Beam Design and Deflections

Criteria for Design

Allowable normal stress or normal stress from LRFD should not be exceeded:

$$F_b \ or \ \phi F_n \geq f_b = \frac{Mc}{I}$$

Knowing M and F_b, the minimum section modulus fitting the limit is:

$$S_{req'd} \geq \frac{M}{F_b}$$

Besides strength, we also need to be concerned about *serviceability*. This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

Determining Maximum Bending Moment

Drawing V and M diagrams will show us the maximum values for design. Remember:

$$V = \Sigma(-w)dx \qquad \frac{dV}{dx} = -w \qquad \frac{dM}{dx} = V$$
$$M = \Sigma(V)dx$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a *non-prismatic* member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, M(x) across a beam of constant material and cross section then the curvature will change:

$$\frac{1}{R} = \frac{M(x)}{EI}$$

The slope of the n.a. of a beam, θ, will be tangent to the radius of curvature, R:

$$\theta = slope = \frac{1}{EI}\int M(x)dx$$

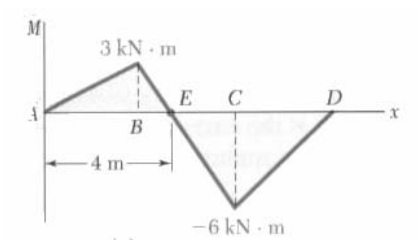
The equation for deflection, y, along a beam is:

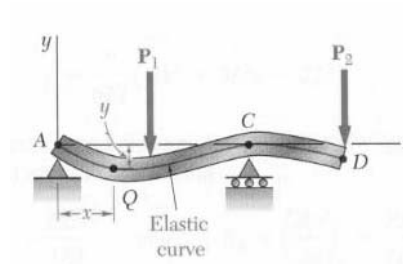
$$y = \frac{1}{EI}\int \theta dx = \frac{1}{EI}\iint M(x)dx$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well. (BigBoy Beam freeware: http://forum.simtel.net/pub/pd/33994.html)

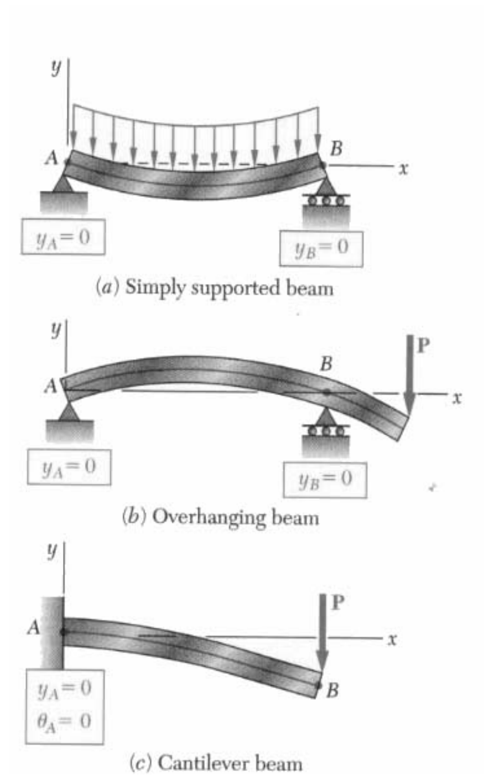
Elastic curve equations can be___________________ONLY if the stresses are in the elastic range.

The deflected shape is roughly the shame shape as the bending moment diagram flipped but is constrained by supports and geometry.





Boundary Conditions



(a) Simply supported beam

(b) Overhanging beam

(c) Cantilever beam

The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.

