

Steel– AISC Load and Resistance Factor Design

Load and Resistance Factor Design

The Manual of Steel Construction LRFD, 3rd ed. by the American Institute of Steel Construction requires that all steel structures and structural elements be proportioned so that no strength limit state is exceeded when subjected to all required **factored** load combinations.

$$\sum \gamma_i R_i \leq \phi R_n$$

where γ = load factor for the type of load
 R = load (dead or live; force, moment or stress)
 ϕ = resistance factor
 R_n = nominal load (ultimate capacity; force, moment or stress)

Nominal strength is defined as the

capacity of a structure or component to resist the effects of loads, as determined by computations using specified material strengths (such as yield strength, F_y , or ultimate strength, F_u) and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions

Load Factors and Load Combinations

Nominal loads that must be considered in design include

D = **dead** load due to the weight of the structural elements and other permanent features supported by the structure, such as permanent partitions.

L = **live** load due to occupancy and movable equipment

L_r = **live roof** load

W = wind load

S = snow load

E = earthquake load

R = initial rainwater load or ice water load exclusive of the ponding contribution

The design strength, ϕR_n , of each structural element or structural assembly must equal or exceed the design strength based on the following combinations of factored nominal loads from ASCE 7 (2005):

$$1.4(D + F)$$

$$1.2(D + F) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$$

$$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W)$$

$$1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R)$$

$$1.2D + 1.0E + L + 0.2S$$

$$0.9D + 1.6W + 1.6 H$$

$$0.9D + 1.0E + 1.6 H$$

Steel Materials

W shapes are preferably in steel grade ASTM A992: $F_y = 50$ ksi, $F_u = 65$ ksi, $E = 30,000$ ksi. ASTM A572 can be specified that has $F_y = 60$ or 65 ksi, $F_u = 75$ or 80 ksi, $E = 30,000$ ksi. ASTM A36 is available for angles and plates with $F_y = 36$ ksi, $F_u = 58$ ksi, $E = 29,000$ ksi.

Pure Flexure

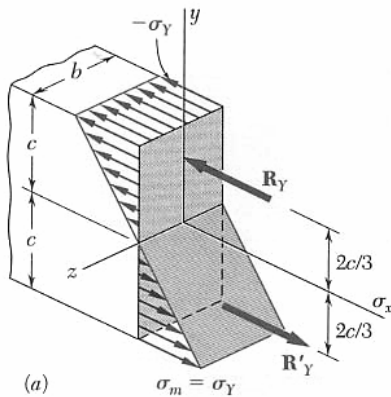
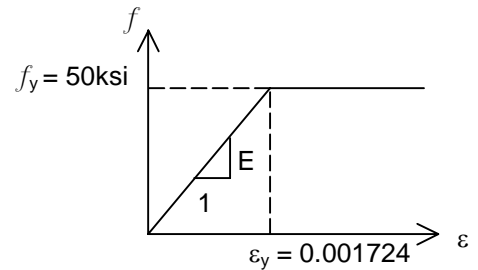
For determining the flexural design strength, $\phi_b M_n$, for resistance to pure bending (no axial load) in most flexural members where the following conditions exist, a single calculation will suffice:

$$\sum \gamma_i R_i = M_u \leq \phi_b M_n = 0.9 F_y Z$$

- where M_u = maximum moment from factored loads
- ϕ_b = resistance factor for bending = 0.9
- M_n = nominal moment (ultimate capacity)
- F_y = yield strength of the steel
- Z = plastic section modulus

Plastic Section Modulus

Plastic behavior is characterized by a yield point and an increase in strain with no increase in stress.



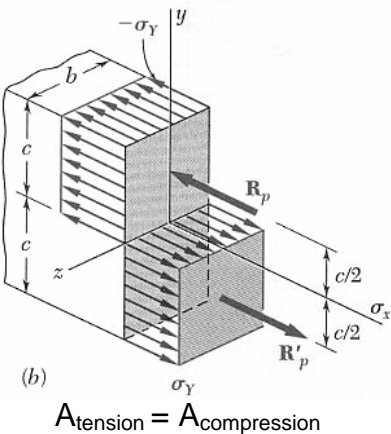
Internal Moments and Plastic Hinges

Plastic hinges can develop when all of the material in a cross section sees the yield stress. Because all the material at that section can strain without any additional load, the member segments on either side of the hinge can rotate, possibly causing instability.

