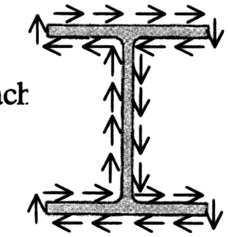
## Shear Flow in Open Sections

The shear flow must wrap around at all edges, and the total torque is distributed among the areas making up the cross section in proportion to the torsional rigidity of each rectangle ($ab^2/3$). The total angle of twist is the sum of the $\phi$ values from each rectangle. $t_i$ is the thickness of each rectangle and $b_i$ is the length of each rectangle.



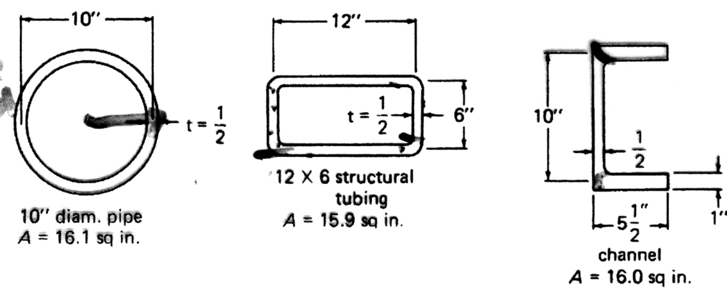
$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\Sigma b_i t_i^3} \qquad \phi = \frac{TL}{\frac{1}{3}G\Sigma b_i t_i^3}$$

Example 1

**Example 8.9.1**

Compare the torsional resisting moment $T$ and the torsional constant $J$ for the sections of Fig. 8.9.4 all having about the same cross-sectional area. The maximum shear stress $\tau$ is 14 ksi.

$\tau_{max} = 14ksi$




**SOLUTION**

(a) Circular thin-wall section.

$$T = \frac{\tau J}{\rho} = \frac{(14ksi)(393.7in^4)}{5.25in}\cdot\frac{1ft}{12in} = 87.5k-ft$$

$$J = \frac{\pi(c_o^4 - c_i^4)}{2} = \frac{\pi((5.25in)^4 - (4.75in)^4)}{2} = 393.7in^4$$

polar moment of inertia

I

(b) Rectangular box section. $\tau = \frac{T}{2a}$

$$T = \tau 2ta = (14ksi)2(0.5in)(72in^2)\cdot\frac{1ft}{12in} = 84k-ft$$

$$a \approx (12in)(6in) = 72in^2$$

(c) Channel section. Since for this open section,

$$\tau_{max} = \frac{Tt_{max}}{\frac{1}{3}\sum b_i t_i^3} = \frac{Tt}{J} \qquad T = \frac{\tau J}{t_{max}} = \frac{(14ksi)(4.08in^4)}{1in}\cdot\frac{1ft}{12in} = 4.8k-ft$$

the maximum shear stress will be in the flange. Also,

$$J = \sum\frac{bt^3}{3} \qquad J = \frac{1}{3}\left[10in(0.5in)^3 + (5.5in)(1in)^3 + (5.5in)(1in)^3\right] = 4.08in^4$$

## Thermal Strains

Physical restraints limit deformations to be the same, or sum to **zero**, or be proportional with respect to the rotation of a rigid body.

We know axial stress relates to axial strain: $\delta = \frac{PL}{AE}$ which relates $\delta$ to P

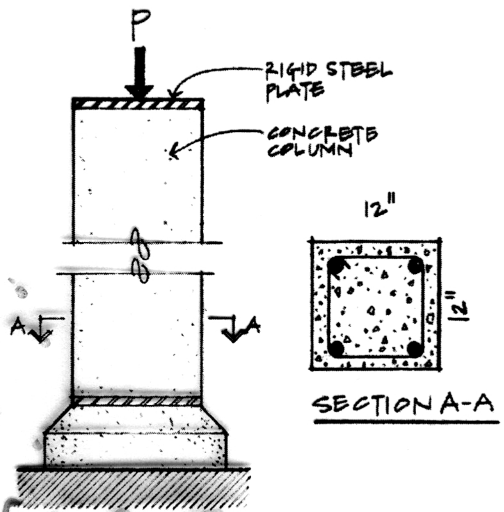
## Example 3

**5.21** A short concrete column measuring 12 in. square is reinforced with four #8 bars ($A_s = 4 \times 0.79$ in.$^2$ = 3.14 in.$^2$) and supports an axial load of 250k. Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$E_c = 3 \times 10^6$ psi and

$E_s = 29 \times 10^6$ psi

$f = \frac{P}{A}$ $P = f \cdot A$




Solution:

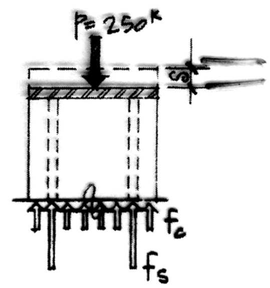
From equilibrium:

2. $[\Sigma F_y = 0] - 250\text{ k} + f_s A_s + f_c A_c = 0$ ← substitute $f \cdot A$ for P

