

**ARCHITECTURAL STRUCTURES I:  
STATICS AND STRENGTH OF MATERIALS**

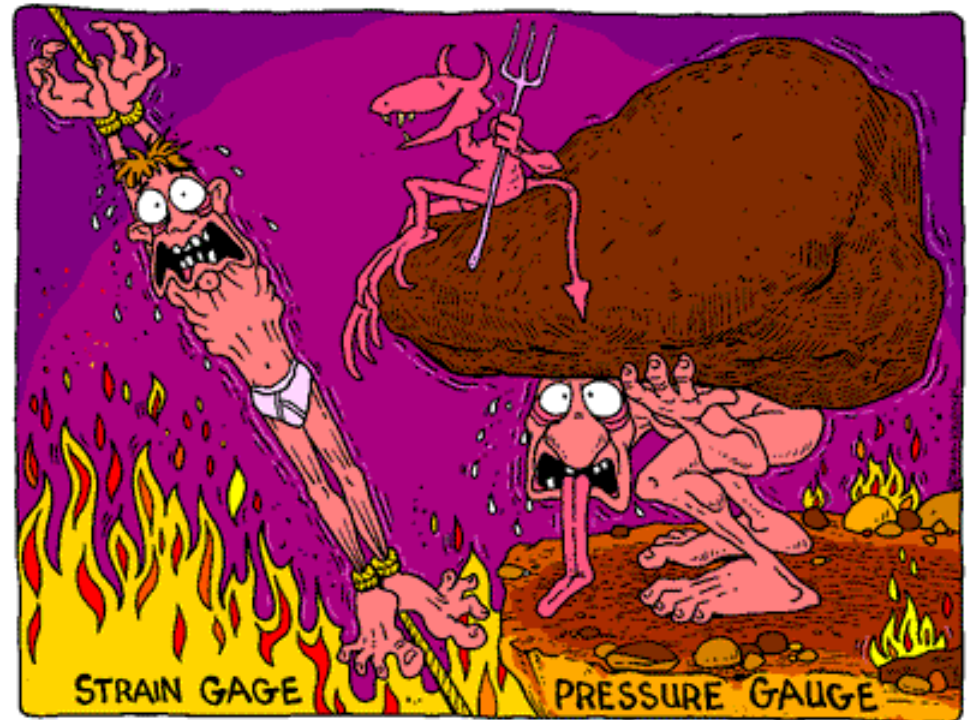
ENDS 231

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FALL 2007

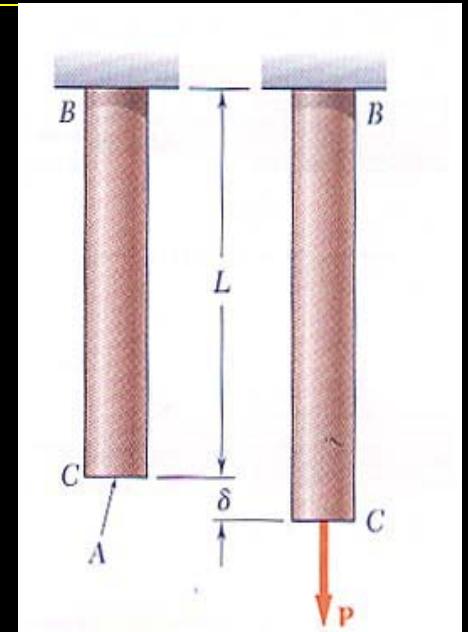
lecture  
**sixteen**

**elasticity  
& strain**



# Deformations

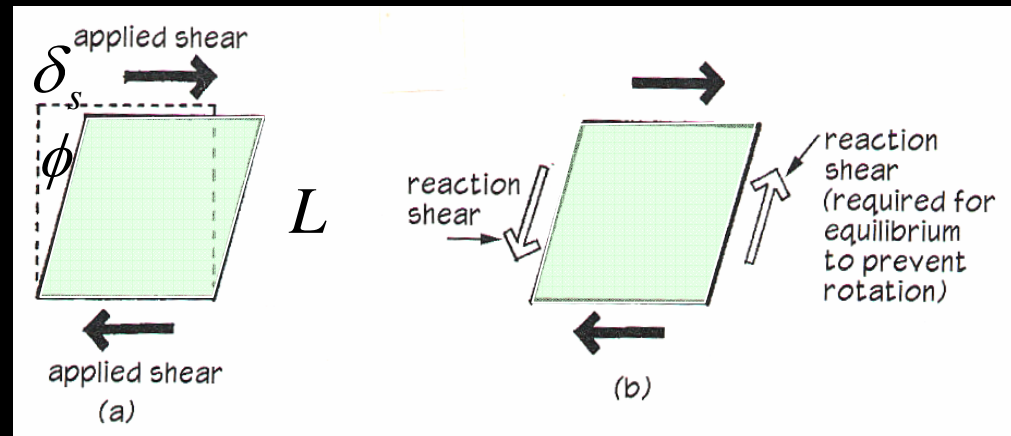
- *materials deform*
- *axially loaded materials change length*
- *normal stress is load per unit area*
- **STRAIN:**
  - *change in length over length*
  - **UNITLESS**



$$\epsilon = \frac{\delta}{L}$$

# Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress:  $\tau$
- strain:  $\gamma$   
– unitless (radians)

