

Columns and Stability

Design Criteria

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be *stable*.

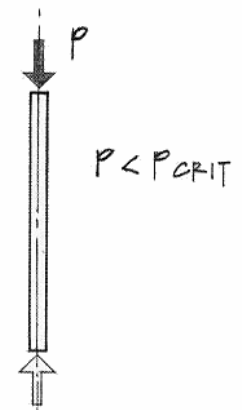
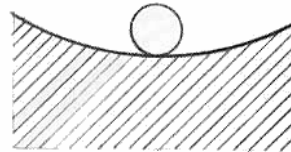
Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

Physics

Recall that things like to be or *prefer* to be in their lowest energy state (potential energy). Examples include water in a water tank. The energy it took to put the water up there is stored until it is released and can flow due to gravity.

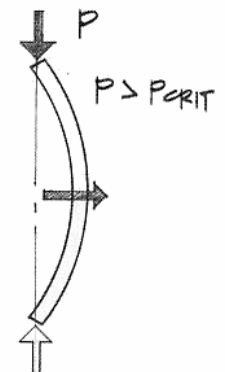
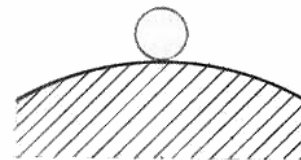
Stable Equilibrium

When energy is added to an object in the form of a push or disturbance, the object will return to it's original position. *Things don't change in the end.*



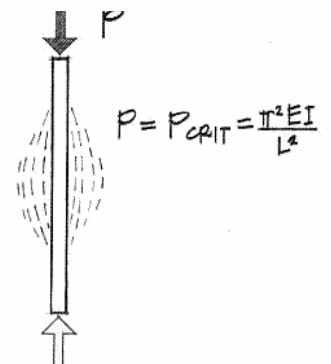
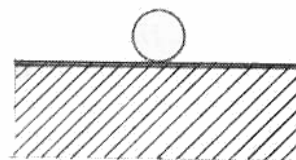
Unstable Equilibrium

When energy is added to an object, the object will move and get more "disturbed". *Things change rapidly.*



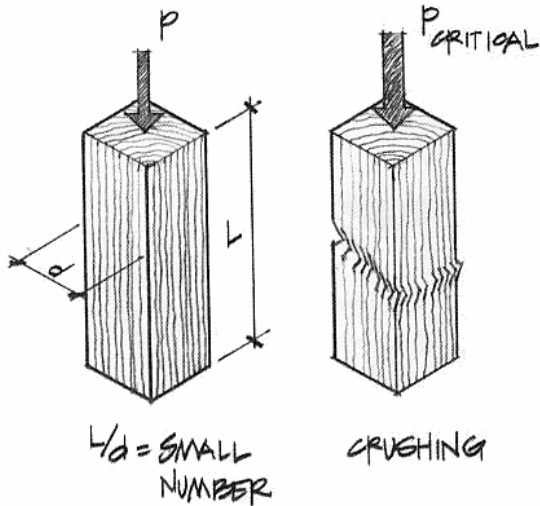
Neutral Equilibrium

When energy is added to an object, the object will move some then stop.. *Things change.*



Column with Axial Loading

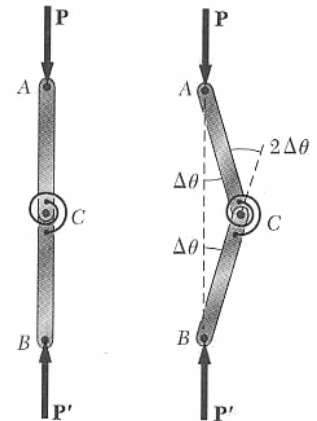
A column loaded centrally can experience unstable equilibrium, called *buckling*, because of how tall and slender they are. This instability is sudden and not good.



Buckling can occur in sheets (like my “memory metal” cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can't resist the moment in it any longer.

Short (stubby) columns will experience crushing before buckling.



Critical Buckling Load

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of $P \cdot \Delta$).

The buckled shape will be in the form of a *sine wave*.

Euler Formula

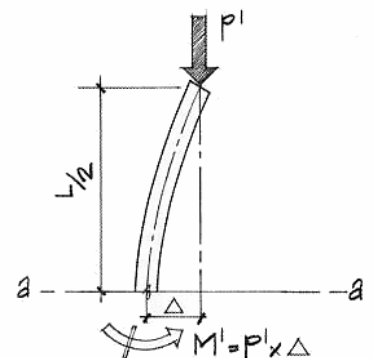
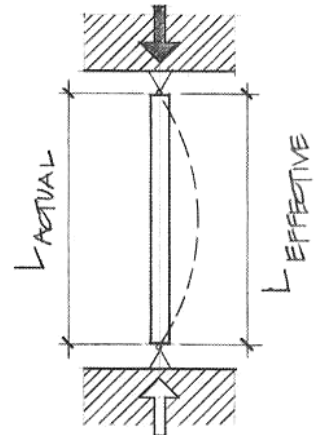
Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

and the critical stress (if less than the normal stress) is:

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EA r^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2}$$

where $I=Ar^2$ and L_e/r is called the slenderness ratio. The smallest I of the section will govern.



