ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

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SPRING 2007

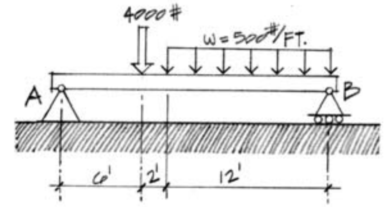
lecture
thirteen

# beam forces – internal

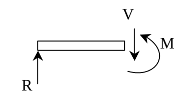


## Beams

- span horizontally
  - floors
  - bridges
  - roofs



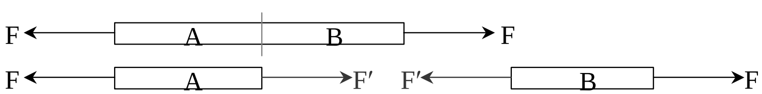

- loaded transversely by gravity loads
- may have internal axial force
- <u>will have</u> internal shear force
- <u>will have</u> internal moment (bending)



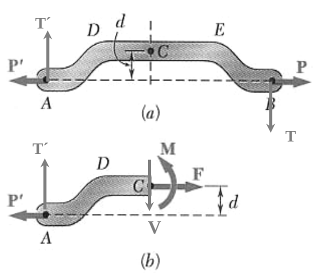



## Internal Forces

- trusses
  - axial only, (compression & tension)



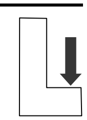

- in general
  - axial force
  - shear force, V
  - bending moment, M

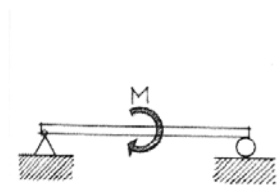




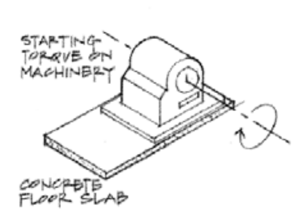

## Beam Loading

- concentrated force
- concentrated <u>moment</u>
  - spandrel beams



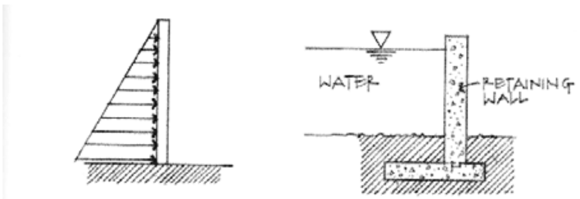



(d) Pure moment.



## Beam Loading

- *uniformly distributed load (line load)*
- *non-uniformly distributed load*
  - *hydrostatic pressure*
  - *wind loads*

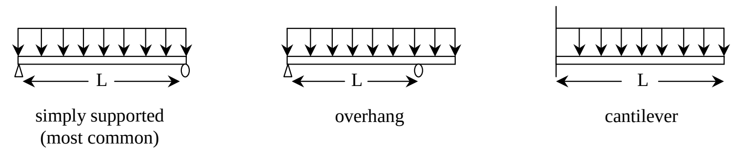



(c) Nonuniformly distributed load.

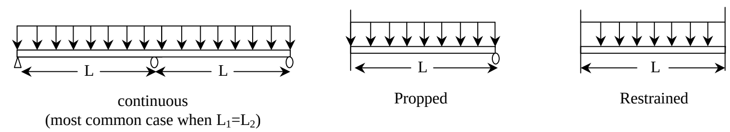


## Beam Supports

- *statically determinate*



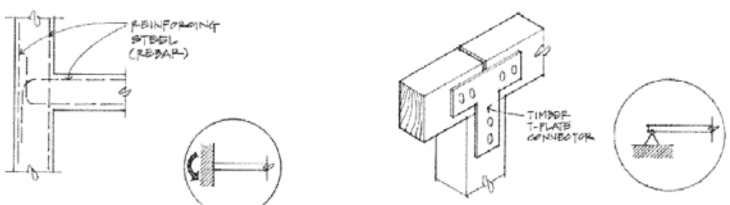

- *statically indeterminate*



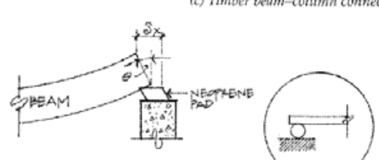



## Beam Supports

- *in the real world, modeled type*




(c) Timber beam–column connection with T-plate.

