ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231

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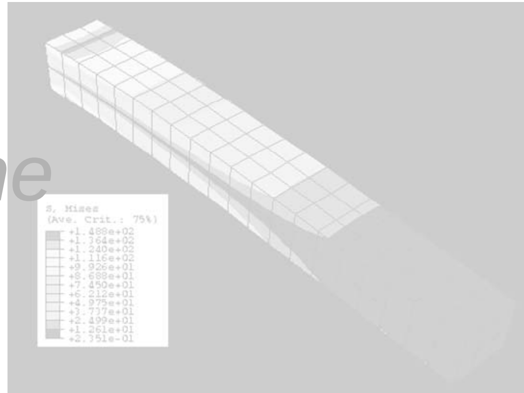
SPRING 2007

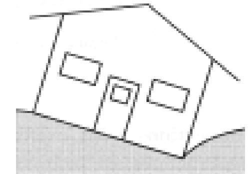
lecture
twenty one

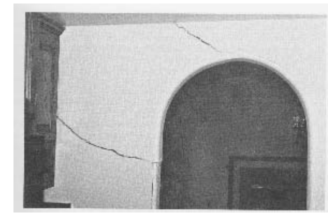


beams: deflection & design

Design for Strength +...

- *strength design*
  - *forces & material*
- *serviceability*
  - *limit deflection and cracking*
  - *control noise & vibration*
  - *no excessive settlement of foundations*
  - *durability*
  - *appearance*
  - *component damage*
  - *ponding*

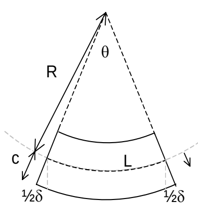


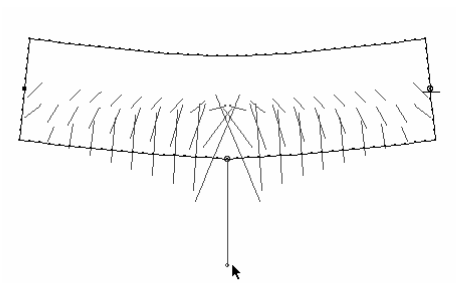


Beam Deformations

- *curvature relates to*
  - *bending moment*
  - *modulus of elasticity*
  - *moment of inertia*

$$\frac{1}{R} = \frac{M}{EI}$$





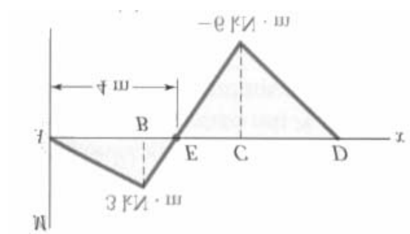
$$curvature = \frac{M(x)}{EI}$$

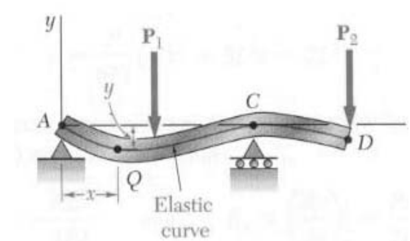
$$\theta = slope = \int \frac{M(x)}{EI} dx$$

$$\Delta = deflection = \int\int \frac{M(x)}{EI} dx$$

Deflected Shape & M(x)

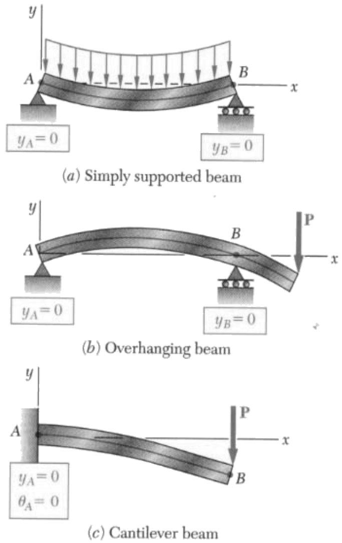
- *-M(x) gives shape indication*
- *boundary conditions must be met*





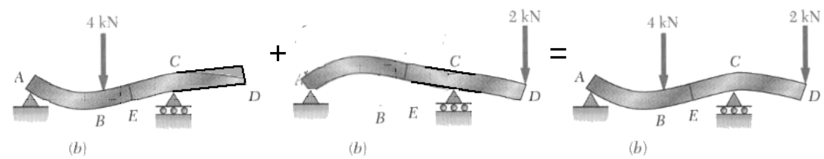
Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- y_{max} at $\frac{dy}{dx} = 0$



Superpositioning

- if w can be superpositioned
  - θ & y can
  - elastic range only!



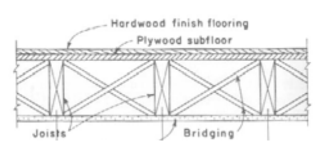
Deflection Limits

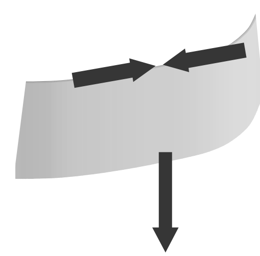
- based on service condition, severity

Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

Lateral Buckling

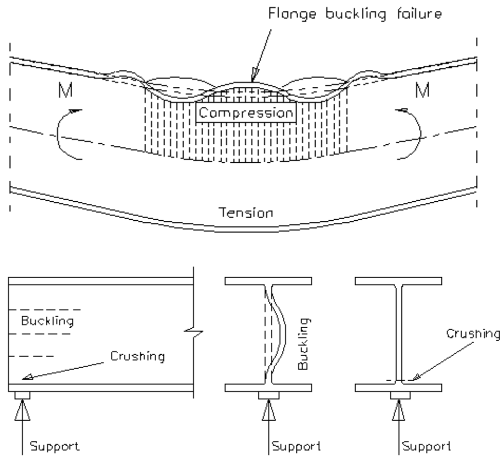
- lateral buckling caused by compressive forces at top coupled with insufficient rigidity
- can occur at low stress levels
- stiffen, brace or bigger I_y





