ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231
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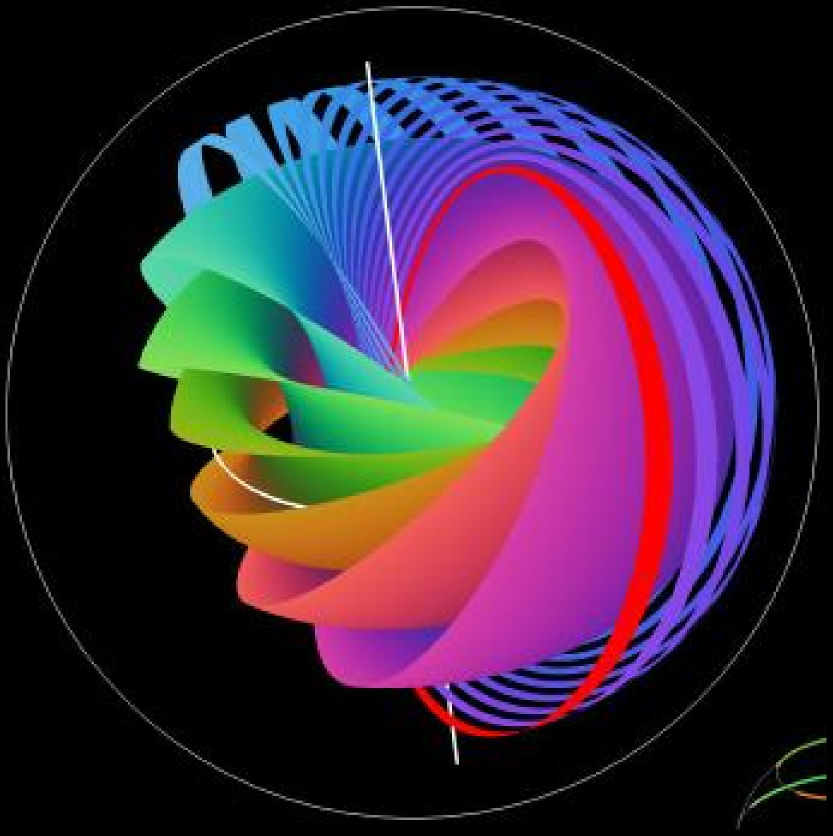
FALL 2007