$A_s = 3.14$ in.$^2$ f unknowns

$A_c = (12'' \times 12'') - 3.14$ in.$^2 \cong 141$ in.$^2$

1 → $3.14 f_s + 141 f_c = 250$ k




you write yourself

From the deformation relationship:

1. $\boxed{\delta_s = \delta_c;\ L_s = L_c}$

$\therefore \frac{\delta_s}{L} = \frac{\delta_c}{L}$

$\delta = \frac{PL}{AE} = \frac{f \cdot L}{E}$ $\Sigma F_y$: $P_c + P_s = 250^k$

and

$\varepsilon_s = \varepsilon_c$

Since

$E = \frac{f}{\varepsilon}$

Substitute

and

$\frac{f_s}{E_s} = \frac{f_c}{E_c}$

→ $f_s = f_c \frac{E_s}{E_c} = \frac{29 \times 10^3 (f_c)}{3 \times 10^3} = 9.67 f_c$

Substituting into the equilibrium equation:

$3.14\,(9.76 f_c) + 141 f_c = 250$

$30.4 f_c + 141 f_c = 250$

$171.4 f_c = 250$

$\boxed{f_c = 1.46\text{ ksi}}$

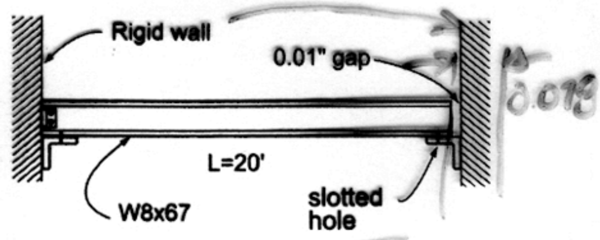
$\therefore f_s = 9.67\,(1.46)$ ksi

$\boxed{f_s = 14.1\text{ ksi}}$ ←

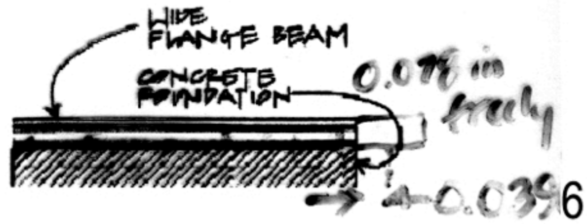
## Example 2 (pg 228)

**Example Problem 6.24 (Figures 6.58 and 6.59)**

A W8×67 steel beam, 20 ft. in length, is rigidly attached at one end of a concrete wall. If a gap of 0.010 in. exists at the opposite end when the temperature is 45°F, what results when the temperature rises to 95°F?

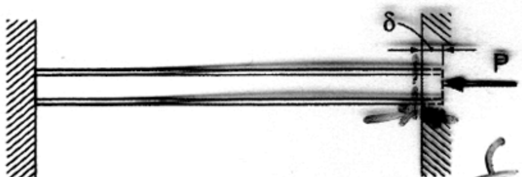



ALSO: If the beam is anchored to a concrete slab, and the steel sees a temperature change of 50° F while the concrete only sees a change of 30° F, determine the compressive stress in the beam.




$\alpha_c = 5.5 \times 10^{-6}\ /°F$ $\quad E_c = 3 \times 10^{6}$ psi

$\alpha_s = 6.5 \times 10^{-6}\ /°F$ $\quad E_s = 29 \times 10^{6}$ psi




$$\delta_T = \alpha\,\Delta T \cdot L$$

$$f \cdot \frac{L}{E} = \delta_P = \frac{PL}{AE} \qquad \Delta T = T_F - T_I$$

1. remove a constraint (take away right wall) apply temperature

$$\delta_{T\,steel} = 6.5\times10^{-6}/°F\,(95° - 45°)\cdot 20\,ft \cdot 12\,\tfrac{in}{ft} = 0.0780 \text{ in}$$

| 0.078 | want

.01 freely

can't

2. $\delta_{restrained} = \delta_T - 0.01$ in

$$\delta_P = 0.078 - 0.01 = 0.068 \text{ in}$$

squish (compression)

$\delta_P = -0.068$ in (shortens)

3. $$f_s = \frac{\delta_P \cdot E}{L} = \frac{-0.068\text{ in} \cdot 29\times10^{6}\ \frac{lb}{in^2}}{20\,ft \cdot 12\ in/ft} = -8{,}216 \text{ psi}$$

$-8.2$ ksi

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PART 2 : concrete expansion

$$\delta_T = \alpha \cdot \Delta T \cdot L = 5.5\times10^{-6}/°F\,(30°F)\cdot 20\,ft \cdot 12\,\tfrac{in}{ft}$$

$$\delta_{T\,concrete} = 0.0396 \text{ in}$$

$$\delta_{restrained\ (steel)} = \delta_{T\,steel} - \delta_{T\,concrete}$$

$\delta_{restrained}$

$$= 0.0780\text{ in} - 0.0396\text{ in} = 0.0384 \text{ in}$$

$\delta_P = -0.0384$ in

$$f_s = \frac{\delta_P \cdot E}{L} = \frac{-0.0384\text{ in} \cdot 29\cdot10^{6}\ \frac{lb}{in^2}}{20\,ft \times 12\ in/ft} = 4{,}640\ lb/in^2$$