ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

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SUMMER 2006

lecture
twenty one

column design

Design Methods

- know
  - loads or lengths
- select
  - section or load
  - adequate for strength and no buckling

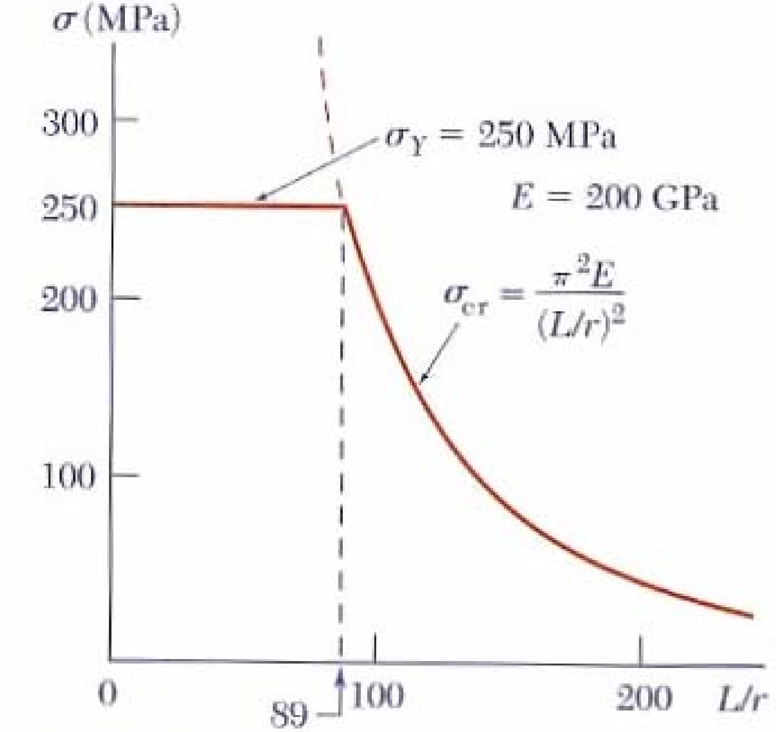


Fig. 10.9

Allowable Stress Design (ASD)

