lecture
eighteen

beams:
deflection & design
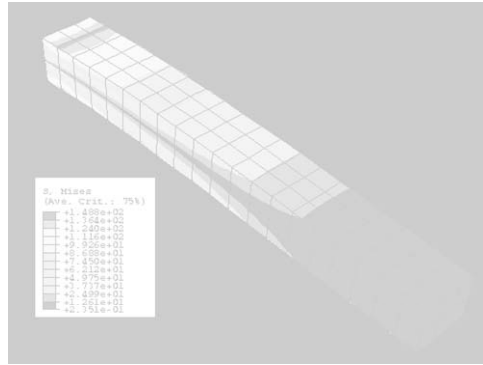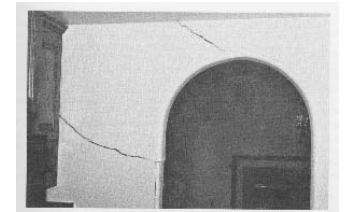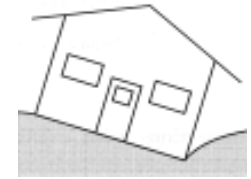
Design for Strength +...

- *strength design*
  - *forces & material*
- *serviceability*
  - *limit deflection and cracking*
  - *control noise & vibration*
  - *no excessive settlement of foundations*
  - *durability*
  - *appearance*
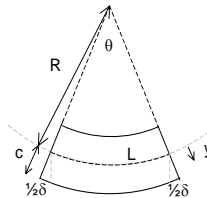 - *component damage*
  - *ponding*

Beam Deformations

- *curvature relates to*
  - *bending moment*
  - *modulus of elasticity*
  - *moment of inertia*

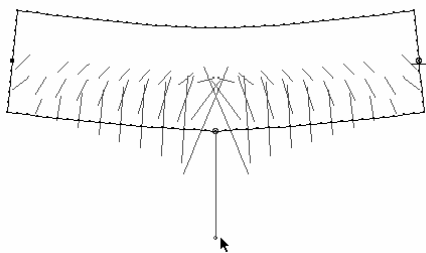$$\frac{1}{R} = \frac{M}{EI}$$

$$curvature = \frac{M(x)}{EI}$$

$$\theta = slope = \int \frac{M(x)}{EI} dx$$
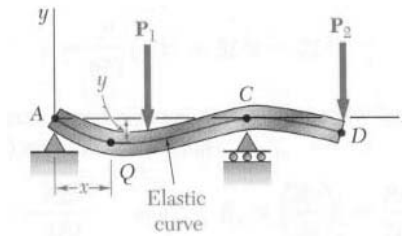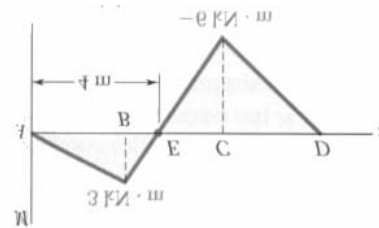
$$\Delta = deflection = \iint \frac{M(x)}{EI} dx$$

Deflected Shape & M(x)

- *-M(x) gives shape indication*
- *boundary conditions must be met*

1

Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$

- at fixed supports: $\theta = 0$

- at inflection points from symmetry: $\theta = 0$
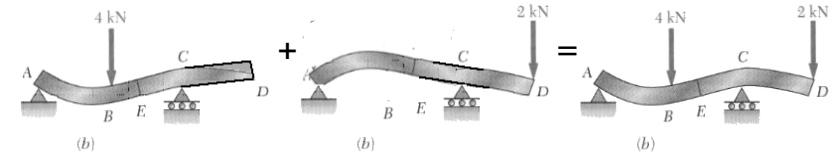
- y_{max} at $\dfrac{dy}{dx} = 0$
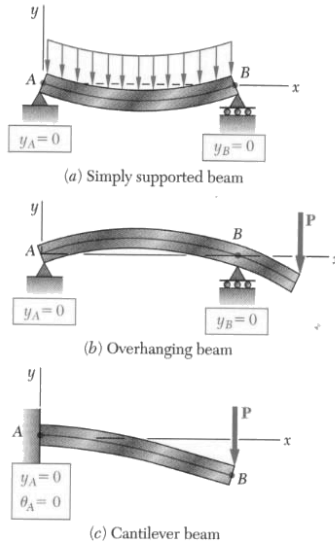
Superpositioning

- if w can be <u>superpositioned</u>
  - θ & y can
  - elastic range only!