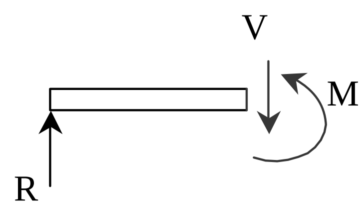



(a) Beam supported by a neoprene pad.



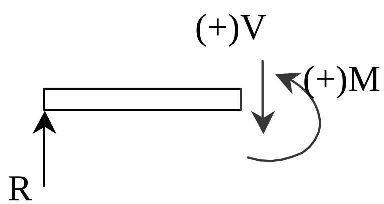
## Internal Forces in Beams

- *like method of sections / joints*
  - *no axial forces*
- *section must be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



## V & M Diagrams

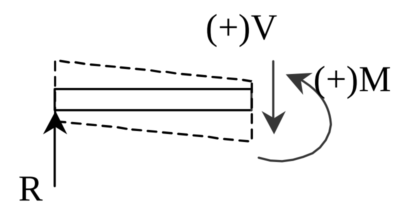
- tool to locate $V_{max}$ and $M_{max}$
- <u>necessary</u> for designing
- have a <u>different sign convention</u> than external forces, moments, and reactions





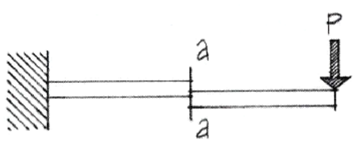

## Sign Convention

- shear force, V:
  - cut section to LEFT
  - if $\Sigma F_y$ is positive by statics, V acts down and is POSITIVE
  - beam has to resist shearing apart by V

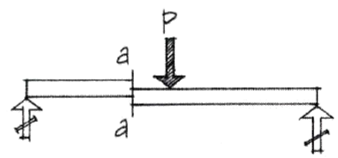




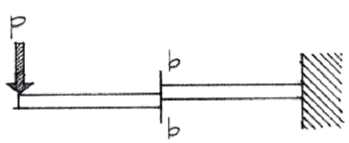

## Shear Sign Convention



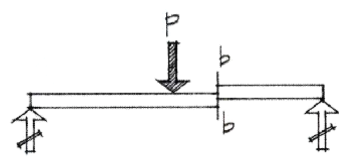

(+) Shear.




(+) Shear.




(–) Shear.

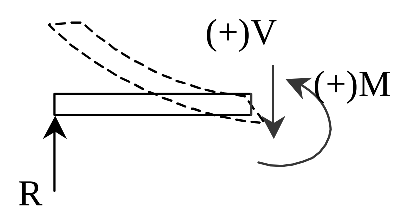



(–) Shear.

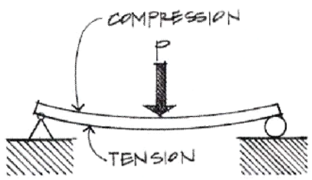


## Sign Convention

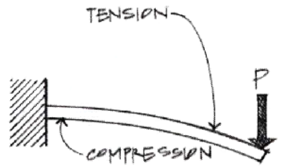
- bending moment, M:
  - cut section to LEFT
  - if $\Sigma M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a "smiley" beam has to resist bending apart by M



# Bending Moment Sign Convention

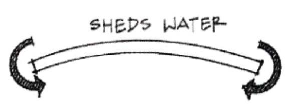



(+) Moment.




(–) Moment.

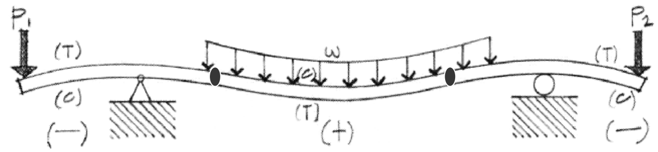

(+) Moment.




(–) Moment.



# Deflected Shape

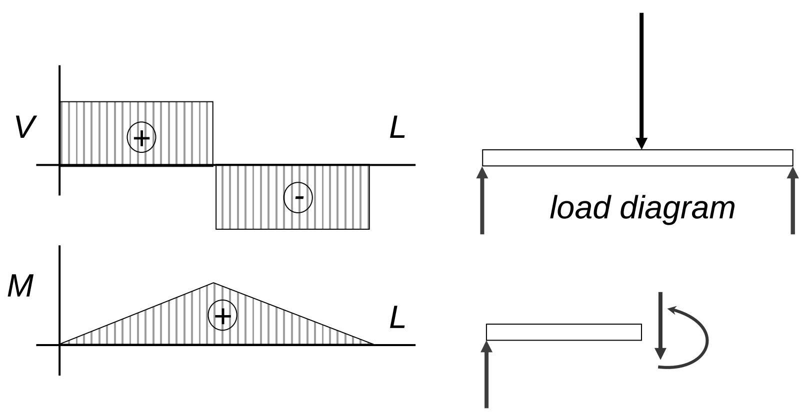



- *positive bending moment*
  - *tension in bottom, compression in top*
- *negative bending moment*
  - *tension in top, compression in bottom*
- *zero bending moment*
  - *inflection point*



