ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

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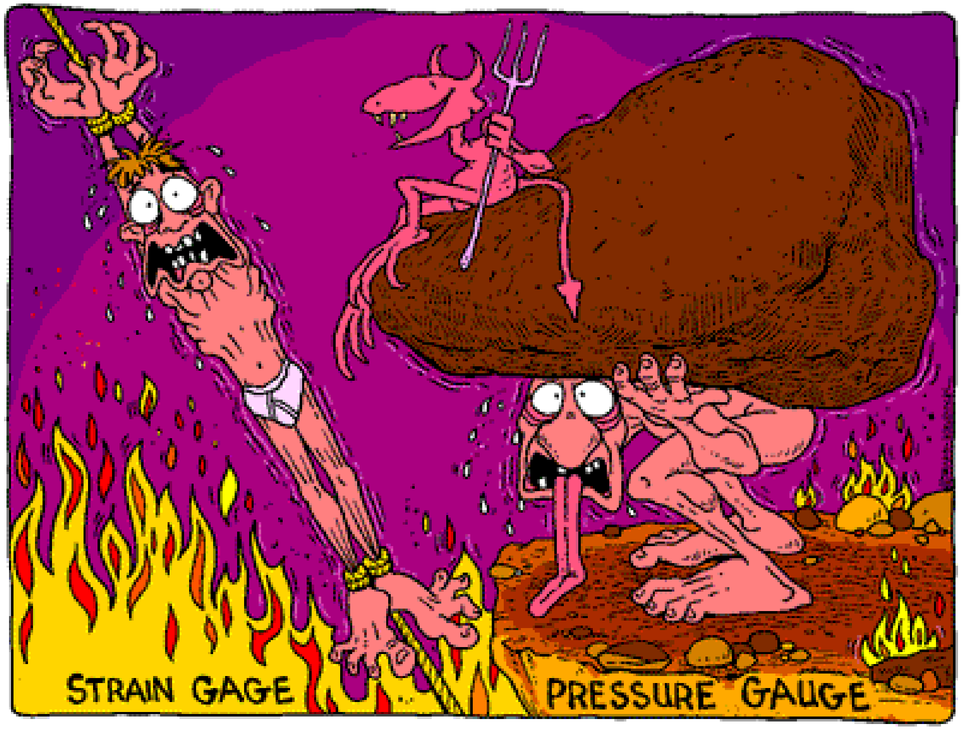
SPRING 2007

lecture

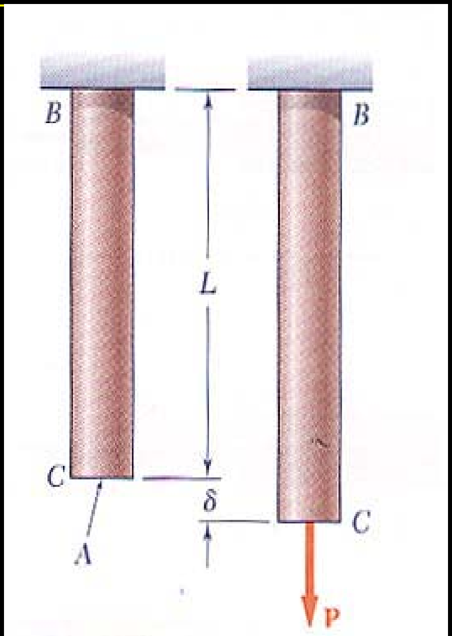
sixteen

elasticity & strain



Deformations

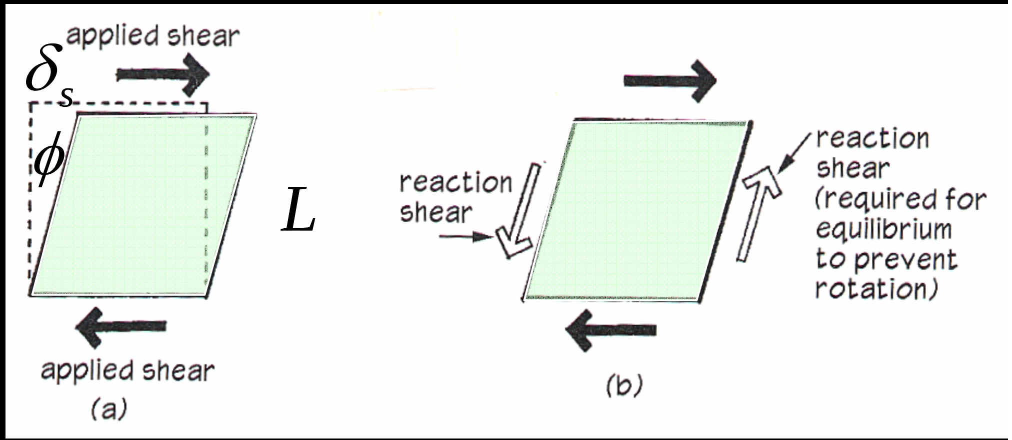
- materials deform
- axially loaded materials change length
- normal stress is load per unit area
- STRAIN:
  - change in length over length
  - UNITLESS



$$\varepsilon = \frac{\delta}{L}$$

Shearing Strain

- *deformations with shear*
- *parallelogram*
- *change in angles*
- *stress:* τ
- *strain:* γ
  - *unitless (radians)*