At Pins, Rollers, Fixed Supports: $y = 0$

At Fixed Supports: $\theta = 0$

At Inflection Points From Symmetry: $\theta = 0$

The Slope Is Zero At The Maximum Deflection y_{max}:.

$$\theta = \frac{dy}{dx} = slope = 0$$

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

$$y_{\max}(x) = \Delta_{actual} \leq \Delta_{allowable} = L/_{value}$$

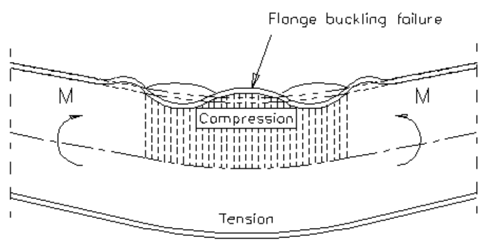
Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

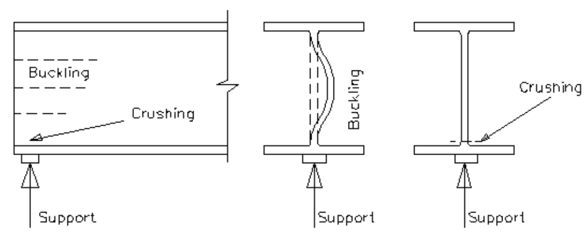
Lateral Buckling

With compression stresses in the top of a beam, a sudden "popping" or _______________ can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger I_y.

Local Buckling in Steel I Beams– Web Crippling or Flange Buckling

Concentrated forces on a steel beam can cause the web to buckle (called __________________). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.





Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

Design Procedure

The intent is to find the most light weight member satisfying the section modulus size.

1. Know F_b (allowable stress) for the material or F_y & F_u for LRFD.
2. Draw V & M, finding M_{max}.
3. Calculate $S_{req'd}$. This step is equivalent to determining $f_b = \frac{M_{max}}{S} \leq F_b$
4. For rectangular beams $S = \frac{bh^2}{6}$
   - For steel or timber: use the section charts to find S that will work *and remember that the beam self weight will increase $S_{req'd.}$* And for steel, the design charts show the lightest section within a grouping of similar S's.
   - For any thing else, try a nice value for b, and calculate h or the other way around.

*****Determine the "updated" V_{max} and M_{max} including the beam self weight, and verify that the updated $S_{req'd}$ has been met.*******

5. Consider lateral stability

6. Evaluate horizontal shear stresses using V_{max} to determine if $f_v \le F_v$

   For I and rectangular beams $$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

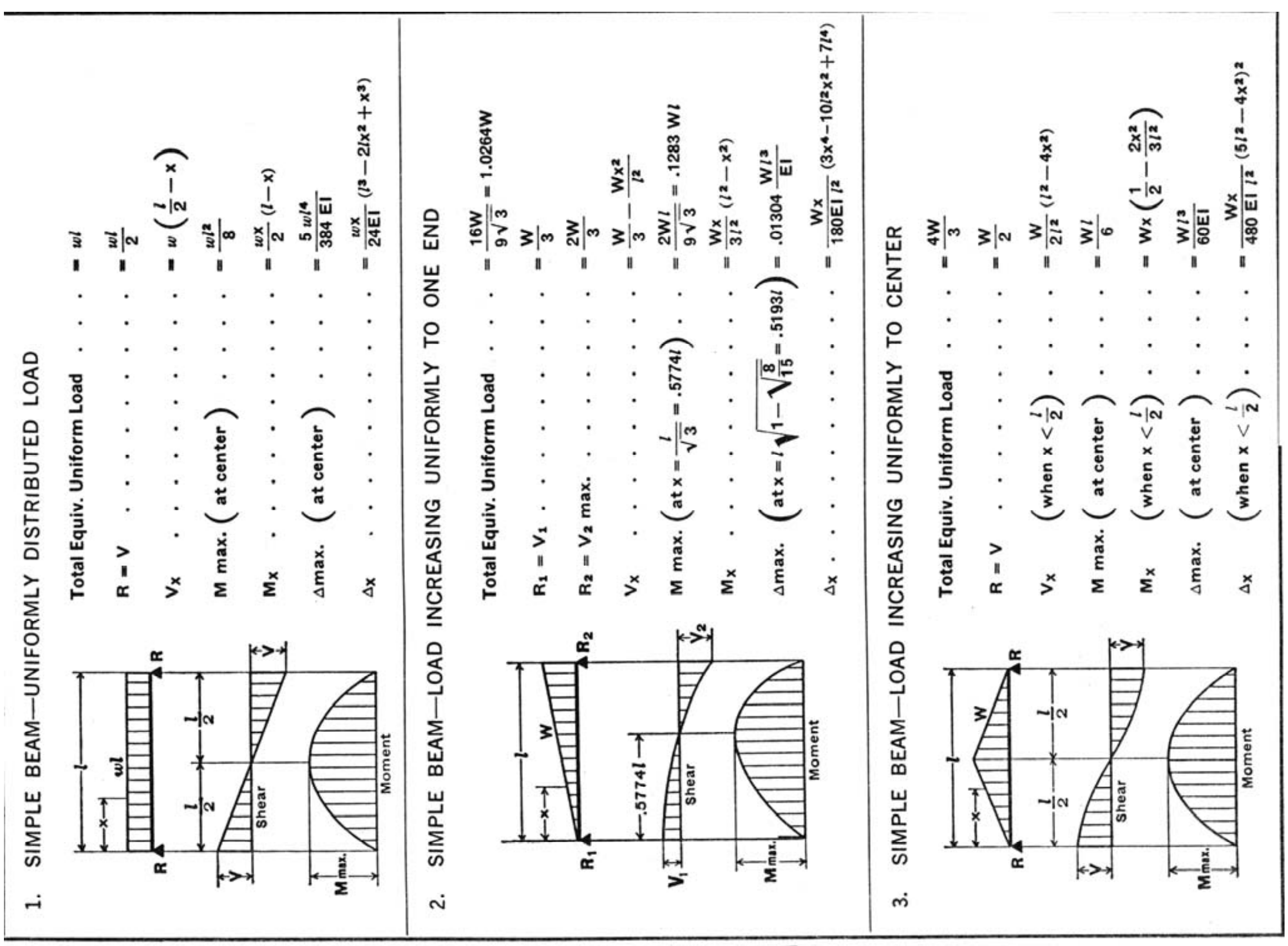
7. Provide adequate bearing area at supports: $$f_p = \frac{P}{A} \le F_p$$

