ARCHITECTURAL STRUCTURES I:
STATICS AND STRENGTH OF MATERIALS
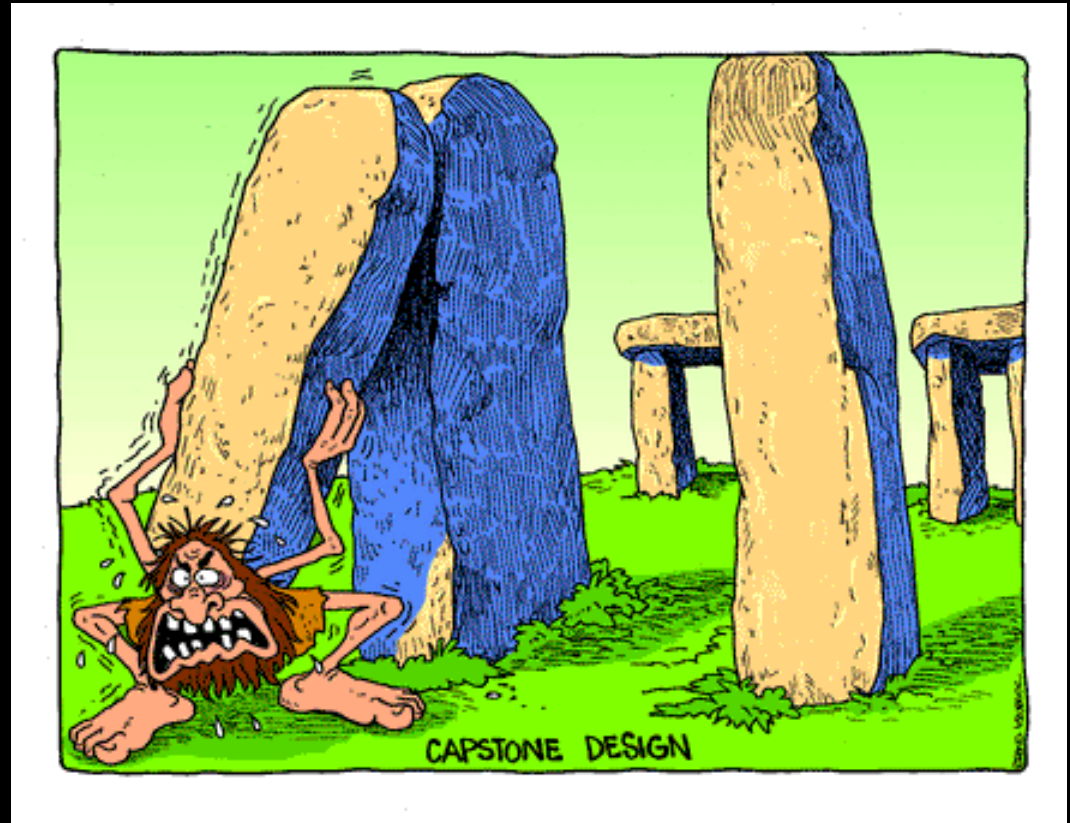
ENDS 231

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SUMMER 2006

lecture
five

rigid body equilibrium
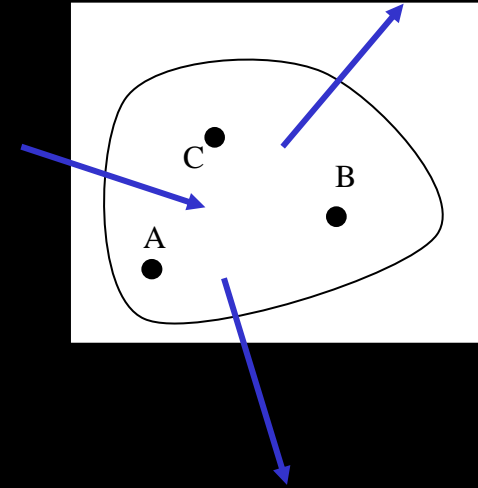

CAPSTONE DESIGN

Equilibrium

- *rigid body*
  - *doesn't deform*
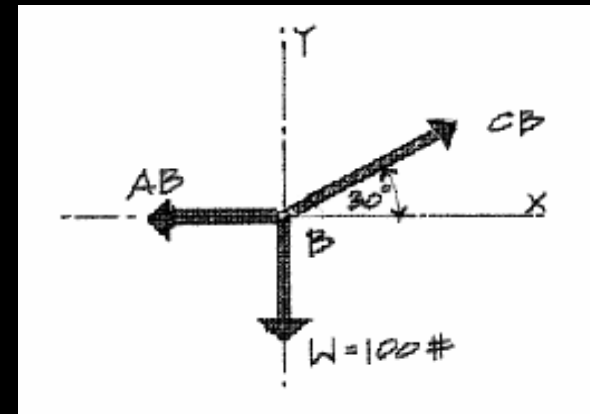 - *coplanar force systems*
- *static:*

