ARCHITECTURAL STRUCTURES I: STATICS AND STRENGTH OF MATERIALS

ENDS 231

DR. ANNE NICHOLS

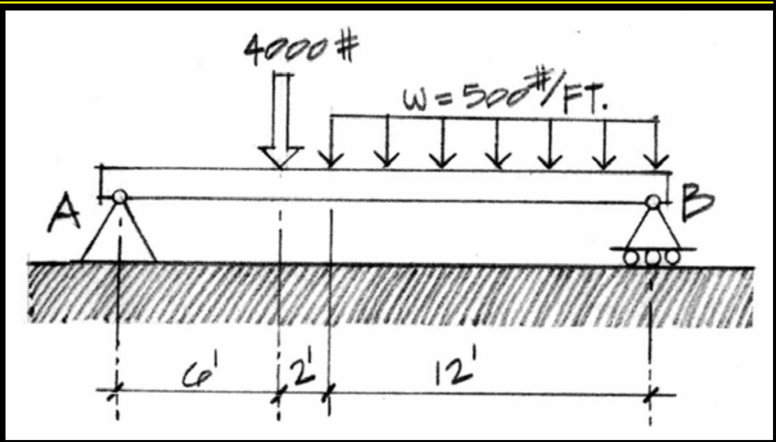
SUMMER 2006

lecture
twelve

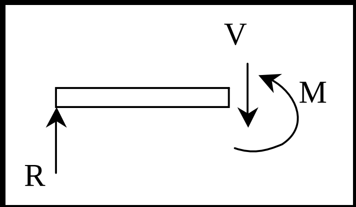
beam forces – internal

Beams

- span horizontally
  - floors
  - bridges
  - roofs

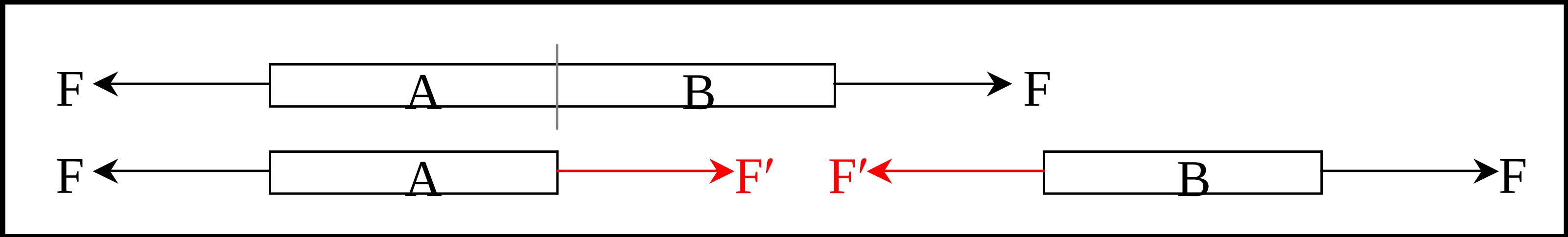


- loaded transversely by gravity loads
- may have internal axial force
- <u>will have</u> internal shear force
- <u>will have</u> internal moment (bending)

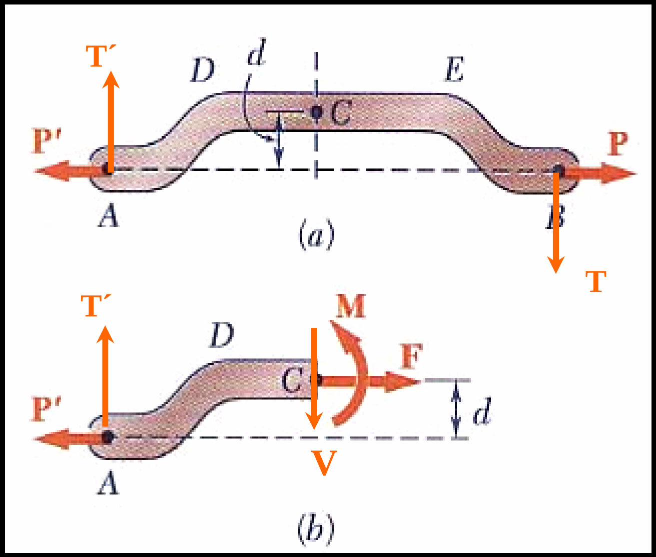


Internal Forces

- trusses
  - axial only, (compression & tension)