Deflection Limits

- based on service condition, severity

Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

Lateral Buckling

- lateral <u>buckling</u> caused by compressive forces at top coupled with insufficient rigidity

- can occur at low stress levels
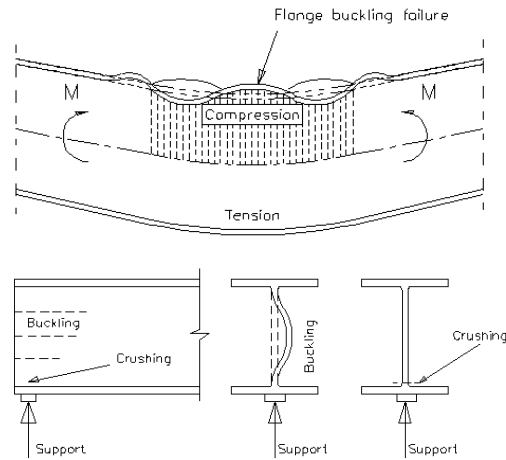
- stiffen, brace or bigger I_y
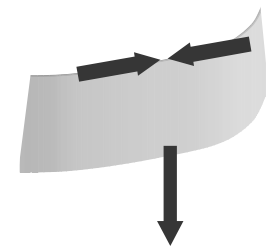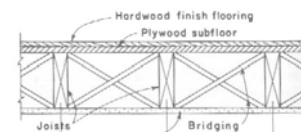
2

Local Buckling

- *steel I beams*
- *flange*
  - *buckle in direction of smaller radius of gyration*
- *web*
  - *force*
  - *"crippling"*

Local Buckling

- *web*
- *flange*

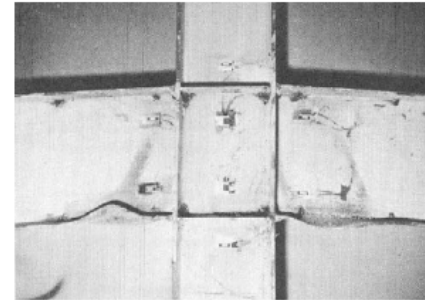Figure 2-5. Flange Local Bending Limit State
(Beedle, L.S., Christopher, R., 1964)

Figure 2-7. Web Local Buckling Limit State
(SAC Project)

Shear in Web
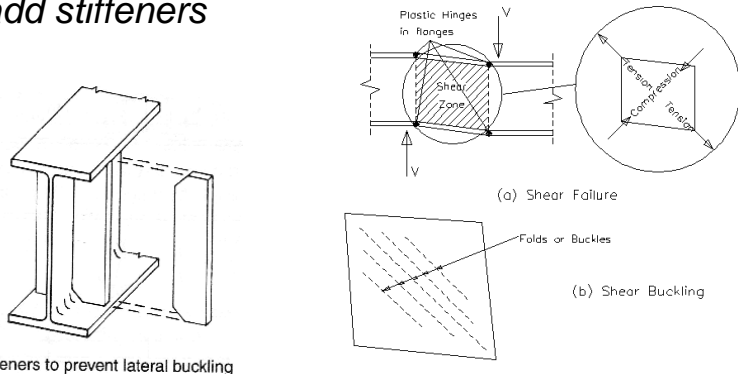
- *panels in plate girders or webs with large shear*
- *buckling in compression direction*
- *add stiffeners*

