Steel & Wood Column Design

Design Aims

If we know the loads, we can select a section that is adequate for strength & buckling.

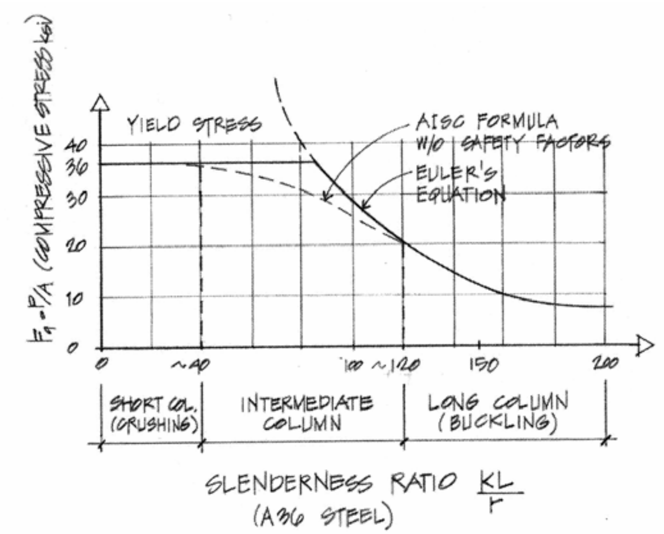
If we know the length, we can find the limiting load satisfying strength & buckling.

Design Code Methodologies

Allowable Stress Design (ASD): the stress in a member must be less than an allowable stress which is equal to the yield stress divided by a factor of safety.

Load and Resistance Factor Design: more efficient method that factors loads for importance and compares the summation to a nominal strength that has been adjusted by a reduction factor.

Allowable Stress Design - Steel



American Institute of Steel Construction (AISC) Manual of ASD, 9th ed:

<u>Long and slender:</u> [$L_e/r \geq C_c$, preferably < 200]

$$F_{allowable} = \frac{F_{cr}}{F.S.} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

The yield limit is idealized into a parabolic curve that blends into the Euler's Formula at C_c.

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

With F_y = 36 ksi, C_c = 126.1

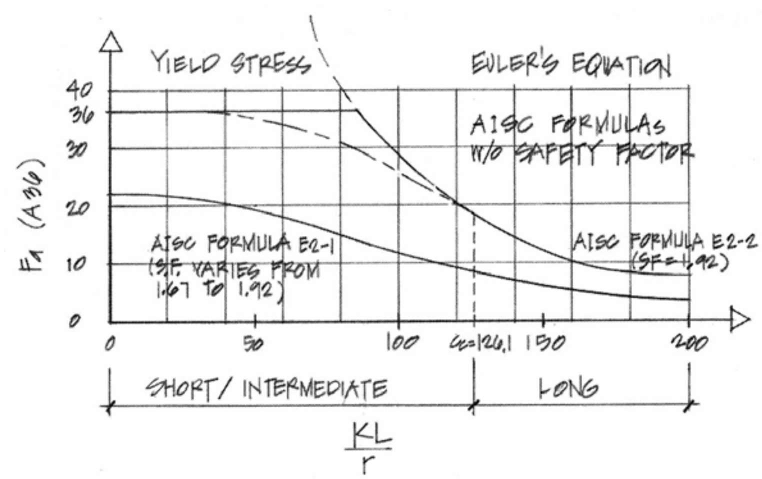
With F_y = 50 ksi, C_c = 107.0

<u>Short and stubby:</u> [$L_e/r < C_c$]

$$F_a = \left[1 - \frac{\left(\frac{KL}{r}\right)^2}{2C_c^2}\right]\frac{F_y}{F.S.}$$

with:

$$F.S. = \frac{5}{3} + \frac{3\left(\frac{KL}{r}\right)}{8C_c} - \frac{\left(\frac{KL}{r}\right)^3}{8C_c^3}$$



Procedure for Analysis

1. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
2. Find F_a as a function of KL/r from Table 10.1 or 10.2 (pp. 361-364)
3. Compute $P_{allowable} = F_a \cdot A$ *or alternatively* compute $f_{actual} = P/A$
4. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; no, it is no good

   or Is $f_{actual} \le F_a$? $\Rightarrow$ yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section.
2. Calculate KL/r for each axis (if necessary). The largest will govern the buckling load.
3. Find F_a as a function of KL/r from Table 10.1 or 10.2 (pp. 361-364)
4. Compute $P_{allowable} = F_a \cdot A$ *or alternatively* compute $f_{actual} = P/A$
5. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*

   or Is $f_{actual} \le F_a$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*
6. Check design efficiency by calculating percentage of stress used = $\frac{P_{actual}}{P_{allowable}} \cdot 100\%$

   If value is between 90-100%, it is efficient.