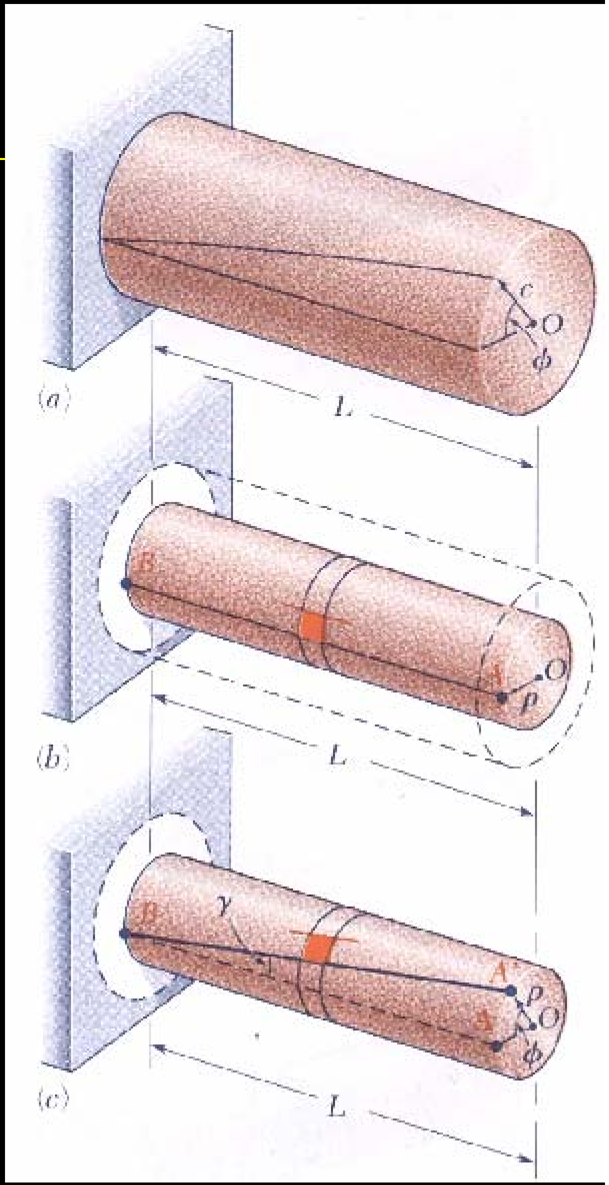


$$\gamma = \frac{\delta_s}{L} = \tan\phi \cong \phi$$

Shearing Strain

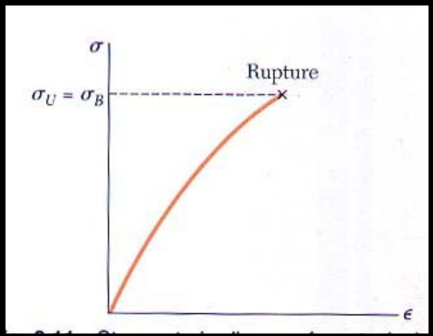
- deformations with torsion
- twist
- change in angle of line
- stress: τ
- strain: γ
  - unitless (radians)

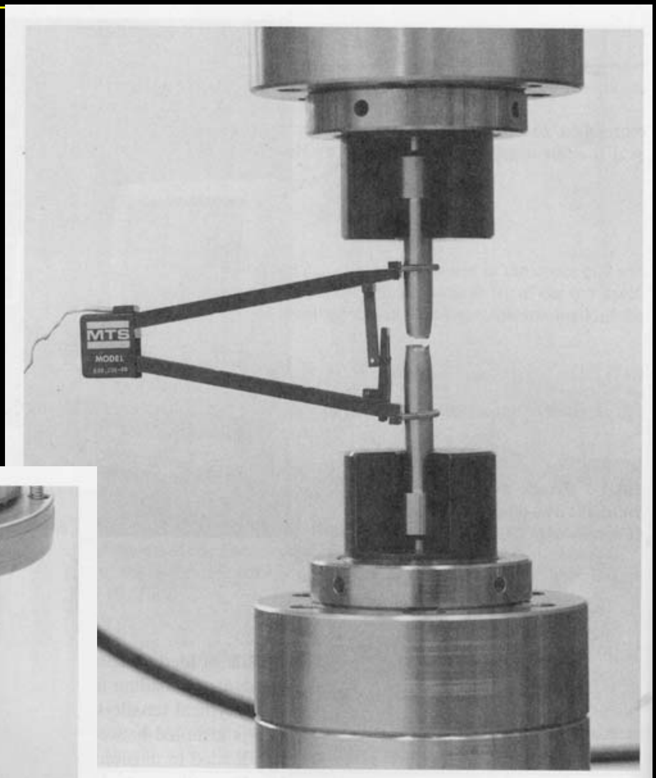
$$\gamma = \frac{\rho\phi}{L}$$



Load and Deformation

- for stress, need P & A
- for strain, need δ & L
  - how?
  - TEST with load and measure
  - plot P/A vs. ε





Material Behavior

- every material has its own response
  - 10,000 psi
  - L = 10 in
  - Douglas Fir vs. steel?

