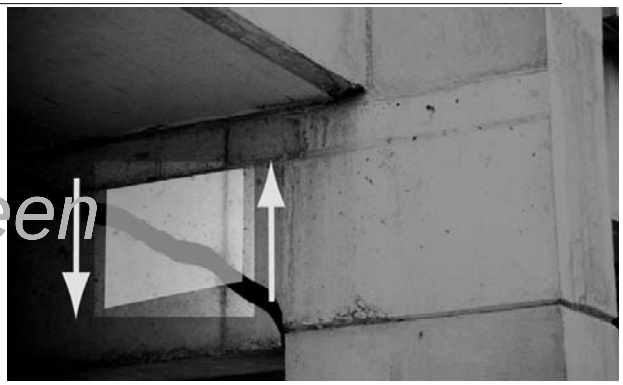
ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231

DR. ANNE NICHOLS

SUMMER 2006

lecture
seventeen

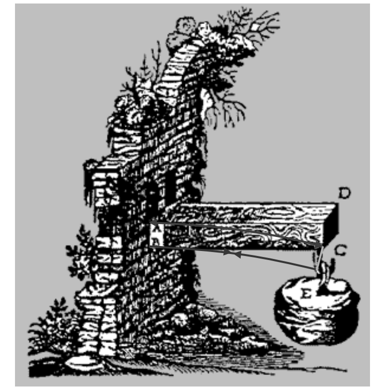


Copyright © Kirk Martini

beams: bending and shear

Beam Bending

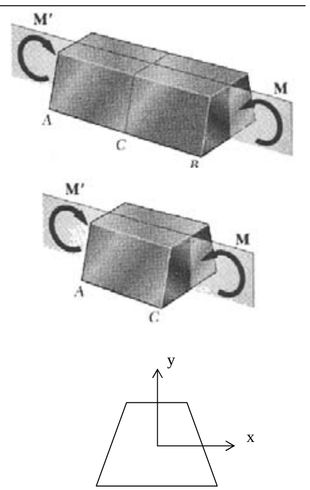
- Galileo
  - relationship between stress and depth^2
- can see
  - top squishing
  - bottom stretching



- what are the stress across the section?

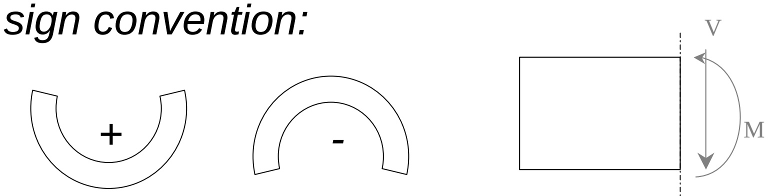
Pure Bending

- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke's law



Bending Moments

- sign convention:



- size of maximum internal moment will govern our design of the section

Normal Stresses

- *geometric fit*
  - *plane sections remain plane*
  - *stress varies linearly*

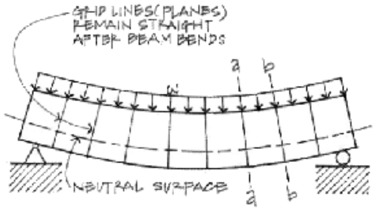


Figure 8.5(b) Beam bending under load.

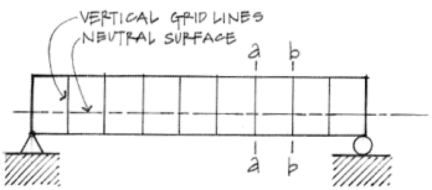
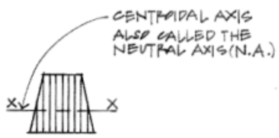


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

Neutral Axis

- *stresses vary linearly*
- *zero stress occurs at the centroid*
- *neutral axis is line of centroids (n.a.)*

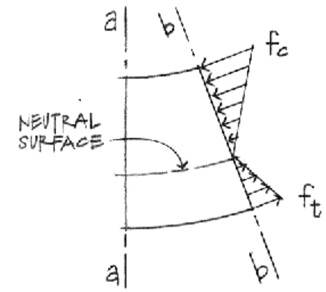


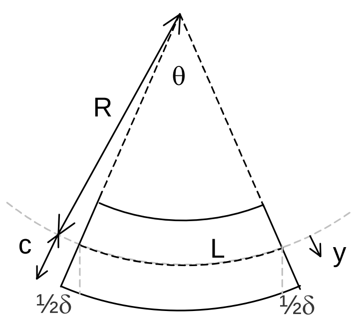
Figure 8.8 Bending stresses on section b-b.