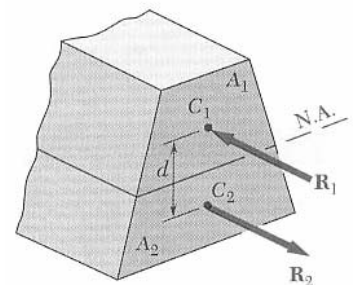
For a rectangular section:

Elastic to f_y : $M_y = \frac{I}{c} f_y = \frac{bh^2}{6} f_y = \frac{b(2c)^2}{6} f_y = \frac{2bc^2}{3} f_y$

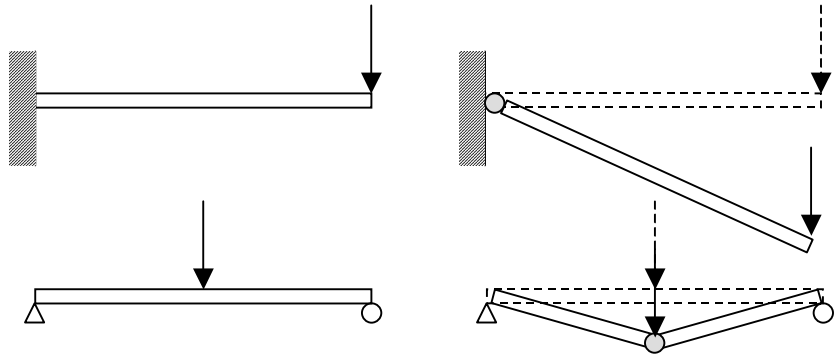
Fully Plastic: $M_{ult} \text{ or } M_p = bc^2 f_y = \frac{3}{2} M_y$



For a non-rectangular section and internal equilibrium at σ_y , the n.a. will not necessarily be at the centroid. The n.a. occurs where the $A_{tension} = A_{compression}$. The reactions occur at the centroids of the tension and compression areas.



Instability from Plastic Hinges



Shape Factor:

The ratio of the plastic moment to the elastic moment at yield:

$$k = \frac{M_p}{M_y}$$

$k = 3/2$ for a rectangle
 $k \approx 1.1$ for an I beam

Plastic Section Modulus

$$Z = \frac{M_p}{f_y} \quad \text{and} \quad k = \frac{Z}{S}$$

Shear

The formulas for the determination of the shear strength on a section are too complex for routine use with the variety of shapes available or possible for steel members. For members that possess an axis of symmetry in the plane of loading, and where web stiffeners are not required, two simplifying assumptions that result in a negligible loss of (theoretical) accuracy are permitted:

1. The contribution of the flanges to shear capacity may be neglected.
2. $\frac{h}{t_w} \leq \frac{418}{\sqrt{F_y}}$ where h equals the clear distance between flanges less the fillet or corner radius for rolled shapes.

With these assumptions, the calculated strength becomes simple. Neglecting the flanges, all symmetrical rolled shapes, box shapes, and built-up sections reduce to an equivalent rectangular section with dimensions $\Sigma t_w d$ and shear strength becomes $\phi_v V_n$:

$$\Sigma \gamma_i R_i = V_u \leq \phi_v V_n = 0.9(0.6 F_{yw} A_w)$$

- where
- V_u = maximum shear from factored loads
 - ϕ_v = resistance factor for shear = 0.9
 - V_n = nominal shear (ultimate capacity)
 - F_{yw} = yield strength of the steel in the web
 - $A_w = t_w d$ = area of the web

Design for Flexure

The nominal flexural strength M_n is the *lowest* value obtained according to the limit states of

1. yielding
2. lateral-torsional buckling
3. flange local buckling
4. web local buckling

For a laterally braced *compact* section (one for which the plastic moment can be reached before local buckling) only the limit state of yielding is applicable. For unbraced compact beams and noncompact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable.

With lateral-torsional buckling the nominal flexural strength is

$$M_n = C_b [\text{constant based on } M\text{'s and } L\text{'s}] \leq M_p \quad (M_u \leq \phi_b M_n)$$

where C_b is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 2M_A + 4M_B + 3M_C}$$

M_{\max} = absolute value of the maximum moment in the unbraced beam segment

M_A = absolute value of the moment at the quarter point of the unbraced beam segment

M_B = absolute value of the moment at the center point of the unbraced beam segment

M_C = absolute value of the moment at the three quarter point of the unbraced beam segment length.