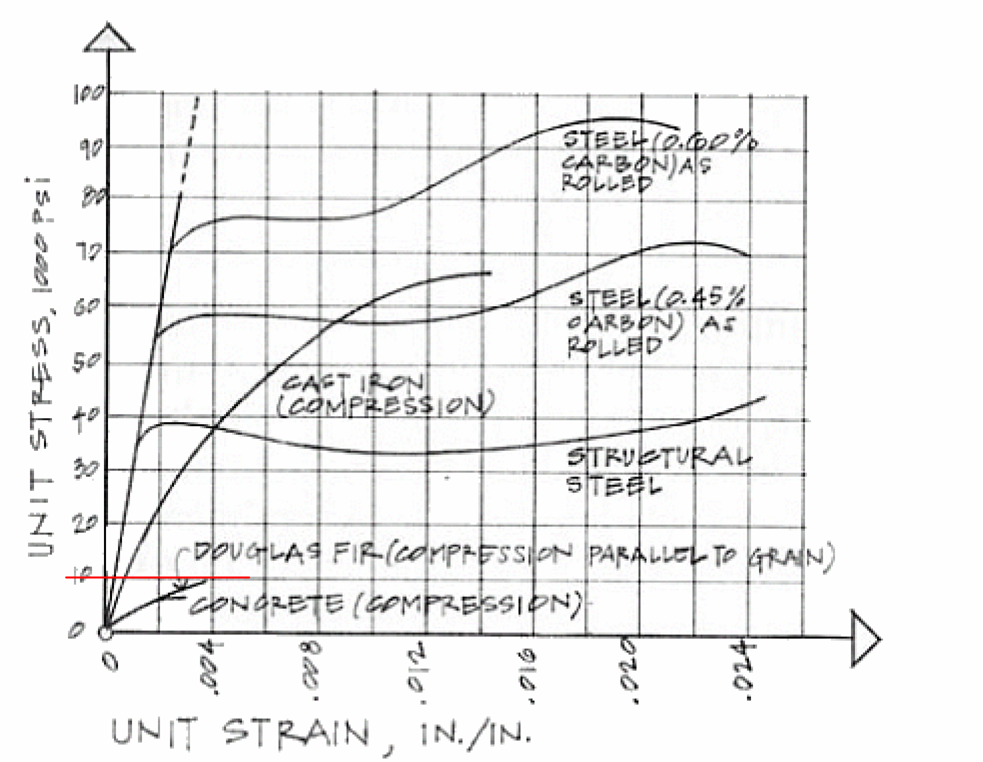


Figure 5.20 Stress-strain diagram for various materials.