Yield Stress and Buckling Stress

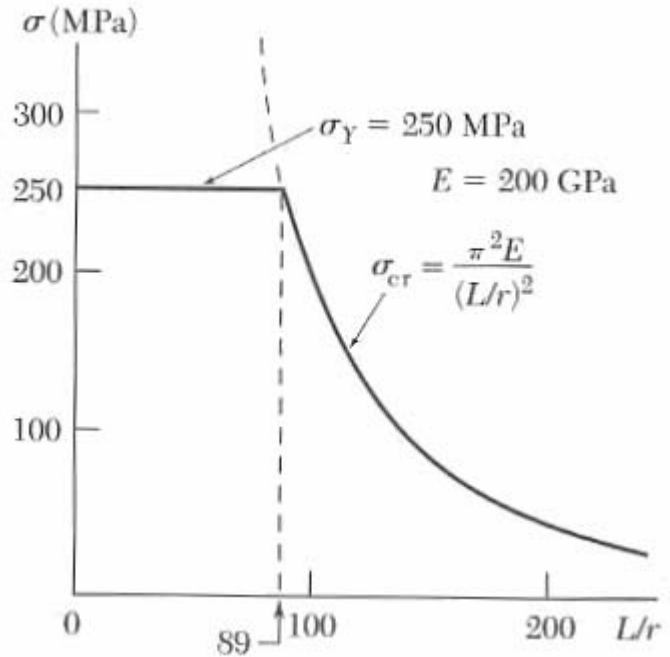
The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler’s formula is used only if the critical stress is less than half of the yield point stress:

$$P_{critical} = \frac{\pi^2 EI_{min}}{(L)^2}; \quad f_{critical} = \frac{P_{critical}}{A} < \frac{F_y}{2}$$

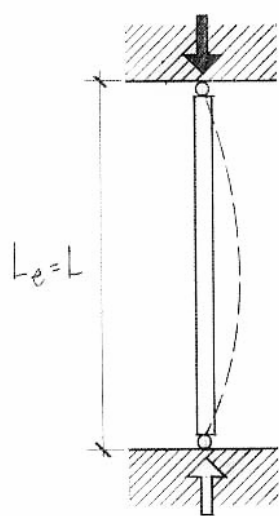
to be used for $\frac{L_e}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$

where C_c is the column slenderness classification constant and is the slenderness ratio of a column for which the critical stress is equal to half the yield point stress.

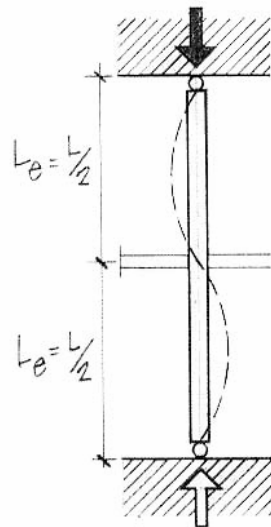


Effective Length and Bracing

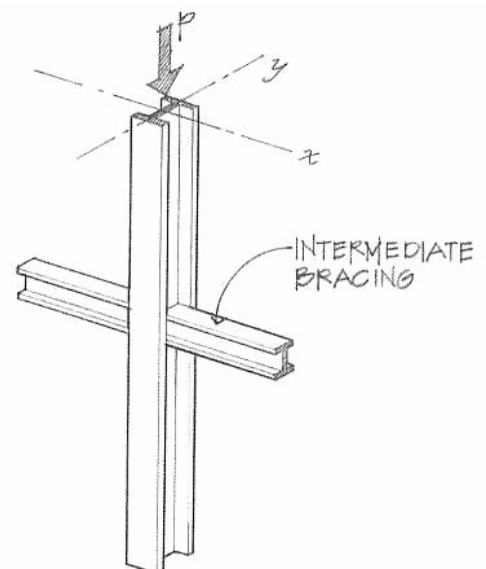
Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K. $L_e = K \cdot L$

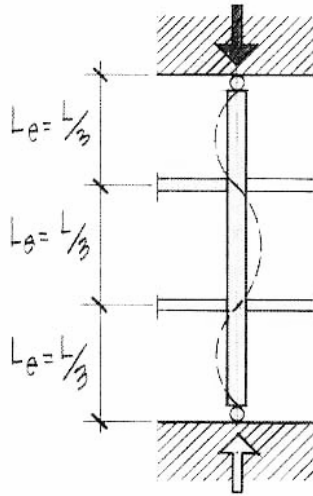


(a) No bracing.