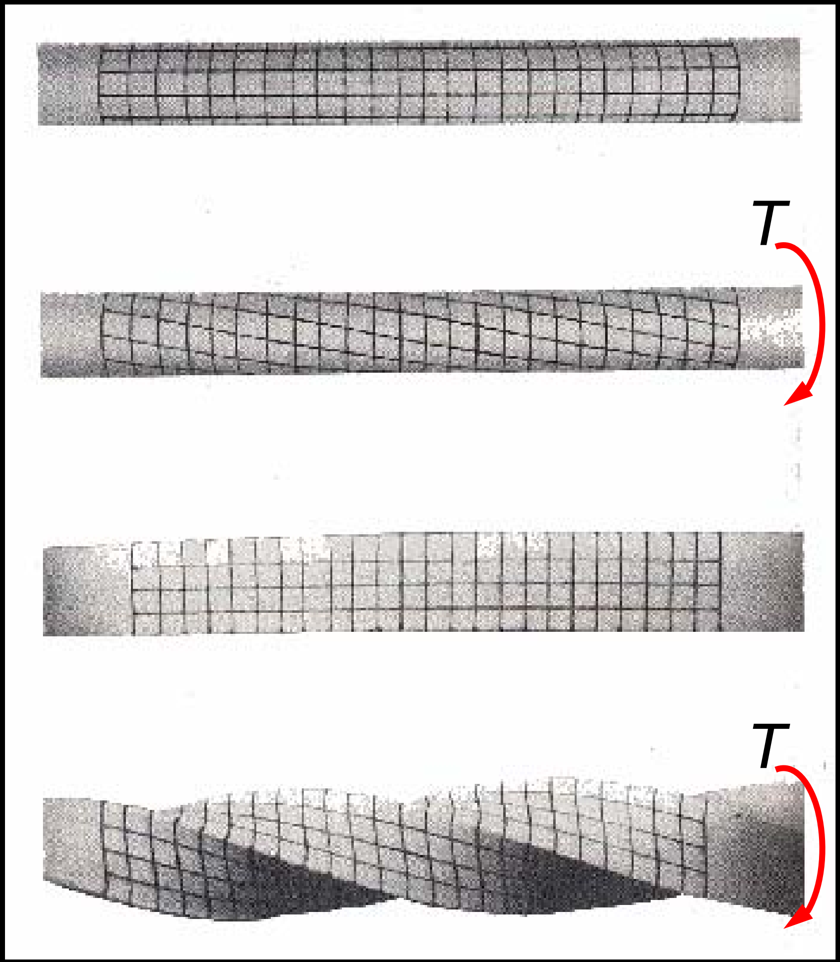
lecture
seventeen



torsion
& thermal effects

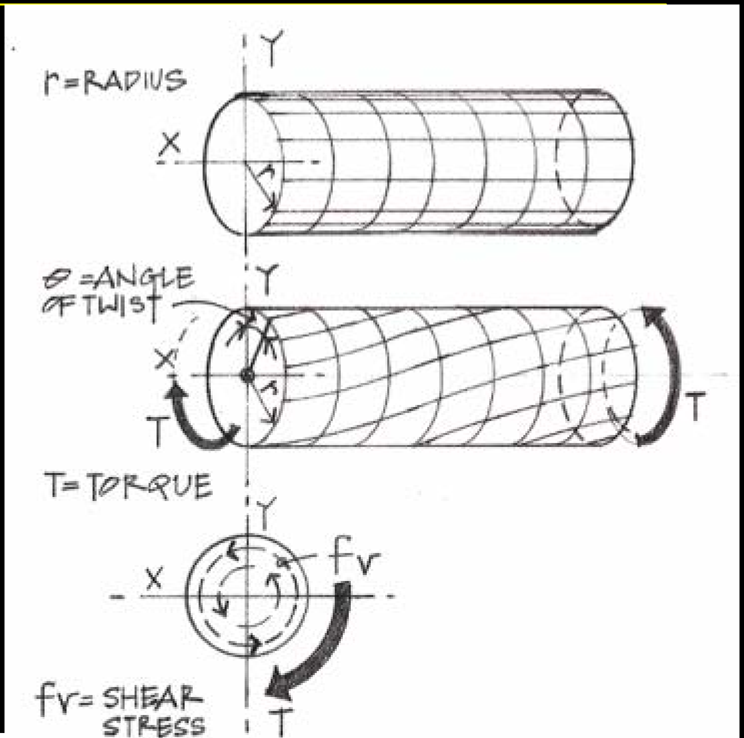
Torsional Stress & Strain

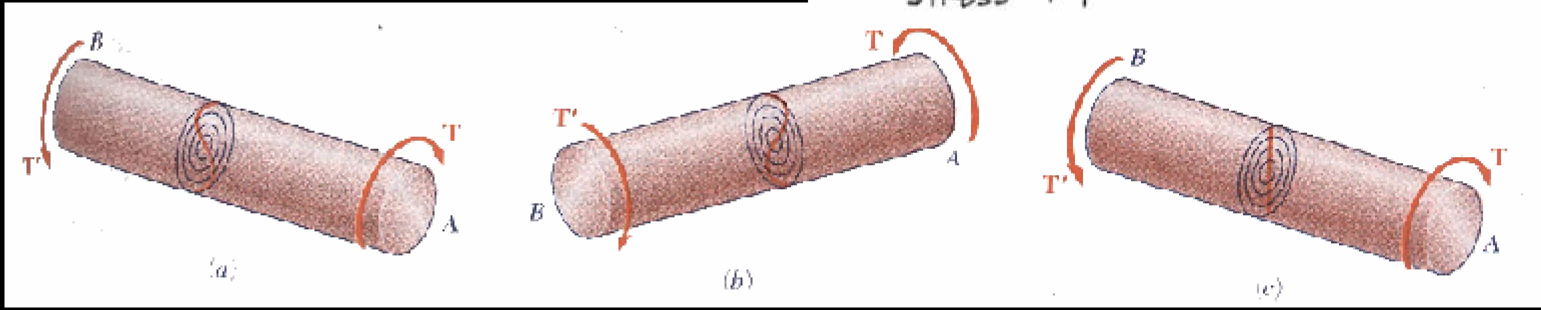
- *can see torsional stresses & twisting of axi-symmetrical cross sections*
  - *torque*
  - *remain plane*
  - *undistorted*
  - *rotates*
- *not true for square sections....*