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$M = \sum M = 0$$
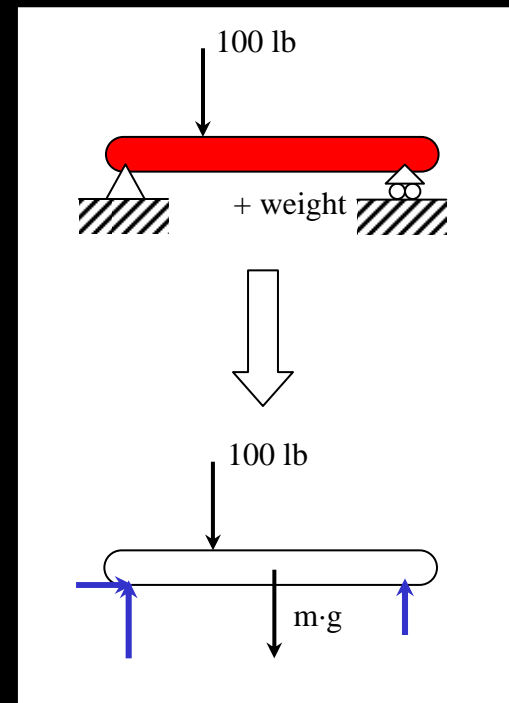
Free Body Diagram

- *FBD (sketch)*

- *tool to see all forces on a body or a point including*
  - *external forces*
  - *weights*
  - *force reactions*
  - *external moments*
  - *moment reactions*
  - *internal forces*

(Example 1)

Free Body Diagram

- *determine body*
- *FREE it from:*
  - *ground*
  - *supports & connections*
- *draw all external forces acting ON the body*
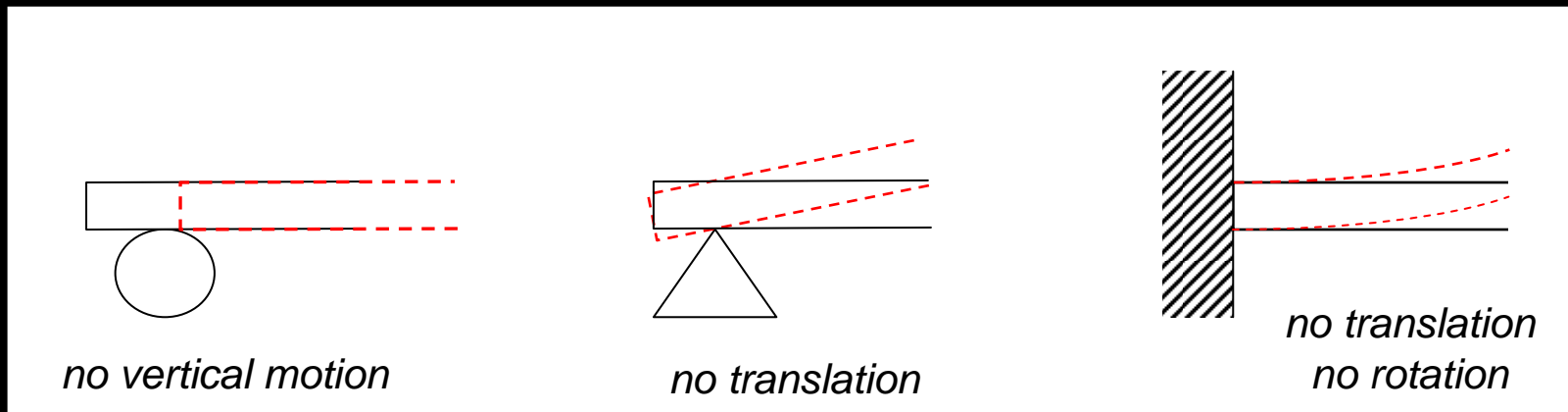 - *reactions*
  - *applied forces*
  - *gravity*
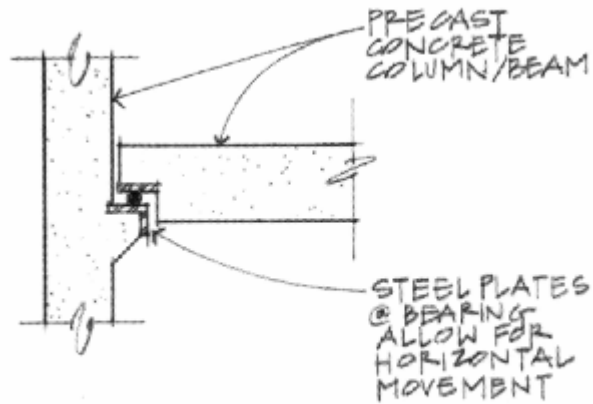
100 lb