8. Evaluate shear due to torsion $$f_v = \frac{T\rho}{J} \text{ or } \frac{T}{c_1 ab^2} \le F_v$$
   (circular section or rectangular)

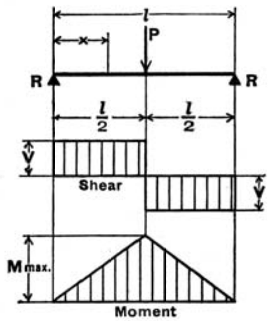
9. Evaluate the deflection to determine if $\Delta_{max\,LL} \le \Delta_{LL-allowed}$ and/or $\Delta_{max\,Total} \le \Delta_{T-allowed}$

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

BEAM DIAGRAMS AND FORMULAS
For Various Static Loading Conditions, AISC ASD 8th ed.

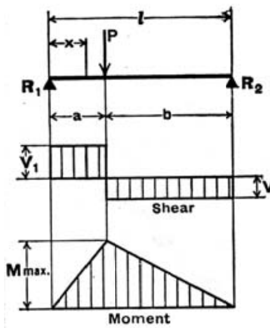


7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



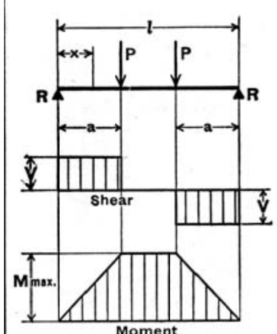
Total Equiv. Uniform Load		$= 2P$
$R = V$		$= \frac{P}{2}$
M max.	(at point of load)	$= \frac{Pl}{4}$
M_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{2}$
Δmax.	(at point of load)	$= \frac{Pl^3}{48EI}$
Δ_x	(when $x < \frac{l}{2}$)	$= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



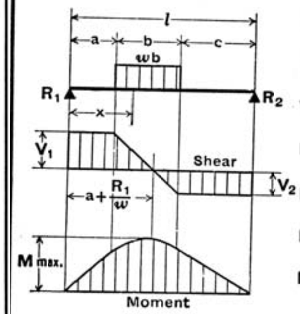
Total Equiv. Uniform Load		$= \frac{8Pab}{l^2}$
$R_1 = V_1$	(max. when $a < b$)	$= \frac{Pb}{l}$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{Pa}{l}$
M max.	(at point of load)	$= \frac{Pab}{l}$
M_x	(when $x < a$)	$= \frac{Pbx}{l}$
Δmax.	(at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)	$= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$
Δa	(at point of load)	$= \frac{Pa^2b^2}{3EIl}$
Δ_x	(when $x < a$)	$= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



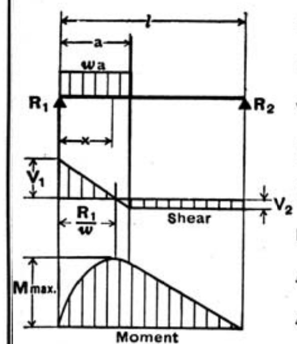
Total Equiv. Uniform Load		$= \frac{8Pa}{l}$
$R = V$		$= P$
M max.	(between loads)	$= Pa$
M_x	(when $x < a$)	$= Px$
Δmax.	(at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_x	(when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x	(when $x > a$ and $< (l - a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



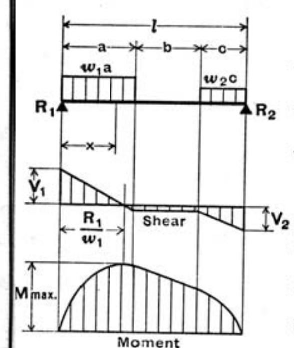
$R_1 = V_1$	(max. when $a < c$)	$= \frac{wb}{2l}(2c + b)$
$R_2 = V_2$	(max. when $a > c$)	$= \frac{wb}{2l}(2a + b)$
V_x	(when $x > a$ and $< (a + b)$)	$= R_1 - w(x - a)$
M max.	(at $x = a + \frac{R_1}{w}$)	$= R_1\left(a + \frac{R_1}{2w}\right)$
M_x	(when $x < a$)	$= R_1x$
M_x	(when $x > a$ and $< (a + b)$)	$= R_1x - \frac{w}{2}(x - a)^2$
M_x	(when $x > (a + b)$)	$= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



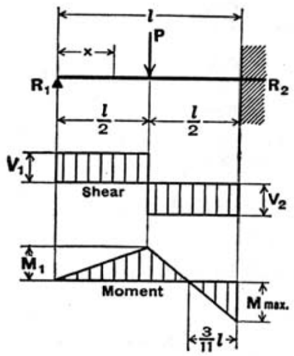
$R_1 = V_1$ max.		$= \frac{wa}{2l}(2l - a)$
$R_2 = V_2$		$= \frac{wa^2}{2l}$
V_x	(when $x < a$)	$= R_1 - wx$
M max.	(at $x = \frac{R_1}{w}$)	$= \frac{R_1^2}{2w}$
M_x	(when $x < a$)	$= R_1x - \frac{wx^2}{2}$
M_x	(when $x > a$)	$= R_2(l - x)$
Δ_x	(when $x < a$)	$= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$
Δ_x	(when $x > a$)	$= \frac{wa^2(l-x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



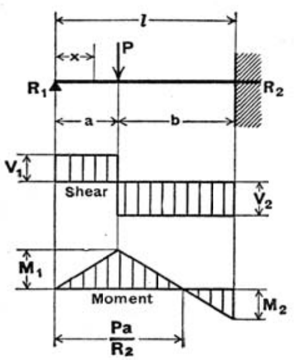
$R_1 = V_1$		$= \frac{w_1a(2l - a) + w_2c^2}{2l}$
$R_2 = V_2$		$= \frac{w_2c(2l - c) + w_1a^2}{2l}$
V_x	(when $x < a$)	$= R_1 - w_1x$
V_x	(when $x > a$ and $< (a + b)$)	$= R_1 - w_1a$
V_x	(when $x > (a + b)$)	$= R_2 - w_2(l - x)$
M max.	(at $x = \frac{R_1}{w_1}$ when $R_1 < w_1a$)	$= \frac{R_1^2}{2w_1}$
M max.	(at $x = l - \frac{R_2}{w_2}$ when $R_2 < w_2c$)	$= \frac{R_2^2}{2w_2}$
M_x	(when $x < a$)	$= R_1x - \frac{w_1x^2}{2}$
M_x	(when $x > a$ and $< (a + b)$)	$= R_1x - \frac{w_1a}{2}(2x - a)$
M_x	(when $x > (a + b)$)	$= R_2(l - x) - \frac{w_2(l - x)^2}{2}$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT CENTER