Shear Stress Distribution

- depend on the deformation
- ϕ = angle of twist
  - measure
- can prove planar section doesn't distort



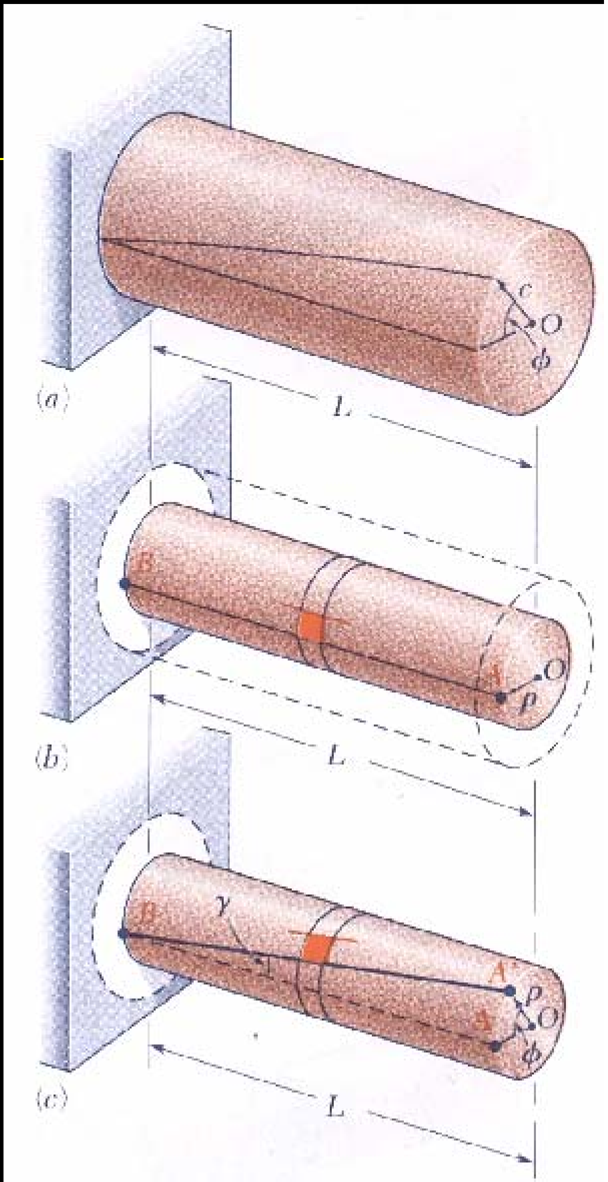


Shearing Strain

- related to ϕ

$$\gamma = \frac{\rho\phi}{L}$$

- ρ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: γ_{max} is at outer diameter

