# Architectural Structures I: Statics and Strength of Materials

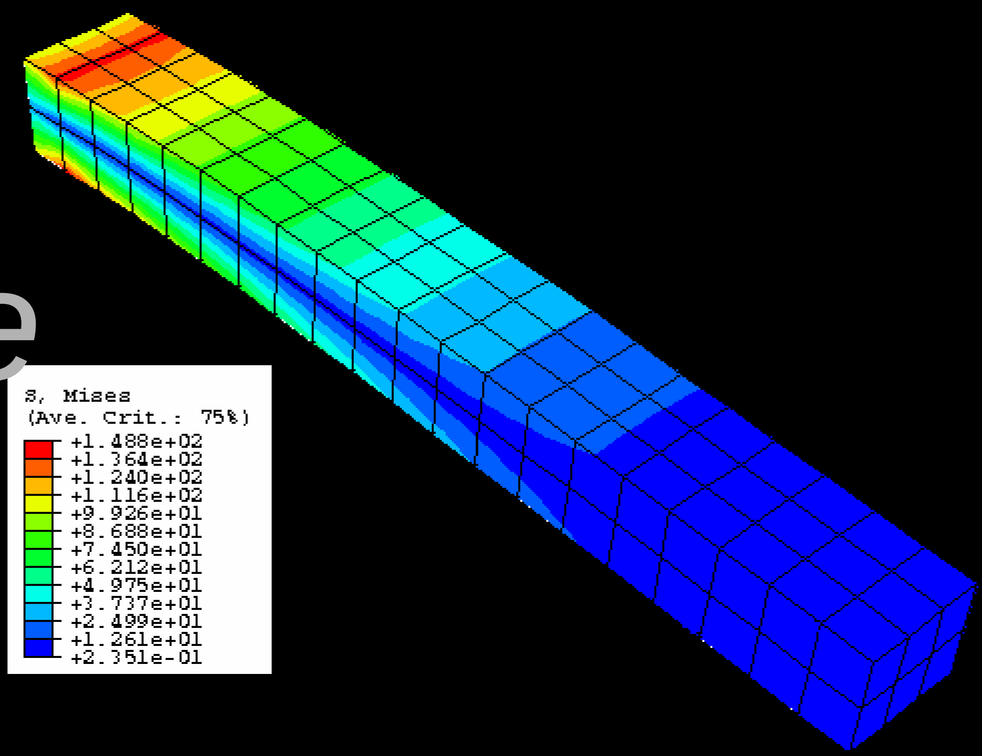
ENDS 231

Dr. Anne Nichols

Fall 2007

lecture

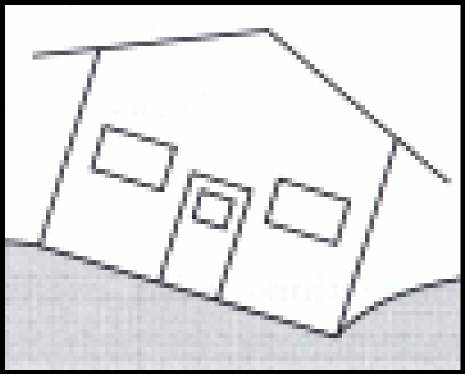
twenty one

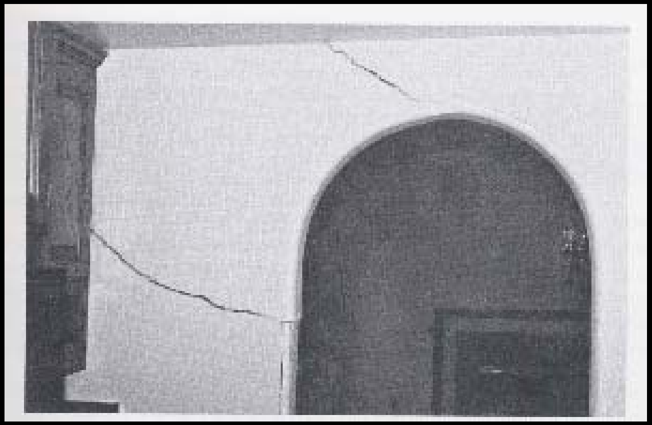



# beams: deflection & design

# Design for Strength +...

- strength design
  - forces & material
- serviceability
  - limit deflection and cracking
  - control noise & vibration
  - no excessive settlement of foundations
  - durability
  - appearance
  - component damage
  - ponding

