ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
ENDS 231

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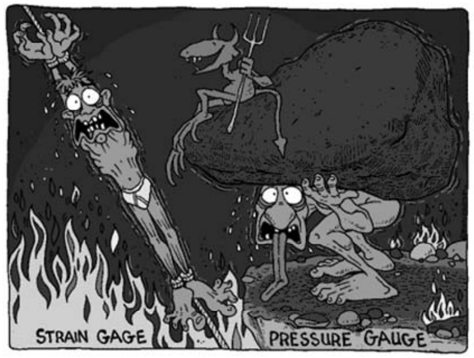
SPRING 2007

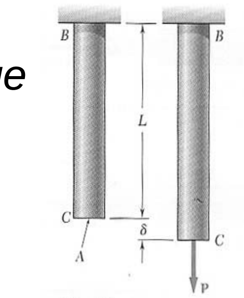
lecture
sixteen



elasticity & strain

Deformations

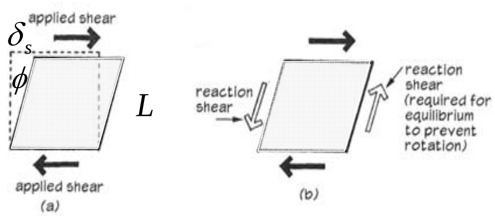
- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS



$$\varepsilon = \frac{\delta}{L}$$

Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: τ
- strain: γ
  - unitless (radians)

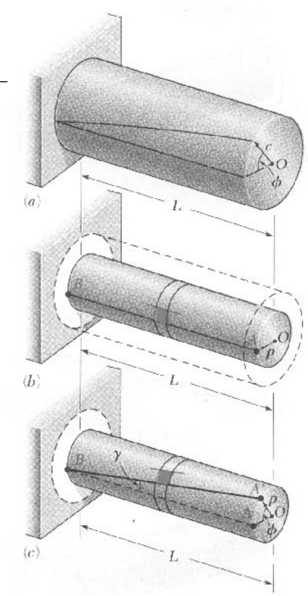


$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

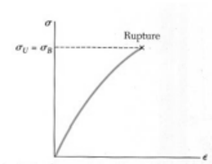
- deformations with torsion
- twist
- change in angle of line
- stress: τ
- strain: γ
  - unitless (radians)

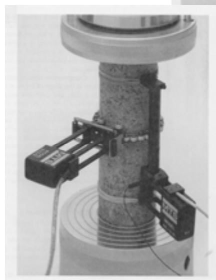
$$\gamma = \frac{\rho\phi}{L}$$

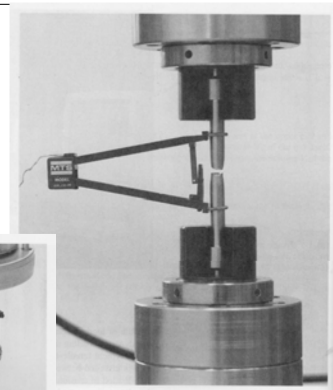


Load and Deformation

- *for stress, need P & A*
- *for strain, need δ & L*
  - *how?*
  - *TEST with load and measure*
  - *plot P/A vs. ε*







Material Behavior

- *every material has its own response*
  - *10,000 psi*
  - *L = 10 in*
  - *Douglas Fir vs. steel?*

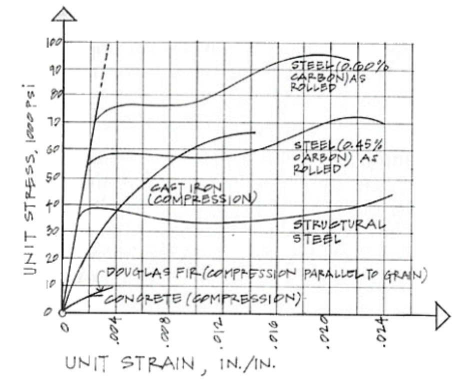


Figure 5.20 Stress-strain diagram for various materials.