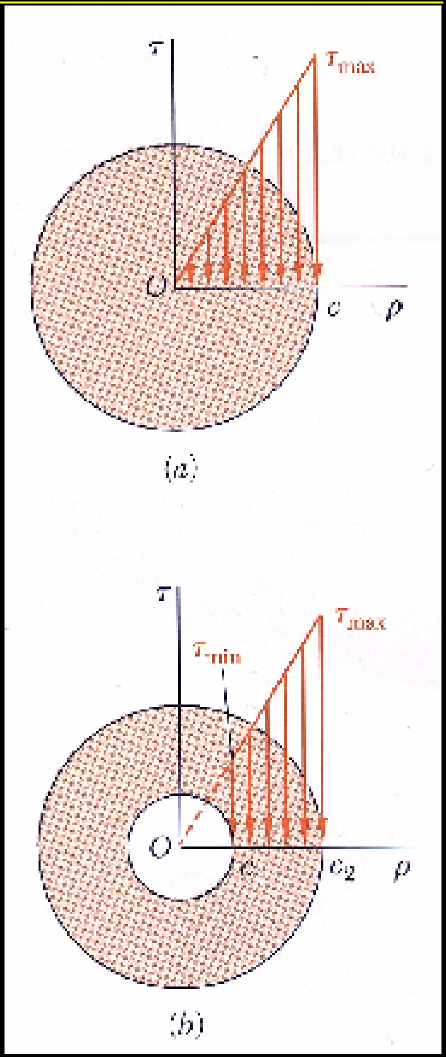


Torsional Stress - Strain

- *know* $f_v = \tau = G \cdot \gamma$ *and* $\gamma = \dfrac{\rho\phi}{L}$
- *so* $\tau = G \cdot \dfrac{\rho\phi}{L}$
- *where G is the* *Shear Modulus*

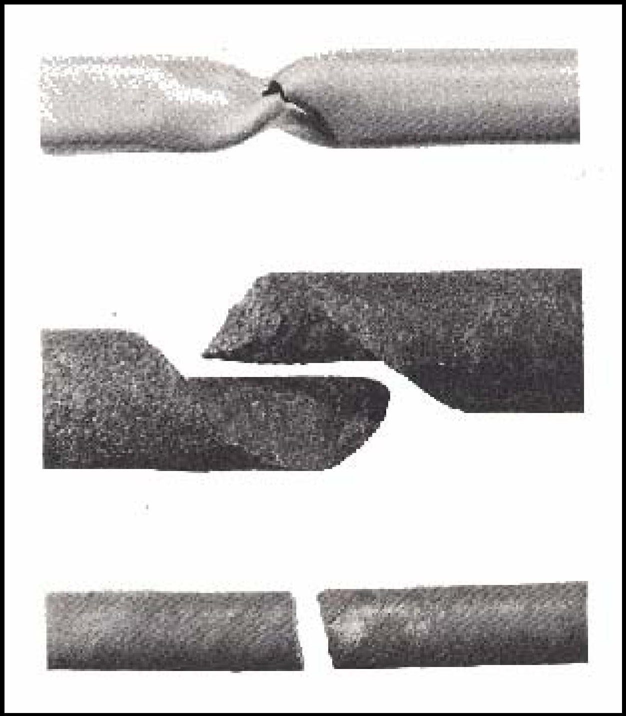
Torsional Stress - Strain

- *from* $$T = \Sigma\tau(\rho)\Delta A$$
- *can derive* $$T = \frac{\tau J}{\rho}$$
  - *where J is the polar moment of inertia*
  - *elastic range* $$\tau = \frac{T\rho}{J}$$



Shear Stress

- *τ_{max} happens at <u>outer diameter</u>*



- *combined shear and axial stresses*
  - *maximum shear stress at 45° "twisted" plane*

Shear strain

- knowing $\tau = G \cdot \frac{\rho\phi}{L}$ and $\tau = \frac{T\rho}{J}$
- solve: $\phi = \frac{TL}{JG}$
- composite shafts: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