Derivation of Stress from Strain

- *pure bending = arc shape*

$$L = R\theta$$

$$L_{outside} = (R+y)\theta$$



$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

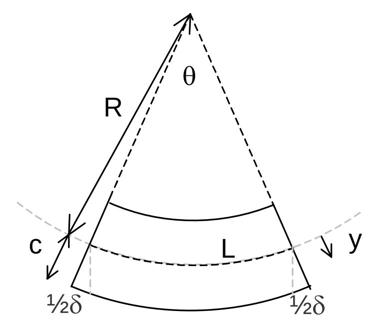
Derivation of Stress

- *zero stress at n.a.*

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{\max} = \frac{Ec}{R}$$

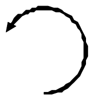
$$f = \frac{y}{c} f_{\max}$$



Bending Moment

- resultant moment from stresses = bending moment!

$$M = \Sigma f y \Delta A$$



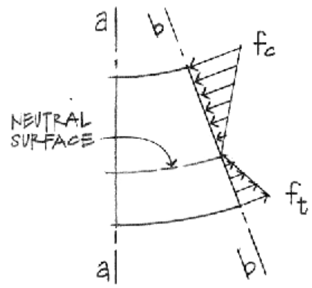


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

$$f_b = \frac{M}{S}$$

maximum bending stress

$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

Transverse Loading and Shear

(*a*) Internal forces (positive shear and positive bending moment)

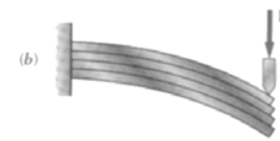
(*b*) Effect of external forces (positive shear)

- perpendicular loading
- internal shear
- along with bending moment

Bending vs. Shear in Design

- bending stresses dominate

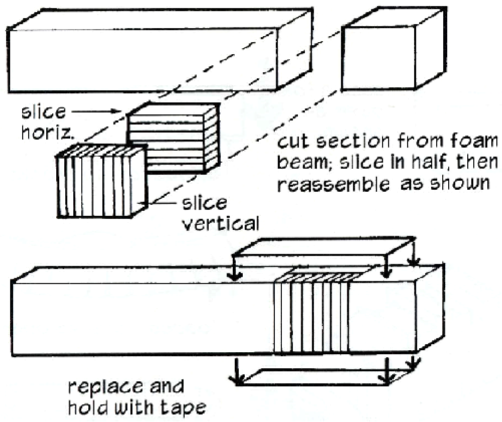
- shear stresses exist horizontally with shear



- <u>no shear stresses</u> with pure bending

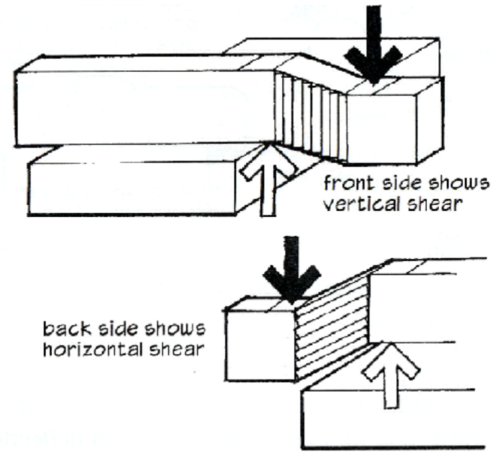
Shear Stresses

- *horizontal & vertical*



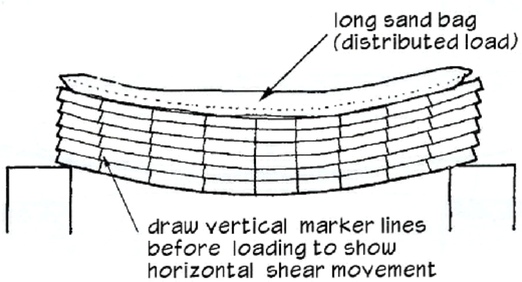
Shear Stresses

- *horizontal & vertical*



Beam Stresses

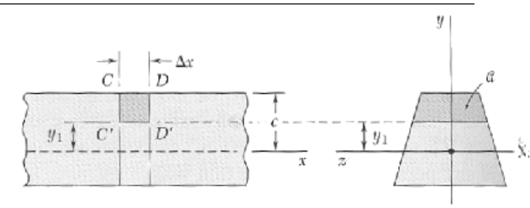
- *horizontal with bending*

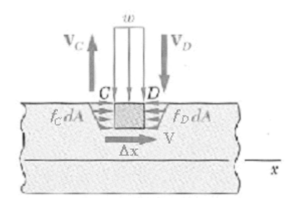


Equilibrium

- *horizontal force V needed*

$$V_{longitudinal} = \frac{V_T Q}{I}\Delta x$$

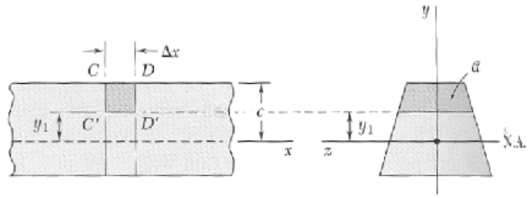




- *Q is a moment area*

Moment of Area

- Q is a moment area with respect to the n.a. of area <u>above or below</u> the horizontal
- Q_{max} at y=0 <u>(n.a.)</u>