   If values is less than 90%, *pick a smaller section and go back to step 2.*

The critical load with respect to the slenderness ratio is presented in chart format in ASD, 8th ed, as well as the allowable stress charts for compression members.

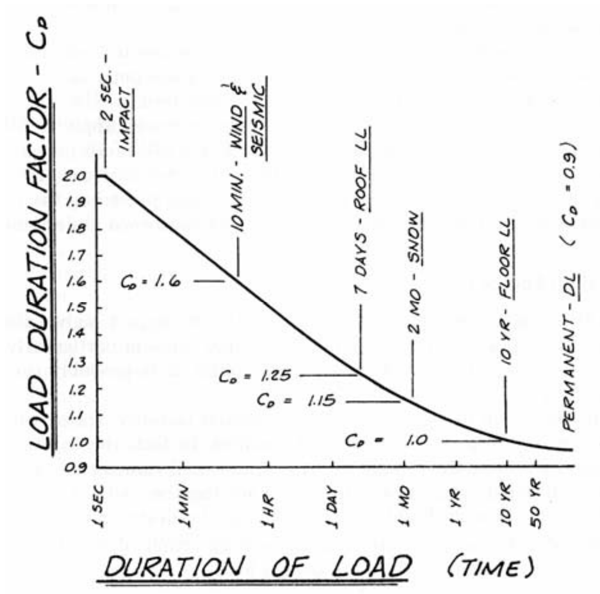
Allowable Stress Design - Wood

National Design Specification for Wood Construction (1992):

Any slenderness ratio, $L_e/d \le 50$:

$$f_c = \frac{P}{A} \le F'_c \qquad\qquad F'_c = F_c(C_D)(C_M)(C_t)(C_F)(C_p)$$

The curve uses factors to replicate the combination curve:



where:

- F_c' = allowable compressive stress parallel to the grain
- F_c = compressive strength parallel to the grain
- C_D = load duration factor
- C_M = wet service factor (1.0 for dry)
- C_t = temperature factor
- C_F = size factor
- Cp = column stability factor off chart

For preliminary column design:

$$F_c' = F_c^* C_p = (F_c C_D) C_p$$

Procedure for Analysis

1. Calculate L_e/d_{min}
2. Obtain F'$_c$

   compute $F_{cE} = \frac{K_{cE} E}{\left(\frac{l_e}{d}\right)^2}$ with K_{cE} =0.3 for sawn, = 0.418 for glu-lam
3. Compute $F_c^* \cong F_c C_D$ with C_D = 1, normal, C_D =1.25 for 7 day roof...
4. Calculate F_{cE}/F_c^* and get C_p from Appendix A, Table 14 (pp. 413-414)
5. Calculate $F_c' = F_c^* C_p$
6. Compute $P_{allowable} = F'_c \cdot A$ *or alternatively* compute $f_{actual} = P/A$
7. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; no, it is no good

   or Is $f_{actual} \le F'_c$? $\Rightarrow$ yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section
2. Calculate L_e/d_{min}
3. Obtain F'$_c$

   compute $F_{cE} = \frac{K_{cE} E}{\left(\frac{l_e}{d}\right)^2}$ with K_{cE} =0.3 for sawn, = 0.418 for glu-lam
4. Compute $F_c^* \cong F_c C_D$ with C_D = 1, normal, C_D =1.25 for 7 day roof...
5. Calculate F_{cE}/F_c^* and get C_p from Appendix A, Table 14 (pp. 413-414)

6. Calculate $F'_c = F_c^* C_p$
7. Compute $P_{allowable} = F'_c \cdot A$ *or alternatively* compute $f_{actual} = P/A$
8. Is the design satisfactory?

