ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS

ENDS 231
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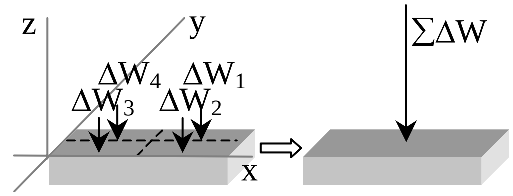
SUMMER 2006

lecture
ten

centers of gravity- centroids

Center of Gravity

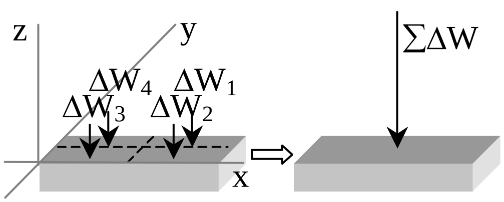
- location of equivalent weight
- determined with calculus



- sum element weights $\boldsymbol{W} = \int dW$

Center of Gravity

- "average" x & y from moment



$$\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}\boldsymbol{W} \Rightarrow \quad \bar{x} = \frac{\sum(x\Delta W)}{\boldsymbol{W}}$$

"bar" means average

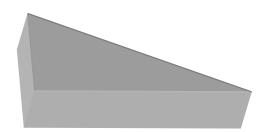
$$\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}\boldsymbol{W} \Rightarrow \quad \bar{y} = \frac{\sum(y\Delta W)}{\boldsymbol{W}}$$

Centroid

- "average" x & y of an area
- for a volume of constant thickness
  - $\Delta W = \gamma t \Delta A$ where γ is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{\boldsymbol{A}}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{\boldsymbol{A}}$$

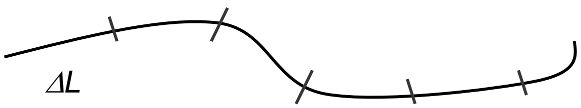


Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{\boldsymbol{L}}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{\boldsymbol{L}}$$

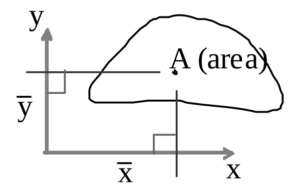


1st Moment Area

- math concept
- the moment of an area about an axis

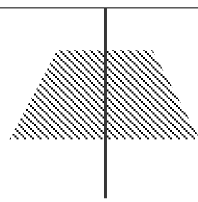
$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$

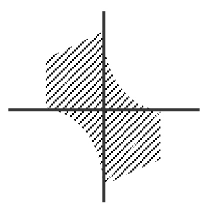


Symmetric Areas

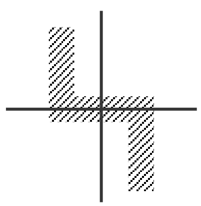
- symmetric about an axis



- symmetric about a center point

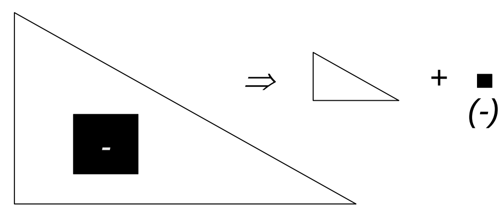


- mirrored symmetry



Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\bar{x}$ and $\bar{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
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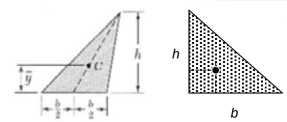
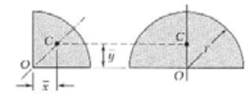
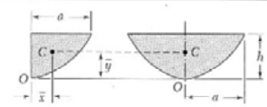
Area Centroids

- Table 7.1 – pg. 242

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$
Triangular area		$\frac{b}{3}$ right triangle only	$\frac{h}{3}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area		0	$\frac{4r}{3\pi}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$
Parabolic area		0	$\frac{3h}{5}$