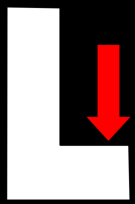


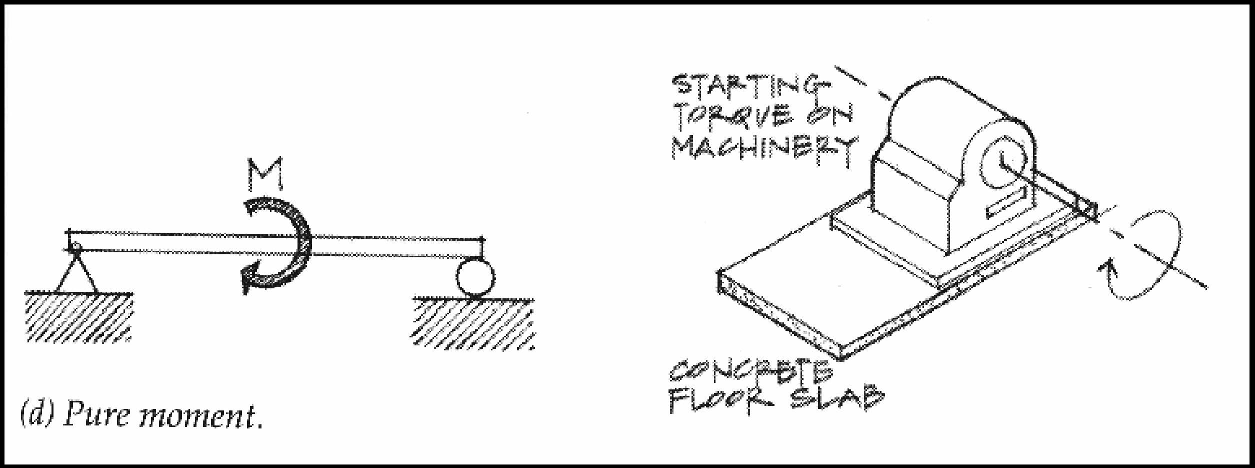
- in general
  - axial force
  - shear force, V
  - bending moment, M



Beam Loading

- concentrated force
- concentrated <u>moment</u>
  - spandrel beams

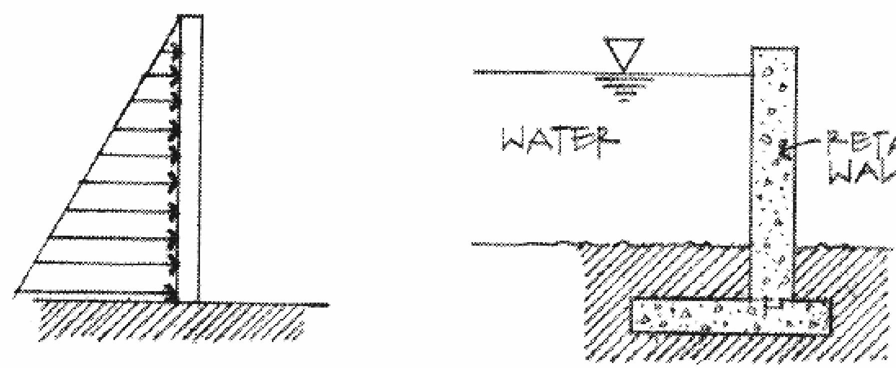




(d) Pure moment.

