads*
  - *start and end of distributed loads*
  - *concentrated moments*
- *free ends*
  - *zero forces*



Method 2: Semigraphical

- *by knowing*
  - *area under loading curve = change in V*
  - *area under shear curve = change in M*
  - *concentrated forces cause "jump" in V*
  - *concentrated moments cause "jump" in M*

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx \qquad M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Method 2

- *relationships*

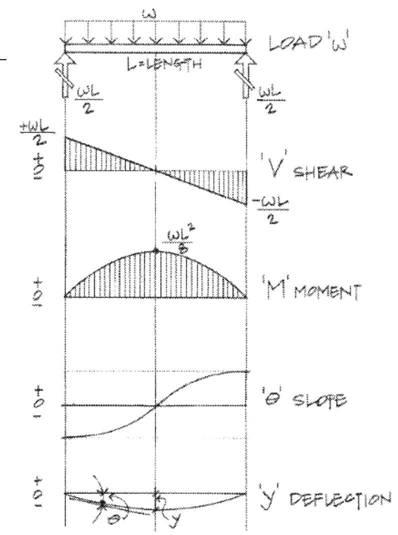
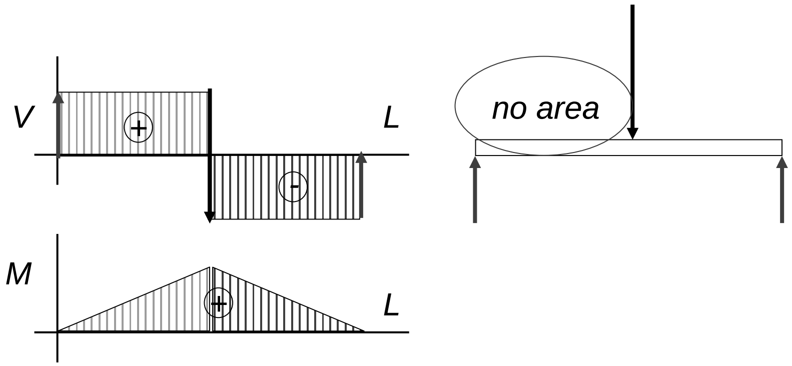


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

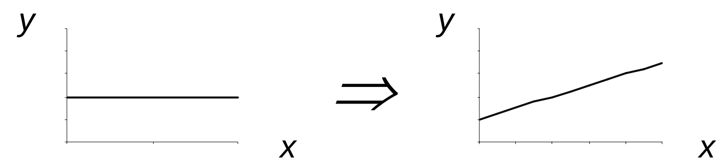
Method 2: Semigraphical

- M_{max} occurs where V = 0 (calculus)



Curve Relationships

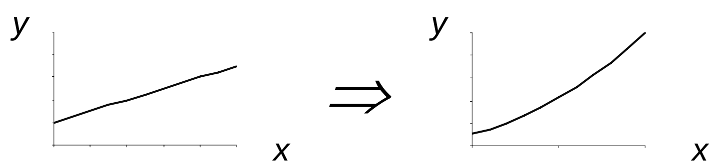
- integration of functions
- line with 0 slope, integrates to sloped



- ex: load to shear, shear to moment

Curve Relationships

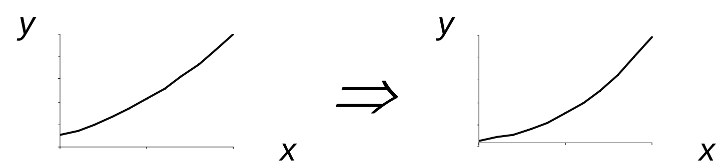
- line with slope, integrates to parabola



- ex: load to shear, shear to moment

Curve Relationships

- parabola, integrates to 3rd order curve



- ex: load to shear, shear to moment

Basic Procedure

1. Find reaction forces & moments

   Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load

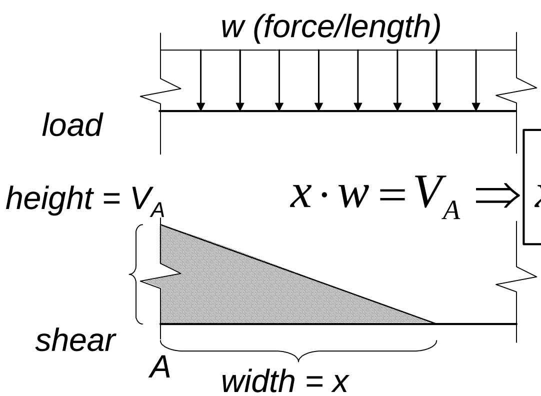
Basic Procedure

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V

Triangle Geometry

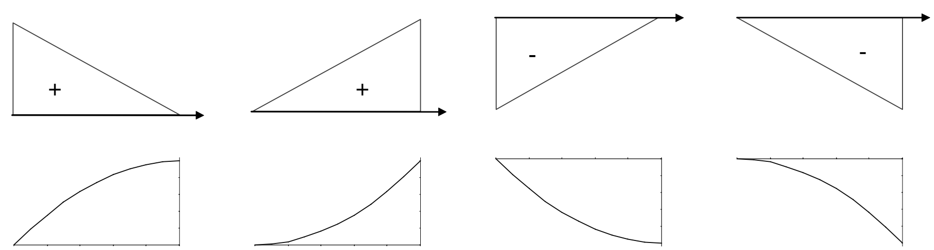
- slope of V is w (-w:1)



$$x \cdot w = V_A \Rightarrow \boxed{x = V_A / w}$$

Parabolic Shapes

- cases



up fast, then slow | up slow, then fast | down fast, then slow | down slow, then fast