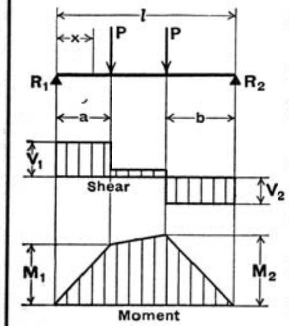
Total Equiv. Uniform Load		$= \frac{3P}{2}$
$R_1 = V_1$		$= \frac{5P}{16}$
$R_2 = V_2$ max.		$= \frac{11P}{16}$
M max.	(at fixed end)	$= \frac{3Pl}{16}$
M_1	(at point of load)	$= \frac{5Pl}{32}$
M_x	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{5Px}{16}$
M_x	$\left(\text{when } x > \frac{l}{2}\right)$	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax.	$\left(\text{at } x = l\sqrt{\frac{1}{5}} = .4472l\right)$	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δx	(at point of load)	$= \frac{7Pl^3}{768EI}$
Δx	$\left(\text{when } x < \frac{l}{2}\right)$	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δx	$\left(\text{when } x > \frac{l}{2}\right)$	$= \frac{P}{96EI}(x-l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT ANY POINT



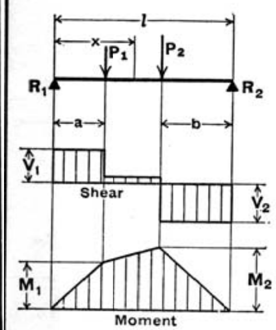
$R_1 = V_1$		$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$		$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1	(at point of load)	$= R_1a$
M_2	(at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x	(when $x < a$)	$= R_1x$
M_x	(when $x > a$)	$= R_1x - P(x - a)$
Δmax.	$\left(\text{when } a < .414l \text{ at } x = l\frac{l^2+a^2}{3l^2-a^2}\right)$	$= \frac{Pa}{3EI}\frac{(l^2-a^2)^3}{(3l^2-a^2)^2}$
Δmax.	$\left(\text{when } a > .414l \text{ at } x = l\sqrt{\frac{a}{2l+a}}\right)$	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l+a}}$
Δa	(at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δx	(when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δx	(when $x > a$)	$= \frac{Pa}{12EIl^3}(l-x)^2(3l^2x - a^2x - 2a^2l)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



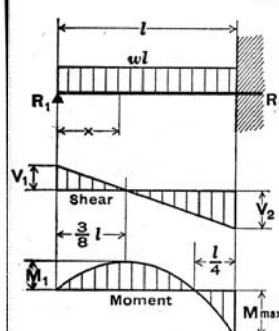
$R_1 = V_1$	(max. when $a < b$)	$= \frac{P}{l}(l - a + b)$
$R_2 = V_2$	(max. when $a > b$)	$= \frac{P}{l}(l - b + a)$
V_x	(when $x > a$ and $< (l-b)$)	$= \frac{P}{l}(b - a)$
M_1	(max. when $a > b$)	$= R_1a$
M_2	(max. when $a < b$)	$= R_2b$
M_x	(when $x < a$)	$= R_1x$
M_x	(when $x > a$ and $< (l-b)$)	$= R_1x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



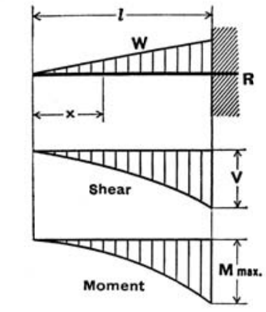
$R_1 = V_1$		$= \frac{P_1(l-a) + P_2b}{l}$
$R_2 = V_2$		$= \frac{P_1a + P_2(l-b)}{l}$
V_x	(when $x > a$ and $< (l-b)$)	$= R_1 - P_1$
M_1	(max. when $R_1 < P_1$)	$= R_1a$
M_2	(max. when $R_2 < P_2$)	$= R_2b$
M_x	(when $x < a$)	$= R_1x$
M_x	(when $x > a$ and $< (l-b)$)	$= R_1x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— UNIFORMLY DISTRIBUTED LOAD