Beam Loading

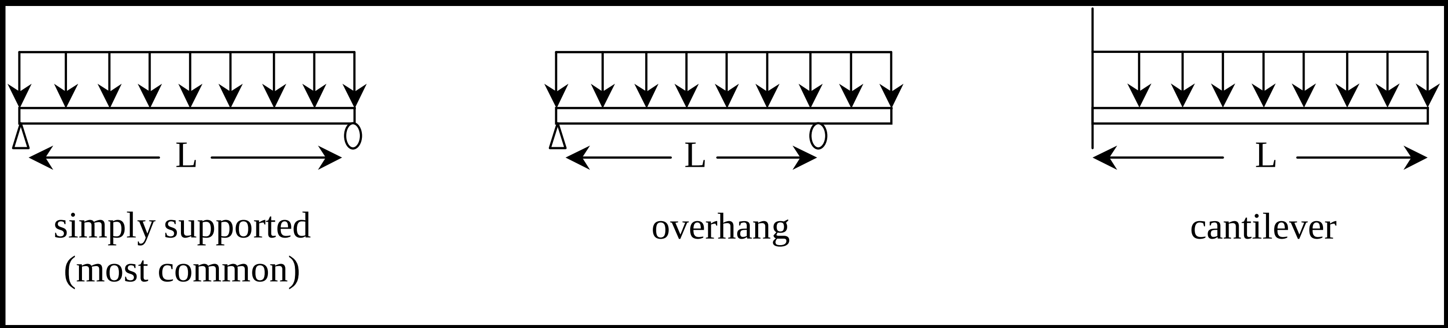
- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure
  - wind loads



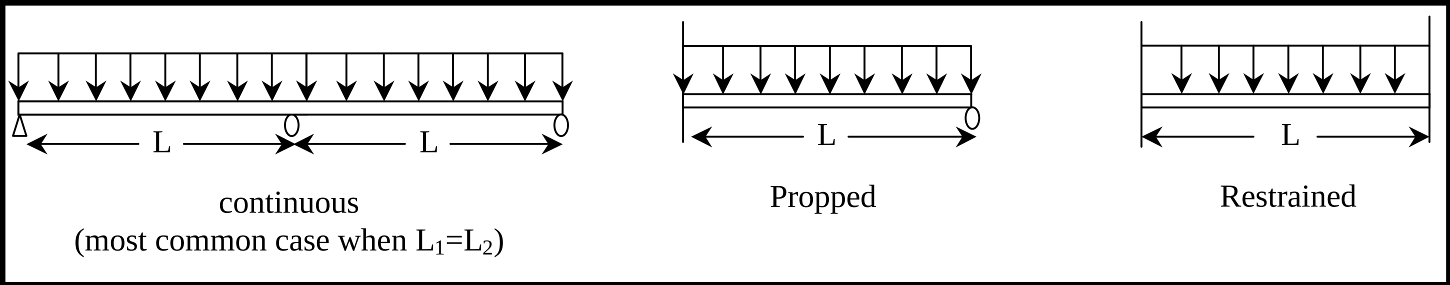
(c) Nonuniformly distributed load.