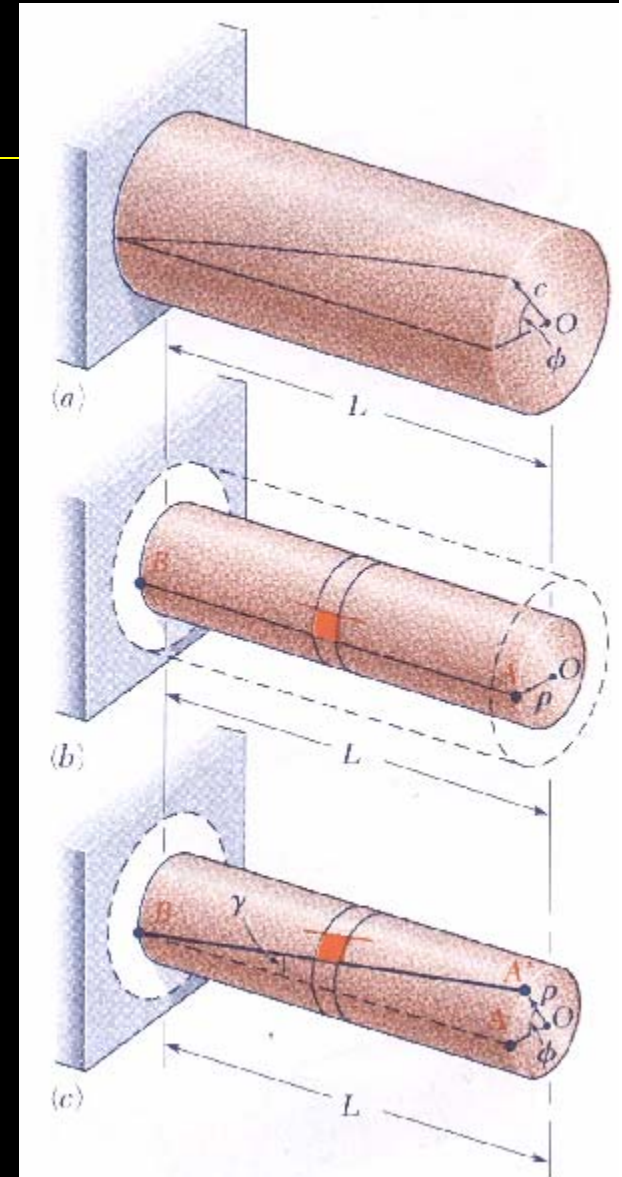


$$\gamma = \frac{\delta_s}{L} = \tan \phi \cong \phi$$

# Shearing Strain

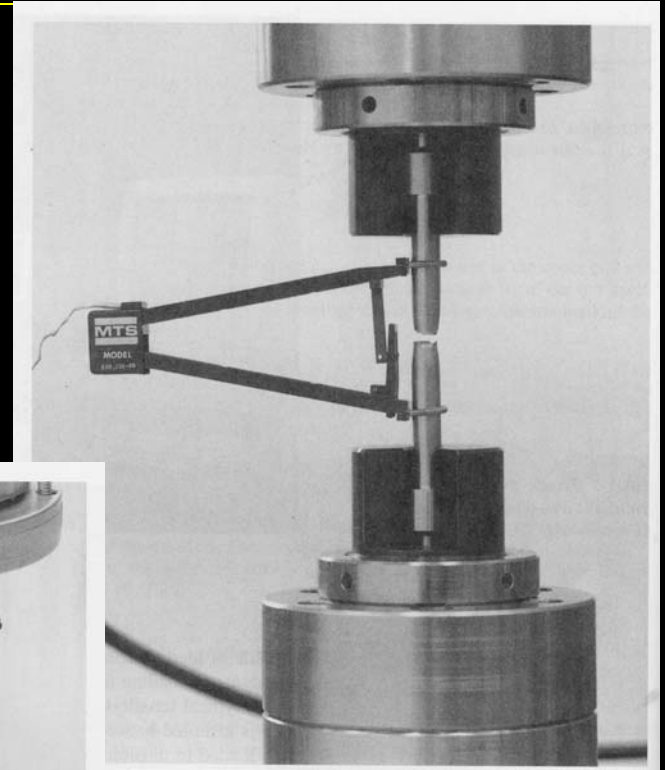
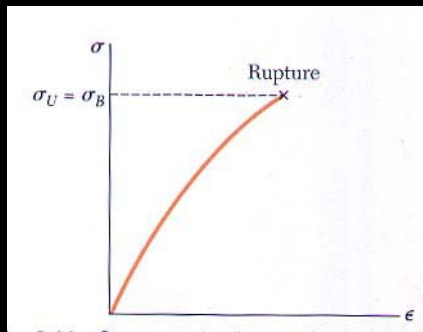
- *deformations with torsion*
- *twist*
- *change in angle of line*
- *stress:*  $\tau$
- *strain:*  $\gamma$
- *unitless (radians)*

$$\gamma = \frac{\rho\phi}{L}$$



# Load and Deformation

- for stress, need  $P$  &  $A$
- for strain, need  $\delta$  &  $L$ 
  - how?
  - TEST with load and measure
  - plot  $P/A$  vs.  $\epsilon$



# Material Behavior

- every material has its own response
  - 10,000 psi
  - $L = 10$  in
  - Douglas Fir vs. steel?