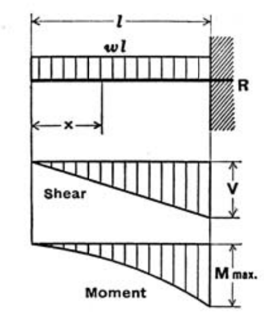
Total Equiv. Uniform Load		$= wl$
$R_1 = V_1$		$= \frac{3wl}{8}$
$R_2 = V_2$ max.		$= \frac{5wl}{8}$
V_x		$= R_1 - wx$
M max.		$= \frac{wl^2}{8}$
M_1	$\left(\text{at } x = \frac{3}{8}l\right)$	$= \frac{9}{128}wl^2$
M_x		$= R_1x - \frac{wx^2}{2}$
Δmax.	$\left(\text{at } x = \frac{l}{16}(1 + \sqrt{33}) = .4215l\right)$	$= \frac{wl^4}{185EI}$
Δx		$= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



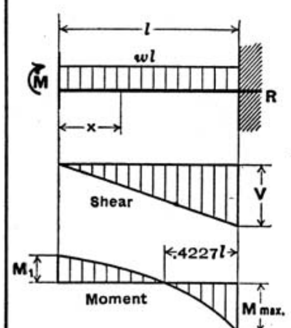
Total Equiv. Uniform Load	$= \frac{8}{3} W$
$R = V$	$= W$
V_x	$= W \frac{x^2}{l^2}$
M max. (at fixed end)	$= \frac{Wl}{3}$
M_x	$= \frac{Wx^3}{3l^2}$
Δmax. (at free end)	$= \frac{Wl^3}{15EI}$
Δx	$= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



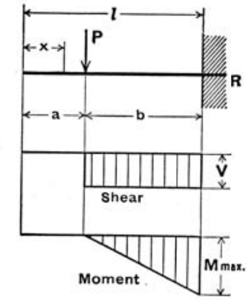
Total Equiv. Uniform Load	$= 4wl$
$R = V$	$= wl$
V_x	$= wx$
M max. (at fixed end)	$= \frac{wl^2}{2}$
M_x	$= \frac{wx^2}{2}$
Δmax. (at free end)	$= \frac{wl^4}{8EI}$
Δx	$= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



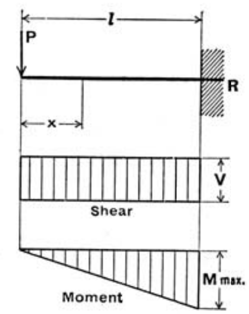
Total Equiv. Uniform Load	$= \frac{8}{3} wl$
$R = V$	$= wl$
V_x	$= wx$
M max. (at fixed end)	$= \frac{wl^2}{3}$
M_1 (at deflected end)	$= \frac{wl^2}{6}$
M_x	$= \frac{w}{6}(l^2 - 3x^2)$
Δmax. (at deflected end)	$= \frac{wl^4}{24EI}$
Δx	$= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



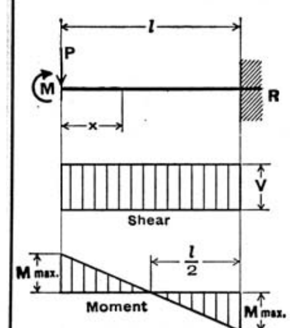
Total Equiv. Uniform Load	$= \frac{8Pb}{l}$
$R = V$	$= P$
M max. (at fixed end)	$= Pb$
M_x (when $x > a$)	$= P(x - a)$
Δmax. (at free end)	$= \frac{Pb^2}{6EI}(3l - b)$
Δa (at point of load)	$= \frac{Pb^3}{3EI}$
Δx (when $x < a$)	$= \frac{Pb^2}{6EI}(3l - 3x - b)$
Δx (when $x > a$)	$= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load	$= 8P$
$R = V$	$= P$
M max. (at fixed end)	$= Pl$
M_x	$= Px$
Δmax. (at free end)	$= \frac{Pl^3}{3EI}$
Δx	$= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