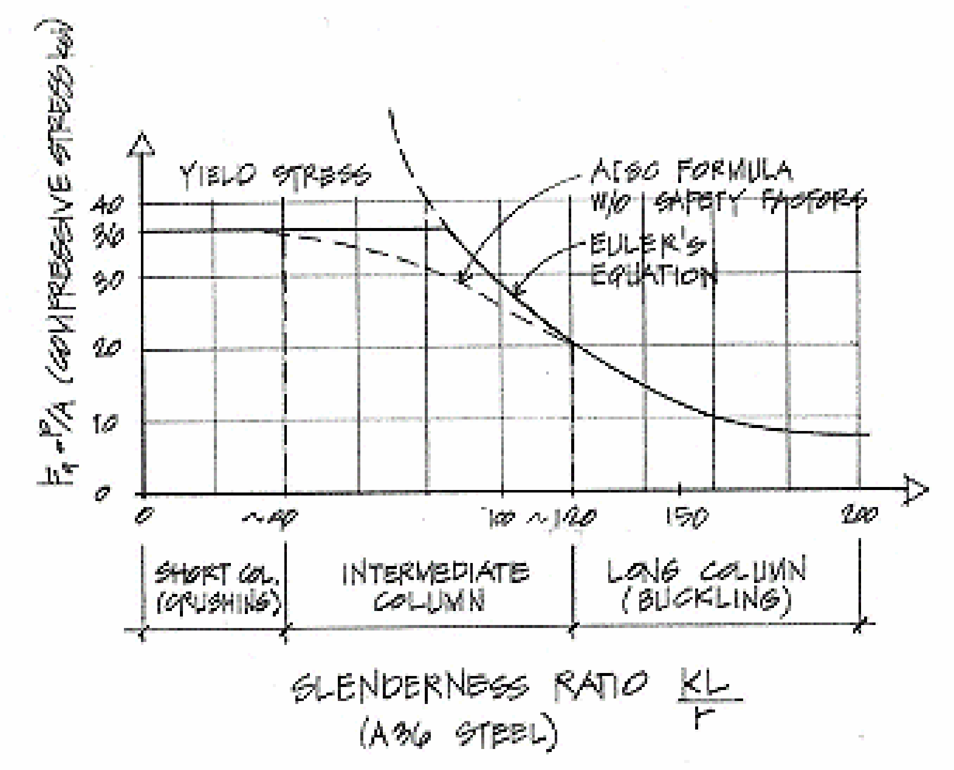
- AICS 9th ed

$$F_a = \frac{f_{critical}}{F.S.} = \frac{12\pi^2 E}{23\left(\frac{Kl}{r}\right)^2}$$

- slenderness ratio $\dfrac{Kl}{r}$
  - for $kl/r \geq C_c$ $= 126.1$ with $F_y = 36$ ksi
    $= 107.0$ with $F_y = 50$ ksi

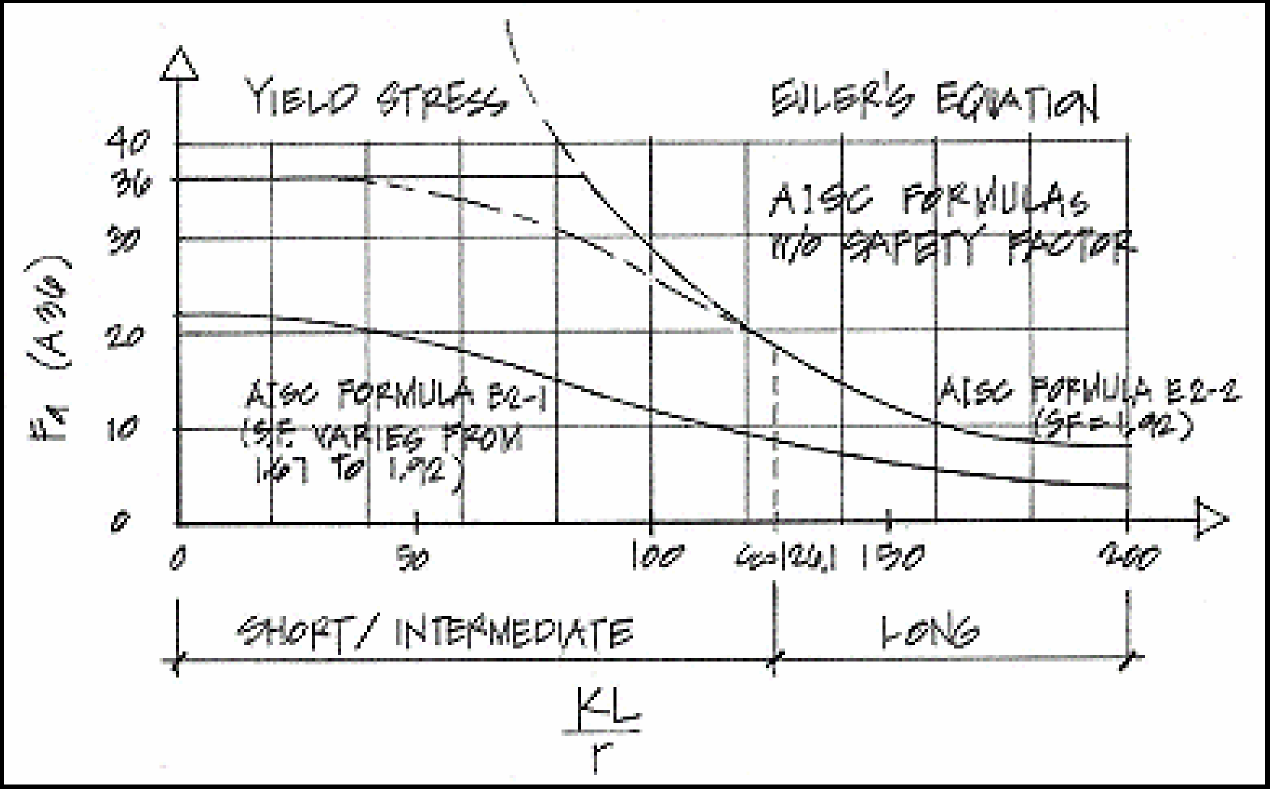
C_c and Euler's Formula

- *$Kl/r < C_c$*
  - *short and stubby*
  - *parabolic transition*
- *$Kl/r > C_c$*
  - *Euler's relationship*
  - *< 200 preferred*