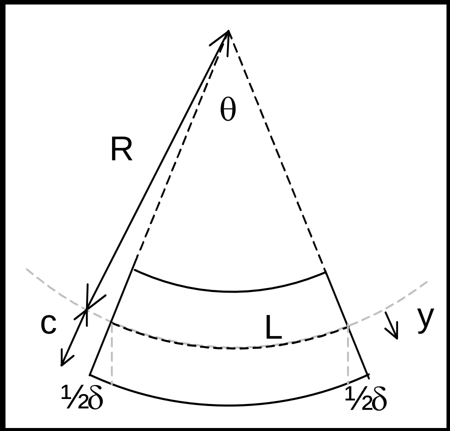


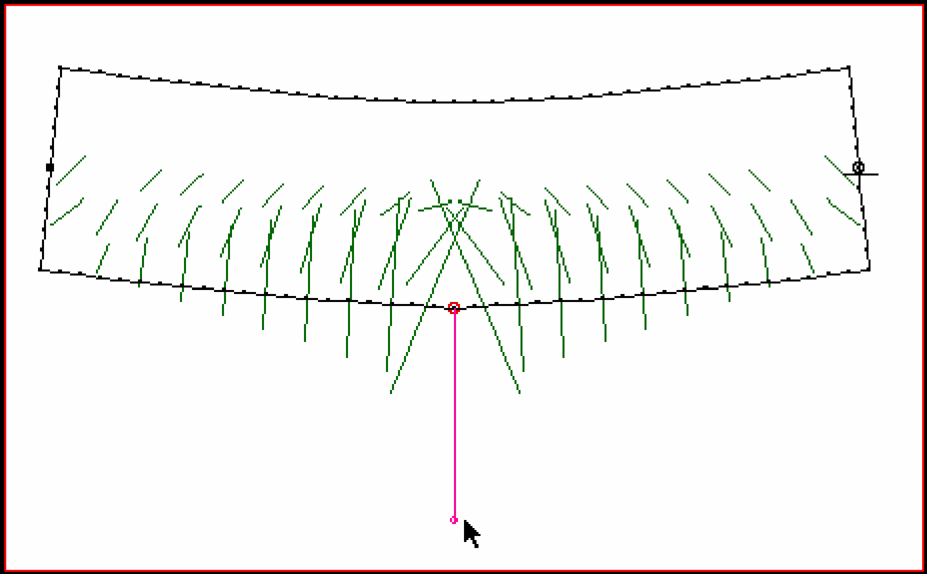


# Beam Deformations

- curvature relates to
  - bending moment
  - modulus of elasticity
  - moment of inertia

$$\frac{1}{R} = \frac{M}{EI}$$



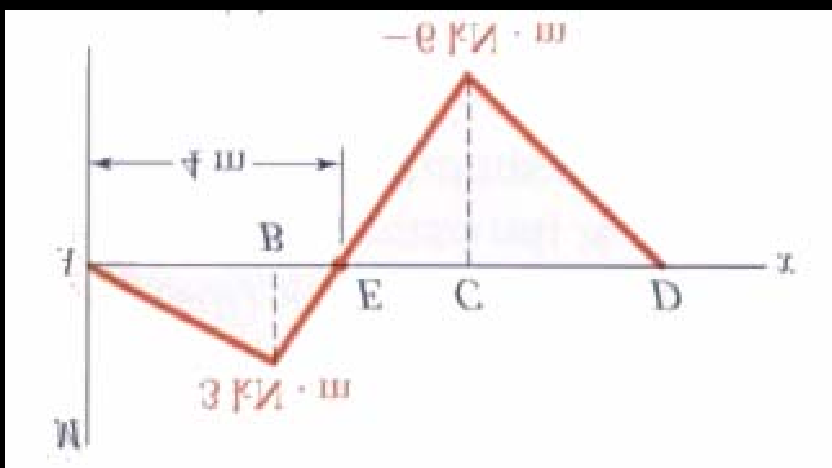
$$curvature = \frac{M(x)}{EI}$$

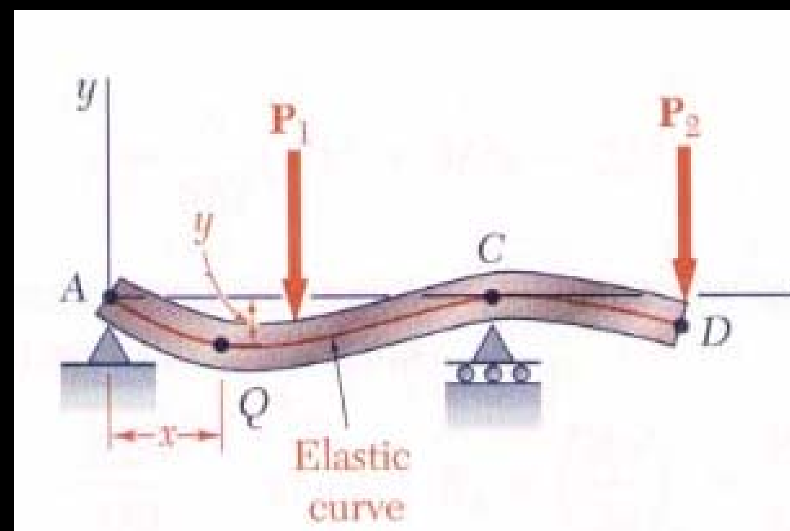
$$\theta = slope = \int \frac{M(x)}{EI} dx$$