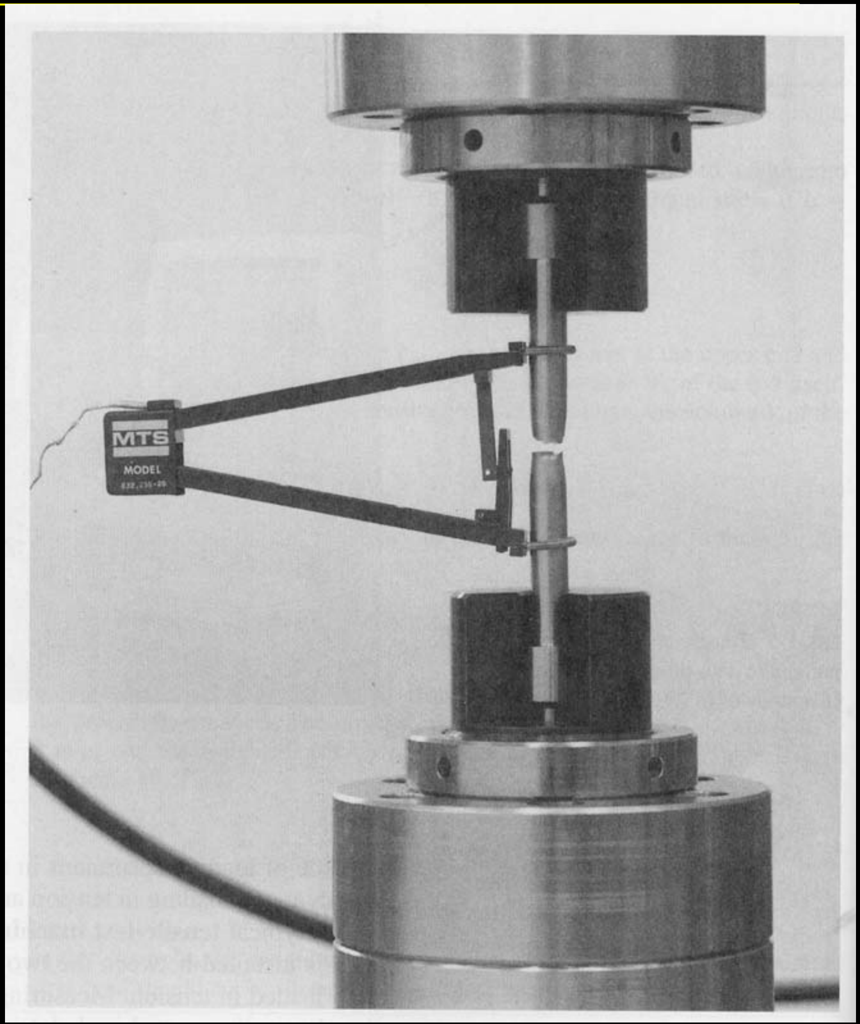
Behavior Types

- *ductile - "necking"*
- *true stress*

$$f = \frac{P}{A}$$

- *engineering stress*
  - *(simplified)*

$$f = \frac{P}{A_o}$$



Behavior Types

- brittle

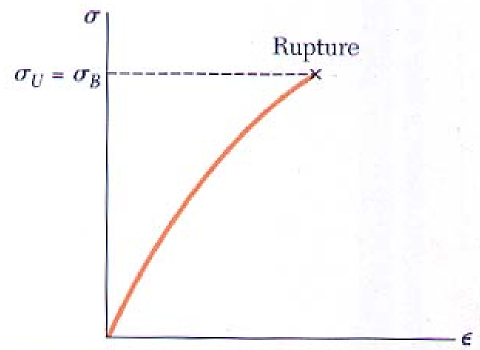


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- semi-brittle

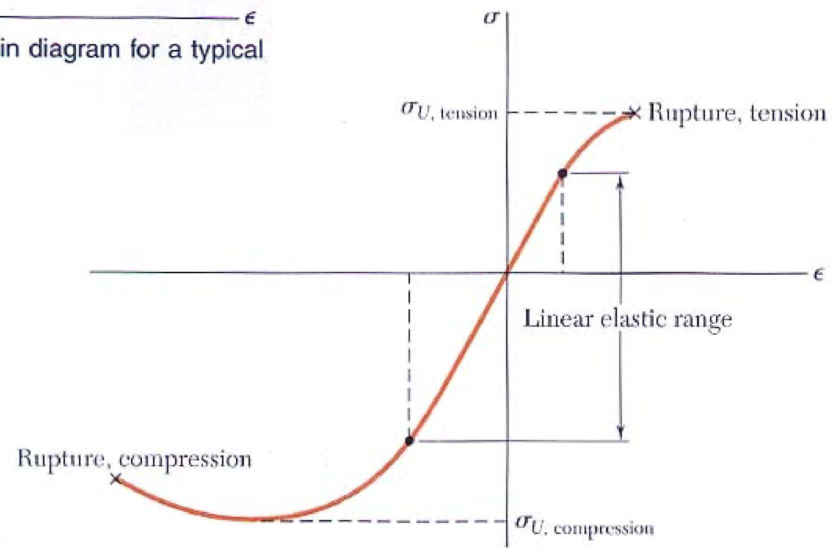


Fig. 2.14 Stress-strain diagram for concrete.

Stress to Strain

- important to us in f-ε diagrams:
  - straight section
  - LINEAR-ELASTIC
  - recovers shape (no permanent deformation)

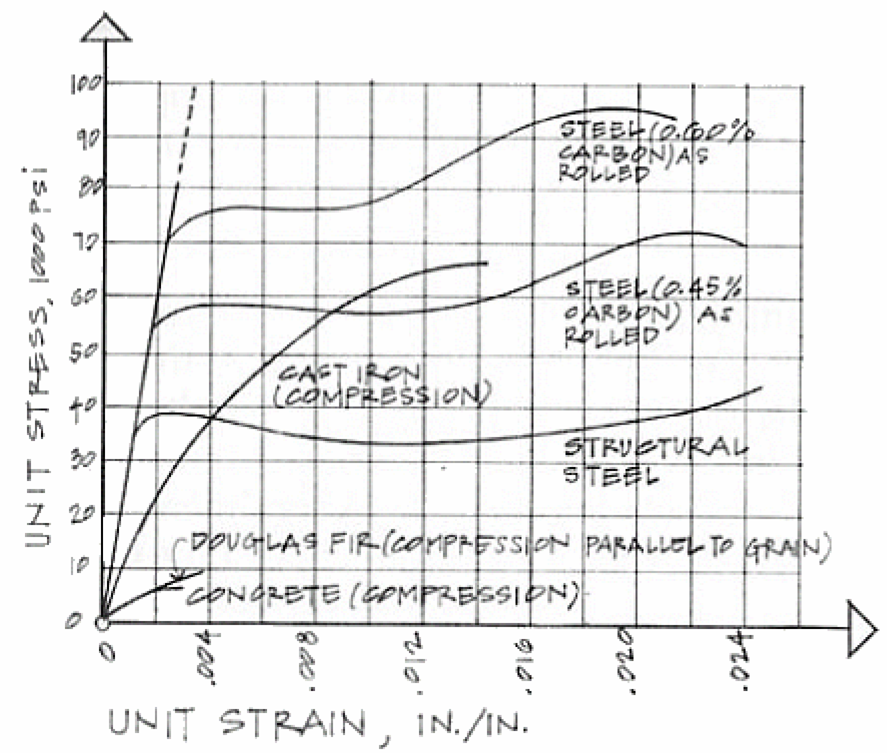
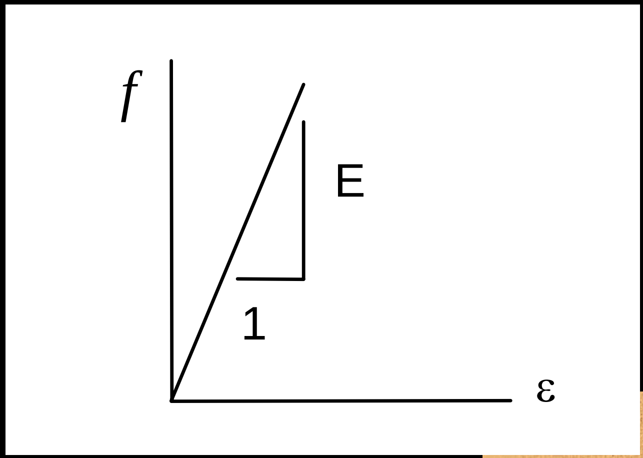


Figure 5.20 Stress-strain diagram for various materials.

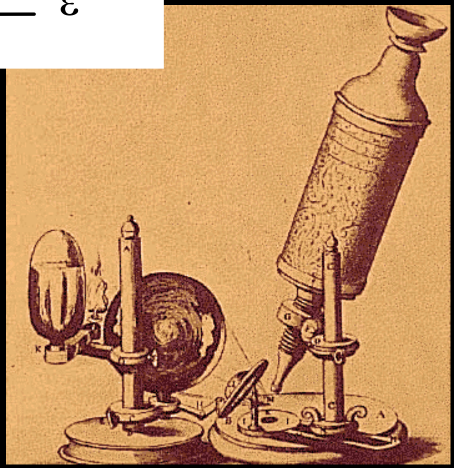
Hooke's Law

- straight line has constant slope
- Hooke's Law

$$f = E \cdot \varepsilon$$



- E
  - Modulus of elasticity
  - Young's modulus
  - units just like stress



Stiffness

- ability to resist strain
- steels
  - same E
  - different <u>yield points</u>
  - different <u>ultimate strength</u>

