



eccentric loading: beam-columns

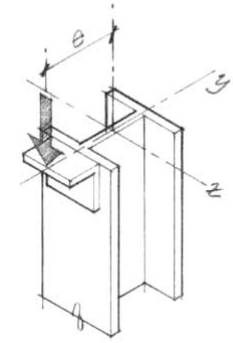
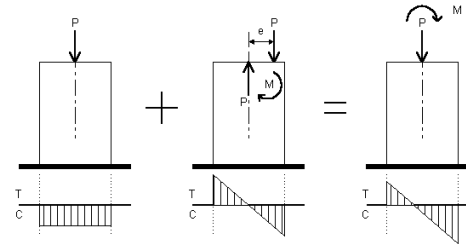
Column Eccentricity 1
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Centric & Eccentric Loading

- centric
 - allowable stress from strength or buckling
- eccentric
 - combined stresses



Column Eccentricity 5
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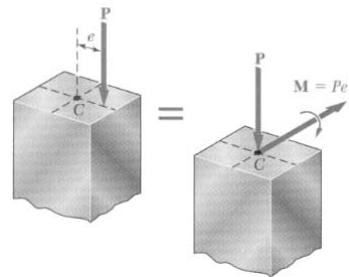
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Eccentric Loading

– axial + bending

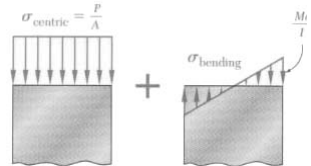
$$f_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$



– design

$$f_{\max} \leq F_{cr} = \frac{f_{cr}}{F.S.}$$



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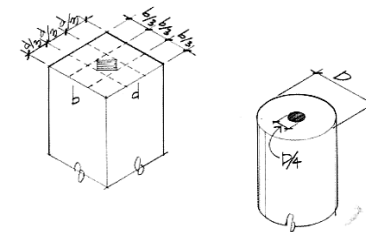
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Eccentric Loading

– find e such that the minimum stress = 0

$$f_{\min} = \frac{P}{A} - \frac{(Pe)c}{I} = 0$$

– area defined by e from centroid is the kern



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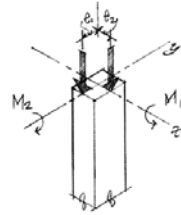
Eccentric Loading

– when there is eccentricity in two directions

$$M_1 = P \cdot e_1 \quad M_2 = P \cdot e_2$$

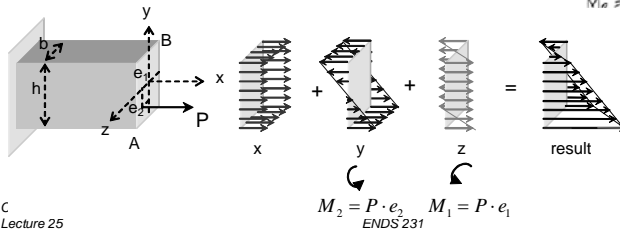
$$f_{\max} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I}$$

– biaxial bending



$$M_1 = P \cdot e_1 \text{ (ABOUT THE } z\text{-axis)}$$

$$M_2 = P \cdot e_2 \text{ (ABOUT THE } y\text{-axis)}$$



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$M_2 = P \cdot e_2$
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$M_1 = P \cdot e_1$

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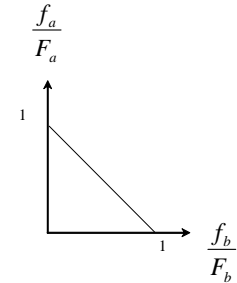
Stress Limit Conditions

– ASD interaction formula

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$

– with biaxial bending

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$



interaction diagram

Column Eccentricity 9
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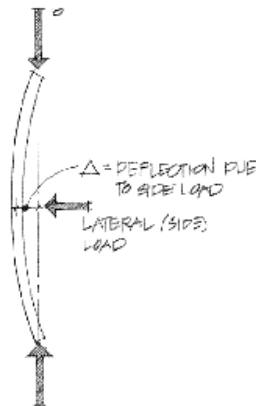
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Stress Limit Conditions

– in reality, as the column flexes,
the moment increases

– P-Δ effect

$$\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0$$



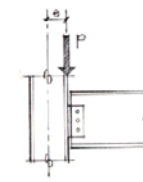
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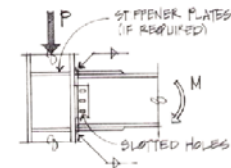
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Design

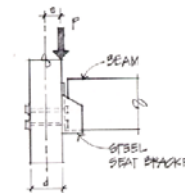
- satisfy
 - strength
 - stability
- pick
 - section



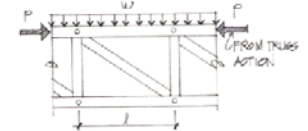
(a) Framed beam (shear) connection.
 $e = \text{Eccentricity}; M = P \times e$



(b) Moment connection (rigid frame).
 $M = \text{Moment due to beam bending}$



(c) Timber beam-column connection.
 $e = d/2 = \text{eccentricity}; M = P \times e$



(d) Upper chord of a truss—compression plus bending.
 $M = \frac{P L^2}{8}$

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Design

• ASD Steel

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

C_m – modification factor for end conditions
 = 0.6 – 0.4(M_1/M_2) or 0.85 restrained

F'_e – allowable buckling strength

() term – magnification factor for $P-\Delta$

Design

• LRFD Steel

$$\text{– for } \frac{P_u}{\phi_c P_n} \geq 0.2 : \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

$$\text{– for } \frac{P_u}{\phi_c P_n} < 0.2 : \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

ϕ_c - resistance factor for compression = 0.85

ϕ_b - resistance factor for bending = 0.9

Design

• Wood

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{bx}}{F'_{bx} \left[1 - \frac{f_c}{F_{cEx}} \right]} \leq 1.0$$

() term – magnification factor for $P-\Delta$

F'_{bx} – allowable bending strength

Design Steps Knowing Loads

1. assume limiting stress

- buckling, axial stress, combined stress

2. solve for r , A or S

3. pick trial section

4. analyze stresses

5. section ok?

6. stop when section is ok