$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

C_c and Euler's Formula



Short / Intermediate

- $L_e/r < C_c$

$$F_a = \left[1 - \frac{\left(Kl/r \right)^2}{2C_c^2} \right] \frac{F_y}{F.S.}$$

  - where

$$F.S. = \frac{5}{3} + \frac{3\left(Kl/r \right)}{8C_c} - \frac{\left(Kl/r \right)^3}{8C_c^3}$$

Procedure for Analysis

1. *calculate KL/r*
   - *biggest of KL/r with respect to x axes and y axis*
2. *find F_a from Table 10.1 or 10.2*
   - *pp. 361 - 364*
3. *compute $P_{allowable} = F_a \cdot A$*
   - *or find $f_{actual} = P/A$*
4. *is $P \leq P_{allowable}$? (or is $f_{actual} \leq F_a$?)*
   - *yes: ok*
   - *no: overstressed and no good*

Procedure for Design

1. *guess a size (pick a section)*
2. *calculate KL/r*
   - *biggest of KL/r with respect to x axes and y axis*
3. *find F_a from Table 10.1 or 10.2*
   - *pp. 361 - 364*
4. *compute $P_{allowable} = F_a \cdot A$*
   - *or find $f_{actual} = P/A$*

Procedure for Design (cont'd)

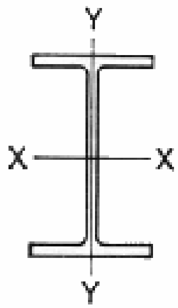
5. *is $P \le P_{allowable}$? (or is $f_{actual} \le F_a$?)*
   - *yes: ok*
   - *no: pick a bigger section and **go back to step 2.***
6. *check design efficiency*
   - *percentage of stress =* $\frac{P_{actual}}{P_{allowable}} \cdot 100\%$
   - *if between 90-100%: good*
   - *if < 90%: pick a smaller section and **go back to step 2.***

Column Charts

Table C-50
Allowable Stress
For Compression Members of 50-ksi Specified Yield Stress Steel[a]

$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)	$\frac{Kl}{r}$	F_a (ksi)
1	29.94	41	25.69	81	18.81	121	10.20	161	5.76
2	29.87	42	25.55	82	18.61	122	10.03	162	5.69
3	29.80	43	25.40	83	18.41	123	9.87	163	5.62
4	29.73	44	25.26	84	18.20	124	9.71	164	5.55
5	29.66	45	25.11	85	17.99	125	9.56	165	5.49
6	29.58	46	24.96	86	17.79	126	9.41	166	5.42
7	29.50	47	24.81	87	17.58	127	9.26	167	5.35
8	29.42	48	24.66	88	17.37	128	9.11	168	5.29
9	29.34	49	24.51	89	17.15	129	8.97	169	5.23
10	29.26	50	24.35	90	16.94	130	8.84	170	5.17
11	29.17	51	24.19	91	16.72	131	8.70	171	5.11
12	29.08	52	24.04	92	16.50	132	8.57	172	5.05
13	28.99	53	23.88	93	16.29	133	8.44	173	4.99
14	28.90	54	23.72	94	16.06	134	8.32	174	4.93
15	28.80	55	23.55	95	15.84	135	8.19	175	4.88
16	28.71	56	23.39	96	15.62	136	8.07	176	4.82
17	28.61	57	23.22	97	15.39	137	7.96	177	4.77

