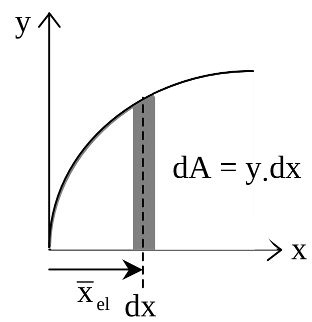
Moments of Inertia

- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.

- *Definition:* Moment of Inertia; the second area moment



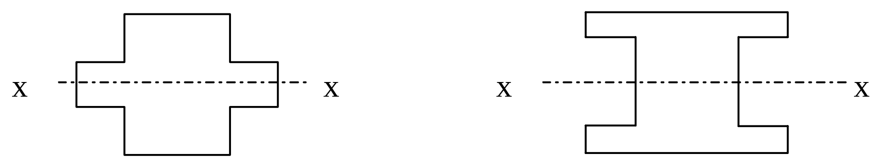
$$I_y = \int x^2 dA \qquad I_x = \int y^2 dA$$

We can define a single integral using a narrow strip:

for I_x,, strip is parallel to x for I_y, strip is parallel to y

**I can be negative if the area is negative (a hole or subtraction).*

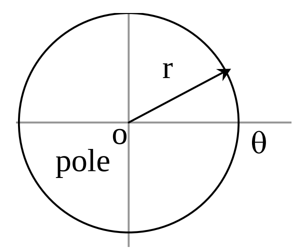
- A shape that has area at a greater distance away from an axis *through its centroid* will have a _______________ value of I.



- Just like for center of gravity of an area, the moment of inertia can be determined with respect to *any* reference____________.

- *Definition:* Polar Moment of Inertia; the second area moment using polar coordinate axes

$$J_o = \int r^2 dA = \int x^2 dA + \int y^2 dA$$
$$J_o = I_x + I_y$$



- *Definition*: Radius of Gyration; the distance from the moment of inertia axis for an area at which the entire area could be considered as being concentrated at.

$$I_x = r_x^{\,2} A \Rightarrow r_x = \sqrt{\frac{I_x}{A}}$$ radius of gyration in x

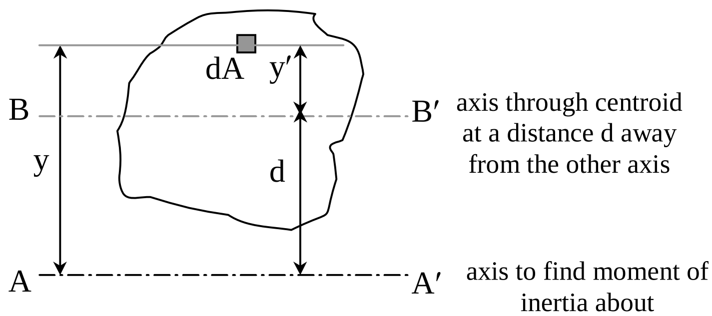
$$r_y = \sqrt{\frac{I_y}{A}}$$ radius of gyration in y

$$r_o = \sqrt{\frac{J_o}{A}}$$ polar radius of gyration, and $r_o^2 = r_x^2 + r_y^2$

The Parallel-Axis Theorem

- *The moment of inertia of an area with respect to any axis not through its centroid is equal to the moment of inertia of that area with respect to its own parallel centroidal axis plus the product of the area and the square of the distance between the two axes.*

$$I = \int y^2 dA = \int (y'\text{-}d)^2 dA$$
$$= \int y'^2 dA + 2d\int y' dA + d^2 \int dA$$



but $\int y' dA = 0$, because the centroid is on this axis, resulting in:

$I_x = I_{cx} + Ad_y^{\,2}$ (text notation) *or* $I_x = \bar{I}_x + Ad_y^{\,2}$

where I_{cx} (*or* $\bar{I}_x$)is the moment of inertia about the centroid of the area about an *x* axis and d_y is the y distance between the parallel axes

Similarly	$I_y = \bar{I}_y + Ad_x^{\,2}$	Moment of inertia about a *y* axis
	$J_o = \bar{J}_c + Ad^2$	Polar moment of Inertia
	$r_o^2 = \bar{r}_c^2 + d^2$	Polar radius of gyration
	$r^2 = \bar{r}^2 + d^2$	Radius of gyration

* *I can be negative again if the area is negative (a hole or subtraction).*
** *If* $\bar{I}$ is *not given in a chart, but* $\bar{x}$ & $\bar{y}$ *are: YOU MUST CALCULATE* $\bar{I}$ *WITH* $\bar{I} = I - Ad^2$

Composite Areas:

