#### **ARCHITECTURAL STRUCTURES I:**

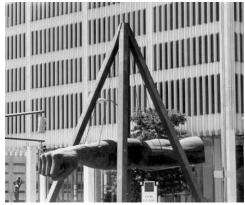
STATICS AND STRENGTH OF MATERIALS

ENDS 23

DR. ANNE NICHOLS

SPRING 2007

lecture eleven



# centers of gravity- centroids

 Centroids 1
 Architectural Structures I
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 Lecture 11
 ENDS 231

## Center of Gravity

"average" x & y from moment

$$\begin{array}{c|c}
z & y \\
\Delta W_4 & \Delta W_1 \\
\Delta W_3 & \Delta W_2 \\
\hline
 & x
\end{array}$$

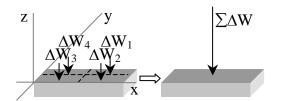
$$\sum M_{y} = \sum_{i=1}^{n} x_{i} \Delta W_{i} = \overline{x} \mathbf{W} \implies \overline{x} = \frac{\sum (x \Delta W)}{\mathbf{W}}$$

$$\sum M_{x} = \sum_{i=1}^{n} y_{i} \Delta W_{i} = \overline{y} \mathbf{W} \implies \overline{y} = \frac{\sum (y \Delta W)}{\mathbf{W}}$$

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## Center of Gravity

- location of equivalent weight
- determined with calculus



• sum element weights  $W = \int dW$ 

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#### Centroid

- "average" x & y of an area
- for a volume of constant thickness
  - $-\Delta W = \gamma t \Delta A$  where  $\gamma$  is weight/volume
  - center of gravity = centroid of area

$$\overline{x} = \frac{\sum (x\Delta A)}{A}$$

$$\overline{y} = \frac{\sum (y\Delta A)}{A}$$

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#### Centroid

• for a line, sum up length

$$\overline{x} = \frac{\sum (x\Delta L)}{L}$$

$$\overline{y} = \frac{\sum (y\Delta L)}{L}$$





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Centroids 11

Lecture 11

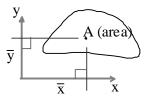
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#### 1st Moment Area

- math concept
- the moment of an area about an axis

$$Q_x = \overline{y}A$$

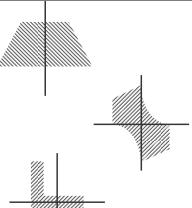




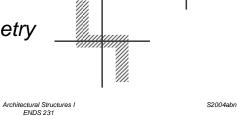
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## Symmetric Areas

• symmetric about an axis

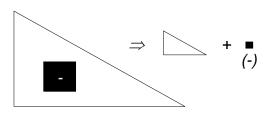


- symmetric about a center point
- mirrored symmetry



## Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



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### Basic Procedure

- 1. Draw reference origin (if not given)
- 2. Divide into basic shapes (+/-)
- 3. Label shapes
- 4. Draw table

5. Fill in table

Component	Area	$\overline{x}$	$\bar{x}A$	$\bar{y}$	$\overline{y}A$
Σ					

- 6. Sum necessary columns
- 7. Calculate  $\bar{x}$  and  $\bar{y}$

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### Area Centroids

• Table 7.1 – pg. 242

Shape		X	У
Triangular area	1	$\frac{b}{3}$ right triangle only	h 3
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		3 <i>a</i> 8	3h 5
Parabolic area		0	$\frac{3h}{5}$

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