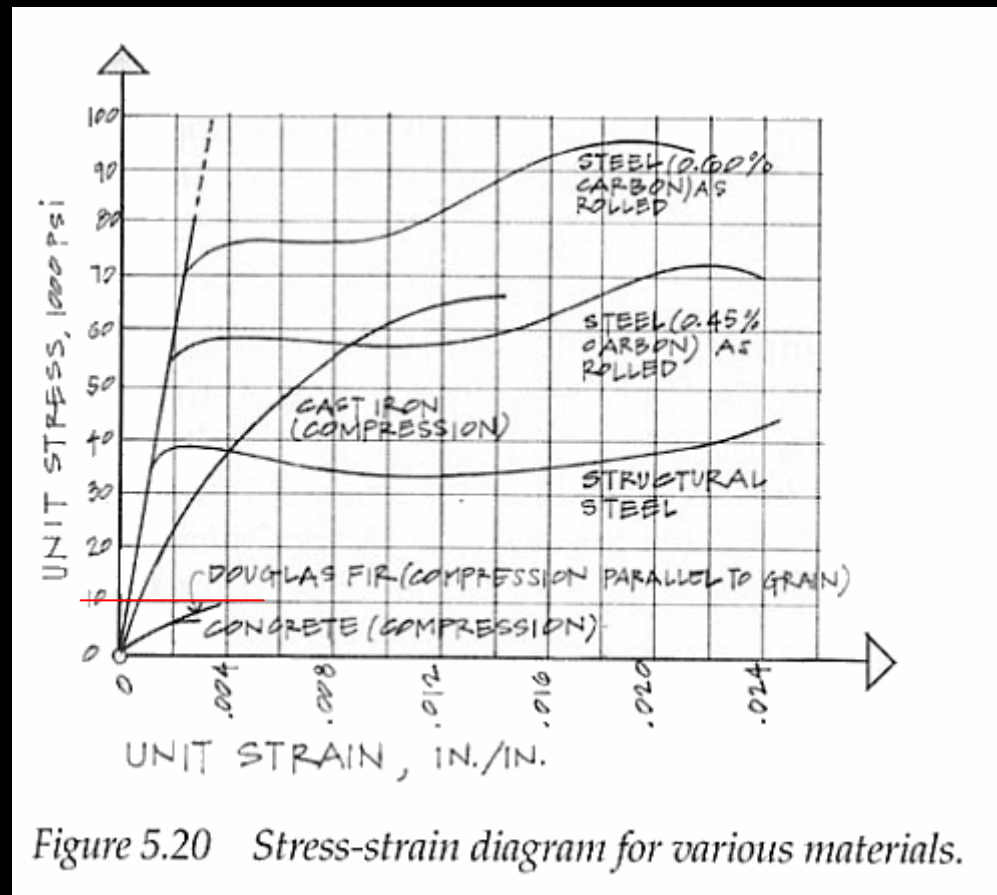


Figure 5.20 Stress-strain diagram for various materials.



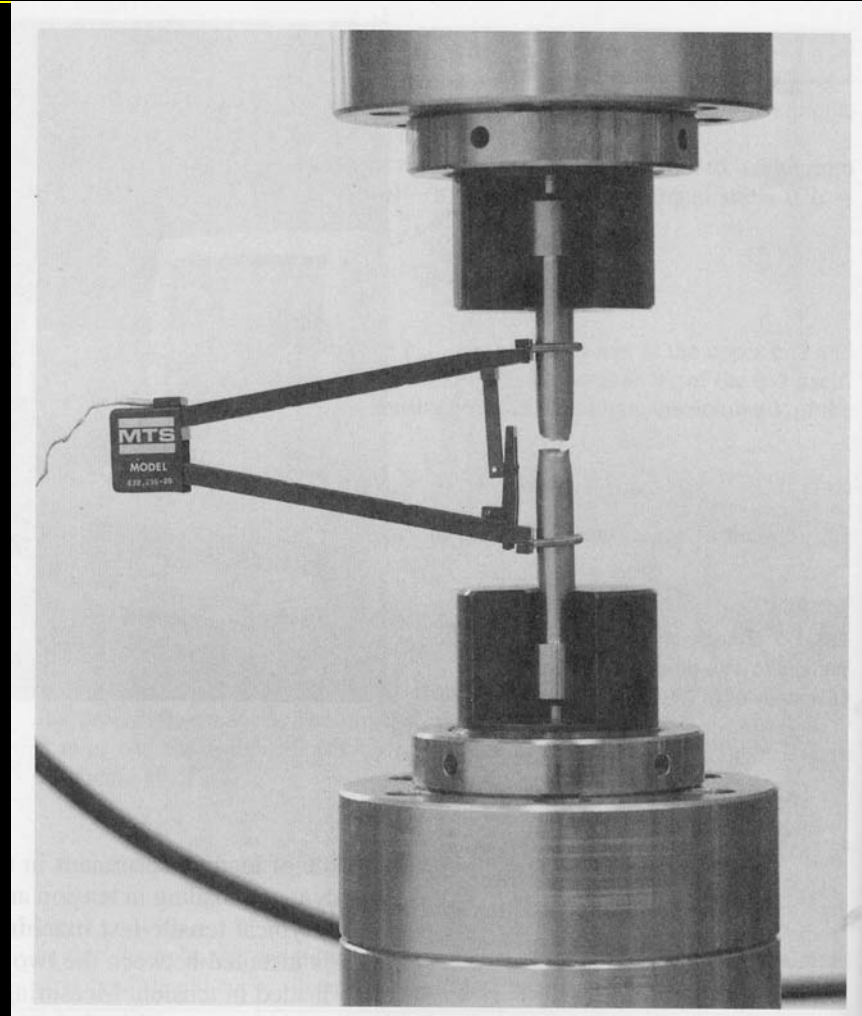
# Behavior Types

- ductile - “necking”
- true stress

$$f = \frac{P}{A}$$

- engineering stress  
– (simplified)