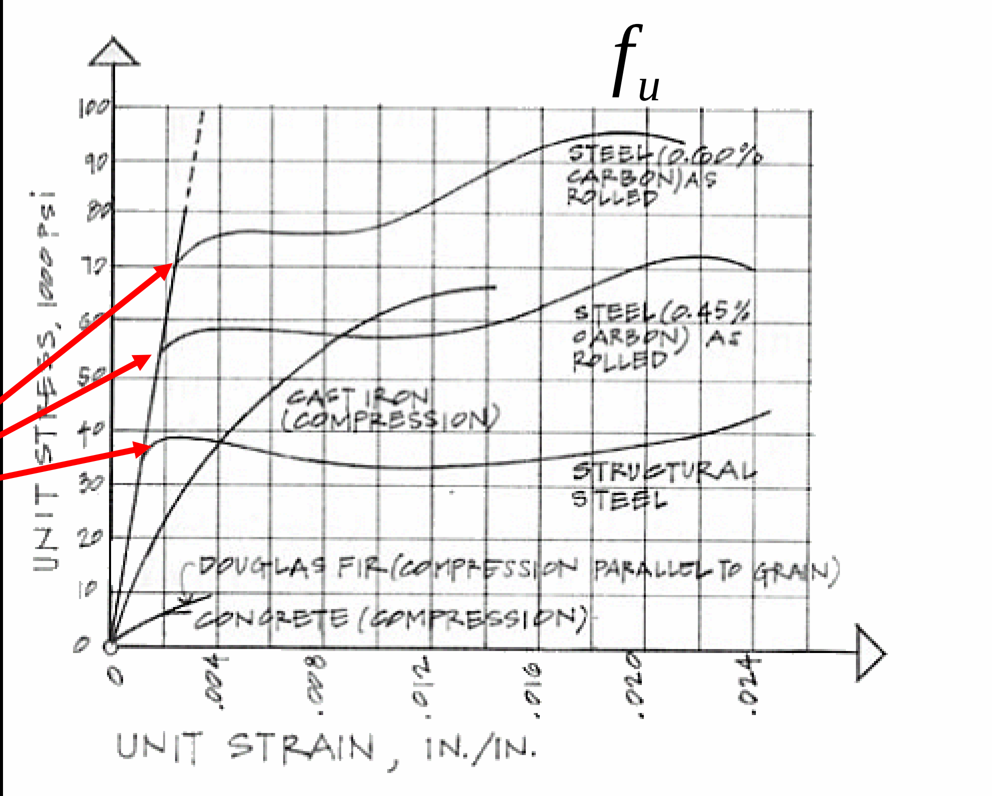
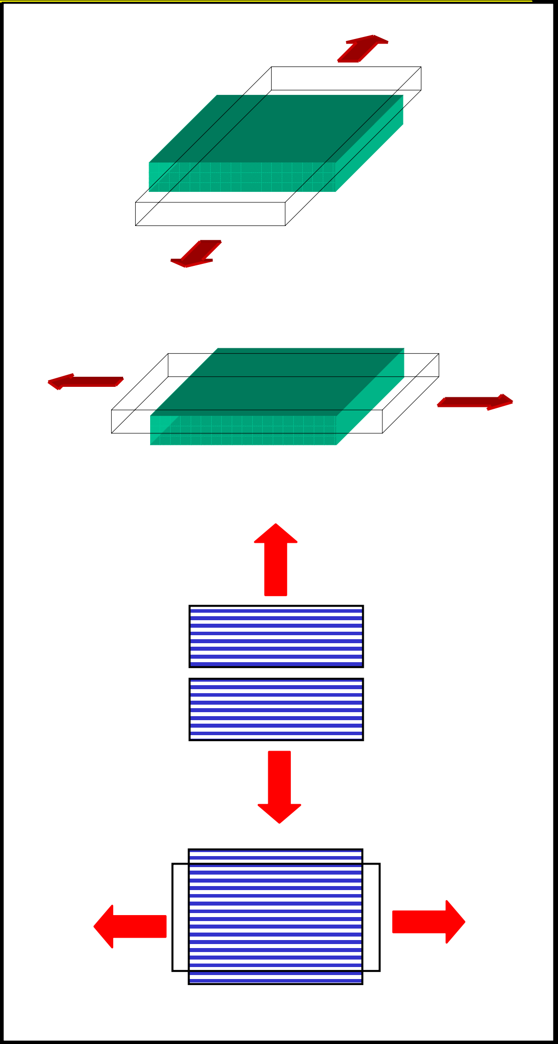


Figure 5.20 Stress-strain diagram for various materials.

Isotropy & Anisotropy

- ISOTROPIC
  - materials with E same at any direction of loading
  - ex. steel

- ANISOTROPIC
  - materials with different E at any direction of loading
  - ex. wood



Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles

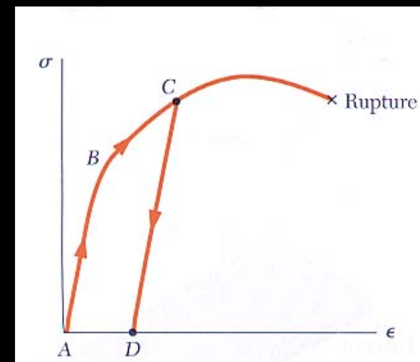


Fig. 2.18

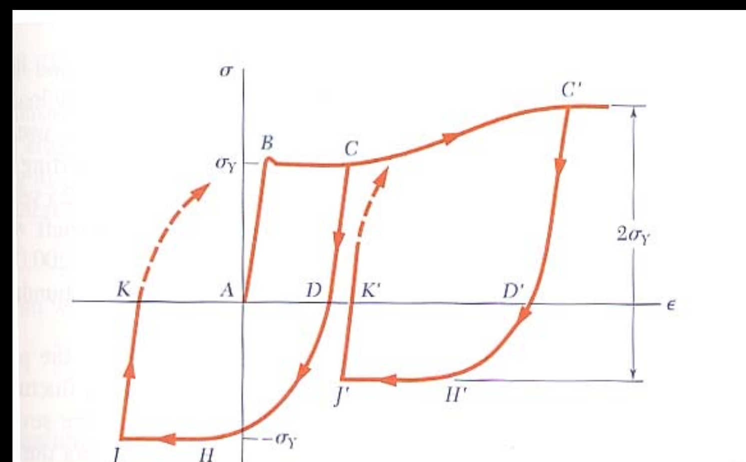


Fig. 2.20

Plastic Behavior

- *ductile*

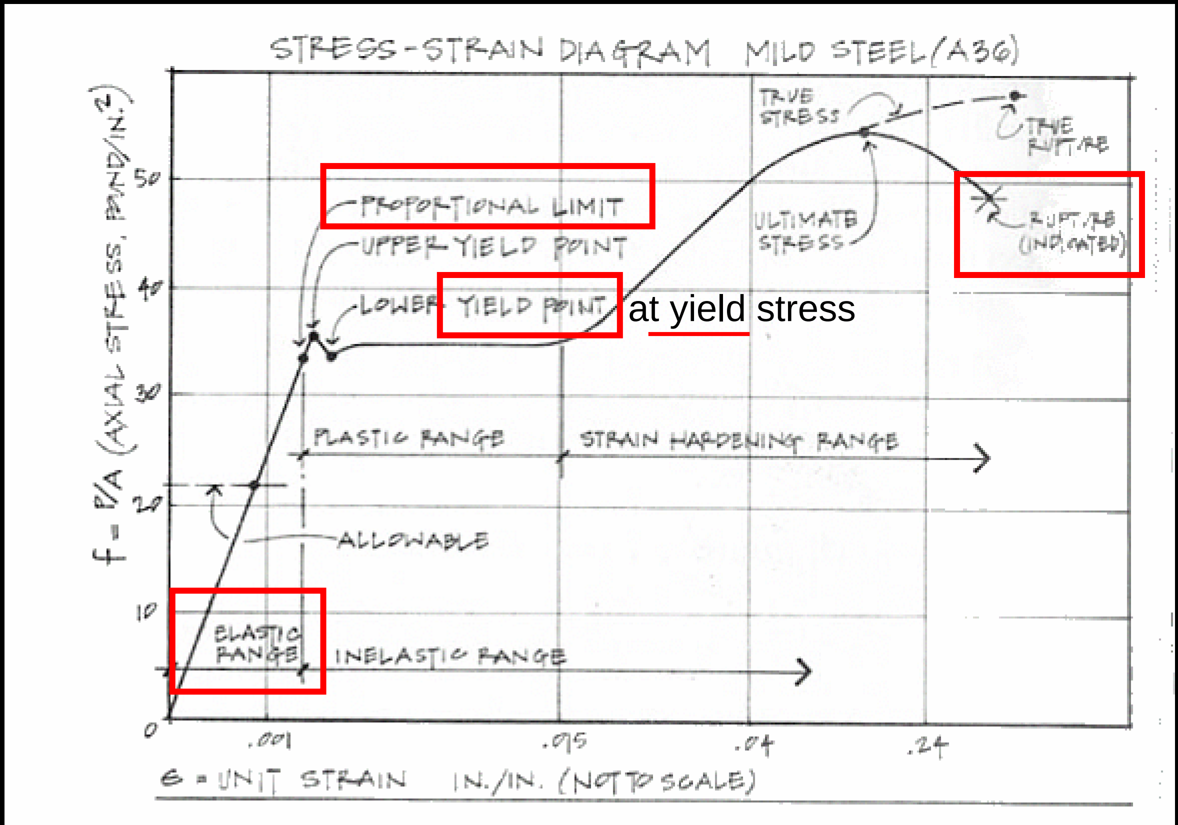
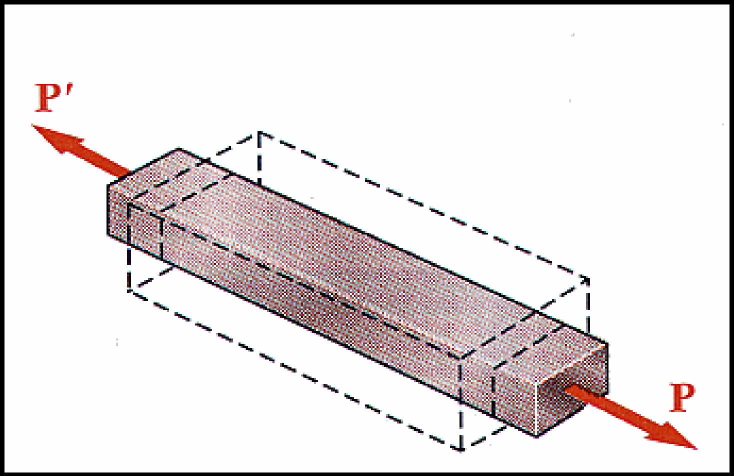