Column Charts



COLUMNS

W shapes

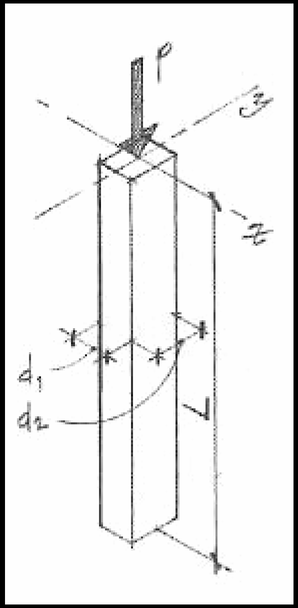
Allowable axial loads in kips

	F_y = 36 ksi / F_y = 50 ksi											
Designation	W8											
Wt./ft.	67		58		48		40		35		31	
F_y	36	50	36	50	36	50	36	50	36	50	36	50
0	426	591	369	513	305	423	253	351	222	309	197	274
6	387	525	336	455	276	375	229	310	201	272	178	241
7	379	510	328	442	270	363	223	300	197	264	174	234
8	370	494	320	428	263	352	218	290	191	255	170	226
9	360	477	312	413	256	339	212	279	186	246	165	217
10	350	459	303	397	249	326	205	268	180	236	160	208
11	339	440	293	380	241	312	199	256	174	225	154	199
12	328	420	283	363	233	297	192	244	168	214	149	189
13	316	399	273	344	224	282	184	231	162	202	143	179
14	304	378	263	325	215	266	177	217	155	190	137	168
15	292	355	251	305	206	249	169	203	148	177	131	156
16	279	331	240	284	196	232	160	188	141	164	124	145
17	265	307	228	263	186	214	152	172	133	150	117	132

…spect to least radius of gyration r_y

Wood Columns

- slenderness ratio = $L/d_{min} = L/d_1$
  - d_1 = smaller dimension
  - $\ell_e/d \leq 50$ (max)