+ weight

100 lb

m·g

Free Body Diagram

- **include relevant geometry**
  - guidelines helpful to see moment arms
- **name and/or color the unknown**
  - forces
  - moments
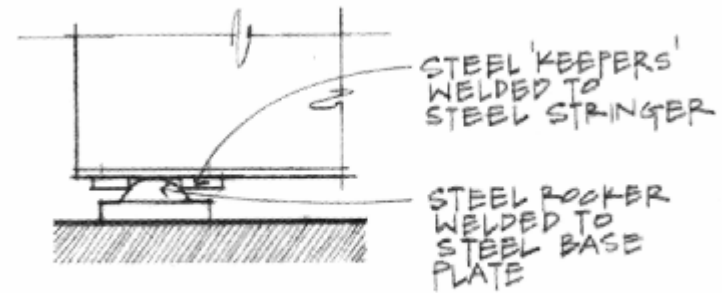 - angles
- **solve up to 3 equations**

Reactions

- *result of applying force*

- *unknown size*

- *connection or support type*
  - *known direction*
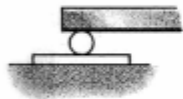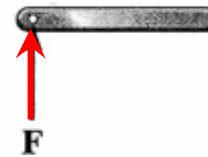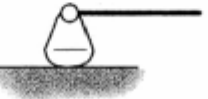 - *related to motion prevented*

no vertical motion   no translation   no translation
                                                     no rotation

Supports and Connections

Supports and Connections

STEEL COLUMN/
BEAM
CONNECTION,
BOLTED.

smooth pin or hinge

F_y F_x

Pin

CAST-IN-PLACE
CONCRETE
COLUMN/BEAM
CONNECTION.

NOTE:
OVERLAPPING
STEEL
REINFORCEMENT
MAKES ASSEMBLY
MONOLITHIC.

fixed support

F_y M F_x

Fixed
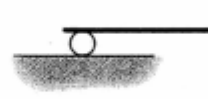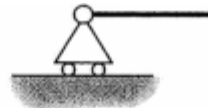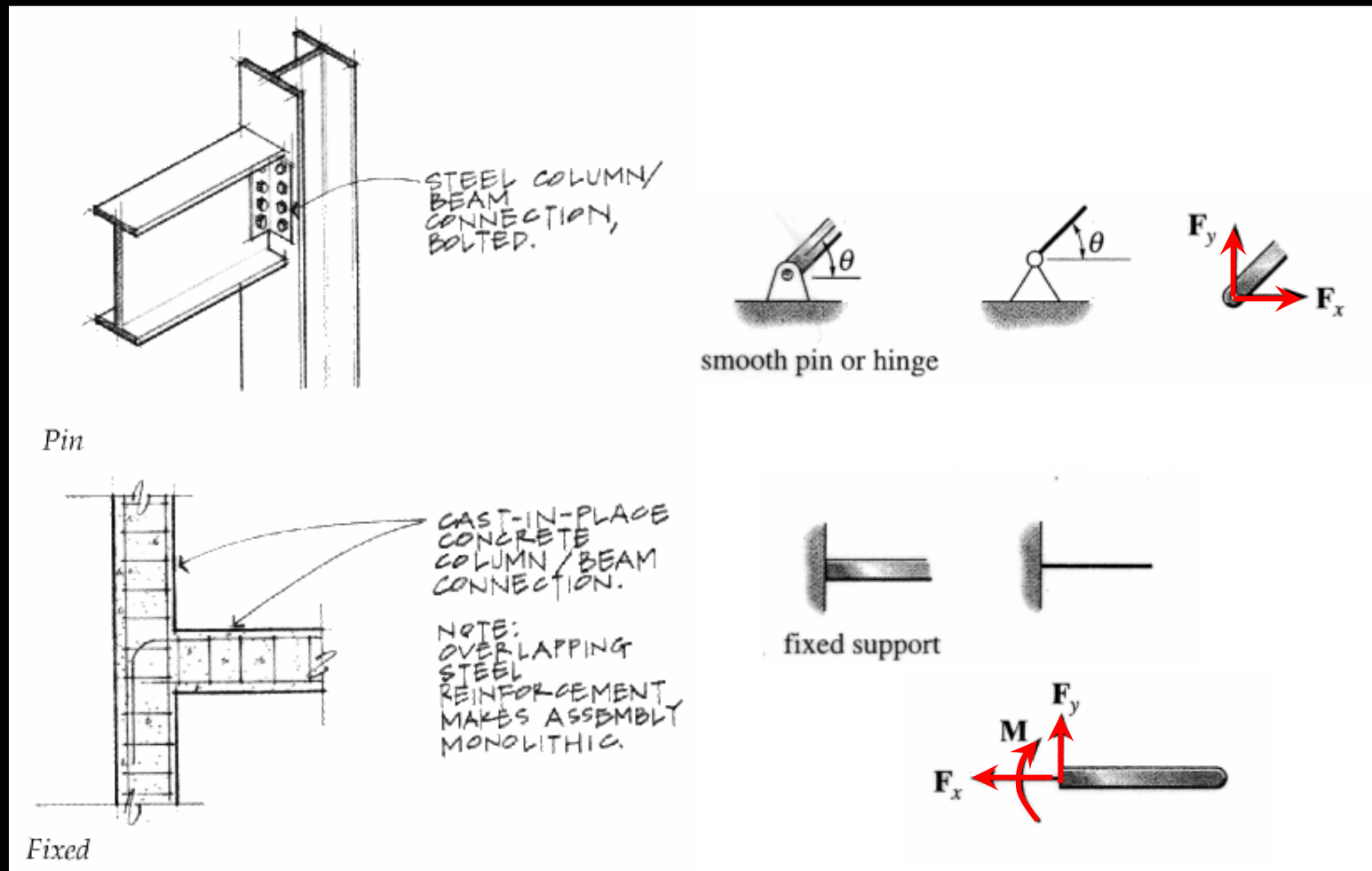
FBD Example