Beam Supports

- *statically determinate*

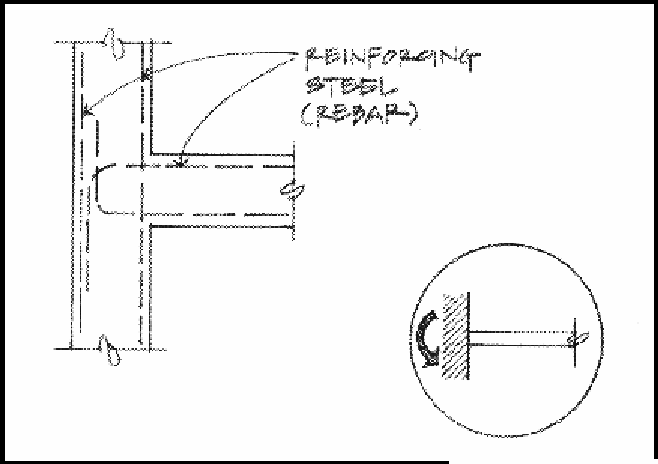


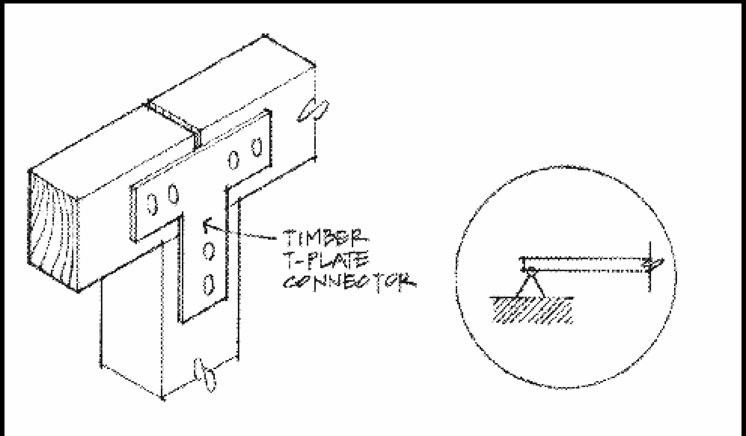
- *statically indeterminate*



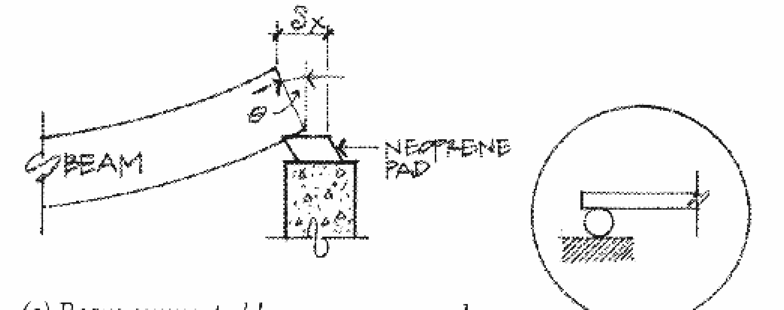
Beam Supports

- *in the real world, modeled type*





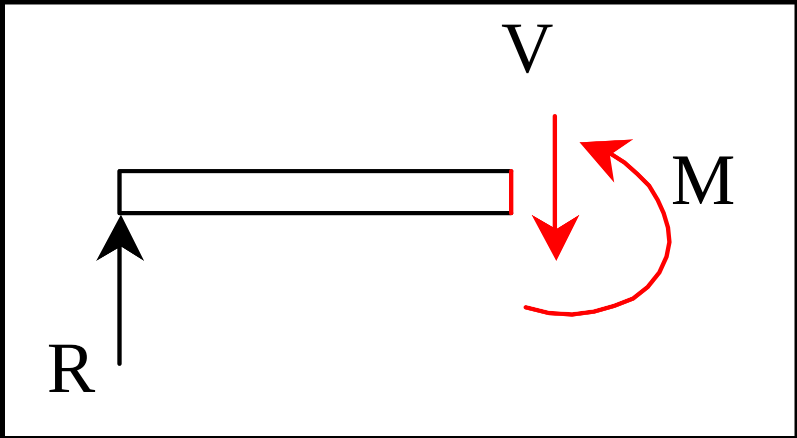
(c) Timber beam–column connection with T-plate.



(a) Beam supported by a neoprene pad.

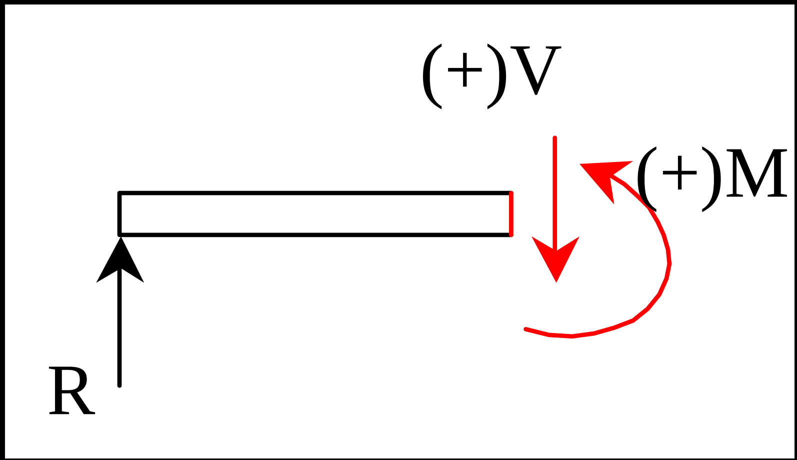
Internal Forces in Beams

- *like method of sections / joints*
  - *no axial forces*
- *section <u>must</u> be in equilibrium*
- *want to know where biggest internal forces and moments are for designing*