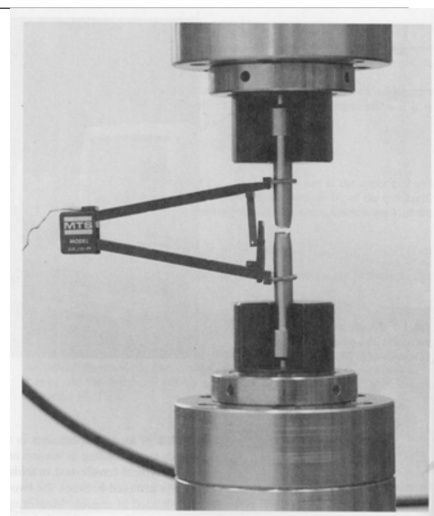
Behavior Types

- *ductile - "necking"*
- *true stress*

$$f = \frac{P}{A}$$

- *engineering stress*
  - *(simplified)*

$$f = \frac{P}{A_o}$$



Behavior Types

- *brittle*

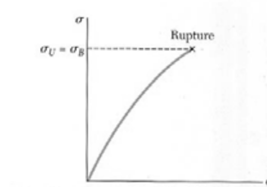


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

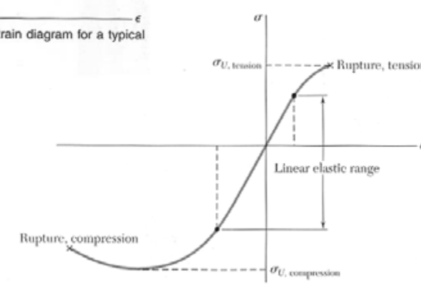


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- *important to us in f-ε diagrams:*
  - *straight section*
  - *LINEAR-ELASTIC*
  - *recovers shape (no permanent deformation)*

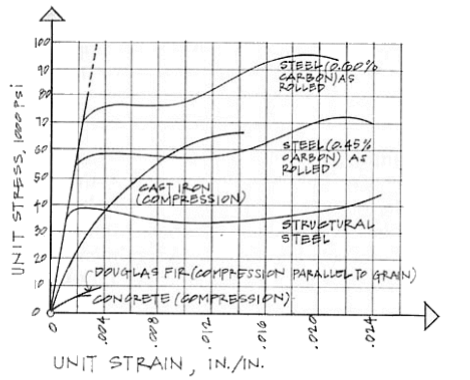
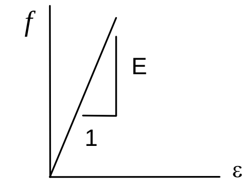


Figure 5.20 Stress-strain diagram for various materials.

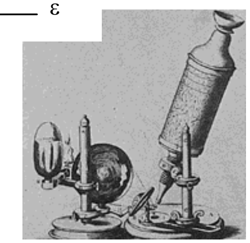
Hooke's Law

- *straight line has constant slope*
- *Hooke's Law*

$$f = E \cdot \varepsilon$$



- *E*
  - *Modulus of elasticity*
  - *Young's modulus*
  - *units just like stress*



Stiffness

- *ability to resist strain*
- *steels*
  - *same E*
  - *different yield points*
  - *different ultimate strength*

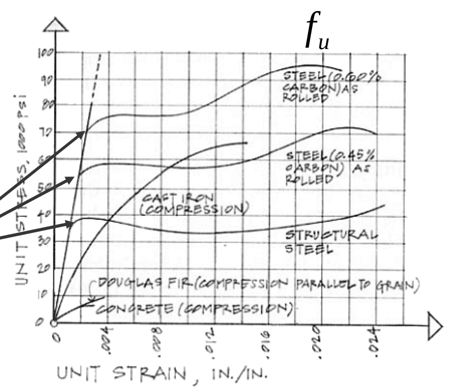
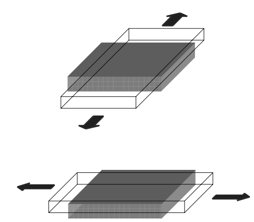


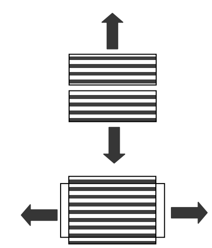
Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

- *ISOTROPIC*
  - *materials with E same at any direction of loading*
  - *ex. steel*



- *ANISOTROPIC*
  - *materials with different E at any direction of loading*
  - *ex. wood*



Elastic, Plastic, Fatigue

- *elastic springs back*
- *plastic has permanent deformation*
- *fatigue caused by reversed loading cycles*

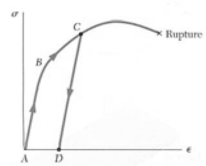


Fig. 2.18

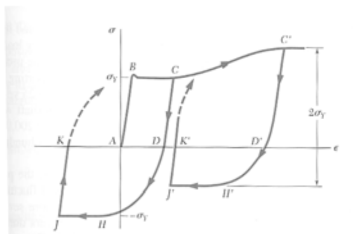


Fig. 2.20

Plastic Behavior

- *ductile*

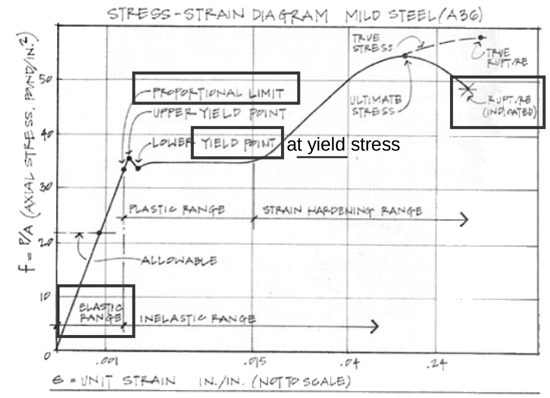


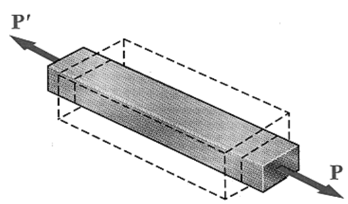
Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- *or "what happens to the cross section with axial stress"*