$$f = \frac{P}{A_o}$$



# Behavior Types

- *brittle*

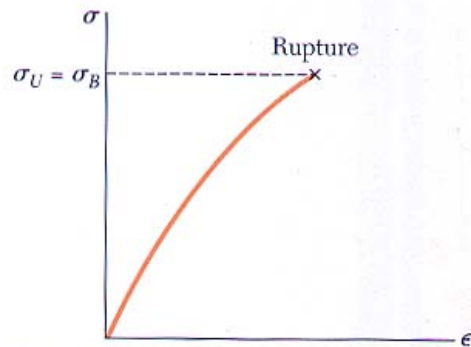


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

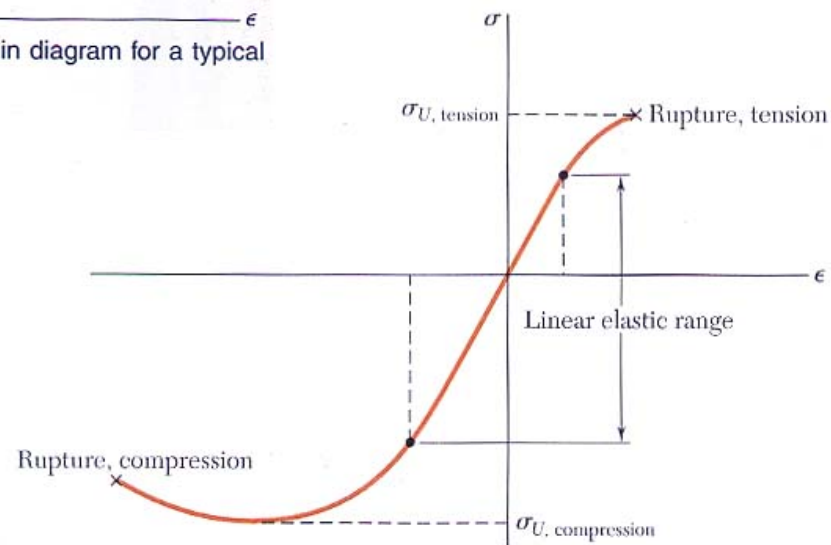


Fig. 2.14 Stress-strain diagram for concrete.



# Stress to Strain

- important to us in  $f$ - $\epsilon$  diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

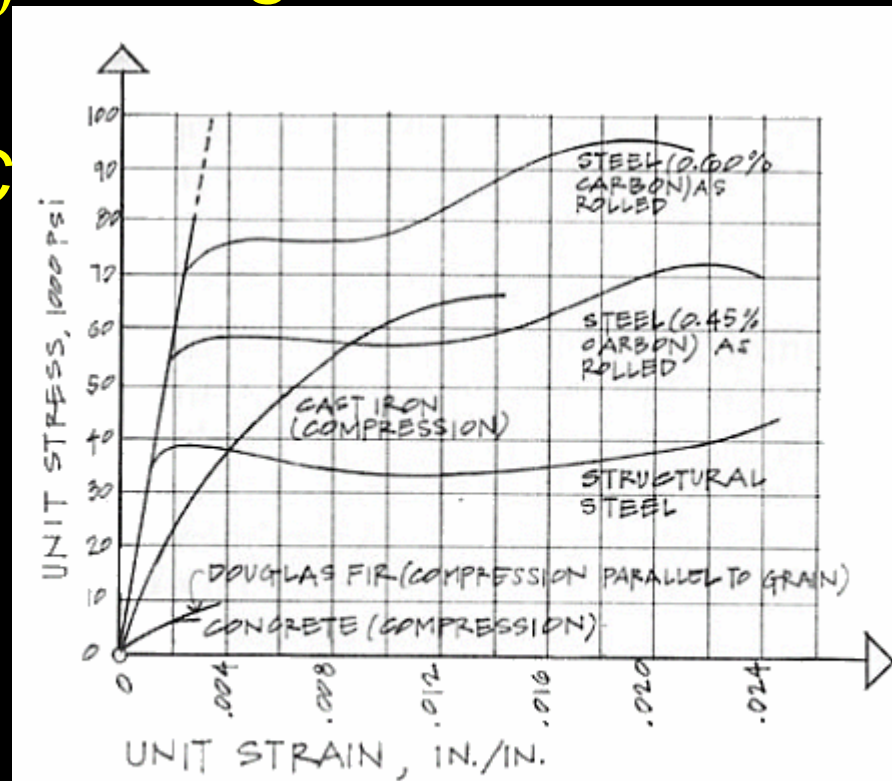


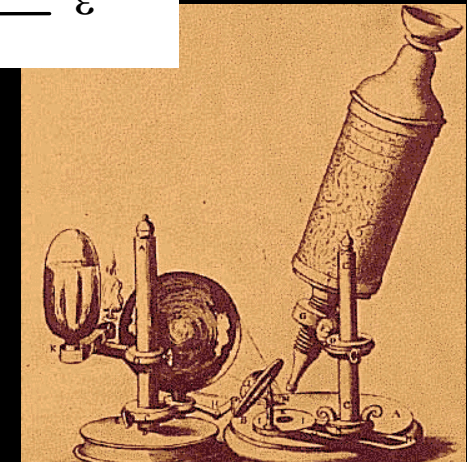
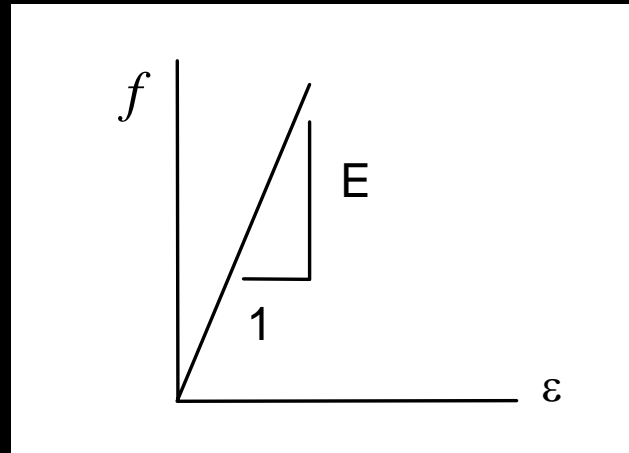
Figure 5.20 Stress-strain diagram for various materials.