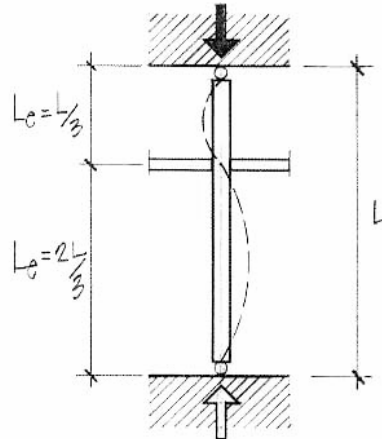


(b) Braced at midpoint.





(c) Third-point bracing.



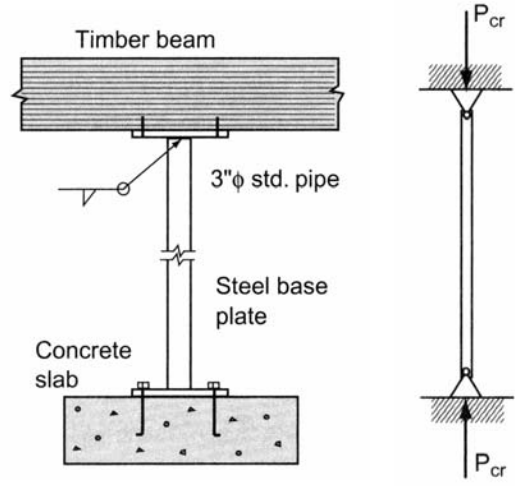
(d) Asymmetric bracing.

Buckled shape of column shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0
End conditions code	Rotation fixed, Translation fixed Rotation free, Translation fixed Rotation fixed, Translation free Rotation free, Translation free					

Example 1 (pg 346)

Example Problem 10.1: Short and Long Columns—Modes of Failure (Figures 10.11 and 10.12)

Determine the critical buckling load for a 3" ϕ standard weight steel pipe column that is 16 ft. tall and pin connected. Assume that $E = 29 \times 10^6$ psi

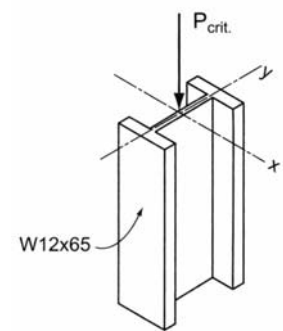


Example 2 (pg 346)

Example Problem 10.2 (Figure 10.13)

Determine the critical buckling stress for a 30-foot-long, W12x65 steel column. Assume simple pin connections at the top and bottom.

$F_y = 36$ ksi (A36 steel); $E = 29 \times 10^3$ ksi



Example 3 (pg357)

Example Problem 10.8 (Figures 10.33 and 10.34a, b)

Determine the buckling load capacity of a 2x4 stud 12 feet high if blocking is provided at midheight. Assume $E = 1.2 \times 10^6$ psi.

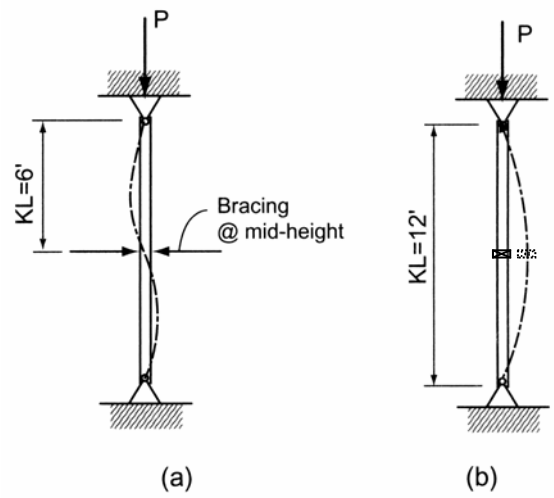
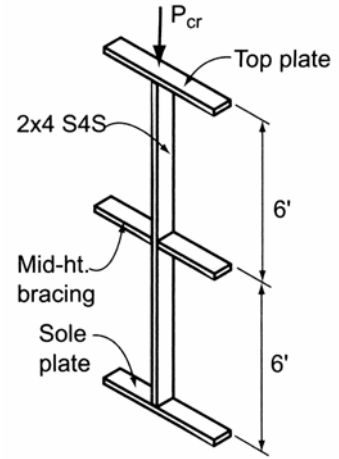


Figure 10.34 (a) Weak axis. (b) Strong axis.