- *500 lb known*

- *pin – A_x, A_y*

- *smooth surface – B at 4:3*

- *3 equations*

- *sum moments at*
  - *A?*
  - *B?   (B_x)*
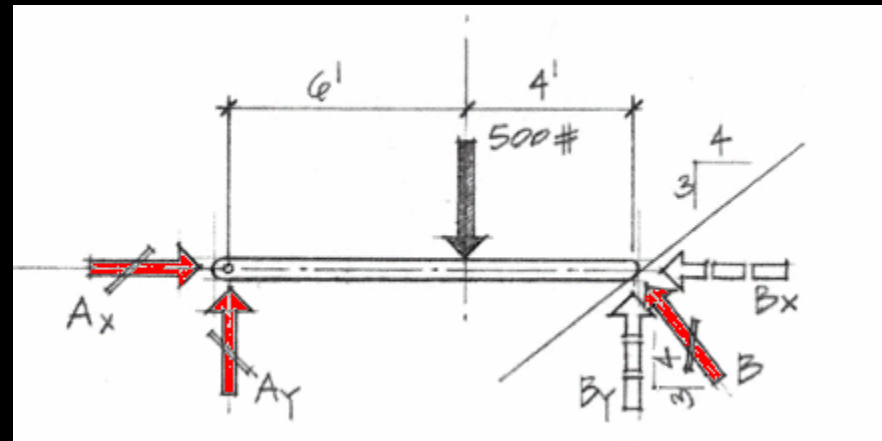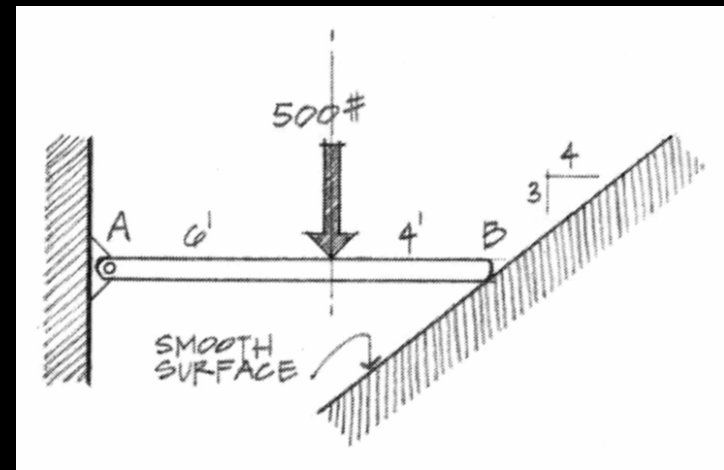
Moment Equations