# Hooke's Law

- *straight line has constant slope*
- *Hooke's Law*

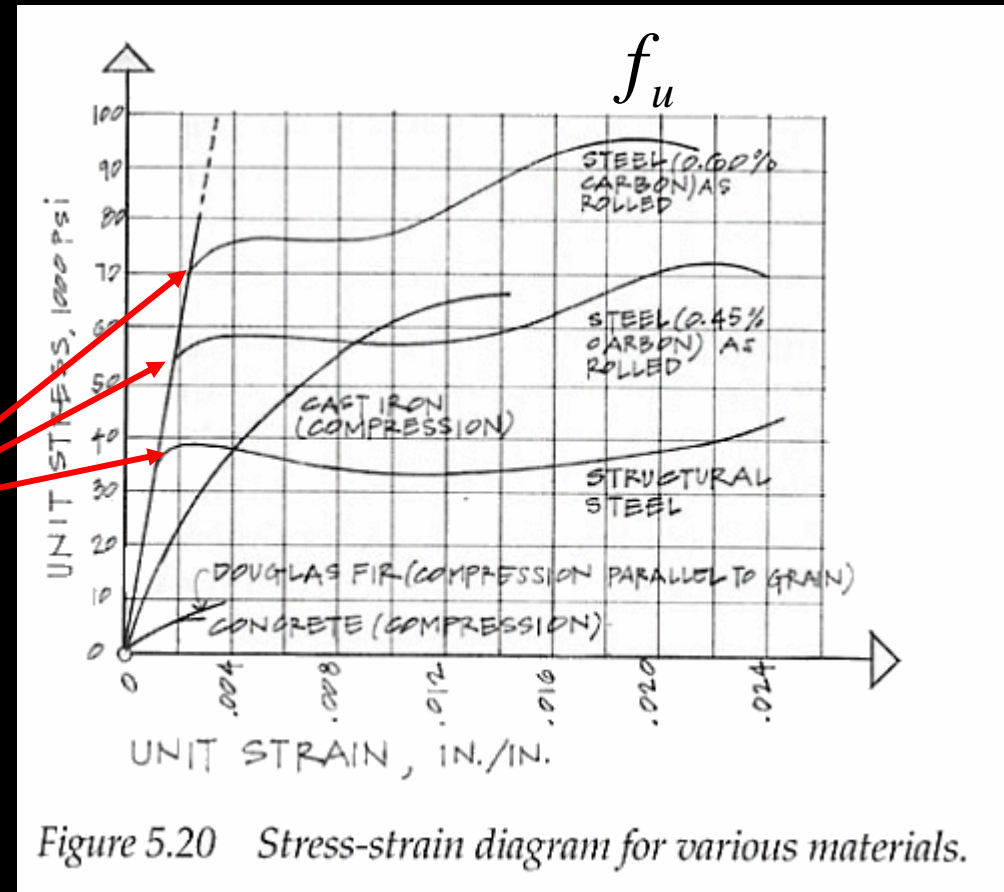
$$f = E \cdot \varepsilon$$

- *E*
  - *Modulus of elasticity*
  - *Young's modulus*
  - *units just like stress*



# Stiffness

- *ability to resist strain*
- *steels*
  - *same  $E$*
  - *different yield points*
  - *different ultimate strength*



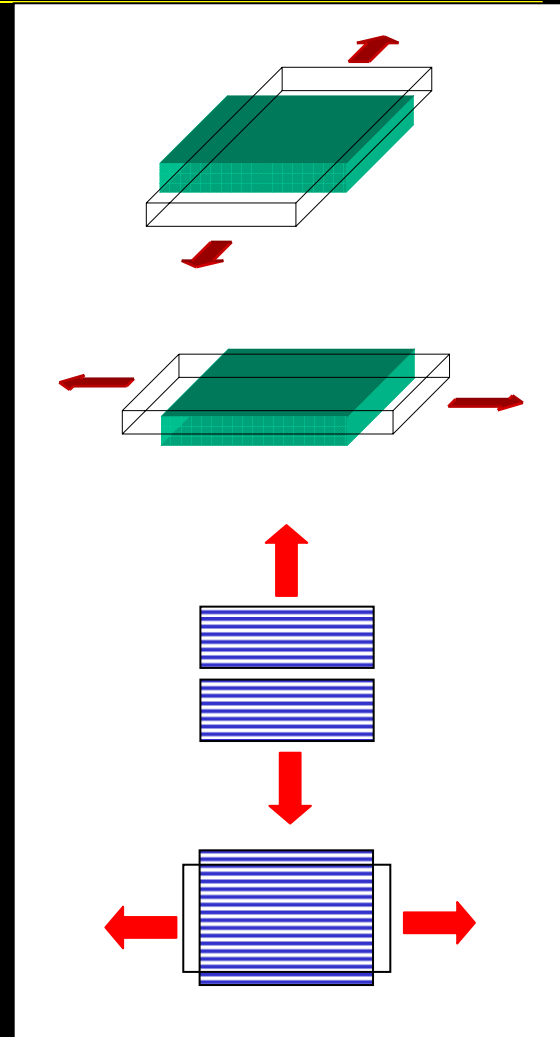
# Isotropy & Anisotropy

- **ISOTROPIC**

- materials with  $E$  same at any direction of loading
- ex. steel

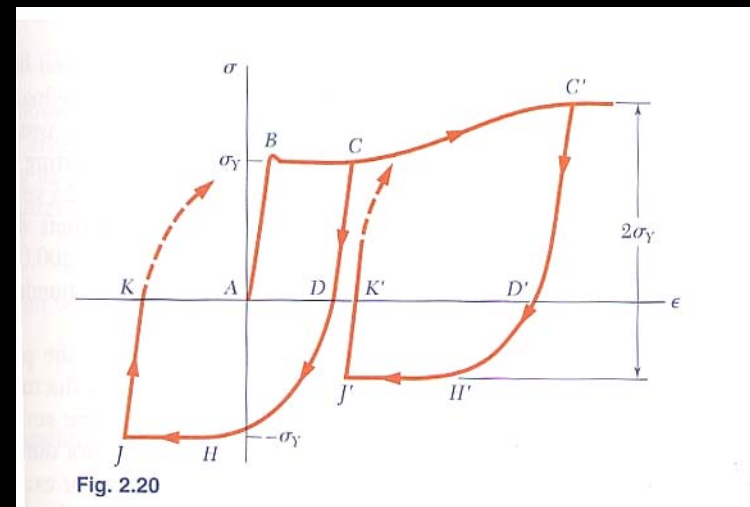
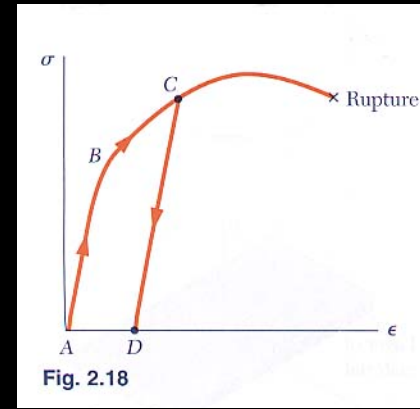
- **ANISOTROPIC**