$$\Delta = deflection = \int\int \frac{M(x)}{EI} dx$$

# Deflected Shape & M(x)

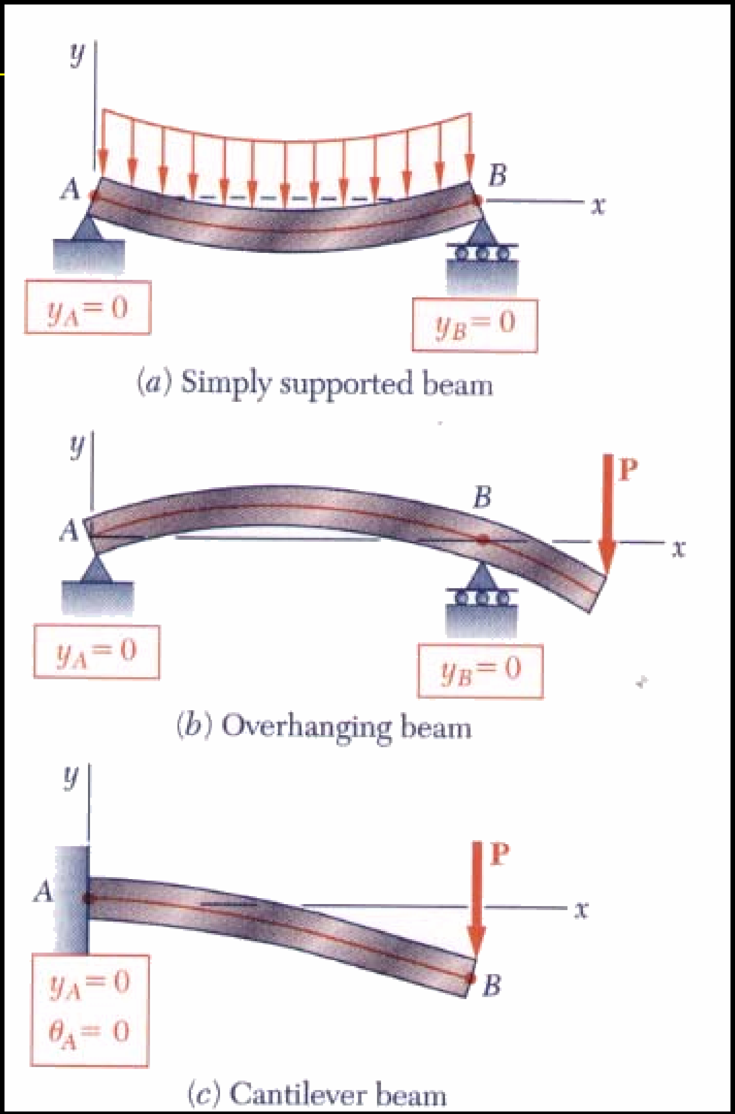
- *-M(x) gives shape indication*
- *boundary conditions must be met*



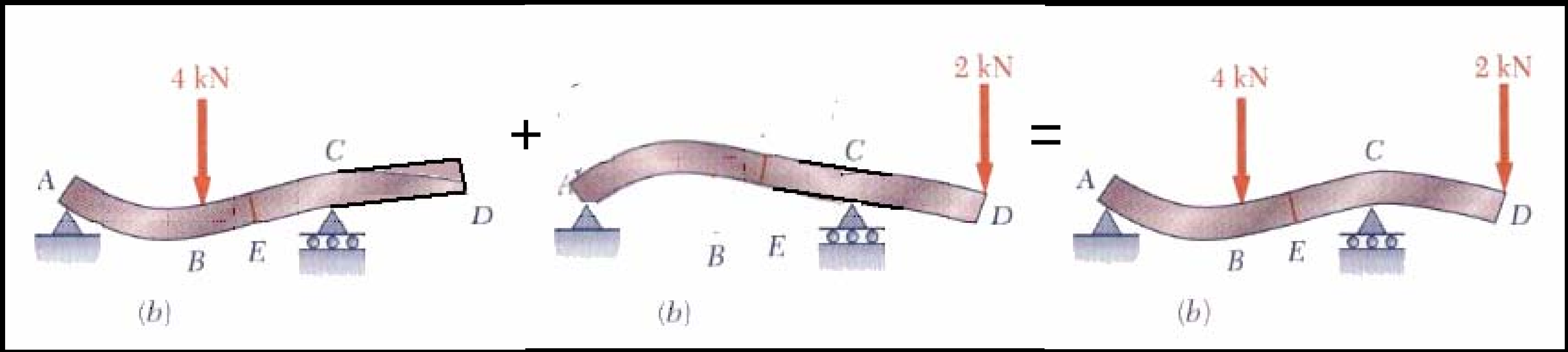
# Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- $y_{max}$ at $\dfrac{dy}{dx} = 0$