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Lateral Strain

- *or "what happens to the cross section with axial stress"*



$$\varepsilon_x = \frac{f_x}{E}$$

$$f_y = f_z = 0$$

- *strain in lateral direction*
  - *negative*
  - *equal for isometric materials*

$$\varepsilon_y = \varepsilon_z$$

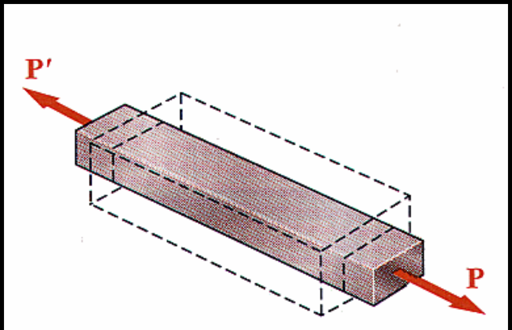
Poisson's Ratio

- constant relationship between longitudinal strain and lateral strain

$$\mu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E}$$

- sign! $0 < \mu < 0.5$



Calculating Strain

- *from Hooke's law*

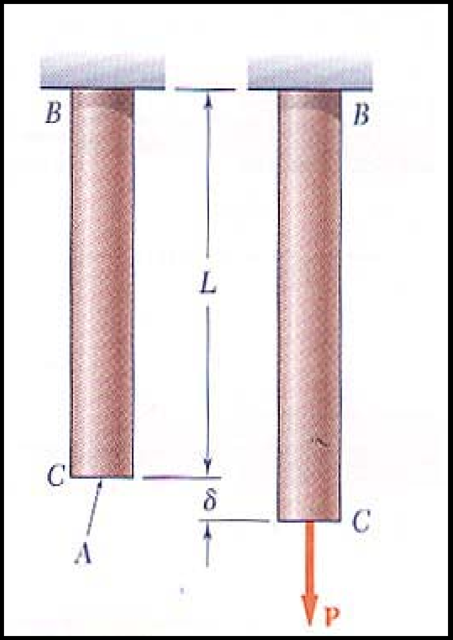
$$f = E \cdot \varepsilon$$

- *substitute*

$$\frac{P}{A} = E \cdot \frac{\delta}{L}$$

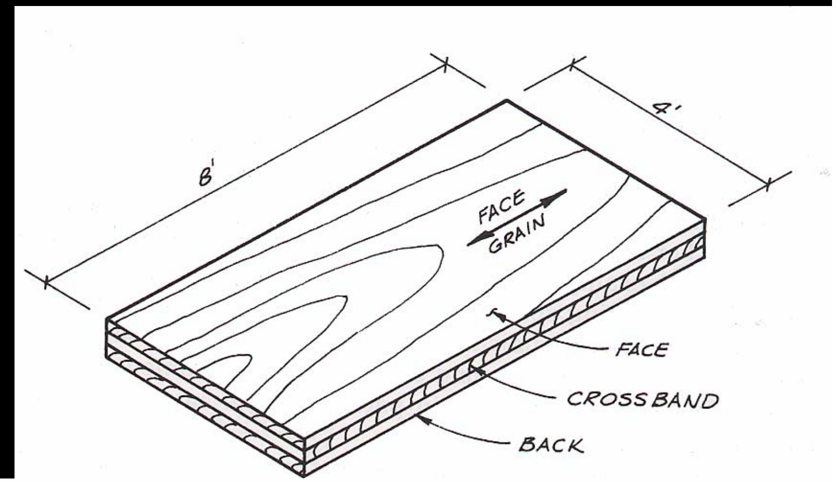
- *get* $\Rightarrow$

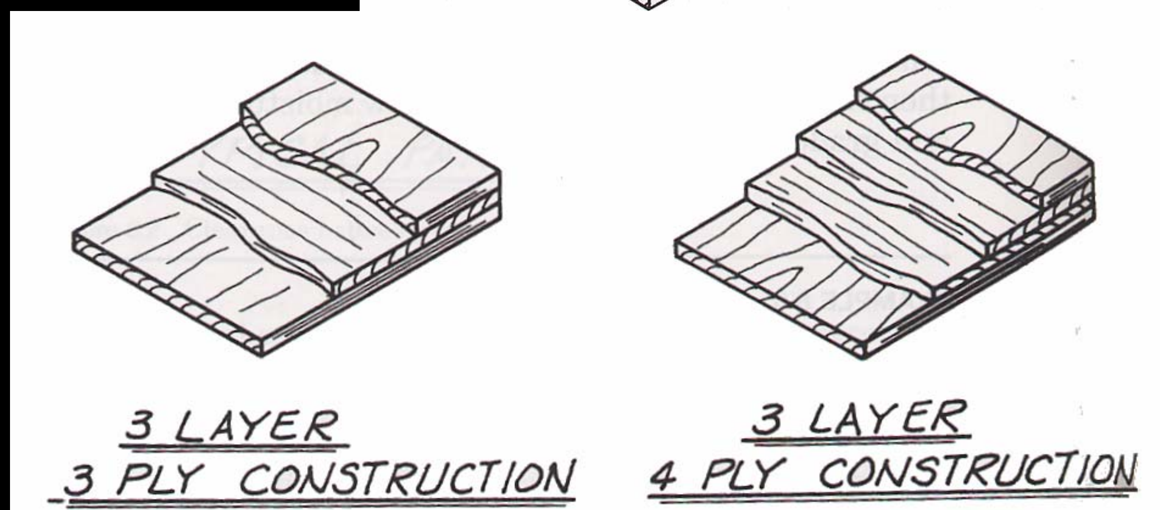
$$\delta = \frac{PL}{AE}$$



Orthotropic Materials

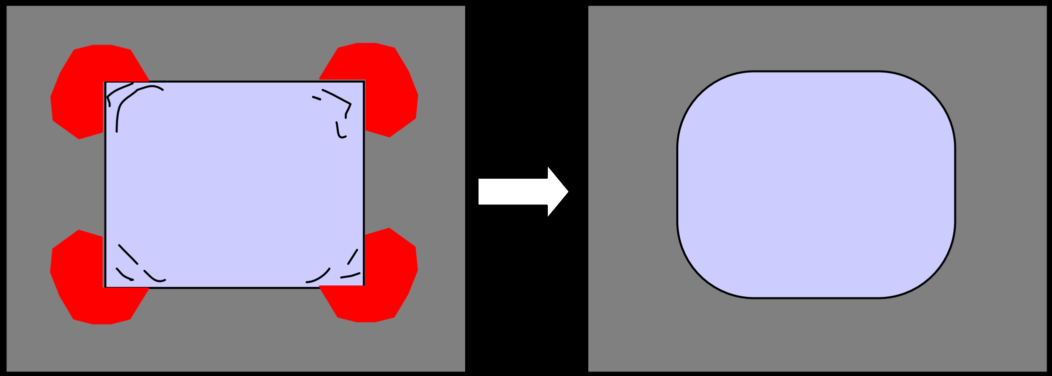
- non-isometric
- directional values of E and μ
- ex:
  - plywood
  - laminates
  - polymer composites