Beam design charts show $\phi_b M_n$ for unbraced length (L_b) of the compression flange in one-foot increments from 1 to 50 ft. for values of the bending coefficient $C_b = 1$. For values of $1 < C_b \leq 2.3$, the required flexural strength M_u can be reduced by dividing it by C_b . L_p , the limiting laterally unbraced length for full plastic flexural strength when $C_b = 1$, is indicated by a solid dot (●) in the beam design moment charts, while L_r , the limiting laterally unbraced length for inelastic lateral-torsional buckling, is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used.

NOTE: the self weight is not included in determination of $\phi_b M_n$

Example 1

EXAMPLE 5.4. W-shape flexural member design (selection using plots of $\phi_b M_p$ vs. L_b for strong-axis bending).

Given: Select an ASTM A992 W-shape flexural member ($F_y = 50$ ksi, $F_u = 65$ ksi) for a required flexural strength M_u of 150 kip-ft, a required shear strength V_u of ~28 kips, and a deflection limit of 1 in. For the deflection calculations, assume the load is a uniformly distributed service load of 2 kips/ft and the length of the simple span is 20 ft. For the strength calculations, assume the beam is braced at the ends and midpoint only ($L_b = 10$ ft).

Solution: From Table 5-5, for an unbraced length $M_u = 150$ kip-ft and $L_b = 10$ ft, a W16x31 with $C_b = 1$ has $\phi_b M_{nx} \approx 150$ kip-ft. Since $C_b > 1$, the actual flexural strength will be higher, so the W16x31 is **o.k.** for flexural design strength.

From Table 5-17, Diagram 1, the maximum deflection Δ_{max} occurs at mid-span and can be calculated as:

$$\Delta_{max} = \frac{5wL^4}{384EI} + \Delta_{max-P} = \frac{Pl^3}{48EI} = \frac{3^{kips} (20ft)^3 (12in/ft)^3}{48(30,000ksi)(375in^4)} = 0.077in$$

$$= \frac{5(2^{kips/ft} + 0.031^{kips/ft})(20ft)^4 (12in/ft)^3}{384(30,000ksi)(375in^4)} = 0.650in$$

$$= 0.650 in. + 0.077 in = 0.727 in. < 1. \text{ o.k.}$$

$$\phi_v V_n = 0.9(0.6F_{yw}A_w) = 0.9(0.6)(50ksi)(15.9in)(0.275in) = 118kips > 28kips$$

Thus, the W16x31 flexural member is **o.k.**

Comments: Note that end connection limit states, such as block shear rupture and bolt bearing strength must also be checked.

Assume that for the design moment calculation:

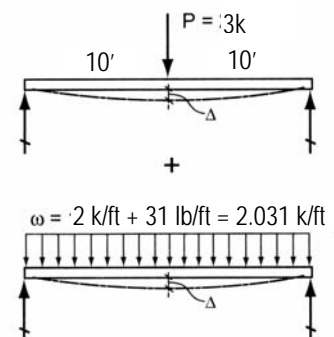
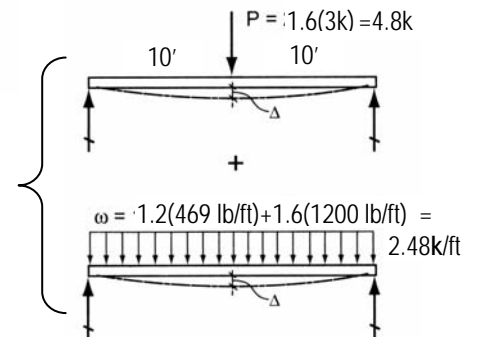
- Dead load = 469 lb/ft
- Live load = 1200 lb/ft
- Live point load at midspan = 3 kips

DESIGN LOADS:
(before self weight is included as a dead load)

$$M_u = \frac{wl^2}{8} + \frac{Pl}{4} = \frac{2.48k/ft(20ft)^2}{8} + \frac{4.8k(20ft)}{4} = 148k-ft$$

$$V_u = \frac{wl}{2} + \frac{P}{2} = \frac{2.48k/ft(20ft)}{2} + \frac{4.8k}{2} = 27.2k$$

$$(M_{u+self\ weight} = 150\ k\text{-ft}, V_{u+self\ weight} = 27.6\ k)$$



Beam Design Moments ($\phi_b=0.9, C_b=1.0, F_y=50$ ksi)