# Superpositioning

- *if w can be superpositioned*
  - *$\theta$ & y can*
  - *elastic range only!*



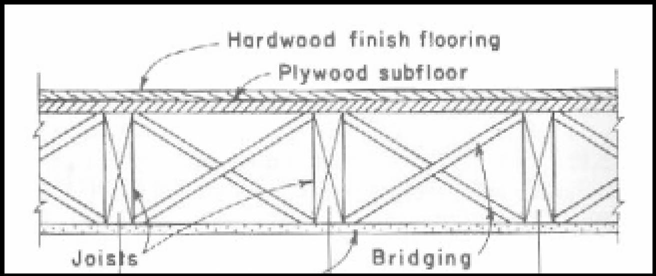
# *Deflection Limits*

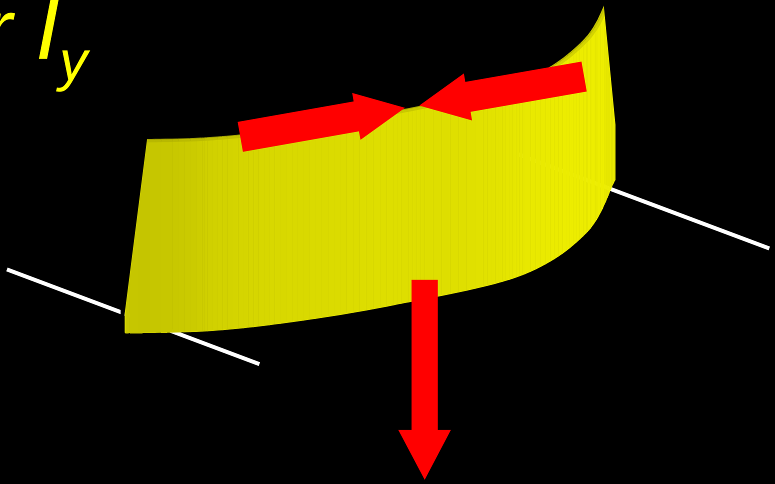
- *based on service condition, severity*

| Use | LL only | DL+LL |
| --- | --- | --- |
| *Roof beams:* | | |
| *Industrial* | *L/180* | *L/120* |
| *Commercial* | | |
| *plaster ceiling* | *L/240* | *L/180* |
| *no plaster* | *L/360* | *L/240* |
| *Floor beams:* | | |
| *Ordinary Usage* | *L/360* | *L/240* |
| *Roof or floor (damageable elements)* | | *L/480* |

