ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

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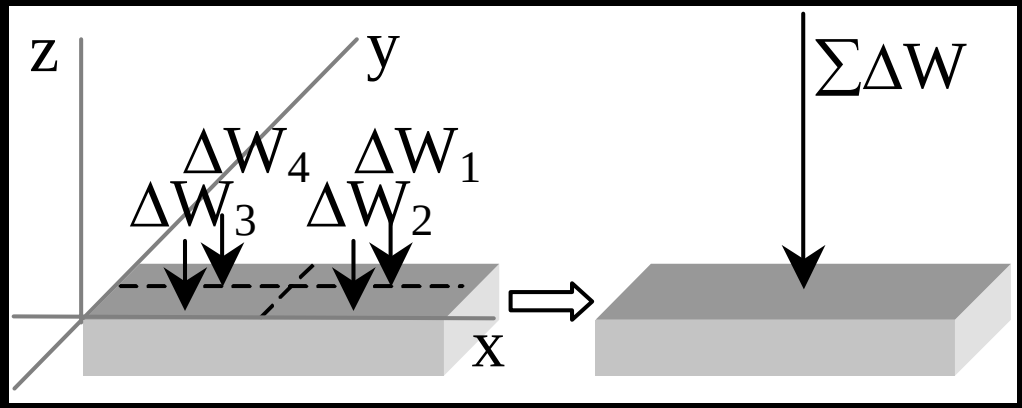
FALL 2007

lecture
eleven

centers of gravity- centroids

Center of Gravity

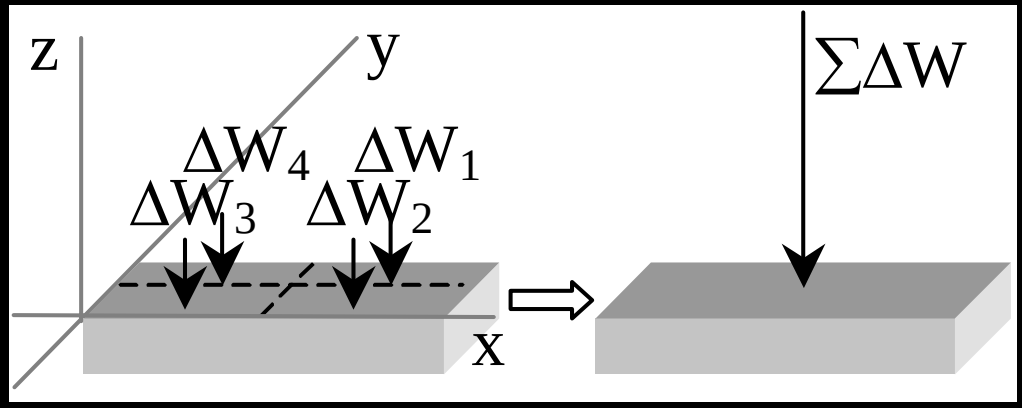
- *location of equivalent weight*
- *determined with calculus*



- *sum element weights* $W = \int dW$

Center of Gravity

- *"average" x & y from moment*



$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x} W \Rightarrow \quad \bar{x} = \frac{\sum (x \Delta W)}{W}$$

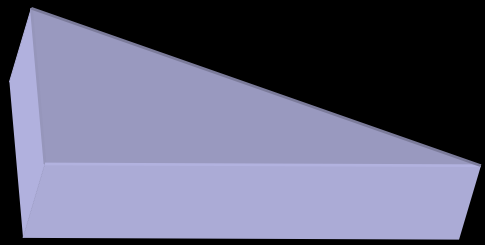
"bar" means average

$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y} W \Rightarrow \quad \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- *"average" x & y of an area*
- *for a volume of constant thickness*
  - $\Delta W = \gamma t \Delta A$ *where* γ *is weight/volume*
  - *center of gravity = centroid of area*