- *sum moments at intersection where the most forces intersect*

- *multiple moment equations may not be useful*

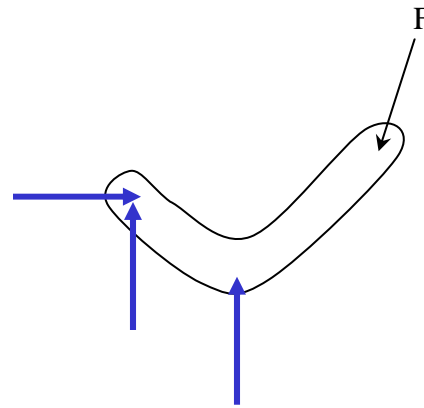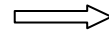- *combos:*

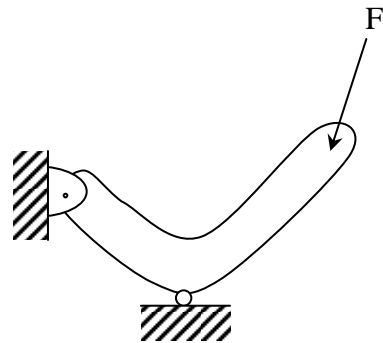$$\sum F_x = 0 \qquad \sum F = 0 \qquad \sum M_1 = 0$$

$$\sum F_y = 0 \qquad \sum M_1 = 0 \qquad \sum M_2 = 0$$

$$\sum M_1 = 0 \qquad \sum M_2 = 0 \qquad \sum M_3 = 0$$

Recognizing Reactions

F
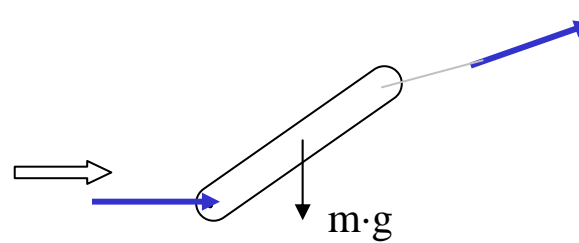
F

3
unknowns

weight

m·g

3
unknowns

Recognizing Reactions

$$\underline{3}$$

unknowns

weight

m·g

F_1 F_2

F_1 F_2

not independent
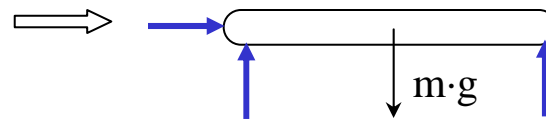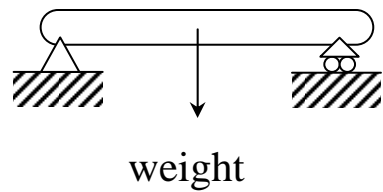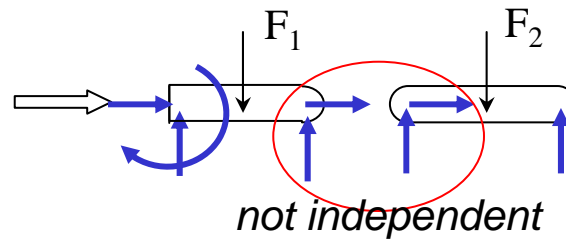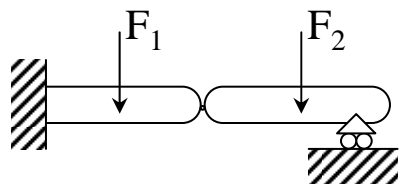
$$\underline{6}$$