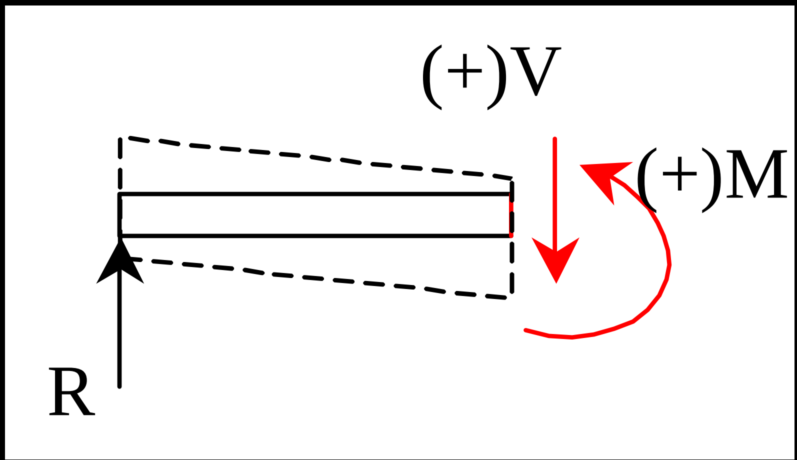
V & M Diagrams

- *tool to locate V_{max} and M_{max}*
- *<u>necessary</u> for designing*
- *have a <u>different sign convention</u> than external forces, moments, and reactions*

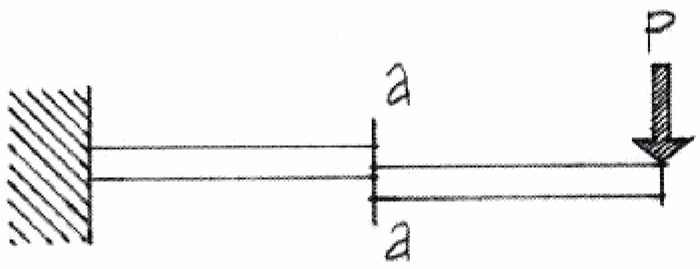


Sign Convention

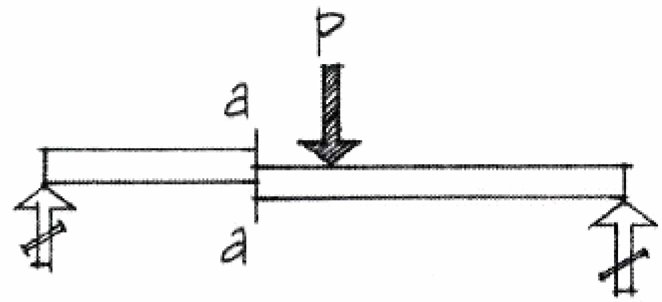
- *shear force, V:*
  - *cut section to LEFT*
  - *if $\sum F_y$ is positive by statics, V acts down and is POSITIVE*
  - *beam has to resist shearing apart by V*



