ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

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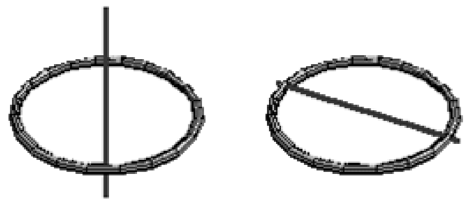
SPRING 2007

lecture
twelve

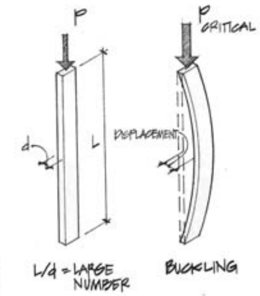


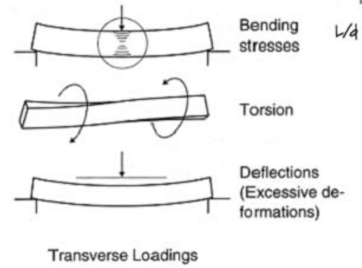
# moment of inertia of an area



## Moments of Inertia

- *$2^{nd}$ moment area*
  - *math concept*
  - *area x (distance)$^2$*
- *need for behavior of*
  - *beams*
  - *columns*



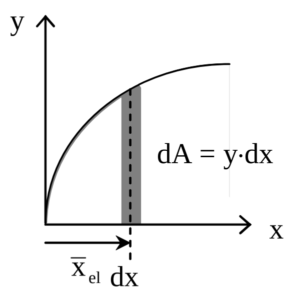



## Moment of Inertia

- *about any reference axis*
- *can be negative*

$$I_y = \int x^2 dA$$

$$I_x = \int y^2 dA$$

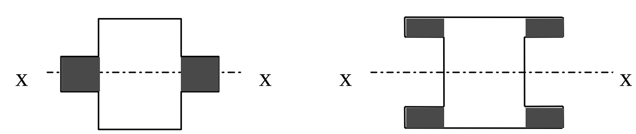



- *resistance to bending and buckling*



## Moment of Inertia

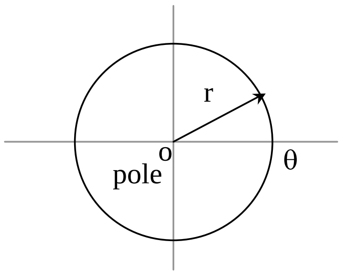
- *larger area away for same distance*
  - *larger $I$*



## Polar Moment of Inertia

- *for round-ish shapes*
- *uses polar coordinates (r and $\theta$)*
- *resistance to twisting*

$$J_o = \int r^2 dA$$






## Radius of Gyration

- *measure of inertia with respect to area*

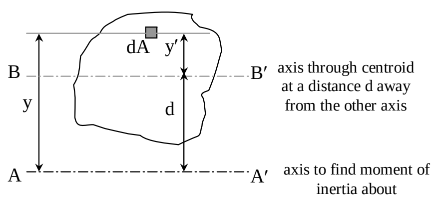
$$r_x = \sqrt{\frac{I_x}{A}}$$




## Parallel Axis Theorem

- *can find composite $I$ once composite centroid is known (basic shapes)*

$$I_x = I_{cx} + Ad_y^{\,2}$$
$$= \underline{\bar{I}_x} + Ad_y^{\,2}$$




$$I = \sum \bar{I} + \sum Ad^2$$

$$\boxed{\bar{I} = I - Ad^2}$$



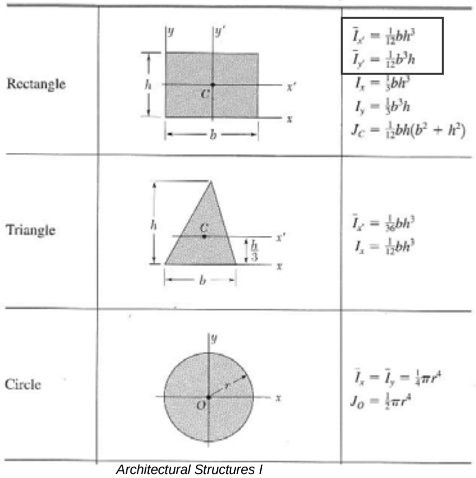
## Basic Procedure

1. *Draw reference origin (if not given)*
2. *Divide into basic shapes (+/-)*
3. *Label shapes*
4. *Draw table with A, $\bar{x}$, $\bar{x}$A, $\bar{y}$, $\bar{y}$A, $\bar{I}$'s, d's, and Ad$^2$'s*
5. *Fill in table and get $\hat{x}$ and $\hat{y}$ for composite*
6. *Sum necessary columns*
7. *Sum $\bar{I}$'s and Ad$^2$'s*

$$(d_x = \hat{x} - \bar{x})$$
$$(d_y = \hat{y} - \bar{y})$$

# *Area Moments of Inertia*

- *Table 7.2 – pg. 252* (*bars refer to centroid*)
  - *x, y*
  - *x', y'*
  - *C*



| | | |
|---|---|---|
| Rectangle | | $\bar{I}_{x'} = \frac{1}{12}bh^3$<br>$\bar{I}_{y'} = \frac{1}{12}b^3h$<br>$I_x = \frac{1}{3}bh^3$<br>$I_y = \frac{1}{3}b^3h$<br>$J_C = \frac{1}{12}bh(b^2 + h^2)$ |
| Triangle | | $\bar{I}_{x'} = \frac{1}{36}bh^3$<br>$I_x = \frac{1}{12}bh^3$ |
| Circle | | $\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$<br>$J_O = \frac{1}{2}\pi r^4$ |