unknowns for
2 bodies

weight

m·g
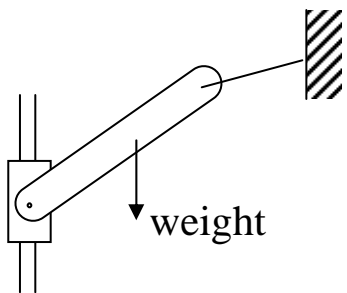
$$\underline{2}$$
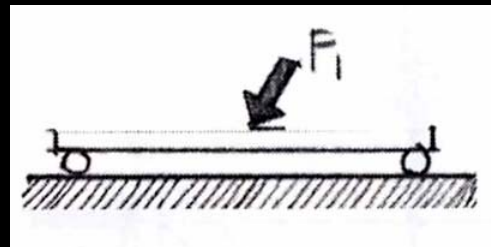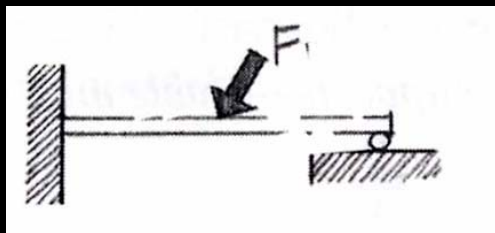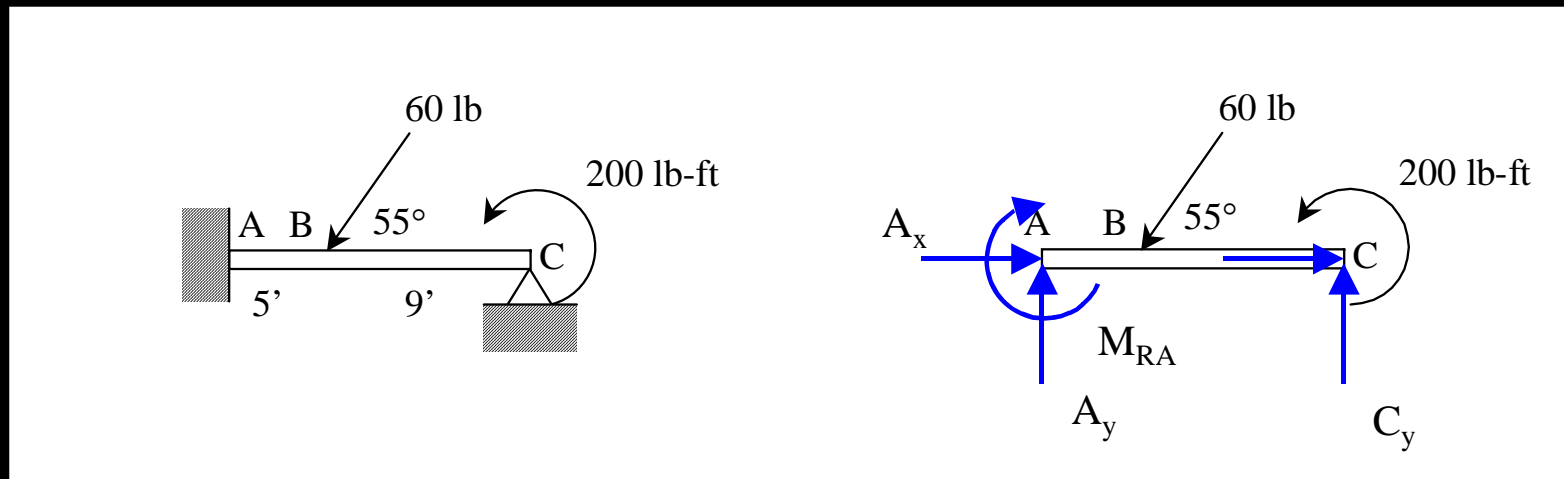
unknowns