- materials with different  $E$  at any direction of loading
- ex. wood is orthotropic



# Elastic, Plastic, Fatigue

- *elastic springs back*
- *plastic has permanent deformation*
- *fatigue caused by reversed loading cycles*



# Plastic Behavior

- ductile

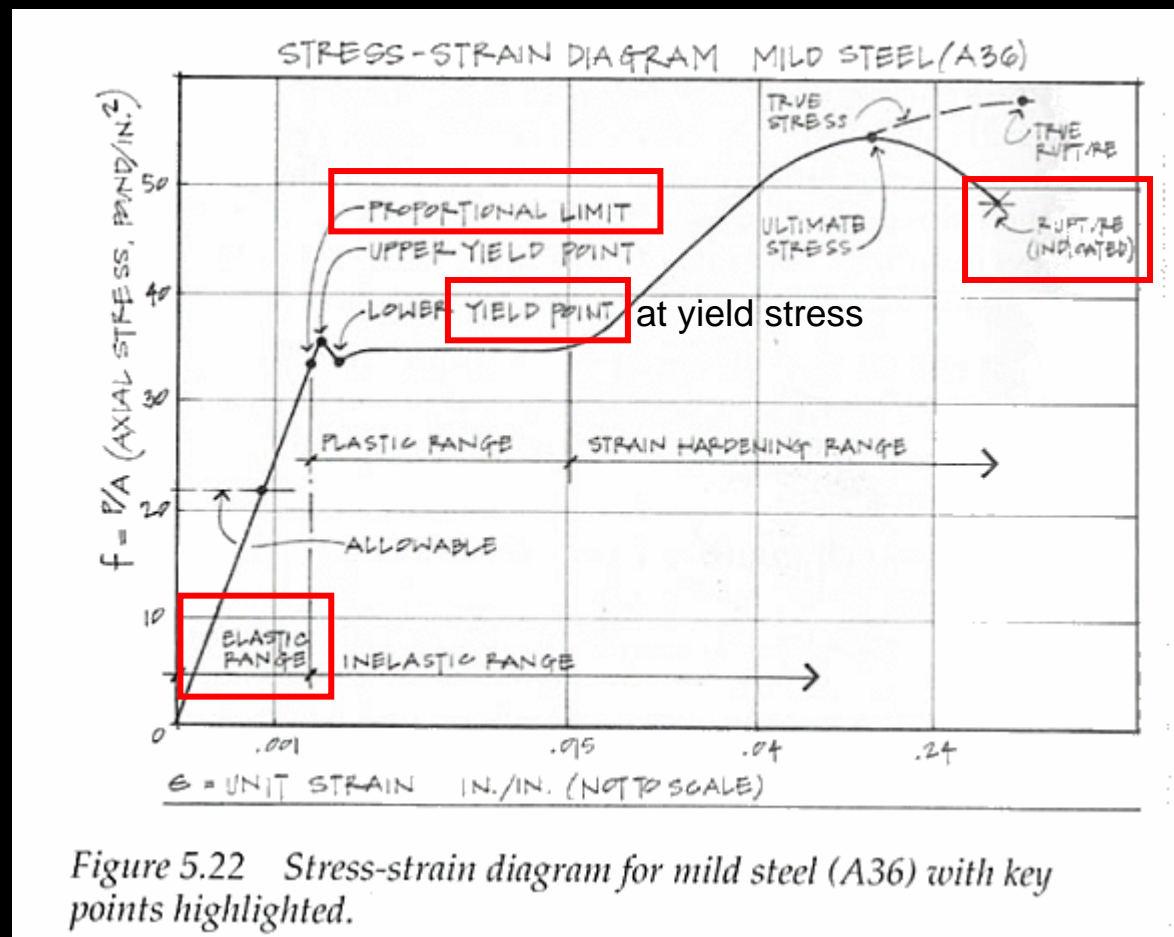


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

# Lateral Strain

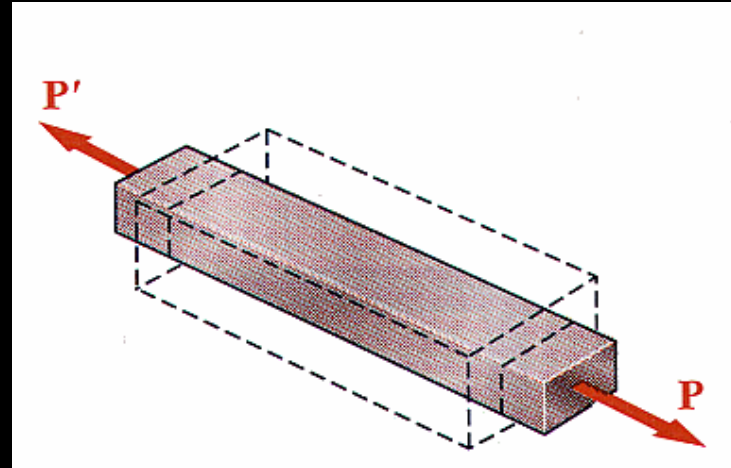
- or “what happens to the cross section with axial stress”

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- strain in lateral direction
  - negative
  - equal for isometric materials

$$\varepsilon_y = \varepsilon_z$$





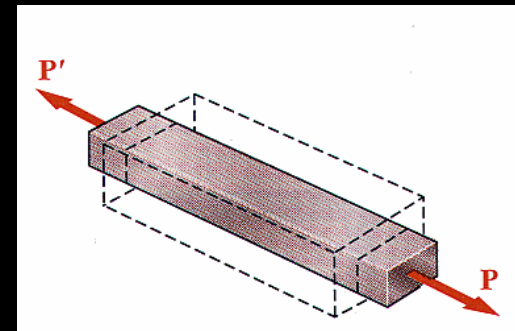
# Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- sign!  $0 < \mu < 0.5$