# Lateral Buckling

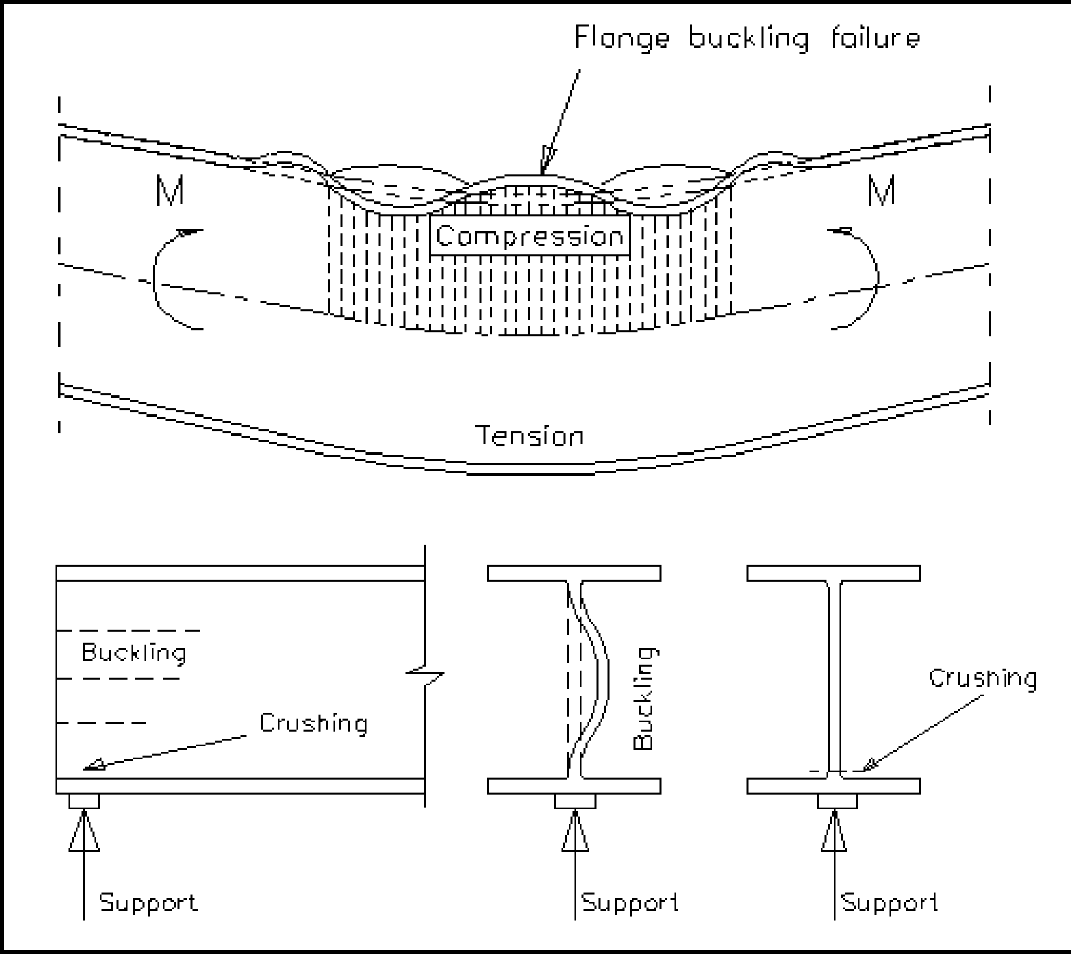
- lateral buckling caused by compressive forces at top coupled with insufficient rigidity
- can occur at low stress levels
- stiffen, brace or bigger $I_y$



# Local Buckling

- *steel I beams*
- *flange*
  - *buckle in direction of smaller radius of gyration*
- *web*
  - *force*
  - *"crippling"*



# Local Buckling

- *web*

*Figure 2-7. Web Local Buckling Limit State (SAC Project)*

- *flange*

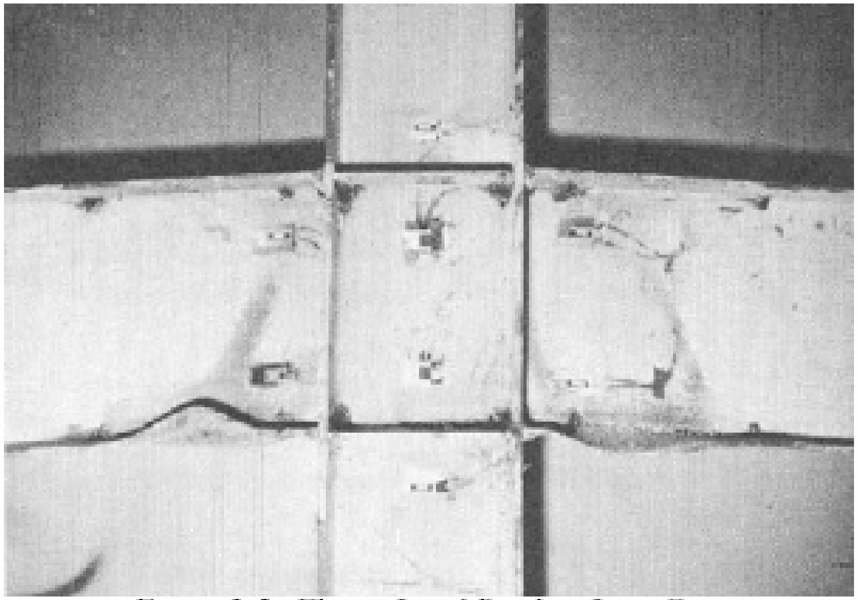
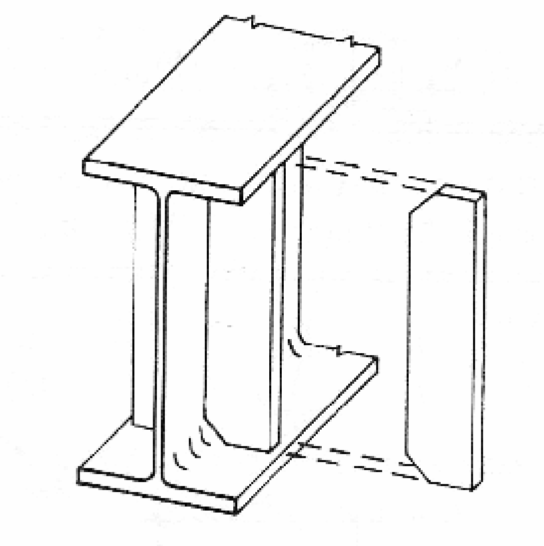