Constraints

- *completely constrained*
  - *doesn't move*
  - *may not be statically determinate*
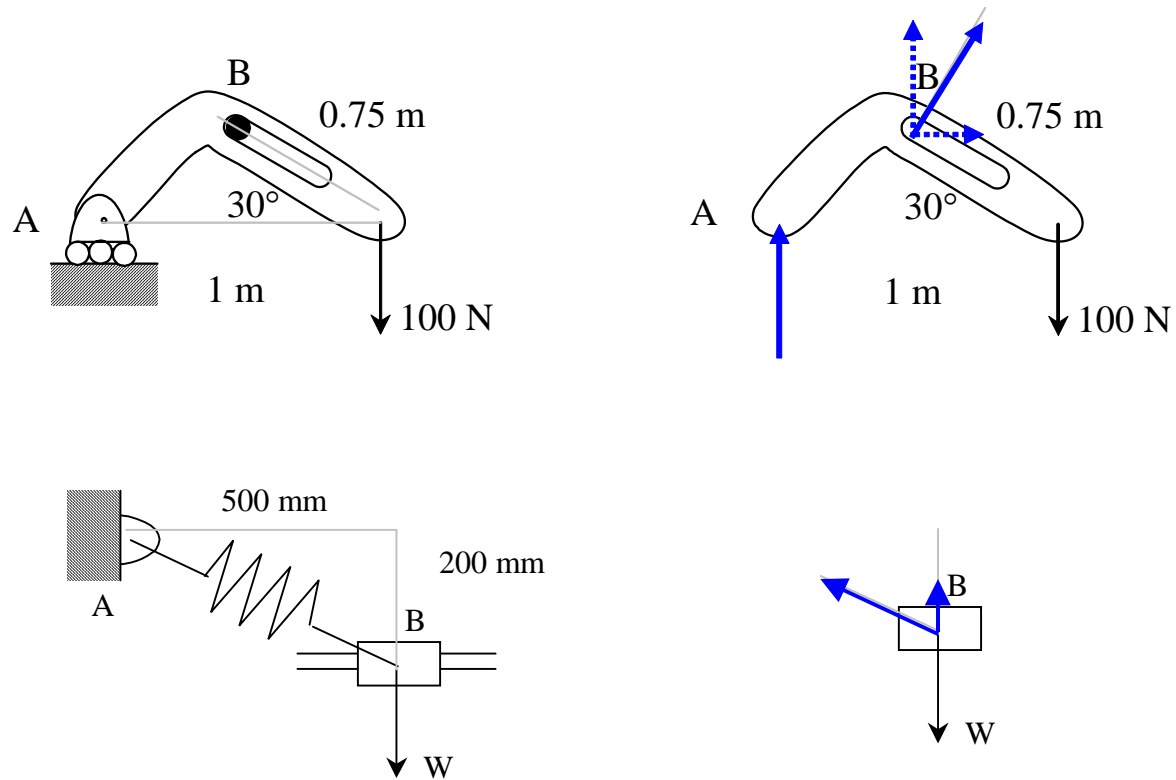- *improperly or partially constrained*
  - *has ≤ unknowns*
  - *can't solve*
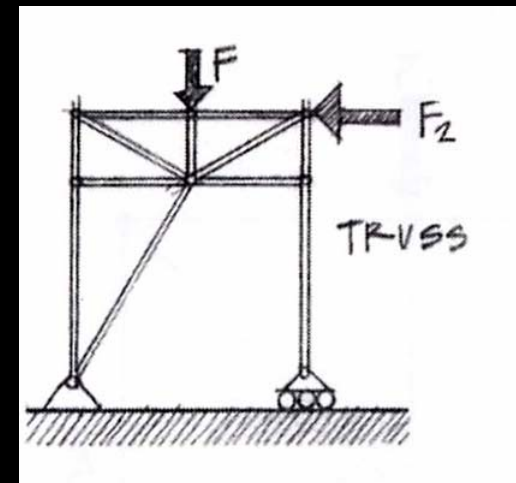
Constraints

- *overconstrained*
  - *won't move*
  - *can't be solved with statics*
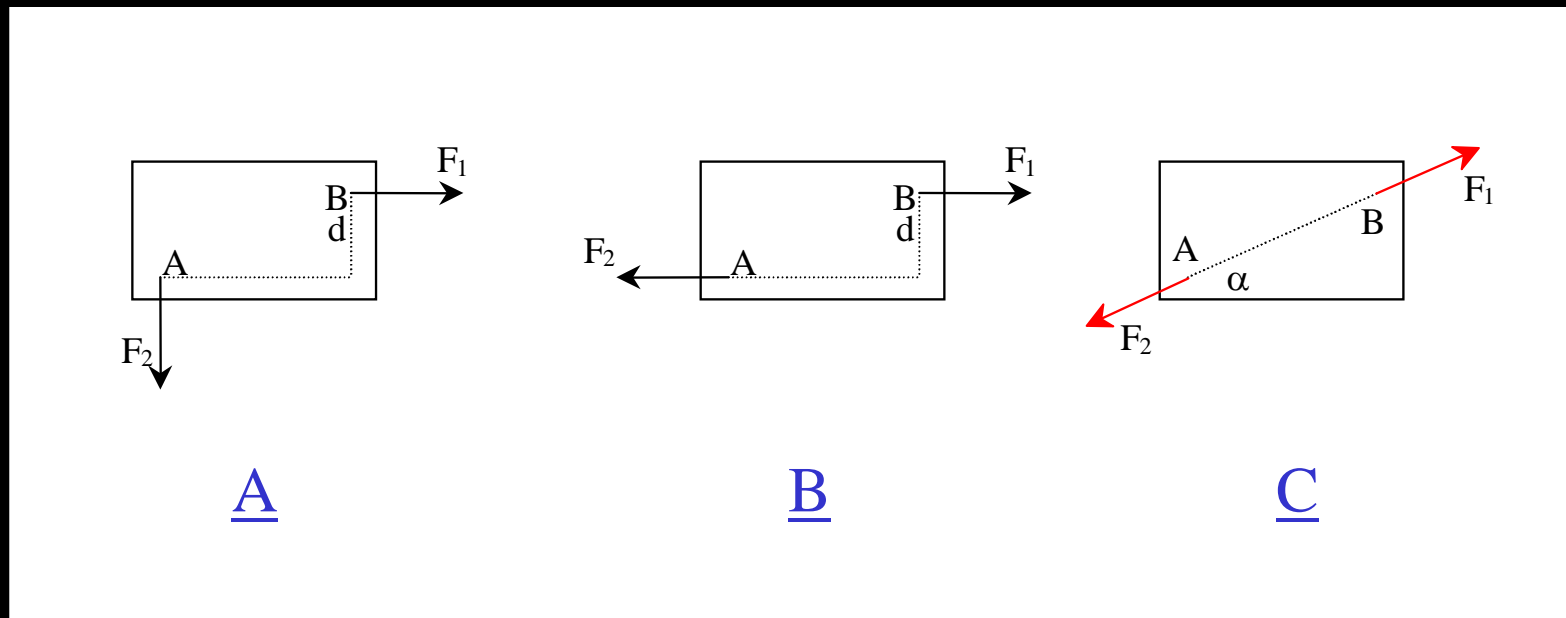 - *statically indeterminate to nth degree*

