ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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SPRING 2007

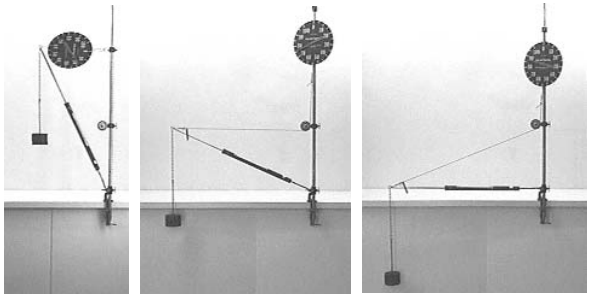
lecture
three

# moments



## Moments

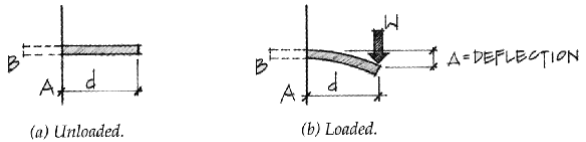
- *forces have the tendency to make a body rotate about an axis*



*– same translation but different rotation*

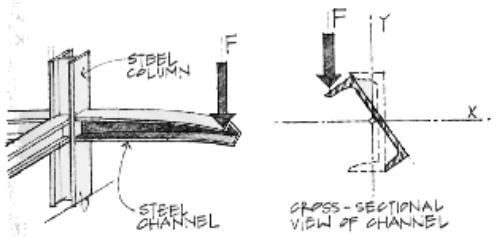


## Moments




(a) Unloaded. (b) Loaded.

Figure 2.33 Moment on a cantilever beam.



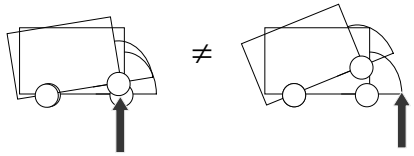

(a) (b)

Figure 2.34 An example of torsion on a cantilever beam.



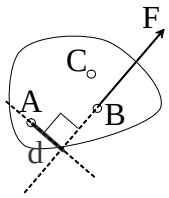
## Moments

- *a force acting at a different point causes a different moment:*



## Moments

- *defined by magnitude and direction*
- *units: N·m, k·ft*
- *direction:*
  - *+ ccw (right hand rule)*
  - *- cw*
- *value found from F and ⊥ distance*




$$M = F \cdot d$$

- *d also called "lever" or "moment" arm*

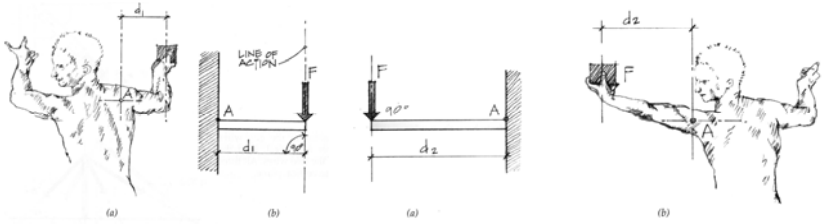


## Moments

- *with same F:*

$$M_A = F \cdot d_1 < M_A = F \cdot d_2$$

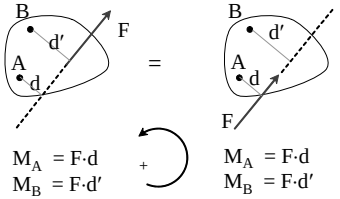
*(bigger)*






## Moments

- *additive with sign convention*
- *can still move the force*
  *<u>along the line of action</u>*



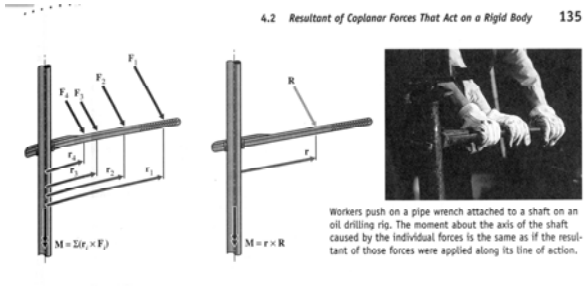



## Moments

- *Varignon's Theorem*
  - *resolve a force into components at a point and finding perpendicular distances*
  - *calculate sum of moments*
  - *equivalent to original moment*
- *makes life easier!*
  - *geometry*
  - *when component runs through point, d=0*