   Is $P \le P_{allowable}$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*

   or Is $f_{actual} \le F'_c$? $\Rightarrow$ yes, it is; *no, pick a bigger section and go back to step 2.*

Load & Resistance Factor Design

American Institute of Steel Construction (AISC) Manual of LRFD, 3rd ed:

$$\Sigma \gamma_i Q_i \le \phi P_n \quad \text{where}$$

γ is a load factor
Q is a load type
ϕ is a resistance factor
P_n is the nominal load capacity (strength)

Load combinations, ex: 1.4D (D is dead load)
1.2D + 1.6L (L is live load)

For compression, $\phi_c = 0.85$ and $P_n = A_g F_{cr}$

where :

A_g is the cross section area and F_{cr} is the critical stress shown below (in Compact Sections).

Compact Sections

Compact sections are defined as sections with flanges continuously connected to the web or webs and the width-thickness rations are less than limiting values given in the manual. This is to avoid local buckling of the flange or the web.

Formula parts depend on $\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}}$ where $\frac{Kl}{r} = \frac{L_e}{r}$,

when $\lambda_c \le 1.5$:

$$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y$$ where F_{cr} is the critical stress

when $\lambda_c > 1.5$:

$$F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right]F_y$$

Sample AISC Table for Allowable Axial Loads (ASD)

COLUMNS
W shapes
Allowable axial loads in kips

$F_y = 36$ ksi
$F_y = 50$ ksi

Designation	W12											
Wt./ft	106		96		87		79		72		65	
F_y	36	50	36	50	36	50	36	50	36	50	36	50†
Effective length in ft *KL* with respect to least radius of gyration r_y												
0	674	936	609	846	553	768	501	696	456	633	413	573
6	637	872	575	788	522	715	473	647	430	589	389	533
7	629	858	568	775	515	703	467	637	424	579	384	524
8	620	844	560	762	508	691	460	626	418	569	378	514
9	611	828	552	748	501	678	453	614	412	558	373	504
10	602	812	544	733	493	665	446	601	406	547	367	494
11	593	795	535	718	485	650	439	588	399	535	361	483
12	583	777	526	701	477	636	431	575	392	522	354	472
13	572	759	516	685	468	620	423	561	385	509	348	460
14	561	740	506	667	459	604	415	546	377	496	341	448
15	550	720	496	649	450	588	407	531	369	482	334	435
16	539	699	486	630	440	570	398	515	361	468	326	422
17	527	678	475	611	430	553	389	499	353	453	319	408
18	514	656	464	591	420	534	379	482	344	438	311	394
19	502	634	452	570	409	515	370	465	336	422	303	380
20	489	611	440	549	398	496	360	447	326	406	294	365
22	462	562	416	505	376	455	339	410	308	372	277	334
24	433	511	390	458	352	412	317	371	288	336	259	301
26	404	457	362	408	327	367	294	329	267	297	240	266
28	372	399	334	356	301	319	270	285	245	258	220	230
30	340	348	304	310	273	278	245	249	222	225	199	201
32	305	306	272	273	244	244	219	219	197	197	176	176
34	271	271	242	242	216	216	194	194	175	175	156	156
36	241	241	215	215	193	193	173	173	156	156	139	139
38	217	217	193	193	173	173	155	155	140	140	125	125
40	196	196	175	175	156	156	140	140	126	126	113	113
Properties												
U	2.59	2.59	2.60	2.60	2.62	2.62	2.63	2.63	2.65	2.65	2.66	2.42
P_{wo} (kips)	185	257	161	223	139	193	122	169	106	148	92	128
P_{wi} (kips/in.)	22	31	20	28	19	26	17	24	15	22	14	20
P_{wb} (kips)	588	693	431	508	354	417	269	317	206	243	154	181
P_{fb} (kips)	221	306	182	253	148	205	122	169	101	140	82	114
L_c (ft)	12.9	10.9	12.8	10.9	12.8	10.9	12.8	10.8	12.7	10.8	12.7	10.7
L_u (ft)	43.3	31.2	39.9	28.7	36.2	26.0	33.3	24.0	30.5	21.9	27.7	20.0
A (in.^{2})	31.2		28.2		25.6		23.2		21.1		19.1	
I_x (in.^{4})	933		833		740		662		597		533	
I_y (in.^{4})	301		270		241		216		195		174	
r_y (in.)	3.11		3.09		3.07		3.05		3.04		3.02	
Ratio r_x/r_y	1.76		1.76		1.75		1.75		1.75		1.75	
B_x Bending factors	0.215		0.215		0.217		0.217		0.217		0.217	
B_y Bending factors	0.633		0.635		0.645		0.648		0.651		0.656	
$a_x/10^6$	139.1		124.3		110.4		98.6		88.6		79.3	
$a_y/10^6$	45.0		40.1		36.0		32.2		29.1		26.0	
$F'_{ex}(K_xL_x)^2/10^2$ (kips)	310		307		300		296		292		289	
$F'_{ey}(K_yL_y)^2/10^2$ (kips)	100		99.0		97.7		96.5		95.8		94.6	

†Flange is noncompact; see discussion preceding column load tables.