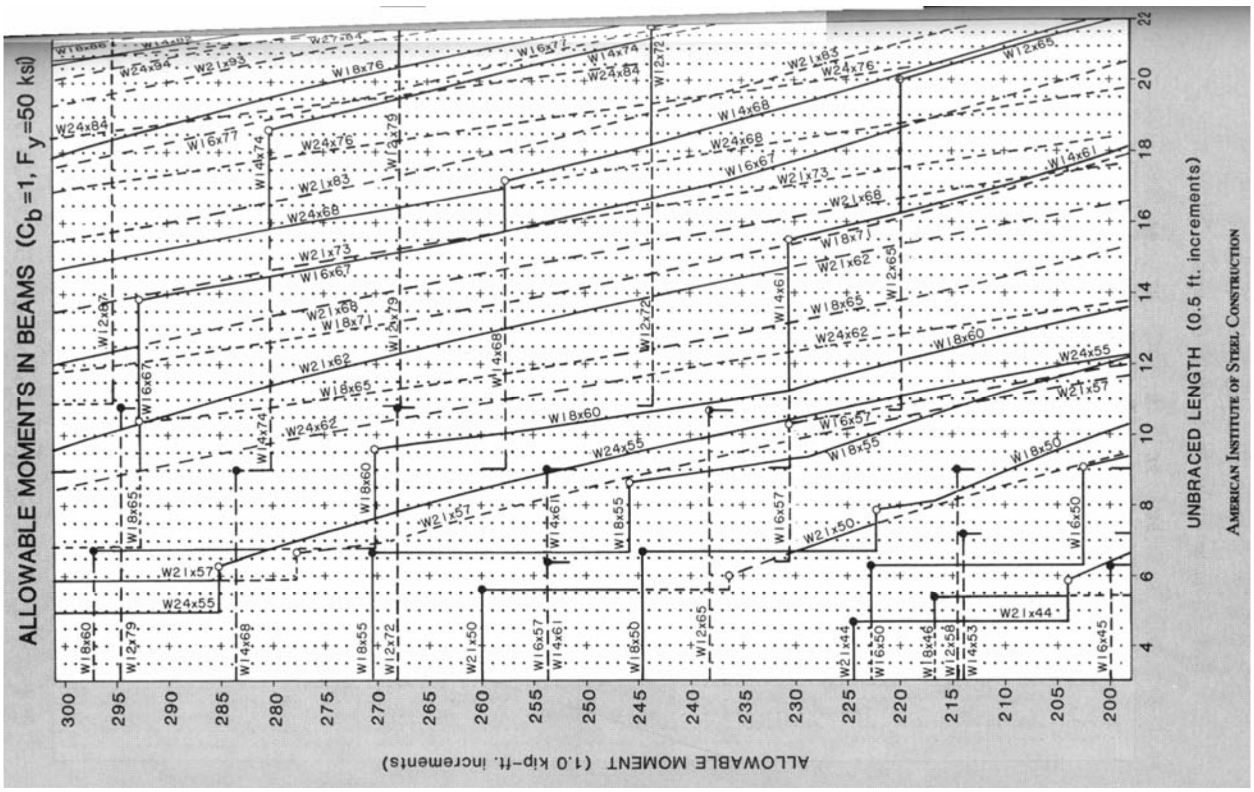
23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load	$= 4P$
$R = V$	$= P$
M max. (at both ends)	$= \frac{Pl}{2}$
M_x	$= P\left(\frac{l}{2} - x\right)$
Δmax. (at deflected end)	$= \frac{Pl^3}{12EI}$
Δx	$= \frac{P(l - x)^2}{12EI}(l + 2x)$

Allowable Moments in Beams with Unbraced Lengths

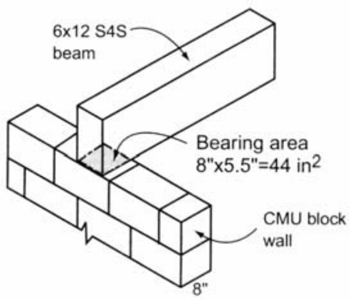
Allowable stresses are reduced when the unbraced length of the compression flange can buckle called L_c. The limiting unbraced length at the lower stresses is called L_u. The maximum moment that can be applied (taking self weight into account) can be plotted against the unbraced length. The limit L_c is indicated by a solid dot (•), while L_u is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used. C_b, which is a modification factor for non-zero moments at the ends, is 1 for simply supported beams (0 moments at the ends).