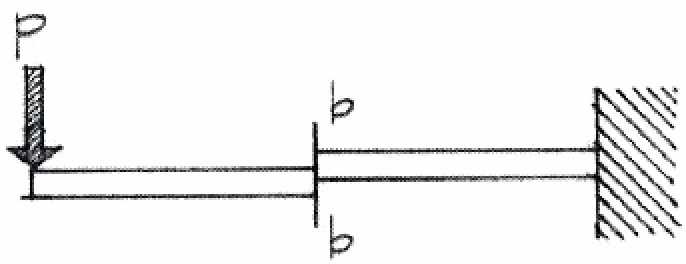
Shear Sign Convention



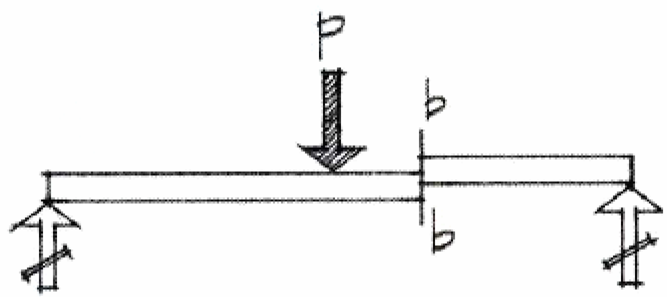
(+) *Shear.*



(+) *Shear.*



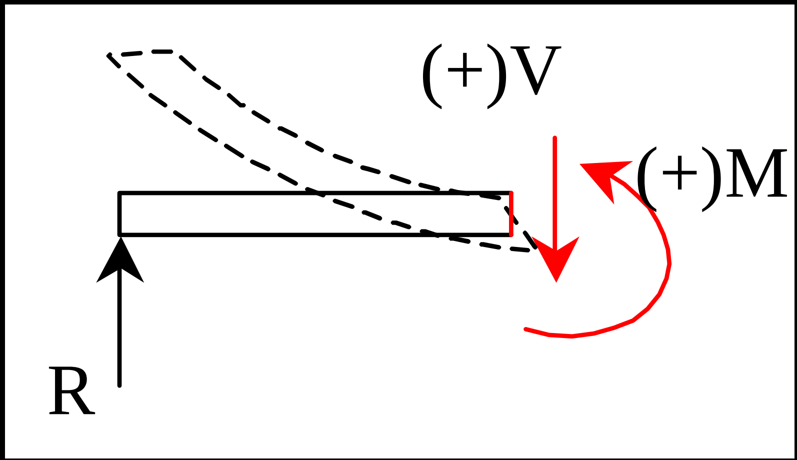
(–) *Shear.*



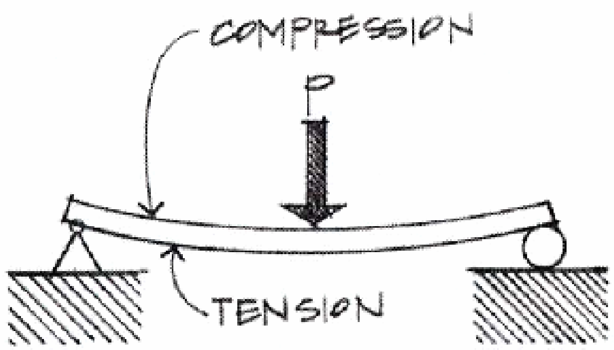
(–) *Shear.*

Sign Convention

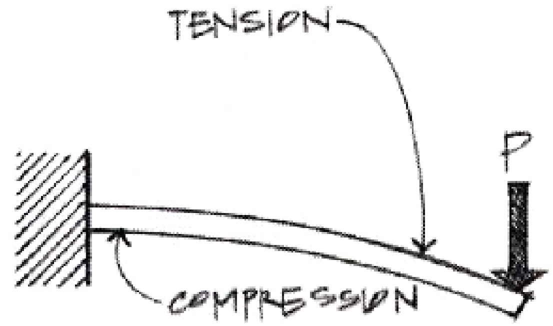
- *bending moment, M:*
  - *cut section to LEFT*
  - *if $\sum M_{cut}$ is clockwise, M acts ccw and is POSITIVE – flexes into a "smiley" beam has to resist bending apart by M*



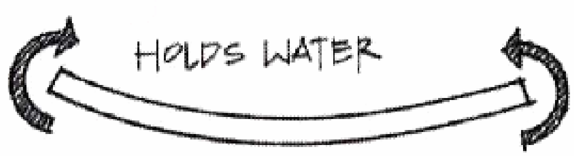
Bending Moment Sign Convention



(+) Moment.



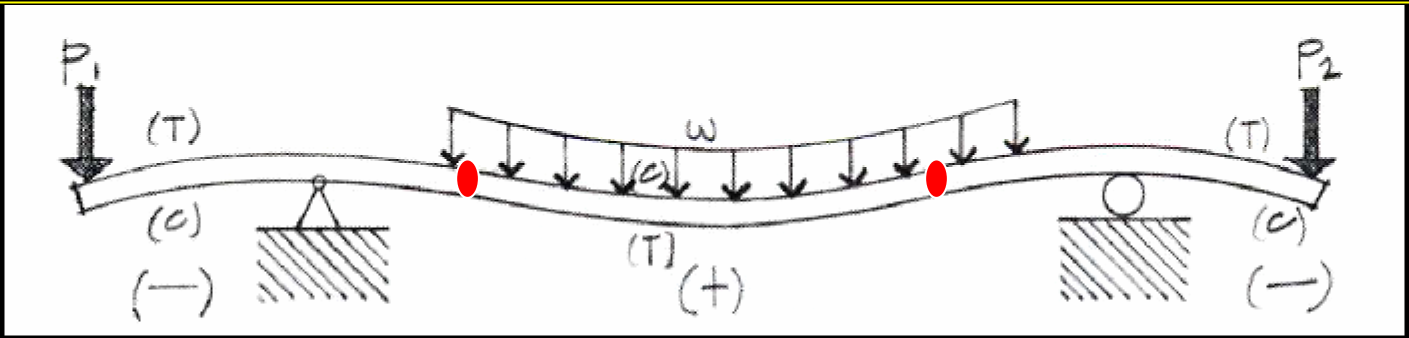
(–) Moment.