# Constructing V & M Diagrams

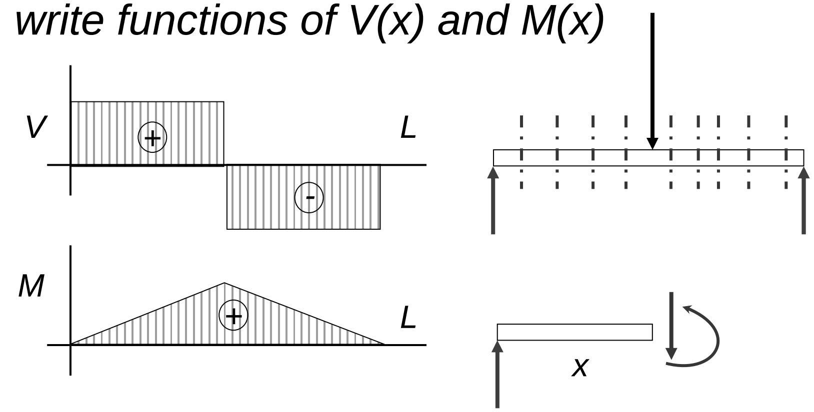
- *along the beam length, plot V, plot M*





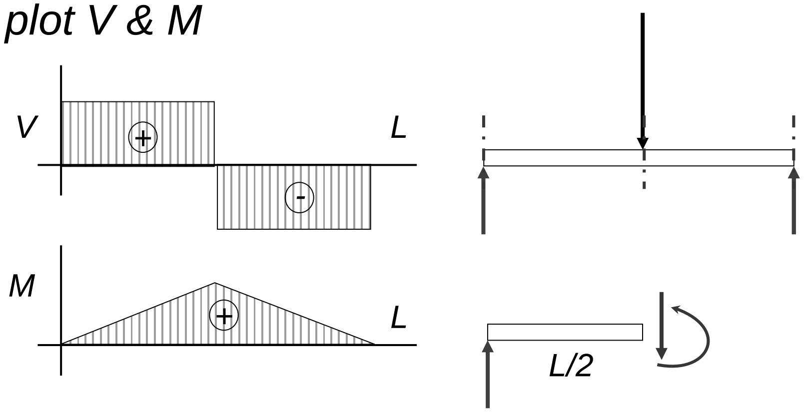

# Mathematical Method

- *cut sections with x as width*
- *write functions of V(x) and M(x)*



# Method 1: Equilibrium

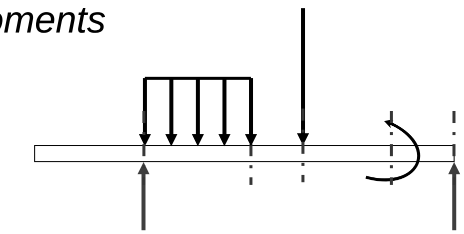
- cut sections at important places
- plot V & M






# Method 1: Equilibrium

- important places
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments
- free ends
  - zero forces





# Method 2: Semigraphical

- by knowing
  - area under loading curve = change in V
  - area under shear curve = change in M
  - concentrated forces cause "jump" in V
  - concentrated moments cause "jump" in M

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx \qquad M_D - M_C = \int_{x_C}^{x_D} V\,dx$$