Stress Concentrations

- why we use f_{ave}
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners



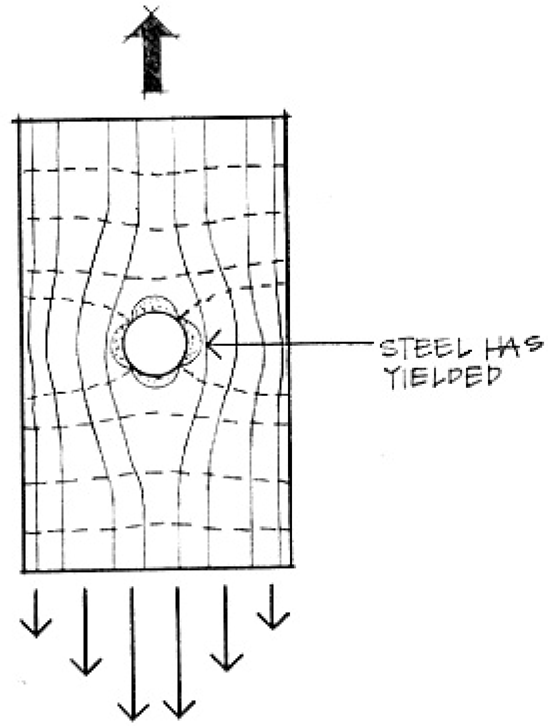
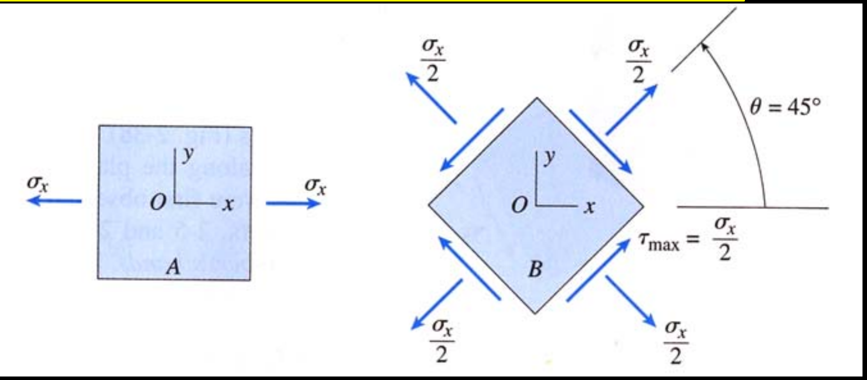


Figure 5.35 Stress trajectories around a hole.

Maximum Stresses

- if we need to know where max f and f_v happen:



$$\theta = 0° \rightarrow \cos\theta = 1 \qquad f_{max} = \frac{P}{A_o}$$

$$\theta = 45° \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-max} = \frac{P}{2A_o} = \frac{f_{max}}{2}$$

Maximum Stresses

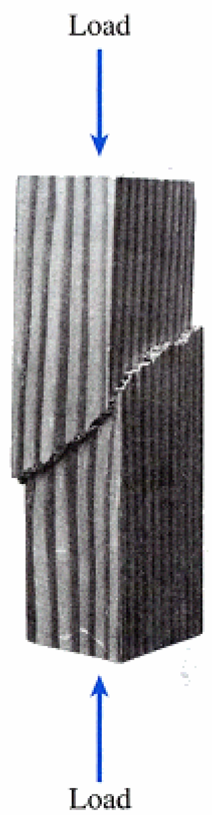
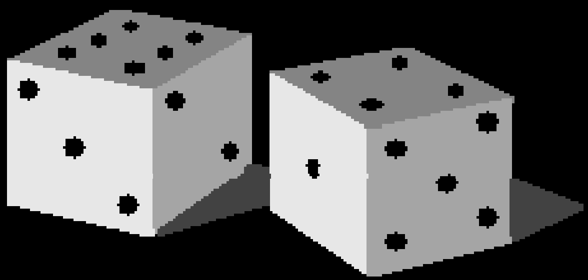


FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

FIG. 2-38 Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension

Design of Members

- *beyond allowable stress...*
- *materials aren't uniform 100% of the time*
  - *ultimate strength or capacity to failure may be different and some strengths hard to test for*
- *RISK & UNCERTAINTY*



$$f_u = \frac{P_u}{A}$$

Factor of Safety

- *accommodate uncertainty with a safety factor:*

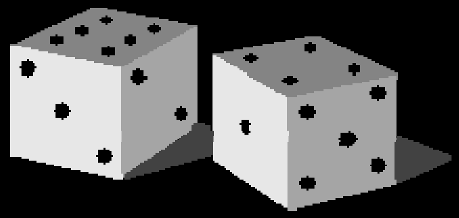
$$allowable\ load = \frac{ultimate\ load}{F.S}$$

- *with linear relation between load and stress:*

$$F.S = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$$

Load and Resistance Factor Design

- *loads on structures are*
  - *not constant*
  - *can be more influential on failure*
  - *happen more or less often*
  - *UNCERTAINTY*



$$R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n$$

ϕ *- resistance factor*

γ *- load factor for (D)ead & (L)ive load*