$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$



- *strain in lateral direction*
  - *negative*
  - *equal for isometric materials*

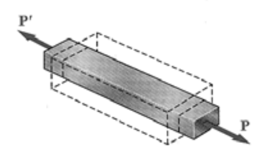
$$\varepsilon_y = \varepsilon_z$$

Poisson's Ratio

- *constant relationship between longitudinal strain and lateral strain*

$$\mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$



- *sign!* $0 < \mu < 0.5$

Calculating Strain

- from Hooke's law

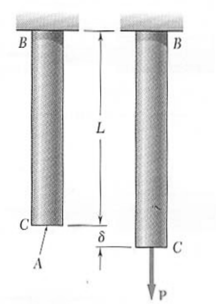
$$f = E \cdot \varepsilon$$

- substitute

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

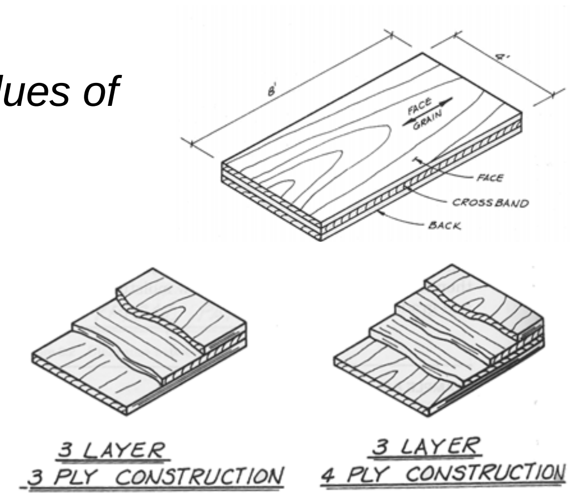
- get $\Rightarrow$

$$\delta = \frac{PL}{AE}$$



Orthotropic Materials

- non-isometric
- directional values of E and μ
- ex:
  – plywood
  – laminates
  – polymer composites



Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
  – sharp notches
  – holes
  – corners

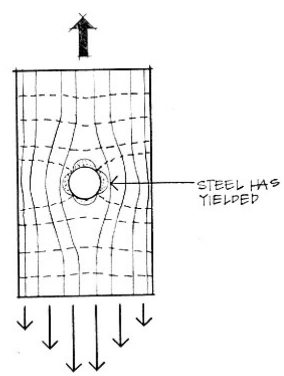
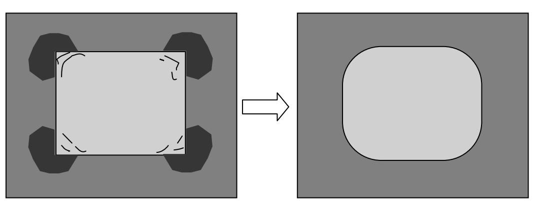
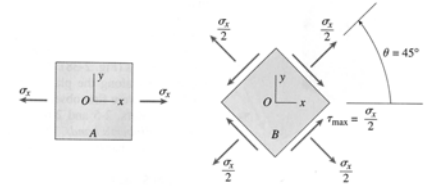


Figure 5.35 Stress trajectories around a hole.



Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0° \rightarrow \cos\theta = 1 \qquad f_{max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

Maximum Stresses

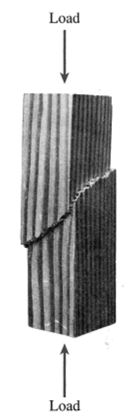
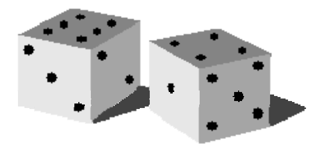


FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- *RISK & UNCERTAINTY*



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

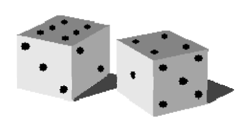
$$\textit{allowable load} = \frac{\textit{ultimate load}}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{\textit{ultimate load}}{\textit{allowable load}} = \frac{\textit{ultimate stress}}{\textit{allowable stress}}$$

Load and Resistance Factor Design

- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*



$$R_u = \gamma_D R_D + \gamma_L R_L \le \phi R_n$$

ϕ *- resistance factor*

γ *- load factor for (D)ead & (L)ive load*