Noncircular Shapes

- *torsion depends on J*
- *plane sections don't remain plane*
- τ_{max} *is still at outer diameter*

$$\tau_{\max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

– *where a is longer side (> b)*

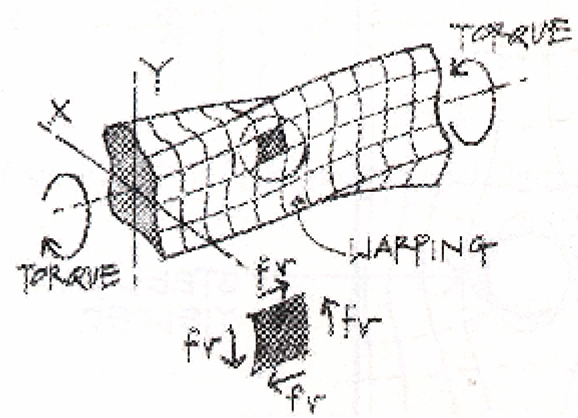
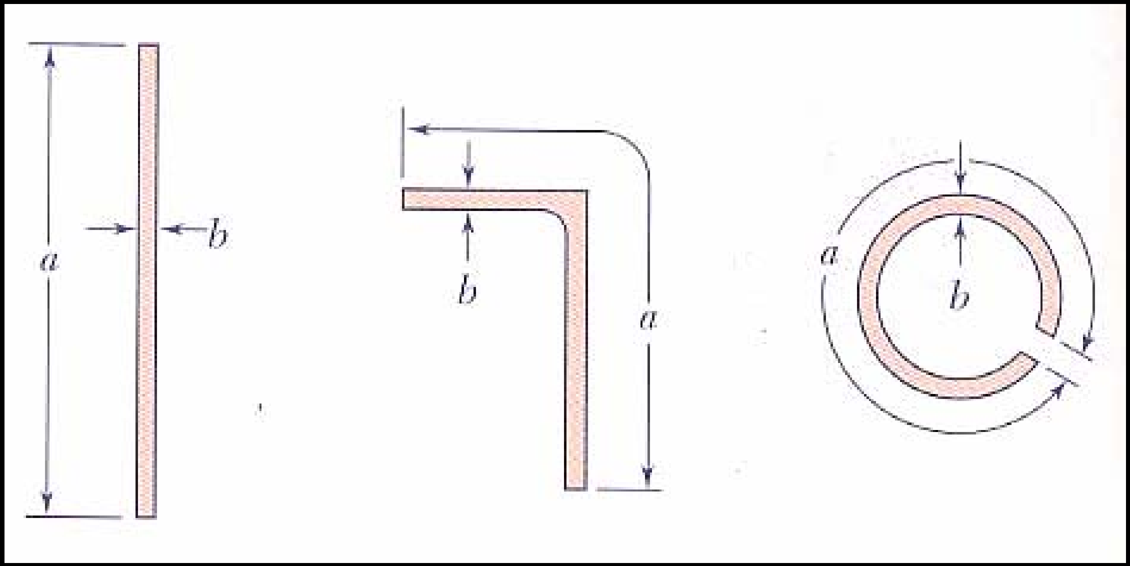


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Open Thin-Walled Sections

- with very large a/b ratios:



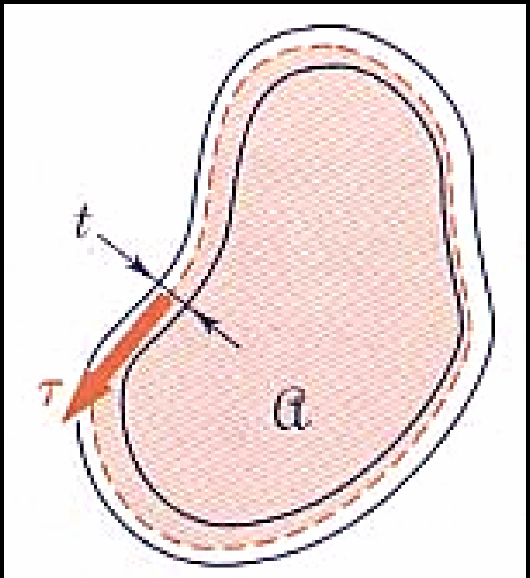
$$\tau_{max} = \frac{T}{\frac{1}{3}ab^2} \qquad \phi = \frac{TL}{\frac{1}{3}ab^3G}$$