## Physics & Moments of a Force

- *moments of a force*
  - *introduced in Physics as "Torque Acting on a Particle"*
  - *and used to satisfy rotational equilibrium*






## Physics and Moments of a Force

- *my Physics book:*

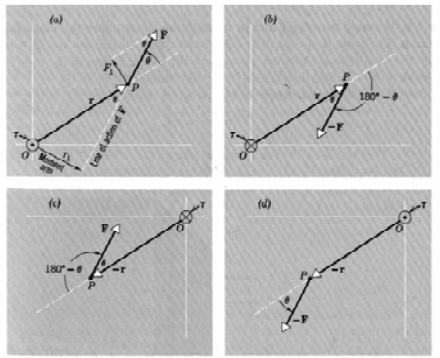
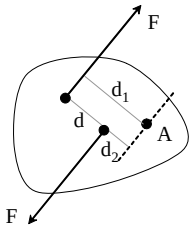



FIGURE 11–2 The plane shown is that defined by **r** and **F** in Fig. 11–1. (*a*) The magnitude of $\boldsymbol{\tau}$ is given by $Fr_\perp$ (Eq. 11–2b) or by $rF_\perp$ (Eq. 11–2c). (*b*) Reversing **F** reverses the direction of $\boldsymbol{\tau}$. (*c*) Reversing **r** reverses the direction of $\boldsymbol{\tau}$. (*d*) Reversing **F** and **r** leaves the direction of $\boldsymbol{\tau}$ unchanged. The directions of $\boldsymbol{\tau}$ are represented by ⊙ (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by ⊗ (perpendicularly into the figure, the symbol representing the tail of an arrow).



## Moment Couples

- *2 forces*
  - *same size*
  - *opposite direction*
  - *distance d apart*
  - *cw or ccw*




$$M = F \cdot d$$

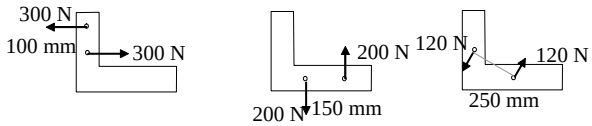
  - *not dependant on point of application*

$$M = -F \cdot d = -F \cdot d_1 + F \cdot d_2$$



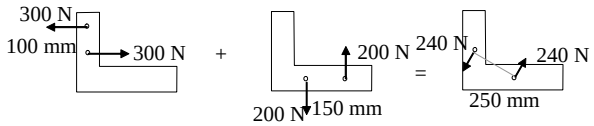
## Moment Couples

- *equivalent couples*
  - *same magnitude and direction*
  - *F & d may be different*



## Moment Couples

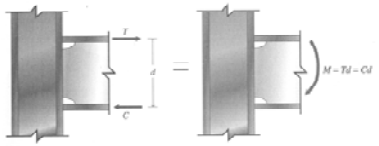
- *added just like moments caused by one force*
- *can <u>replace</u> two couples with a single couple*



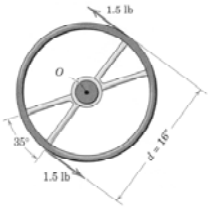



## Moment Couples

- *moment couples in structures*



The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces $T$ and $C$ in the beam flanges form a couple with moment $M$ that is transferred into the column.





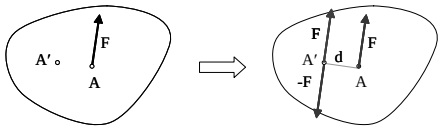

## Equivalent Force Systems

- *two forces at a point is equivalent to the <u>resultant</u> at a point*
- *resultant is equivalent to two components at a point*
- *resultant of equal & opposite forces at a point is zero*
- *put equal & opposite forces at a point (sum to 0)*
- *transmission of a force along action line*



## Force-Moment Systems

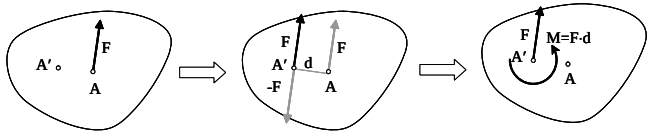
- *single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused*




- *moments are shown as arched arrows*

## *Force-Moment Systems*

- *a force-moment pair can be replaced by a force at another point causing the original moment*






## *Parallel Force Systems*

- *forces are in the same direction*
- *can find resultant <u>force</u>*
- *need to find <u>location</u> for equivalent moments*