ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231
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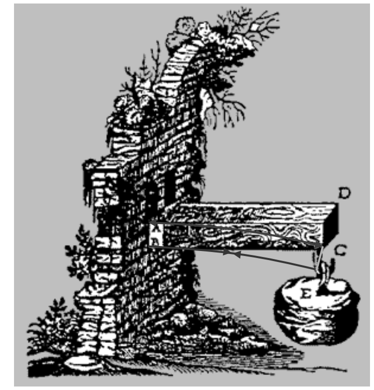
SPRING 2007

lecture
eighteen

beams:
bending and shear

Beam Bending

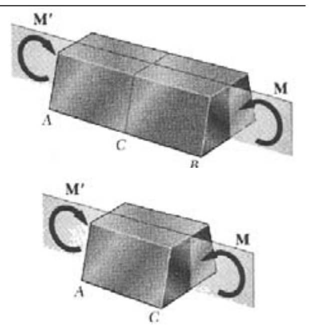
- Galileo
  - relationship between stress and depth^2
- can see
  - top squishing
  - bottom stretching

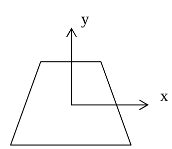


- what are the stress across the section?

Pure Bending

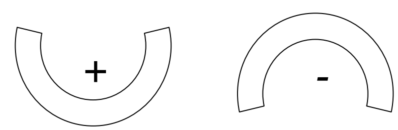
- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke's law

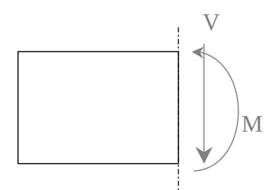




Bending Moments

- sign convention:





- size of maximum internal moment will govern our design of the section

Normal Stresses

- *geometric fit*
  - *plane sections remain plane*
  - *stress varies linearly*

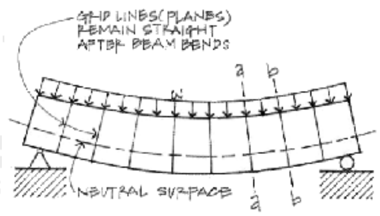


Figure 8.5(b) Beam bending under load.

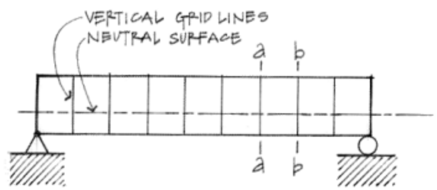
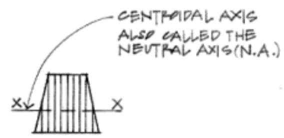


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

Neutral Axis

- *stresses vary linearly*
- *zero stress occurs at the centroid*
- *neutral axis is line of centroids (n.a.)*

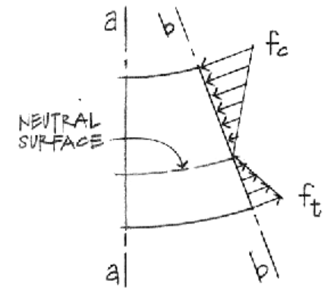


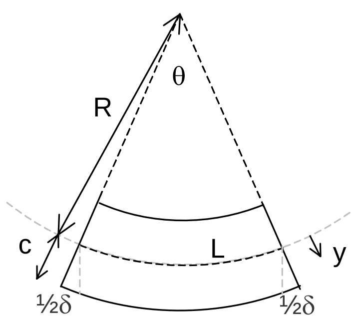
Figure 8.8 Bending stresses on section b-b.

Derivation of Stress from Strain

- *pure bending = arc shape*

$$L = R\theta$$

$$L_{outside} = (R+y)\theta$$



$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

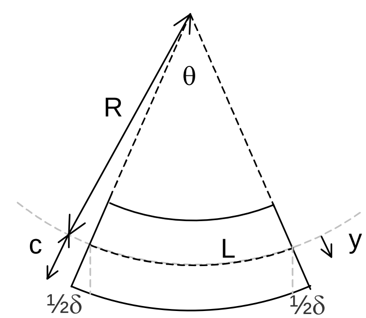
Derivation of Stress

- *zero stress at n.a.*

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\max} = \frac{Ec}{R}$$

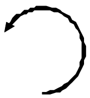
$$f = \frac{y}{c} f_{\max}$$



Bending Moment

- resultant moment from stresses = bending moment!

$$M = \Sigma f y \Delta A$$



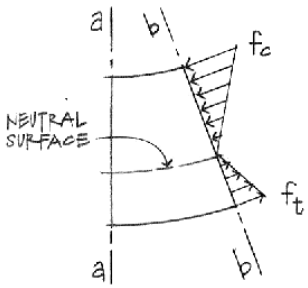


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

$$f_b = \frac{M}{S}$$

maximum bending stress

$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design