# Method 2

- relationships

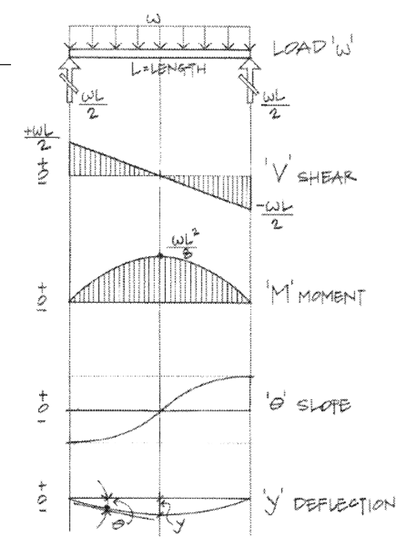
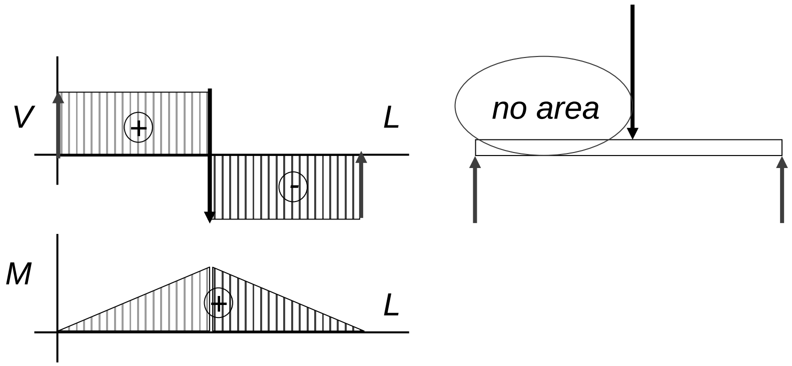



Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

## Method 2: Semigraphical

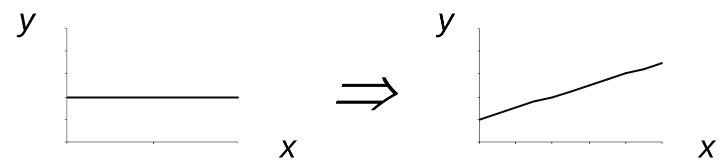
- $M_{max}$ *occurs where V = 0 (calculus)*






## Curve Relationships

- *integration of functions*
- *line with 0 slope, integrates to sloped*

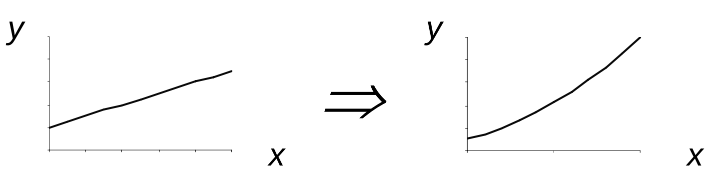



- *ex: load to shear, shear to moment*



## Curve Relationships

- *line with slope, integrates to parabola*

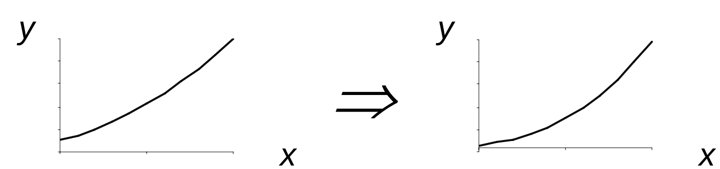



- *ex: load to shear, shear to moment*



## Curve Relationships

- *parabola, integrates to* $3^{rd}$ *order curve*




- *ex: load to shear, shear to moment*

# Basic Procedure

1. Find reaction forces & moments

   Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load



# Basic Procedure

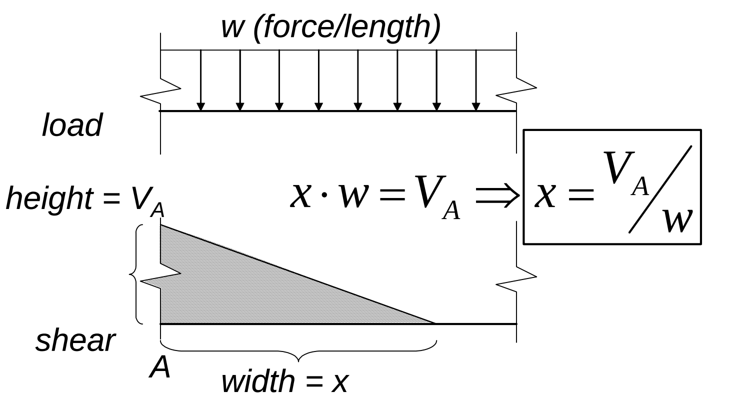
M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment jumps with moment
9. Moment changes with area under V



# Triangle Geometry

- slope of V is w  (-w:1)




$$x \cdot w = V_A \Rightarrow x = \frac{V_A}{w}$$



# Parabolic Shapes

- cases