ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

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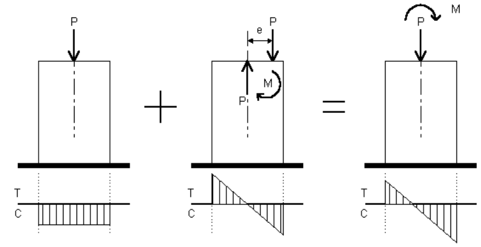
SUMMER 2006

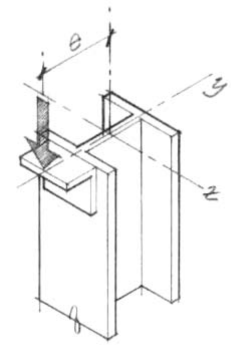
lecture
twenty two

eccentric loading: beam-columns

Centric & Eccentric Loading

- centric
  - allowable stress from strength or buckling
- eccentric
  - combined stresses





Eccentric Loading

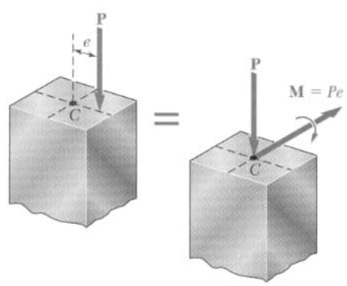
- axial + bending

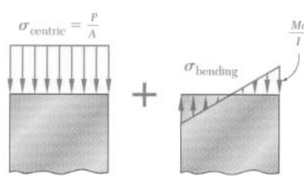
$$f_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$M = P \cdot e$$

- design

$$f_{\max} \le F_{cr} = \frac{f_{cr}}{F.S.}$$



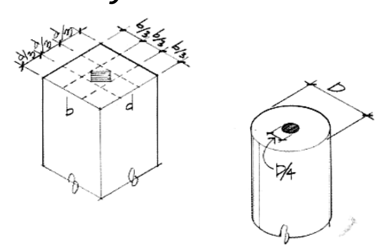


Eccentric Loading

- find e such that the minimum stress = 0

$$f_{\min} = \frac{P}{A} - \frac{(Pe)c}{I} = 0$$

- area defined by e from centroid is the <u>kern</u>



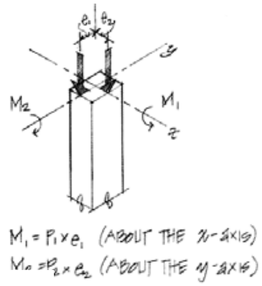
Eccentric Loading

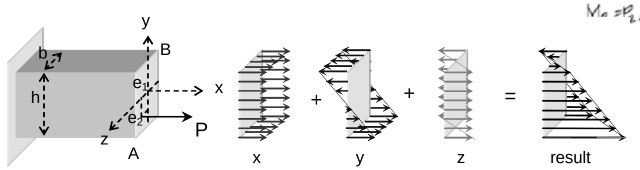
– when there is eccentricity in two directions

$$M_1 = P \cdot e_1 \qquad M_2 = P \cdot e_2$$

$$f_{\max} = \frac{P}{A} + \frac{M_1 y}{I} + \frac{M_2 z}{I}$$

– biaxial bending





$$M_2 = P \cdot e_2 \qquad M_1 = P \cdot e_1$$

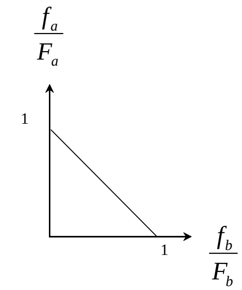
Stress Limit Conditions

– ASD interaction formula

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.0$$

– with biaxial bending

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$



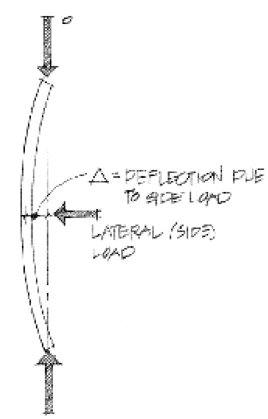
interaction diagram

Stress Limit Conditions

– in reality, as the column flexes, the moment increases

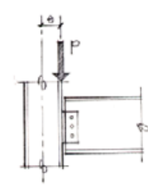
– P-Δ effect

$$\frac{f_a}{F_a} + \frac{f_b \times (Magnification\ \ factor)}{F_{bx}} \le 1.0$$

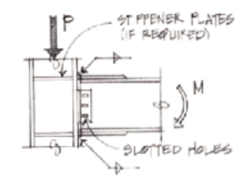


Design

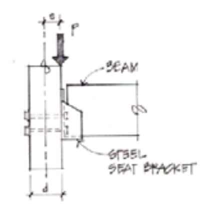
- satisfy
  - strength
  - stability
- pick
  - section



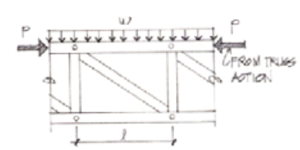
(a) Framed beam (shear) connection.
e = Eccentricity; M = P × e



(b) Moment connection (rigid frame).
M = Moment due to beam bending



(c) Timber beam–column connection.
e = d/2 = eccentricity; M = P × e



(d) Upper chord of a truss—compression plus bending.
$M = \frac{\omega l^2}{8}$

Design

- ASD Steel

$$\frac{f_a}{F_a}+\frac{C_{mx}f_{bx}}{\left(1-\frac{f_a}{F'_{ex}}\right)F_{bx}}+\frac{C_{my}f_{by}}{\left(1-\frac{f_a}{F'_{ey}}\right)F_{by}}\le 1.0$$

C_m – modification factor for end conditions

= 0.6 – 0.4(M_1/M_2) or 0.85 restrained

F'_e – allowable buckling strength

() term – magnification factor for P-Δ

Design

- Wood

$$\left[\frac{f_c}{F'_c}\right]^2+\frac{f_{bx}}{F'_{bx}\left[1-{f_c}/{F_{cEx}}\right]}\le 1.0$$

() term – magnification factor for P-Δ

F'_{bx} – allowable bending strength

Design

- LRFD Steel
  - for $\frac{P_u}{\phi_c P_n}\ge 0.2$: $\frac{P_u}{\phi_c P_n}+\frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}}+\frac{M_{uy}}{\phi_b M_{ny}}\right)\le 1.0$
  - for $\frac{P_u}{\phi_c P_n}< 0.2$: $\frac{P_u}{2\phi_c P_n}+\left(\frac{M_{ux}}{\phi_b M_{nx}}+\frac{M_{uy}}{\phi_b M_{ny}}\right)\le 1.0$

ϕ_c - resistance factor for compression = 0.85

ϕ_b - resistance factor for bending = 0.9

Design Steps Knowing Loads

1. assume limiting stress
   - buckling, axial stress, combined stress
2. solve for r, A or S
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok