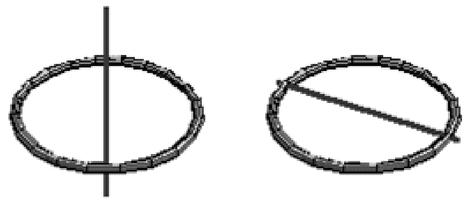
ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
DR. ANNE NICHOLS
FALL 2007

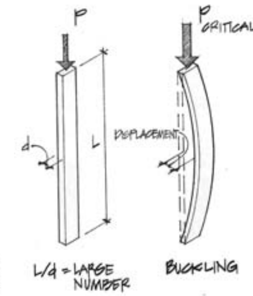
lecture
twelve

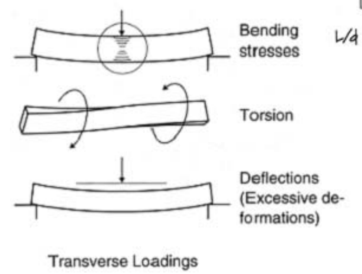


moment of inertia of an area

Moments of Inertia

- 2nd moment area
  - math concept
  - area x (distance)^2
- need for behavior of
  - beams
  - columns



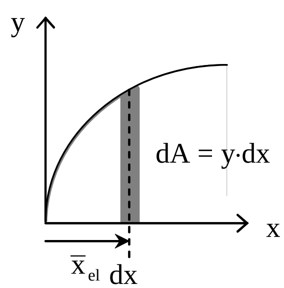


Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

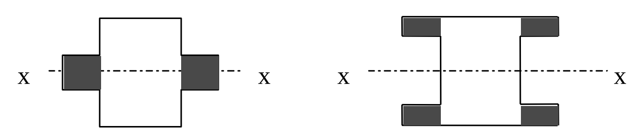
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

Moment of Inertia

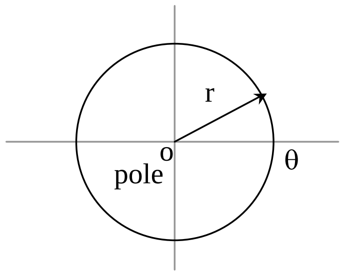
- same area moved away a distance
  - larger I



Polar Moment of Inertia

- *for round-ish shapes*
- *uses polar coordinates (r and θ)*
- *resistance to twisting*

$$J_o = \int r^2 dA$$



Radius of Gyration

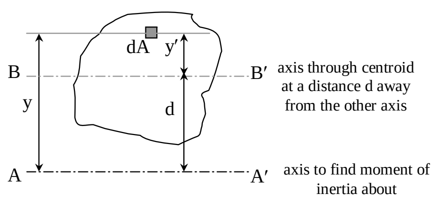
- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$

Parallel Axis Theorem

- *can find composite I once composite centroid is known (basic shapes)*

$$I_x = I_{cx} + Ad_y^{\,2}$$
$$= \bar{I}_x + Ad_y^{\,2}$$



$$I = \sum \bar{I} + \sum Ad^2$$

$$\boxed{\bar{I} = I - Ad^2}$$

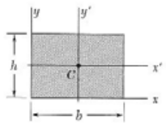
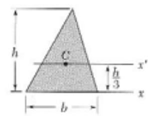
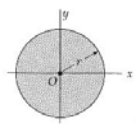
Basic Procedure

1. *Draw reference origin (if not given)*
2. *Divide into basic shapes (+/-)*
3. *Label shapes*
4. *Draw table with A, $\bar{x}$, $\bar{x}$A, $\bar{y}$, $\bar{y}$A, $\bar{I}$'s, d's, and Ad^2's*
5. *Fill in table and get $\hat{x}$ and $\hat{y}$ for composite*
6. *Sum necessary columns*
7. *Sum $\bar{I}$'s and Ad^2's*

$$(d_x = \hat{x} - \bar{x})$$
$$(d_y = \hat{y} - \bar{y})$$

Area Moments of Inertia

- *Table 7.2 – pg. 252* (*bars refer to centroid)*
  - *x, y*
  - *x', y'*
  - *C*

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$