stiffeners to prevent lateral buckling

(a) Shear Failure

(b) Shear Buckling

Shear in Web

- *plate girders and stiffeners*

Beam Design

1. Know F_{all} for the material or F_U for LRFD

2. Draw V & M, finding M_{max}

3. Calculate $S_{req'd}$ $\left(f_b \leq F_b\right)$

4. Determine section size

$$S = \frac{bh^2}{6}$$

Beam Design

4*. Include self weight for M_{max}
   –   and repeat 3 & 4 if necessary

5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper

Beam Design

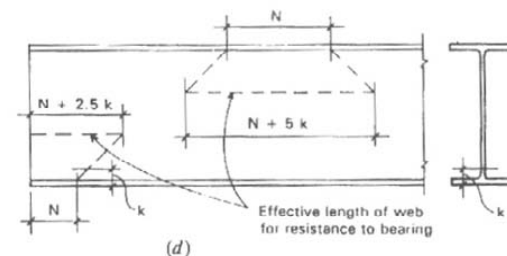6. Evaluate shear stresses - horizontal
   $$\left(f_v \leq F_v\right)$$
   • W and rectangles $$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

   • thin walled sections $$f_{v-\max} = \frac{VQ}{Ib}$$

Beam Design

7. Provide adequate bearing area at supports

$$f_p = \frac{P}{A} \leq F_p$$

N

N + 2.5 k

N + 5 k

N

k

Effective length of web for resistance to bearing

(d)

Beam Design

8. Evaluate torsion

$$\left(f_v \le F_v\right)$$

- circular cross section

$$f_v = \frac{T\rho}{J}$$
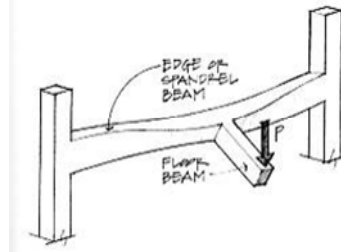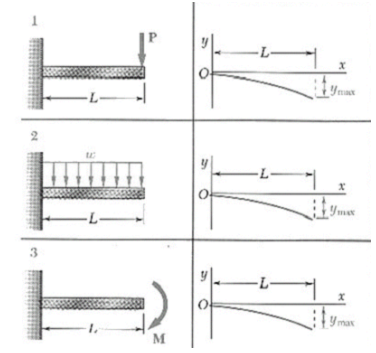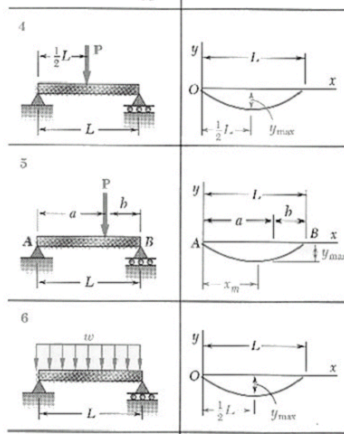
- rectangular

$$f_v = \frac{T}{c_1 ab^2}$$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

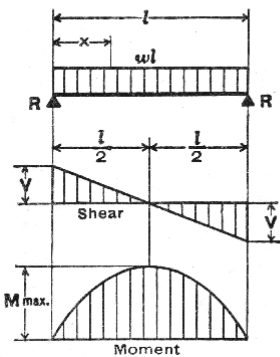| a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Beam Design

9. Evaluate deflections

$$y_{\max}(x) = \Delta_{actual} \le \Delta_{allowable}$$

Beam Design

9. – how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load . . . = wl

$R = V$ . . . . $= \dfrac{wl}{2}$

V_x . . . . $= w\left(\dfrac{l}{2} - x\right)$

M max. (at center) . . . . $= \dfrac{wl^2}{8}$

M_x . . . . $= \dfrac{wx}{2}(l-x)$

Δmax. (at center) . . . . $= \dfrac{5\,wl^4}{384\,EI}$

Δ_x . . . . $= \dfrac{wx}{24EI}(l^3 - 2lx^2 + x^3)$