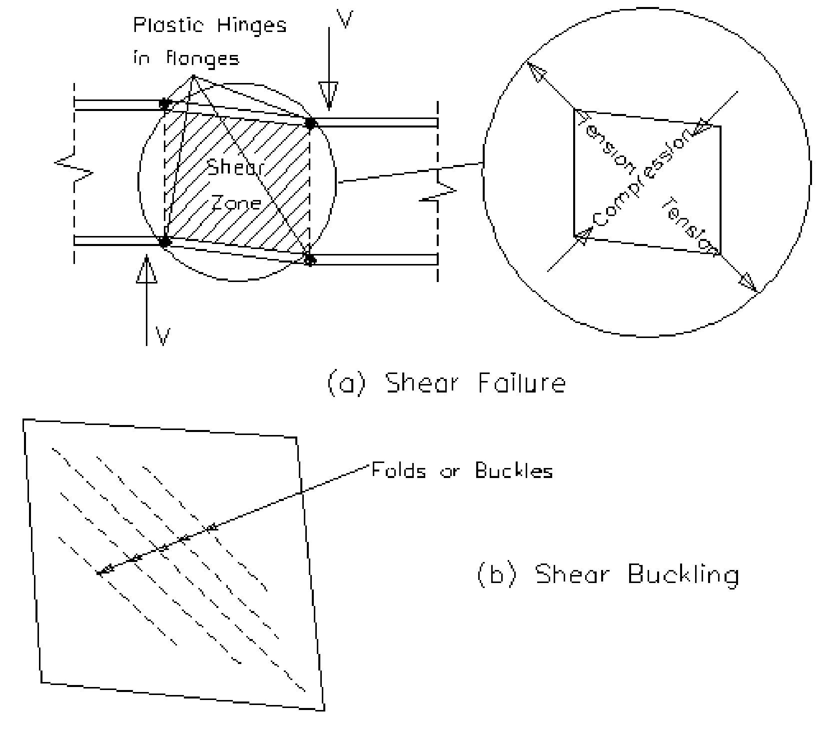
*Figure 2-5. Flange Local Bending Limit State (Beedle, L.S., Christopher, R., 1964)*

# Shear in Web

- panels in plate girders or webs with large shear
- buckling in compression direction
- add stiffeners



stiffeners to prevent lateral buckling

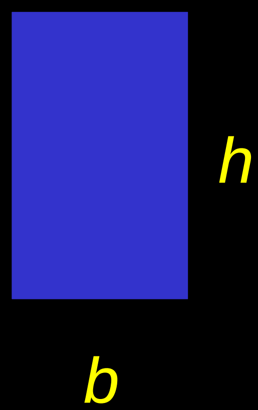


# Shear in Web

- *plate girders and stiffeners*

# *Beam Design*

1. *Know $F_{all}$ for the material or $F_U$ for LRFD*
2. *Draw V & M, finding $M_{max}$*
3. *Calculate $S_{req'd}$* $\left( f_b \le F_b \right)$
4. *Determine section size*




$$S = \frac{bh^2}{6}$$

# *Beam Design*

*4*. Include self weight for $M_{max}$*

– *and repeat 3 & 4 if necessary*

*5. Consider lateral stability*

*Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.*

*Photo: Ken Carper*

# Beam Design

## 6. Evaluate shear stresses - horizontal

- $\left(f_v \leq F_v\right)$
- W and rectangles

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

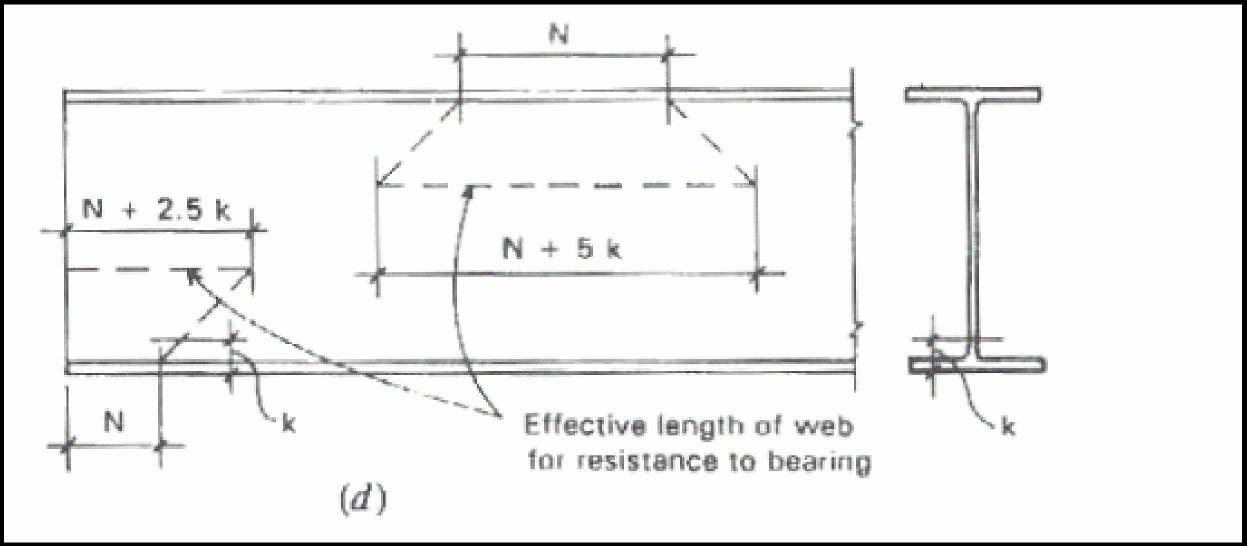
- thin walled sections

$$f_{v-\max} = \frac{VQ}{Ib}$$

# Beam Design

## 7. Provide adequate bearing area at supports

$$f_p = \frac{P}{A} \leq F_p$$



# Beam Design

## 8. Evaluate torsion

$$\left(f_v \le F_v\right)$$

- *circular cross section*

$$f_v = \frac{T\rho}{J}$$

- *rectangular*

$$f_v = \frac{T}{c_1 ab^2}$$

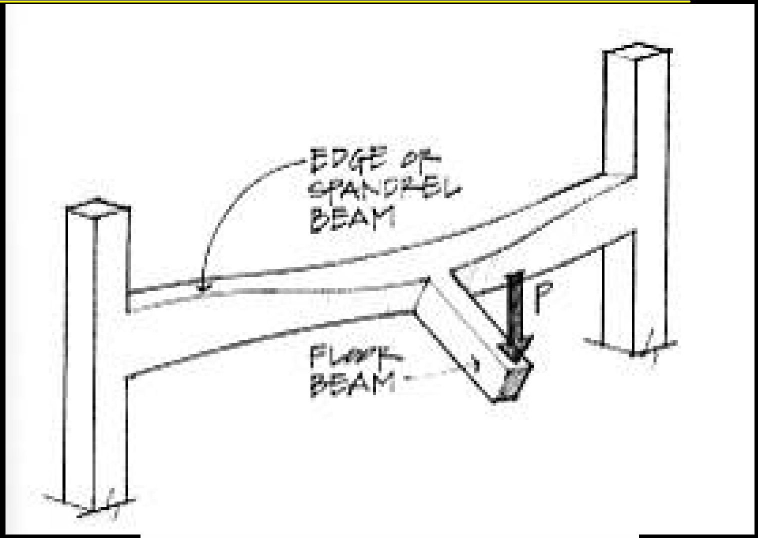
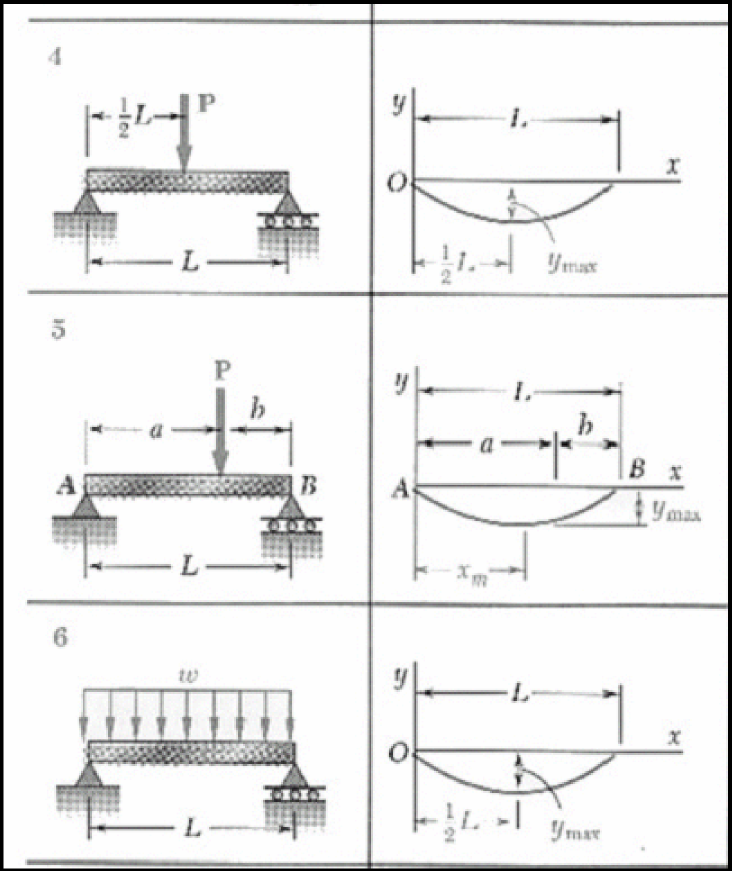