$$f_c = \frac{P}{A} \leq F'_c$$

  - where F'_c is the allowable compressive strength parallel to the grain

Allowable Wood Stress

$$F'_c = F_c(C_D)(C_M)(C_t)(C_F)(C_p)$$

- *where:*

  F_c = *compressive strength parallel to grain*

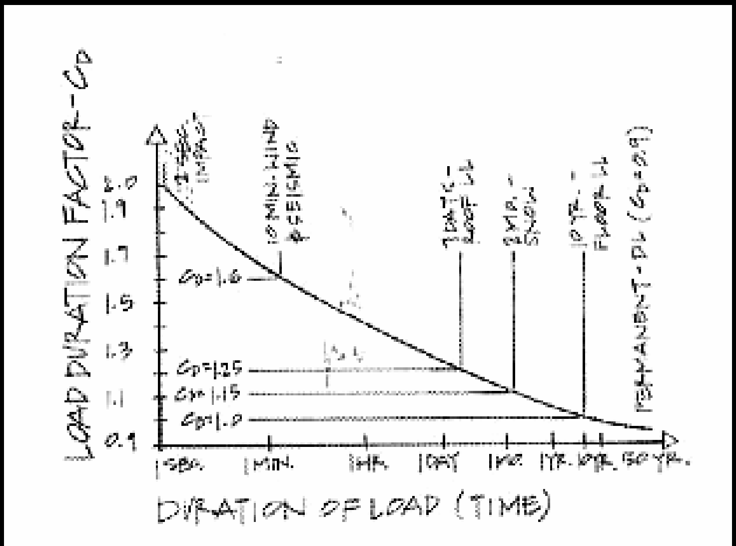
  C_D = *load duration factor*

  C_M = *wet service factor (1.0 dry)*

  C_t = *temperature factor*

  C_F = *size factor*

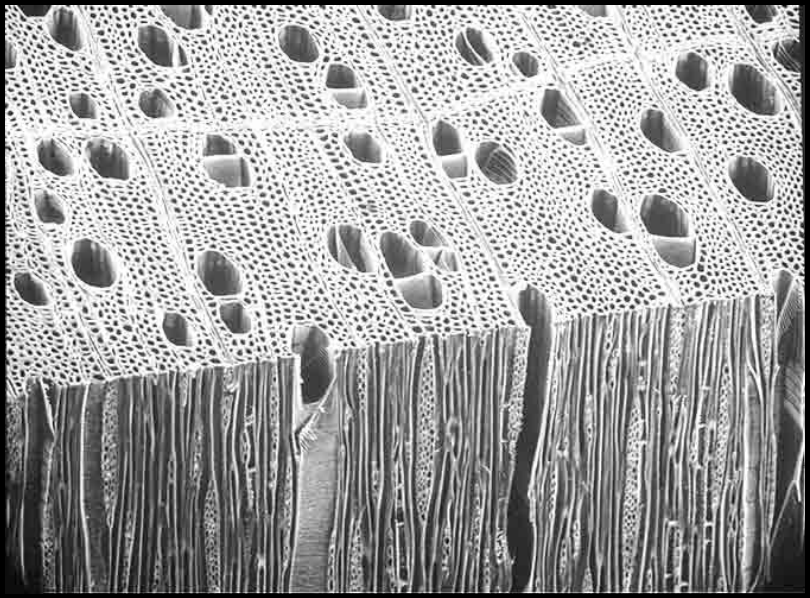
  C_p = *column stability factor*



(Table 10.3)

Strength Factors

- *wood properties and load duration, C_D*
  - *short duration*
    - *higher loads*
  - *normal duration*
    - *> 10 years*



- *stability, C_p*
  - *combination curve - tables*

$$F_c' = F_c^* C_p = \left(F_c C_D\right) C_p$$

C_p Charts – Appendix A, Table 14

Column Stability Factor C_p

"C_p"

$F_c' = C_p \cdot F_c^*$ $\quad F_{CE} = \frac{30\,E}{(L/d)^2}$ for sawn posts $\quad F_{CE} = \frac{418\,E}{(L/d)^2}$ for Glu-Lam posts

F_{CE}/F_c^*	Sawn C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawn C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawn C_p	Glu-Lam C_p	F_{CE}/F_c^*	Sawn C_p	Glu-Lam C_p
0.00	0.000	0.000	0.60	0.500	0.538	1.20	0.750	0.822	2.40	0.894	0.940
0.01	0.010	0.010	0.61	0.506	0.545	1.22	0.755	0.826	2.45	0.897	0.941
0.02	0.020	0.020	0.62	0.512	0.552	1.24	0.760	0.831	2.50	0.899	0.943
0.03	0.030	0.030	0.63	0.518	0.559	1.26	0.764	0.836	2.55	0.901	0.944
0.04	0.040	0.040	0.64	0.524	0.566	1.28	0.769	0.840	2.60	0.904	0.946
0.05	0.049	0.050	0.65	0.530	0.573	1.30	0.773	0.844	2.65	0.906	0.947
0.06	0.059	0.060	0.66	0.536	0.580	1.32	0.777	0.848	2.70	0.908	0.949
0.07	0.069	0.069	0.67	0.542	0.587	1.34	0.781	0.852	2.75	0.910	0.950
0.08	0.079	0.079	0.68	0.548	0.593	1.36	0.785	0.855	2.80	0.912	0.951
0.09	0.088	0.089	0.69	0.553	0.600	1.38	0.789	0.859	2.85	0.914	0.952
0.10	0.098	0.099	0.70	0.559	0.607	1.40	0.793	0.862	2.90	0.916	0.953
0.11	0.107	0.109	0.71	0.564	0.613	1.42	0.796	0.865	2.95	0.917	0.954
0.12	0.117	0.118	0.72	0.569	0.619	1.44	0.800	0.868	3.00	0.919	0.955
0.13	0.126	0.128	0.73	0.575	0.626	1.46	0.803	0.871	3.05	0.920	0.956
0.14	0.136	0.138	0.74	0.580	0.632	1.48	0.807	0.874	3.10	0.922	0.957
0.15	0.145	0.147	0.75	0.585	0.638	1.50	0.810	0.877	3.15	0.923	0.958
0.16	0.154	0.157	0.76	0.590	0.644	1.52	0.813	0.879	3.20	0.925	0.959
0.17	0.164	0.167	0.77	0.595	0.650	1.54	0.816	0.882	3.25	0.926	0.960
0.18	0.173	0.176	0.78	0.600	0.655	1.56	0.819	0.884	3.30	0.927	0.961
0.19	0.182	0.186	0.79	0.605	0.661	1.58	0.822	0.887	3.35	0.929	0.961