# Calculating Strain

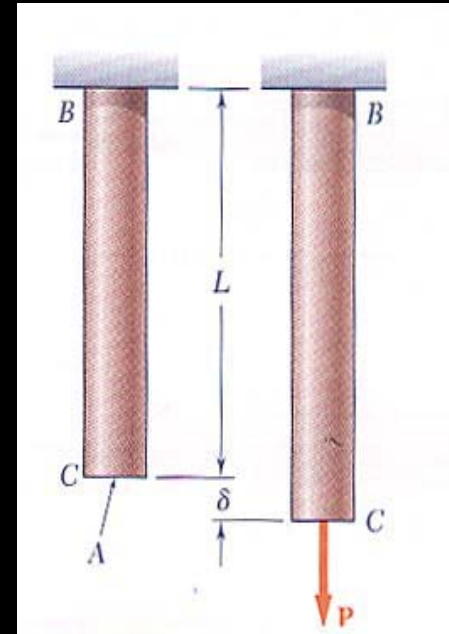
- from Hooke's law

$$f = E \cdot \varepsilon$$

- substitute

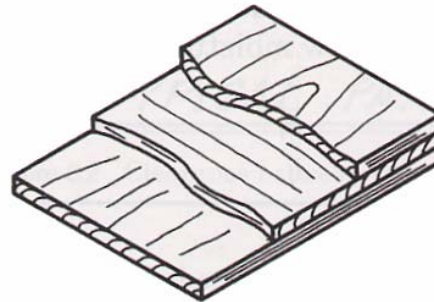
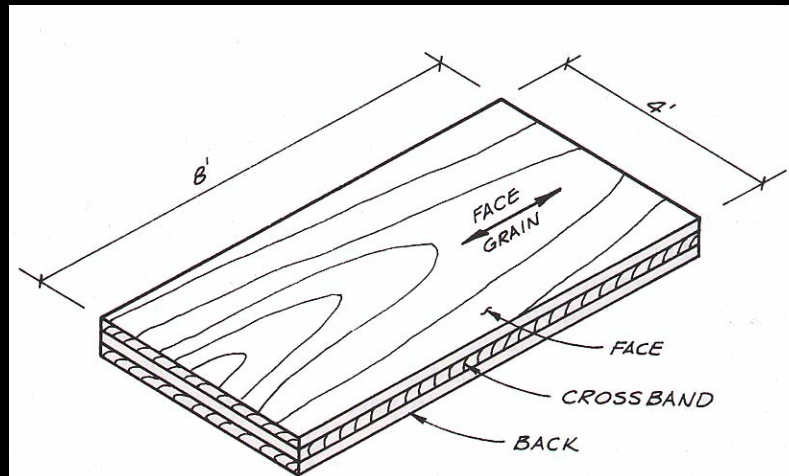
$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

- get  $\Rightarrow$  
$$\delta = \frac{PL}{AE}$$

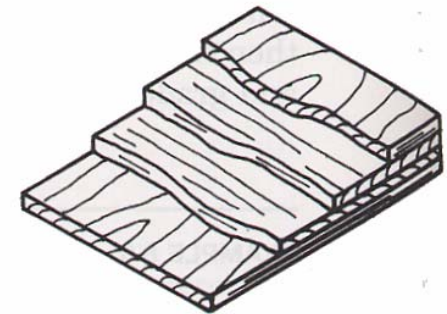


# Orthotropic Materials

- *non-isometric*
- *directional values of  $E$  and  $\mu$*
- *ex:*
  - *plywood*
  - *laminates*
  - *polymer composites*



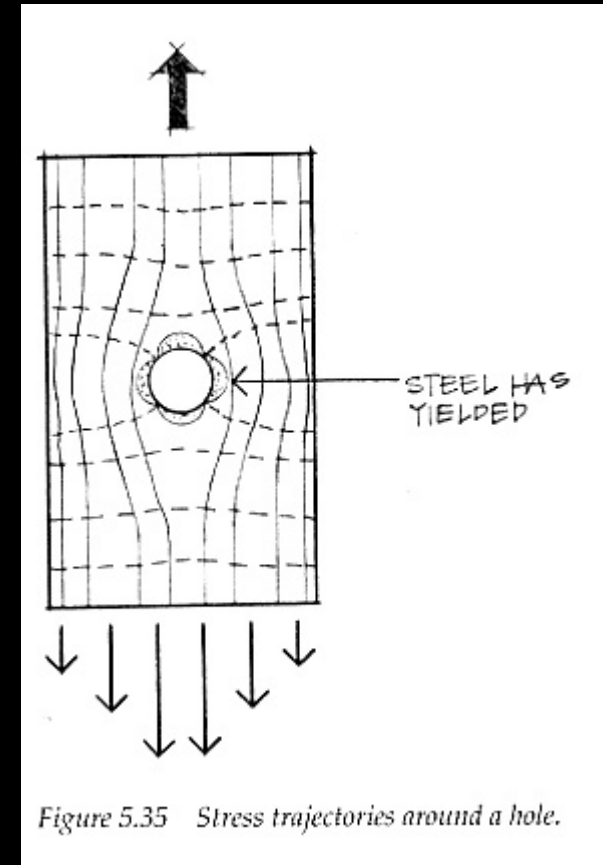
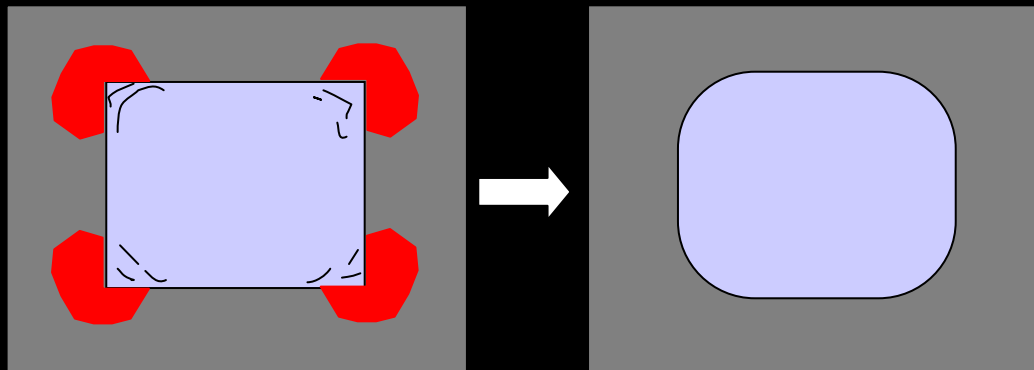
3 LAYER  
3 PLY CONSTRUCTION