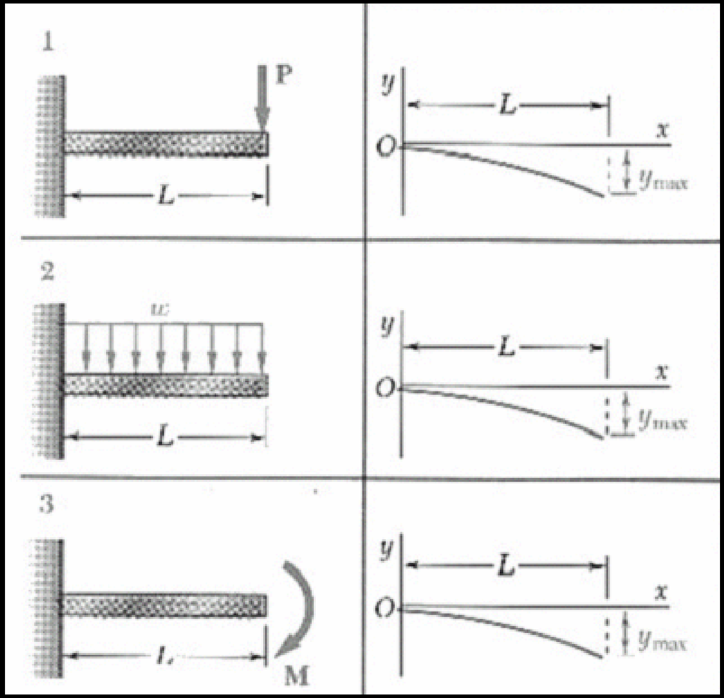

**TABLE 3.1. Coefficients for Rectangular Bars in Torsion**

| $a/b$ | $c_1$ | $c_2$ |
|---|---|---|
| 1.0 | 0.208 | 0.1406 |
| 1.2 | 0.219 | 0.1661 |
| 1.5 | 0.231 | 0.1958 |
| 2.0 | 0.246 | 0.229 |
| 2.5 | 0.258 | 0.249 |
| 3.0 | 0.267 | 0.263 |
| 4.0 | 0.282 | 0.281 |
| 5.0 | 0.291 | 0.291 |
| 10.0 | 0.312 | 0.312 |
| ∞ | 0.333 | 0.333 |

# Beam Design

## 9. Evaluate deflections



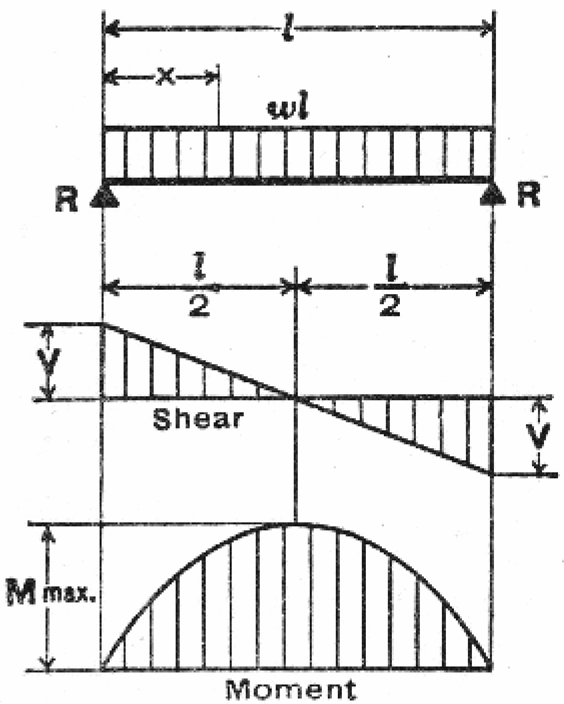




$$y_{\max}(x) = \Delta_{actual} \leq \Delta_{allowable}$$

# Beam Design

## 9. – how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD




Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . . . . . . . $= \dfrac{wl}{2}$

$V_x$ . . . . . . . . . . $= w\left(\dfrac{l}{2} - x\right)$

M max. (at center) . . . . . $= \dfrac{wl^2}{8}$

$M_x$ . . . . . . . . . . $= \dfrac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \dfrac{5\,wl^4}{384\,EI}$

$\Delta x$ . . . . . . . . . . $= \dfrac{wx}{24EI}(l^3 - 2lx^2 + x^3)$