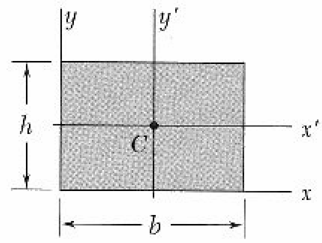
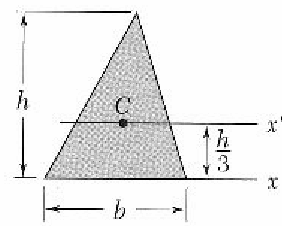
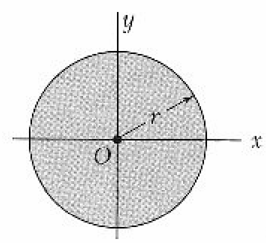
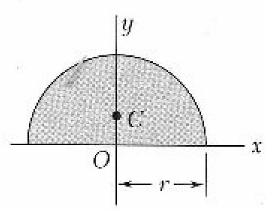
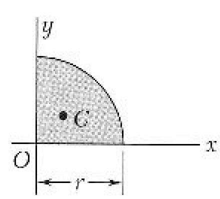
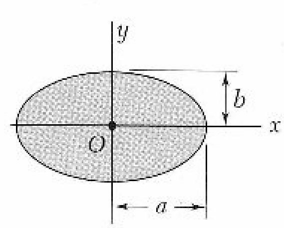
$I = \sum \bar{I} + \sum Ad^2$ where $\bar{I}$ is the moment of inertia about the centroid of the component area
d is the distance from the centroid of the component area to the centroid of the composite area (ie. $d_y = \hat{y} - \bar{y}$)

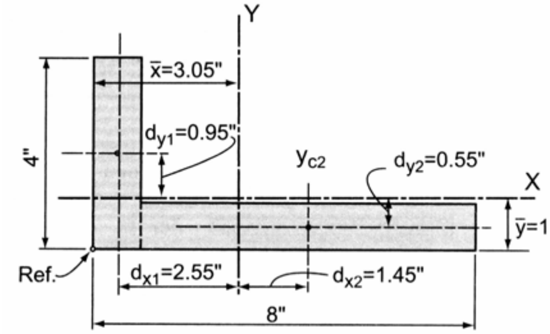
Basic Steps

1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of

   Component, Area, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}_x$, d_y, Ad_y^2, $\bar{I}_y$, d_x, Ad_x^2
5. Fill in the table values needed to calculate $\hat{x}$ and $\hat{y}$ for the composite
6. Fill in the rest of the table values.
7. Sum the moment of inertia ($\bar{I}$’s) and Ad^2 columns and add together.

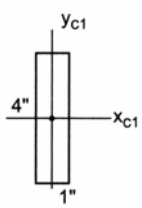
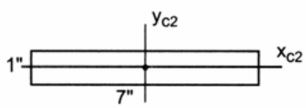
Moments of Inertia of Common Shapes

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ about centroid $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$



Example 1 (pg 257)

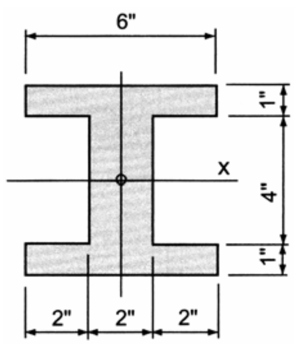
Find the moments of inertia ($\hat{x}$ = 3.05", $\hat{y}$ = 1.05").

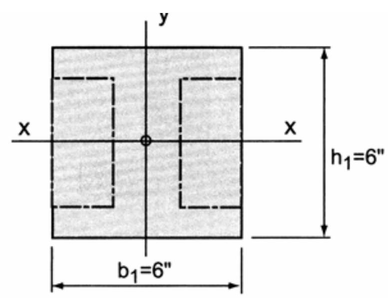
Component	I_{xc} (in.^{4})	d_y (in.)	Ad_y^2 (in.^{4})	I_{yc} (in.^{4})	d_x (in.)	Ad_x^2 (in.^{4})
	$\frac{(1)(4)^3}{12} = 5.33$	0.95	3.61	$\frac{(4)(1)^3}{12} = 0.33$	2.55	26.01
	$\frac{(7)(1)^3}{12} = 0.58$	0.55	2.12	$\frac{(1)(7)^3}{12} = 28.58$	1.45	14.72
	$\sum I_{xc} = 5.91$		$\sum Ad_y^2 = 5.73$	$\sum I_{yc} = 28.91$		$\sum Ad_x = 40.73$

Example 2 (pg 253)

Example Problem 7.6 (Figures 7.24 to 7.26)

Determine the I about the centroidal x-axis.





Example 3 (pg 258)

Example Problem 7.10 (Figures 7.35 and 7.36)

Locate the centroidal x and y axes for the cross-section shown. Use the reference origin indicated and assume that the steel plate is centered over the flange of the wide-flange section. Compute the I_x and I_y about the major centroidal axes.