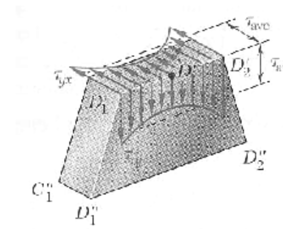


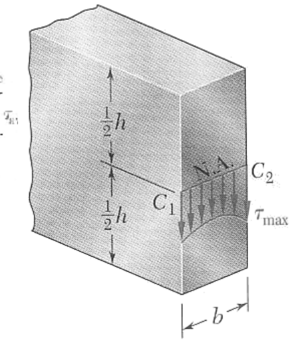
- q is shear flow: $$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$



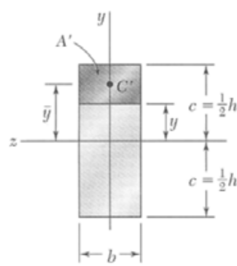


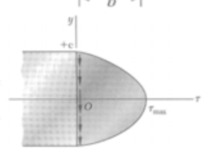
- f_{v-ave} = 0 on the top/bottom
- b min may not be with Q max
- with h/4 ≥b, $f_{v\text{-}max} \leq 1.008\ f_{v\text{-}ave}$

Rectangular Sections

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = \frac{bh^2}{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

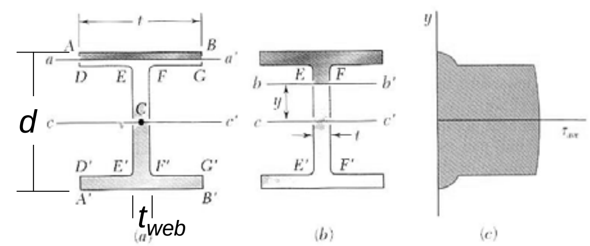




- $f_{v\text{-}max}$ occurs at n.a.

Steel Beam Webs

- W and S sections
  - b varies



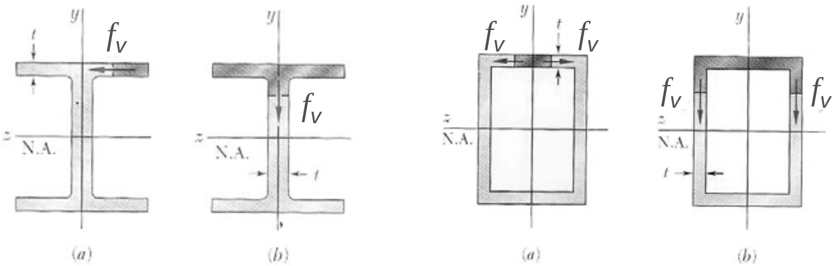
  - stress in flange negligible
  - presume constant stress in web

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

Shear Flow

- *loads applied in plane of symmetry*
- *cut made perpendicular*

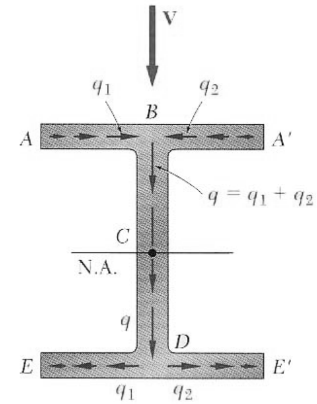
$$q = \frac{VQ}{I}$$



Shear Flow Quantity

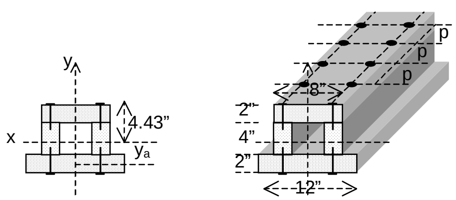
- *sketch from Q*

$$q = \frac{VQ}{I}$$



Connectors Resisting Shear

- *plates with*
  - *nails*
  - *rivets*
  - *bolts*
- *splices*



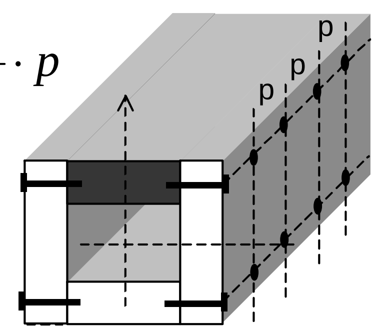
$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

Vertical Connectors

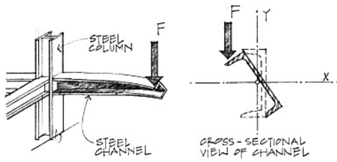
- *isolate an area with vertical interfaces*

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



Unsymmetrical Shear or Section

- *member can bend and twist*
  - *not symmetric*
  - *shear not in that plane*
- *shear center*
  - *moments balance*



(a) (b)