Shear Flow in Closed Sections

- *q is the internal shear force/unit length*

$$\tau = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$

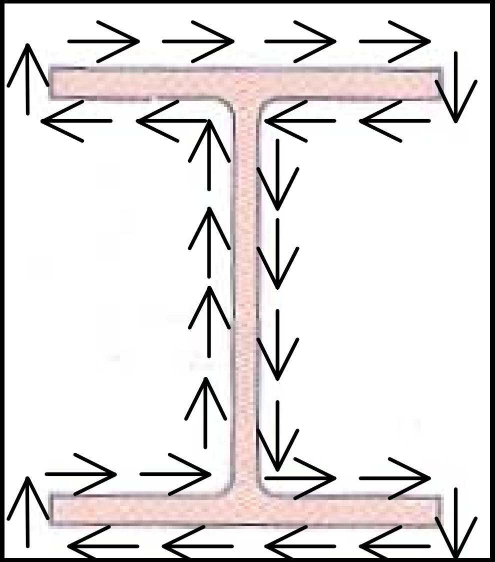


- *$\mathcal{A}$ is the area bounded by the centerline*
- *s_i is the length segment, t_i is the thickness*

Shear Flow in Open Sections

- *each segment has proportion of T with respect to torsional rigidity,*

$$\tau_{\max} = \frac{Tt_{\max}}{\frac{1}{3}\Sigma b_i t_i^3}$$



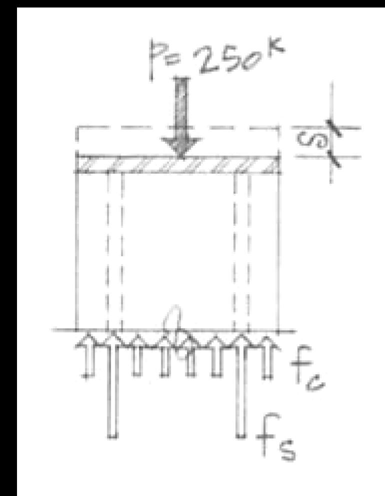
- *total angle of twist:*

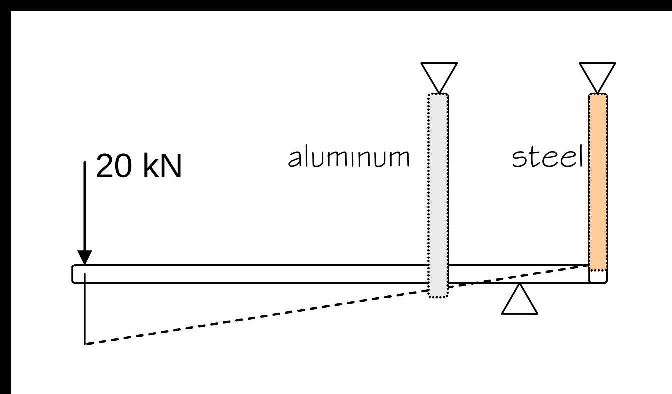
$$\phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

- *I beams - web is thicker, so τ_{max} is in web*

Deformation Relationships

- physical movement
  - axially (same or zero)
  - rotations from axial changes

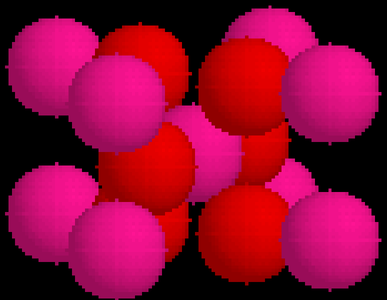




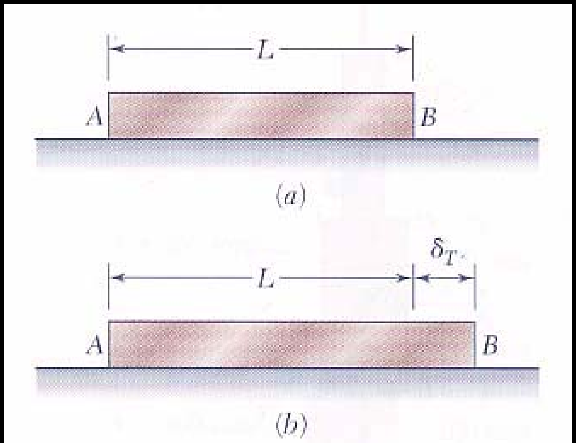
- $\delta = \dfrac{PL}{AE}$ relates δ to P

Deformations from Temperature

- *atomic chemistry reacts to changes in energy*



- *solid materials*
  - *can contract with decrease in temperature*
  - *can expand with increase in temperature*
- *linear change can be measured per degree*



Thermal Deformation

- α *- the rate of strain per degree*
- *UNITS :* $\frac{}{°F}$, $\frac{}{°C}$
- *length change:*