Procedure for Analysis

1. *calculate* L_e/d_{min}
2. *obtain* F'_c
   - *compute* $F_{cE} = \frac{K_{cE}E}{\left(L_e/d\right)^2}$
     - K_{cE} *=0.3 sawn*
     - K_{cE} *= 0.418 glu-lam*
3. *compute* $F_c^* \approx F_c C_D$
4. *calculate* F_{cE}/F_c^* *and get* C_p *(table 14)*
5. *calculate* $F'_c = F_c^* C_p$

Procedure for Analysis (cont'd)

6. *compute $P_{allowable} = F'_c \cdot A$*
   - *or find $f_{actual} = P/A$*
7. *is $P \leq P_{allowable}$? (or $f_{actual} \leq F'_c$?)*
   - *yes: OK*
   - *no: overstressed & no good*

Procedure for Design

1. guess a size (pick a section)
2. calculate L_e/d_{min}
3. obtain F'_c $$F_{cE} = \frac{K_{cE}E}{\left(\frac{L_e}{d}\right)^2}$$
   – compute
     - K_{cE} =0.3 sawn
     - K_{cE} = 0.418 glu-lam
4. compute $F_c^* \approx F_c C_D$
5. calculate F_{cE}/F_c^* and get C_p (table 14)
6. calculate $F'_c = F_c^* C_p$

Procedure for Design (cont'd)

6. *compute $P_{allowable} = F'_c \cdot A$*
   - *or find $f_{actual} = P/A$*
7. *is $P \le P_{allowable}$? (or $f_{actual} \le F'_c$?)*
   - *yes: OK*
   - *no: pick a bigger section and **go back to step 2**.*

LRFD design

- *limit states for failure* $$P_u \leq \phi_c P_n$$

$$\phi_c = 0.85 \quad P_n = F_{cr} A_g$$

1. *yielding* $\lambda_c \leq 1.5$
2. *buckling* $\lambda_c > 1.5$

$$\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}} \qquad L_e/r$$

λ_c – *column slenderness parameter*

A_g - *gross area*

Compact Sections

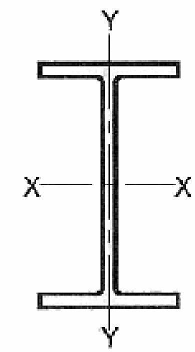
- flanges continuously connected to the web or webs and width-thickness rations < limiting values
  - no local buckling of flange or web
  - for $\lambda_c \leq 1.5$ $$F_{cr} = \left(0.658^{\lambda_c^2}\right)F_y$$
  - for $\lambda_c > 1.5$ $$F_{cr} = \left[\frac{0.877}{\lambda_c^2}\right]F_y$$

Column Charts

$F_y = 50$ ksi

$\phi_c P_n = 0.85 F_{cr} A_g$

Table 4-2 (cont.).
W-Shapes
Design Strength in Axial Compression, $\phi_c P_n$, kips



Shape		W12×										
		106	96	87	79	72	65††	58	53	50	45	40
adius of gyration r_y	0	1330	1200	1090	986	897	812	723	663	621	557	497
	6	1280	1150	1050	947	861	779	680	623	562	504	450
	7	1260	1140	1030	933	848	767	666	610	543	486	434
	8	1240	1120	1010	917	834	754	649	594	521	466	416
	9	1210	1100	994	900	818	739	631	577	497	445	396
	10	1190	1070	973	880	800	723	611	559	472	422	376
	11	1160	1050	950	860	781	706	590	539	445	398	354
	12	1130	1020	926	838	761	687	568	518	418	374	332
	13	1100	995	901	814	740	668	545	496	390	349	310
	14	1070	966	874	790	717	647	521	474	363	324	287