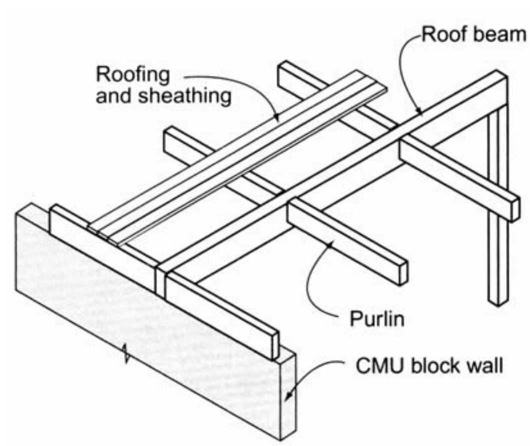


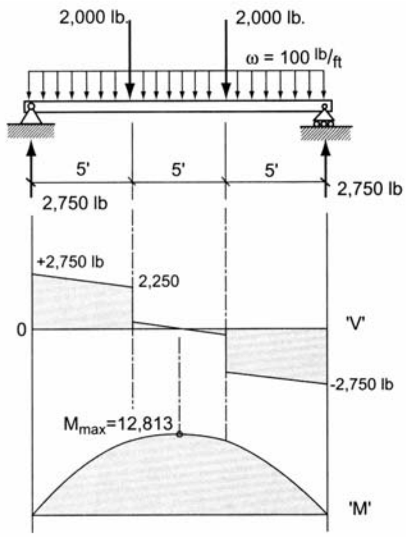
Example 1 (pg 328)

Example Problem 9.15 (Figures 9.73 to 9.75)

Design a Southern pine No. 1 beam to carry the loads shown (roof beam, no plaster). Assume the beam is supported at each end by an 8" block wall. F_b = 1550 psi; F_v = 110 psi; $E = 1.6 \times 10^6$ psi.





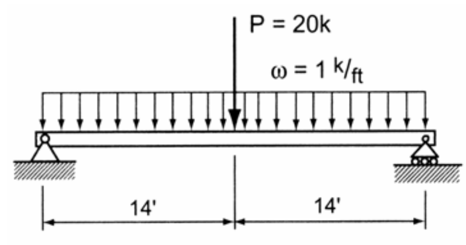

2,000 lb.
2,000 lb.
ω = 100 lb/ft
5'
5'
5'
2,750 lb
2,750 lb
+2,750 lb
2,250
0
'V'
-2,750 lb
Mmax=12,813
'M'

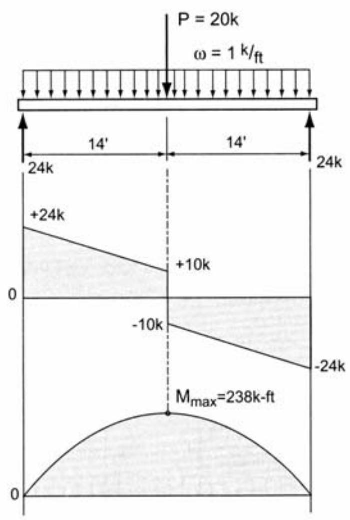
<u>Example 2 (pg 330)</u>

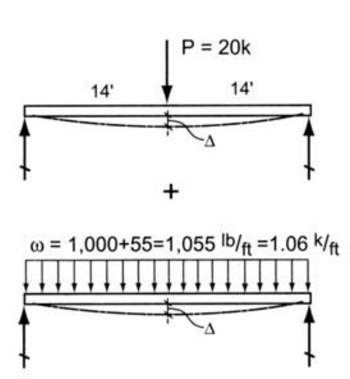
Example Problem 9.16 (Figures 9.76 to 9.78)

A steel beam (A572/50) is loaded as shown. Assuming a deflection requirement of $\Delta_{total} = L/240$ and a depth restriction of 18" nominal, select the most economical section.

$F_b = 30$ ksi; $F_v = 20$ ksi; $E = 30\times10^3$ ksi







Beam Design Flow Chart