$$\delta_T = \alpha(\Delta T)L$$

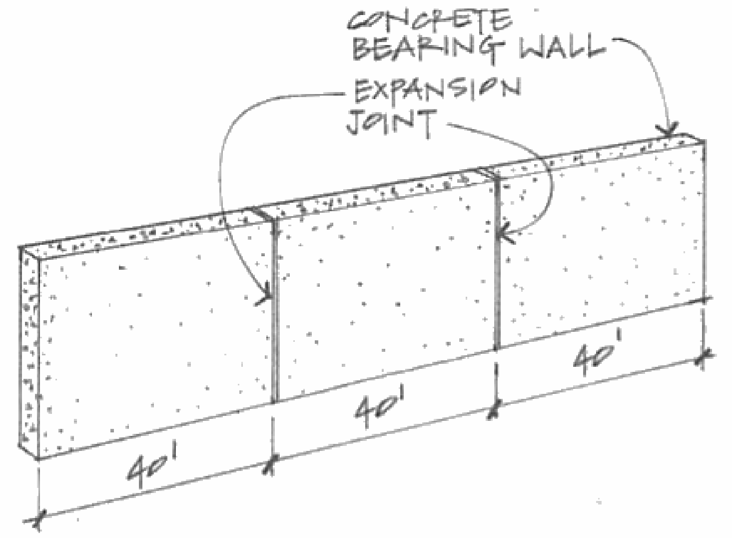
- *thermal strain:*

$$\varepsilon_T = \alpha(\Delta T)$$

  – *no stress when movement allowed*

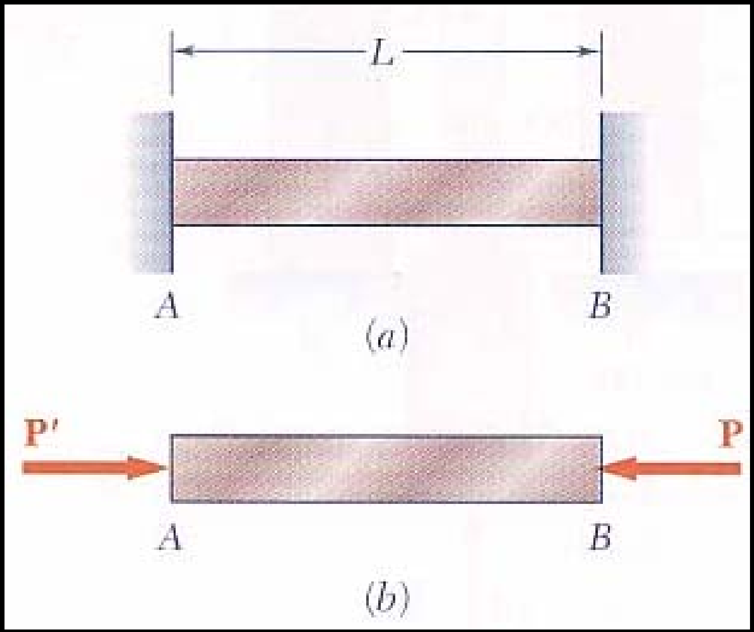
Coefficients of Thermal Expansion

Material	Coefficients (α) [in./in./°F]
Wood	3.0×10^{-6}
Glass	4.4×10^{-6}
Concrete	5.5×10^{-6}
Cast Iron	5.9×10^{-6}
Steel	6.5×10^{-6}
Wrought Iron	6.7×10^{-6}
Copper	9.3×10^{-6}
Bronze	10.1×10^{-6}
Brass	10.4×10^{-6}
Aluminum	12.8×10^{-6}



Stresses and Thermal Strains

- *if thermal movement is restrained* *stresses are induced*

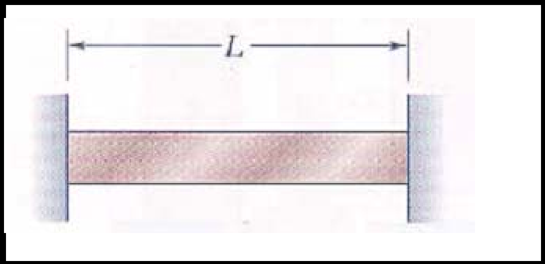


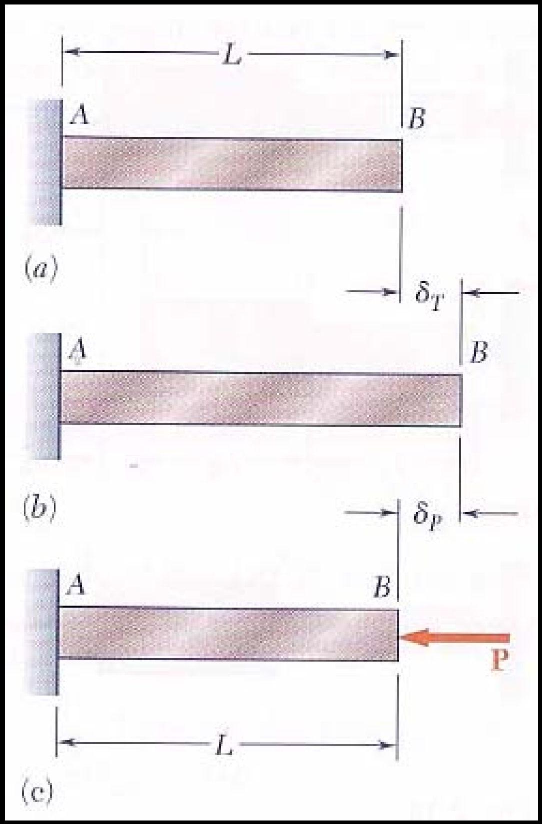
1. *bar pushes on supports*
2. *support pushes back*
3. *reaction causes internal stress*

$$f = \frac{P}{A} = \frac{\delta}{L} E$$

Superposition Method

– *can remove a support to make it look determinant*
– *replace the support with a reaction*
– *enforce the geometry constraint*

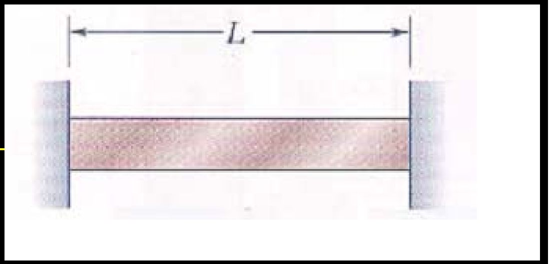




Superposition Method

– *total length change restrained to* ***zero***

constraint: $\delta_P + \delta_T = 0$



$$\delta_p = -\frac{PL}{AE} \qquad \delta_T = \alpha(\Delta T)L$$

sub: $$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$