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Sample AISC Table for LRFD Design Strength in Compression

$F_y = 50$ ksi
$\phi_c P_n = 0.85 F_{cr} A_g$

Table 4-2 (cont.).
W-Shapes
Design Strength in Axial Compression, $\phi_c P_n$, kips

Shape	W12×										
	106	96	87	79	72	65††	58	53	50	45	40
Effective length *KL* (ft) with respect to least radius of gyration r_y											
0	1330	1200	1090	986	897	812	723	663	621	557	497
6	1280	1150	1050	947	861	779	680	623	562	504	450
7	1260	1140	1030	933	848	767	666	610	543	486	434
8	1240	1120	1010	917	834	754	649	594	521	466	416
9	1210	1100	994	900	818	739	631	577	497	445	396
10	1190	1070	973	880	800	723	611	559	472	422	376
11	1160	1050	950	860	781	706	590	539	445	398	354
12	1130	1020	926	838	761	687	568	518	418	374	332
13	1100	995	901	814	740	668	545	496	390	349	310
14	1070	966	874	790	717	647	521	474	363	324	287
15	1040	935	846	764	694	626	496	451	335	299	265
16	1000	904	817	738	670	604	471	428	308	274	243
17	968	871	788	711	645	581	446	404	281	250	222
18	932	838	758	683	620	558	420	381	255	227	201
19	895	805	727	655	594	535	395	357	230	204	181
20	858	771	696	627	569	512	370	334	208	185	163
22	783	703	634	570	517	464	322	290	172	152	135
24	708	635	572	514	465	417	276	247	144	128	113
26	635	569	511	459	415	372	235	210	123	109	96.5
28	565	505	453	406	367	328	202	181	106	94.1	83.2
30	497	443	397	355	321	287	176	158	92.3	82.0	72.5
32	437	390	349	312	282	252	155	139	81.2	72.1	63.7
34	387	345	309	277	250	223	137	123			
36	345	308	276	247	223	199	122	110			
38	310	276	248	221	200	179	110	98.4			
40	279	249	223	200	181	161	99.2	88.9			
Properties											
P_{wo}, kips	242	206	182	156	137	117	112	101	105	90.5	75.2
P_{wi}, kips/in.	30.5	27.5	25.8	23.5	21.5	19.5	18.0	17.3	18.5	16.8	14.8
P_{wb}, kips	609	445	365	278	213	159	125	110	133	98.6	67.4
P_{fb}, kips	276	228	185	152	126	103	115	93.0	115	93.0	74.6
L_p, ft	11.0	10.9	10.8	10.8	10.7	11.9	8.87	8.76	6.92	6.89	6.85
L_r, ft	44.9	41.4	38.4	35.7	33.6	31.7	27.0	25.6	21.5	20.3	19.2
A_g, in.^{2}	31.2	28.2	25.6	23.2	21.1	19.1	17.0	15.6	14.6	13.1	11.7
I_x, in.^{4}	933	833	740	662	597	533	475	425	391	348	307
I_y, in.^{4}	301	270	241	216	195	174	107	95.8	56.3	50.0	44.1
r_y, in.	3.11	3.09	3.07	3.05	3.04	3.02	2.51	2.48	1.96	1.95	1.94
Ratio r_x/r_y	1.76	1.76	1.75	1.75	1.75	1.75	2.10	2.11	2.64	2.64	2.64
$P_{ex}(KL^2)/10^4$	26700	23800	21200	18900	17100	15300	13600	12200	11200	9960	8790
$P_{ey}(KL^2)/10^4$	8620	7730	6900	6180	5580	4980	3060	2740	1610	1430	1260

††Flange is non-compact.
Note: Heavy line indicates Kl/r equal to or greater than 200.

