ARCHITECTURAL **S**TRUCTURES **I**:

STATICS AND STRENGTH OF MATERIALS

ENDS 231

DR. ANNE NICHOLS

SPRING 2007

lecture eighteen

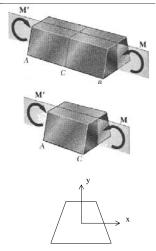


Beam Bending Stress 1 Lecture 18 Architectural Structures I

F2005abri

Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
 - homogeneous materials
 - plane of symmetry
 - follow Hooke's law



Beam Bending

- Galileo
 - relationship between stress and depth²
- can see
 - top squishing
 - bottom stretching



what are the stress across the section?

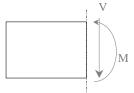
Beam Bending Stress 7 Lecture 17 Architectural Structures I ENDS 231 Su2004abn

Bending Moments

• sign convention:







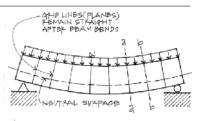
 size of maximum internal moment will govern our design of the section

Beam Bending Stress 9 Lecture 19 Architectural Structures I ENDS 231 S2004abn

S2004abr

Normal Stresses

- geometric fit
 - plane sections remain plane
 - stress varies linearly



CENTROIDAL AXIS

Figure 8.5(b) Beam bending under load.

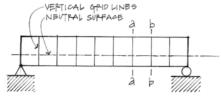


Figure 8.5(a) Beam elevation before loading.

ALSO CALLED THE NEUTRAL AXIS (N.A.)

Beam cross section.

Beam Bending Stress 10

Architectural Structures I **ENDS 231**

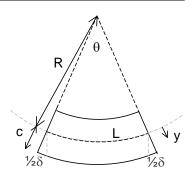
S2004abn

Derivation of Stress from Strain

pure bending = arc shape

$$L = R\theta$$

$$L_{outside} = (R + y)\theta$$



$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

Beam Bending Stress 12 Lecture 19

Architectural Structures I ENDS 231

S2004ahn

Neutral Axis

- stresses vary linearly
- zero stress occurs at the centroid
- neutral axis is line of centroids (n.a.)

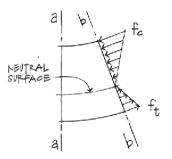


Figure 8.8 Bending stresses on section b-b.

Beam Bending Stress 11 Lecture 19

Architectural Structures I **ENDS 231**

S2004abn

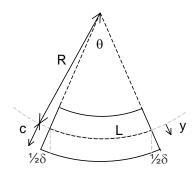
Derivation of Stress

zero stress at n.a.

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\text{max}} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{\text{max}}$$



Beam Bending Stress 13 Lecture 19

Architectural Structures **ENDS 231**

S2004ahn

Bending Moment

resultant moment from stresses = bending moment!

 $M = \Sigma f y \Delta A$



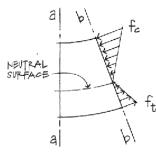


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \Sigma y^2 \Delta A = \frac{f_{max}}{c} I = f_{max} S$$

Beam Bending Stress 14 Lecture 19 Architectural Structures I ENDS 231 S2004abn

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

$$f_b = \frac{My}{I}$$

$$S = \frac{I}{c}$$

curvature

general bending stress se

section modulus

$$f_b = \frac{M}{S}$$

 $S_{required} \ge \frac{M}{F_b}$

maximum bending stress

required section modulus for design

Beam Bending Stress 15 Lecture 19 Architectural Structures ENDS 231 S2004abn