$$\bar{x} = \frac{\Sigma(x\Delta A)}{A}$$



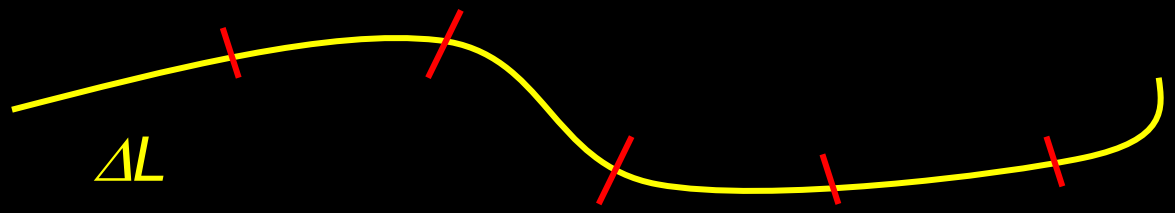
$$\bar{y} = \frac{\Sigma(y\Delta A)}{A}$$

Centroid

- *for a line, sum up length*

$$\bar{x} = \frac{\Sigma(x\Delta L)}{L}$$

$$\bar{y} = \frac{\Sigma(y\Delta L)}{L}$$

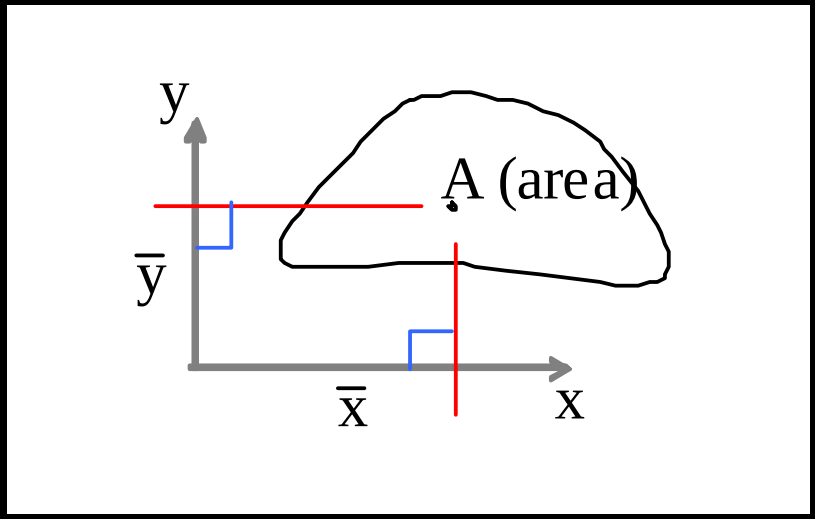


1st Moment Area

- *math concept*
- *the moment of an area about an axis*

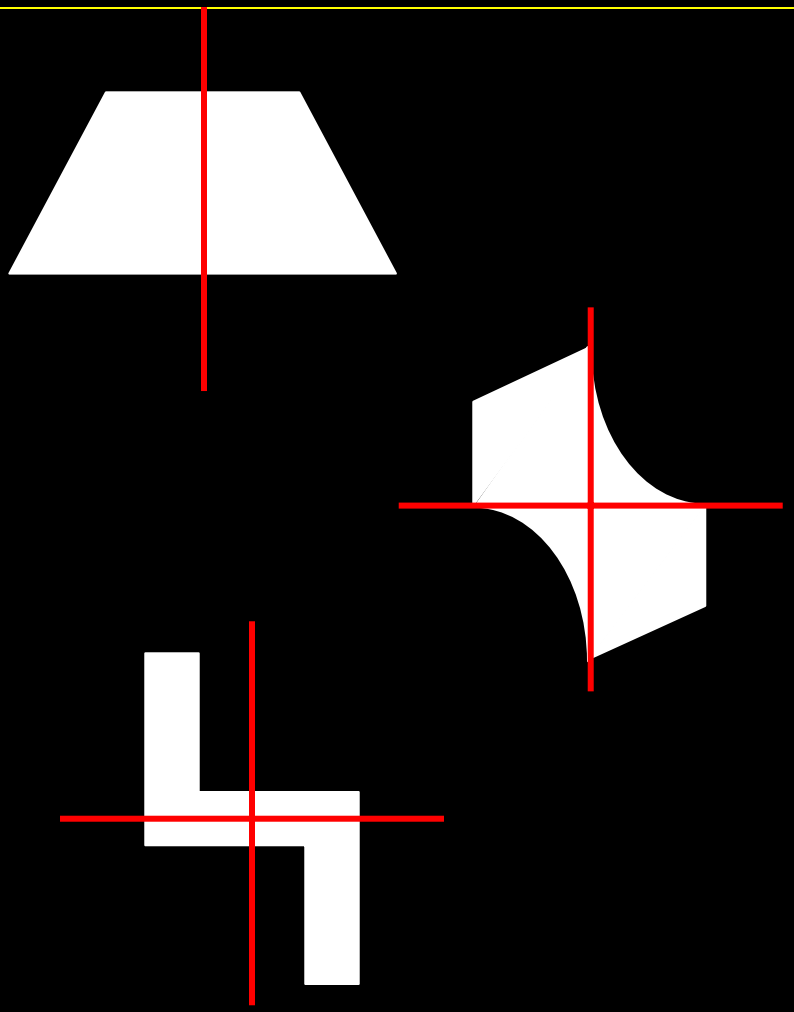
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