3 LAYER  
4 PLY CONSTRUCTION

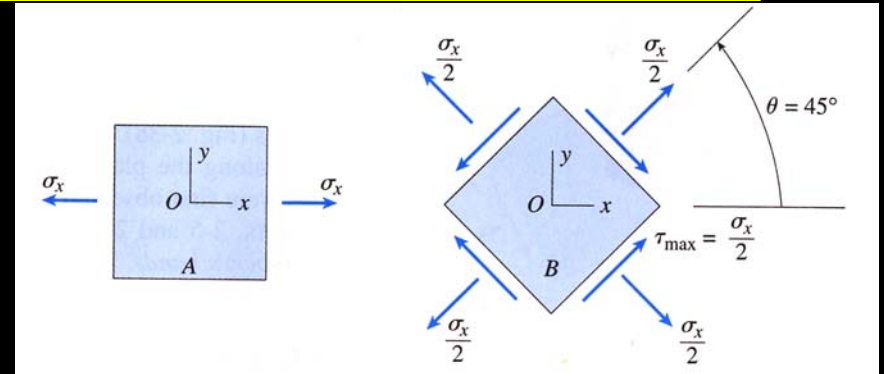
# Stress Concentrations

- why we use  $f_{ave}$
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners



# Maximum Stresses

- if we need to know where max  $f$  and  $f_v$  happen:

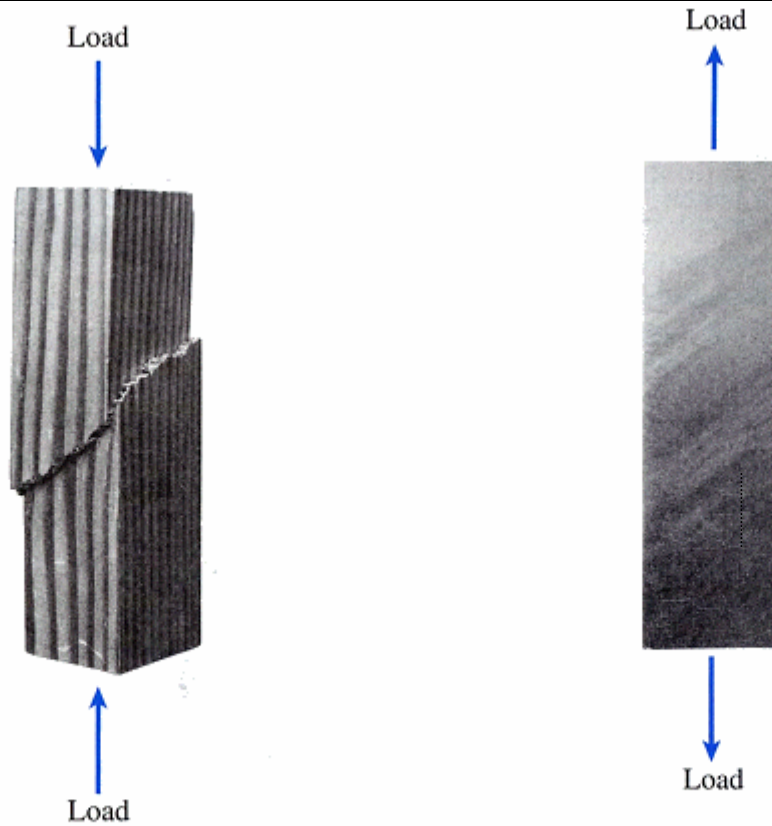


$$\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{\max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_o} = \frac{f_{\max}}{2}$$

# Maximum Stresses



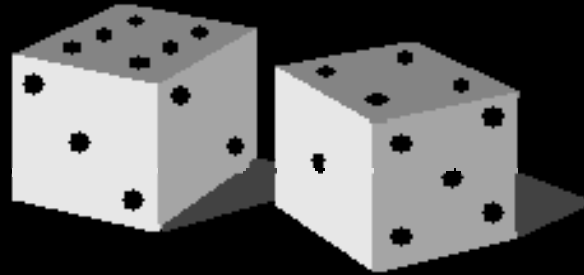
**FIG. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**FIG. 2-38** Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

# Design of Members

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- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- **RISK & UNCERTAINTY**



$$f_u = \frac{P_u}{A}$$



# Factor of Safety

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- *accommodate uncertainty with a safety factor:*

$$\text{allowable load} = \frac{\text{ultimate load}}{F.S}$$

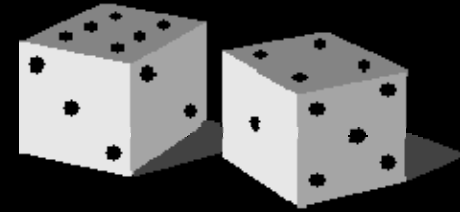
- *with linear relation between load and stress:*

$$F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

# *Load and Resistance Factor Design*

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- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

$\phi$  - *resistance factor*

$\gamma$  - *load factor for (D)ead & (L)ive load*