Partial Constraints
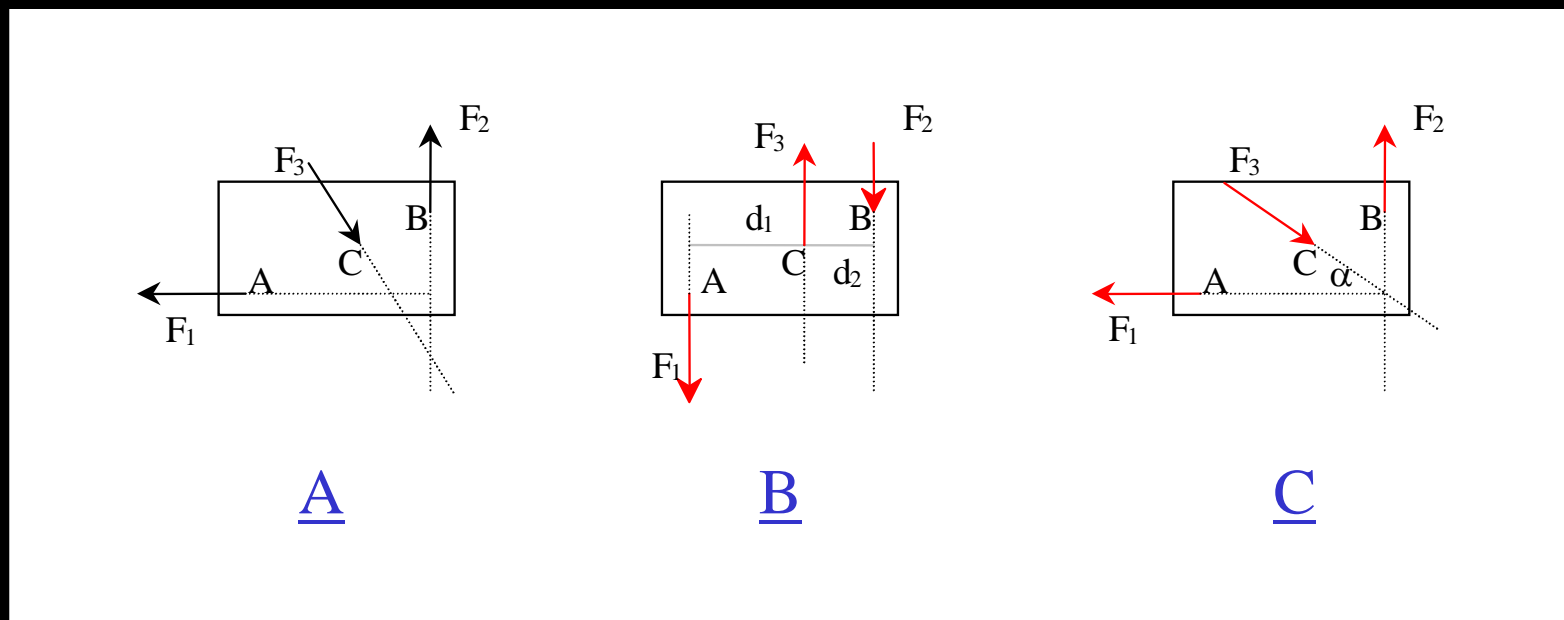
Two Force Rigid Bodies

- *equilibrium:*
  - *forces in line, equal and opposite*

Three Force Rigid Bodies

- *equilibrium:*
  - *concurrent or parallel forces*

Cable Reactions

- *equilibrium:*
  - *more reactions (4) than equations*
  - *but, we have slope relationships*
  - *x component the same everywhere*