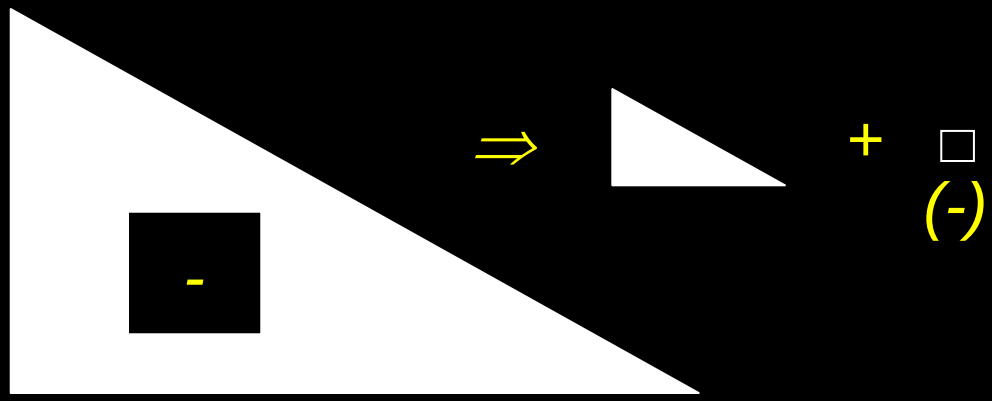
Symmetric Areas

- *symmetric about an axis*
- *symmetric about a center point*
- *mirrored symmetry*



Composite Areas

- *made up of basic shapes*
- *areas can be <u>negative</u>*
- *(centroids can be negative for any area)*



Basic Procedure

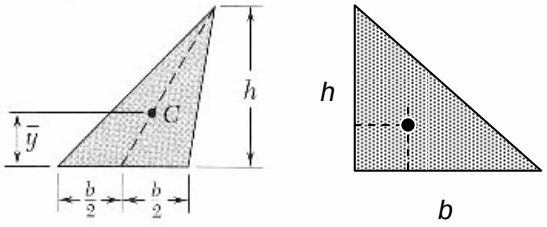
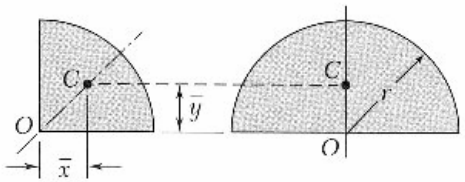
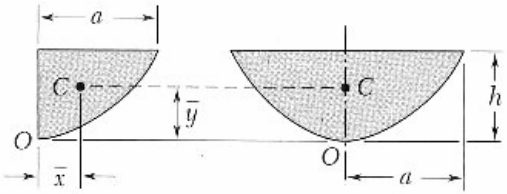
1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\hat{x}$ and $\hat{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
Σ					

Area Centroids

- *Table 7.1 – pg. 242*

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$
Triangular area		$\frac{b}{3}$ *right triangle only*	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$