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Example 1 (pg 367)

Example Problem 10.10 (Figure 10.41)

A 24-ft.-tall, A572 grade 50, steel column (W14×82) with an F_y = 50 ksi has pins at both ends. Its weak axis is braced at midheight, but the column is free to buckle the full 24 ft. in the strong direction. Determine the safe load capacity for this column.

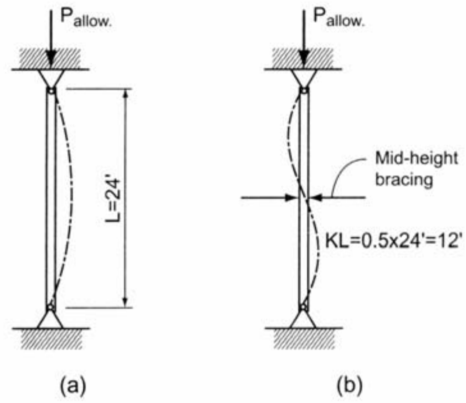


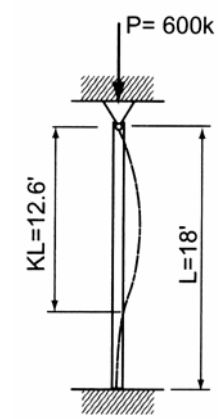
Figure 10.41 (a) Strong axis buckling. (b) Weak axis buckling.

Example 2 (pg 371) + chart method

Example Problem 10.14: Design of Steel Columns (Figure 10.48)

Select the most economical W12 × column 18' in height to support an axial load of 600 kips using A572 grade 50 steel. Assume that the column is hinged at the top but fixed at the base.

ALSO: Select the column using the ASD design charts, and the LRFD charts assuming that the load is a dead load (factor of 1.4)

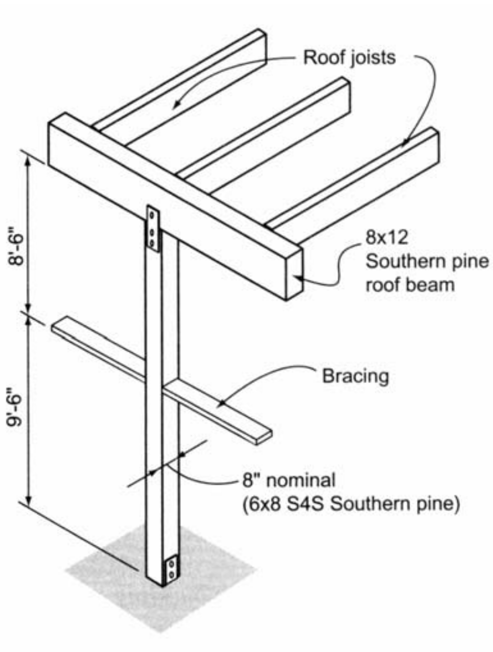


Example 3 (pg 379)

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6×8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

$$F_c = 975 \text{ psi}, \ E = 1.6 \times 10^6 \text{ psi}$$



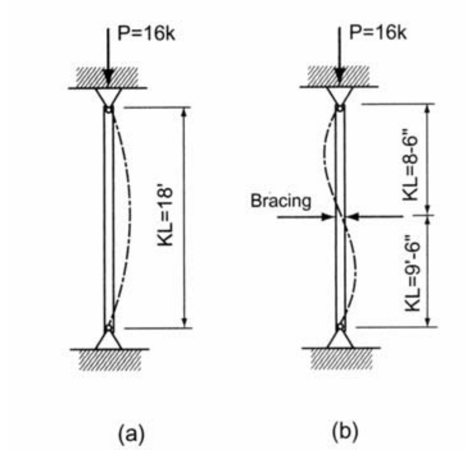


Figure 10.61 (a) Strong axis. (b) Weak axis.

Example 4 (pg 381)

Example Problem 10.20:
Design of Wood Columns(Figure 10.66)

A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming $8\frac{3}{4}$" in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

$8\frac{3}{4}" \times 9"$ ($A = 78.75$ in.^{2})

$8\frac{3}{4}" \times 10\frac{1}{2}"$ ($A = 91.88$ in.^{2})

$8\frac{3}{4}" \times 12"$ ($A = 105.00$ in.^{2})