Local Buckling

- *steel I beams*
- *flange*
  - *buckle in direction of smaller radius of gyration*
- *web*
  - *force*
  - *"crippling"*



Local Buckling

- *flange*

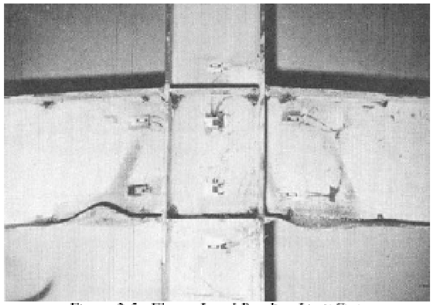


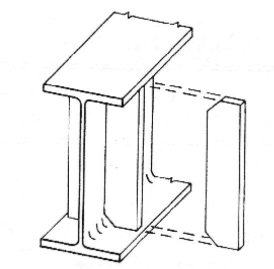
Figure 2-5. Flange Local Bending Limit State (Beedle, L.S., Christopher, R., 1964)

- *web*

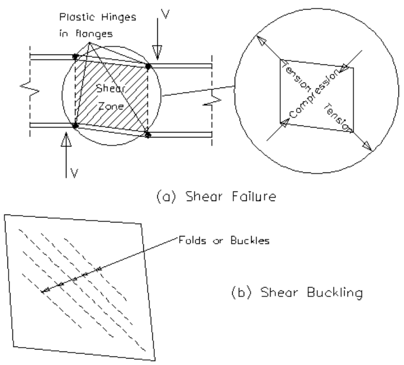
Figure 2-7. Web Local Buckling Limit State (SAC Project)

Shear in Web

- *panels in plate girders or webs with large shear*
- *buckling in compression direction*
- *add stiffeners*



stiffeners to prevent lateral buckling



(a) Shear Failure

(b) Shear Buckling

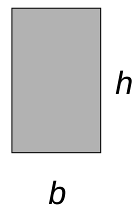
Shear in Web

- *plate girders and stiffeners*

Beam Design

1. Know F_{all} for the material or F_U for LRFD

2. Draw V & M, finding M_{max}



3. Calculate $S_{req'd}$ $\left(f_b \le F_b\right)$

4. Determine section size $S = \frac{bh^2}{6}$

Beam Design

4*. Include self weight for M_{max}
   – and repeat 3 & 4 if necessary

5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper

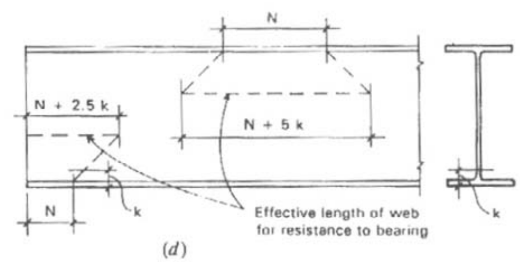
Beam Design

6. Evaluate shear stresses - horizontal $\left(f_v \le F_v\right)$
   - W and rectangles $f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$
   - thin walled sections $f_{v-\max} = \frac{VQ}{Ib}$

Beam Design

7. Provide adequate bearing area at supports

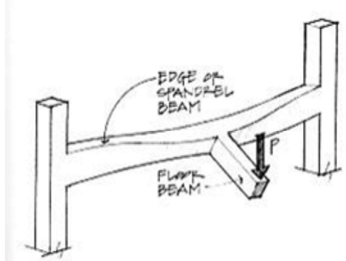
$$f_p = \frac{P}{A} \le F_p$$



Beam Design

8. Evaluate torsion

$$\left(f_v \le F_v\right)$$



- *circular cross section*

$$f_v = \frac{T\rho}{J}$$

- *rectangular*

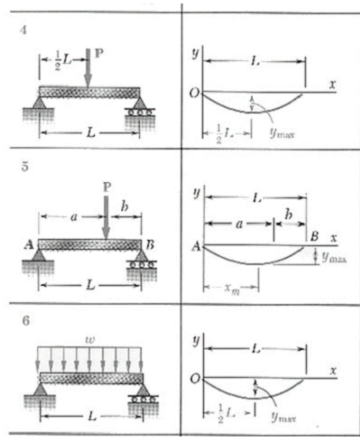
$$f_v = \frac{T}{c_1ab^2}$$

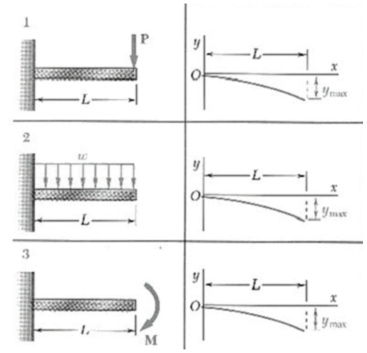
TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Beam Design

9. Evaluate deflections



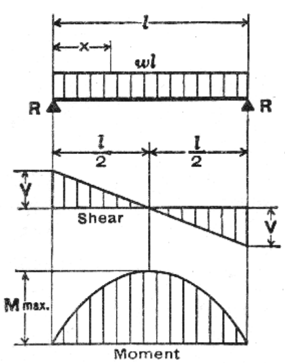


$$y_{\max}(x) = \Delta_{actual} \le \Delta_{allowable}$$

Beam Design

9. – how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

Vx . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . $= \frac{wl^2}{8}$

Mx . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \frac{5\,wl^4}{384\,EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$