(+) Moment.

(–) Moment.

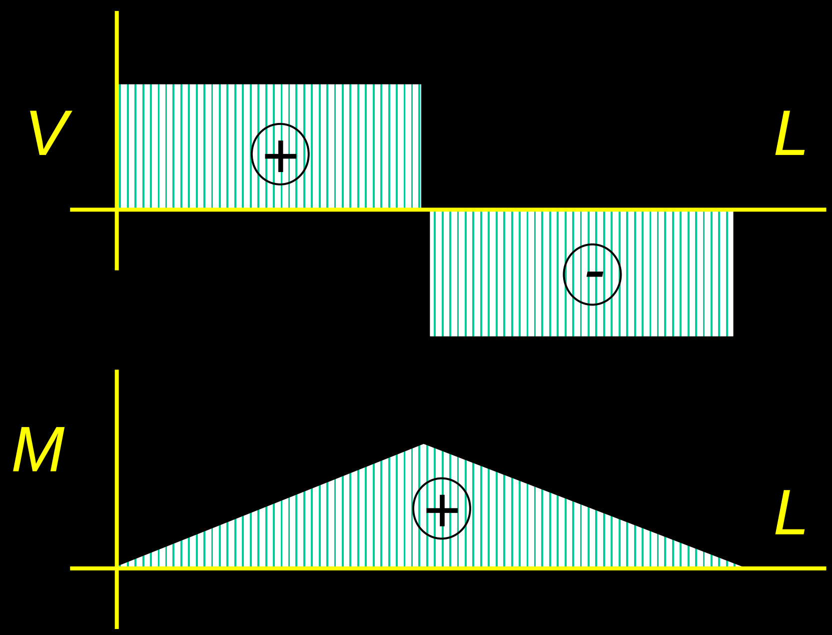
Deflected Shape

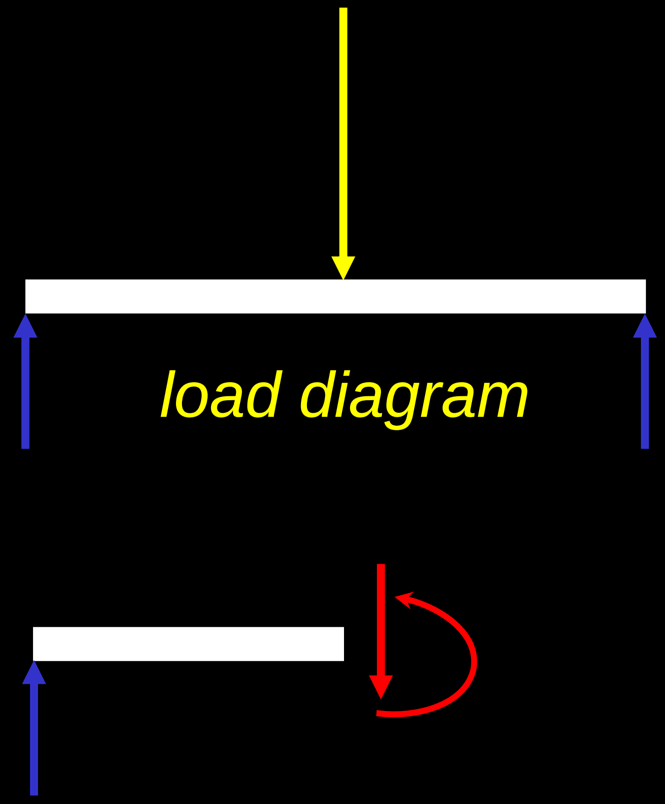


- *positive bending moment*
  - *tension in bottom, compression in top*
- *negative bending moment*
  - *tension in top, compression in bottom*
- *zero bending moment*
  - *inflection point*

Constructing V & M Diagrams

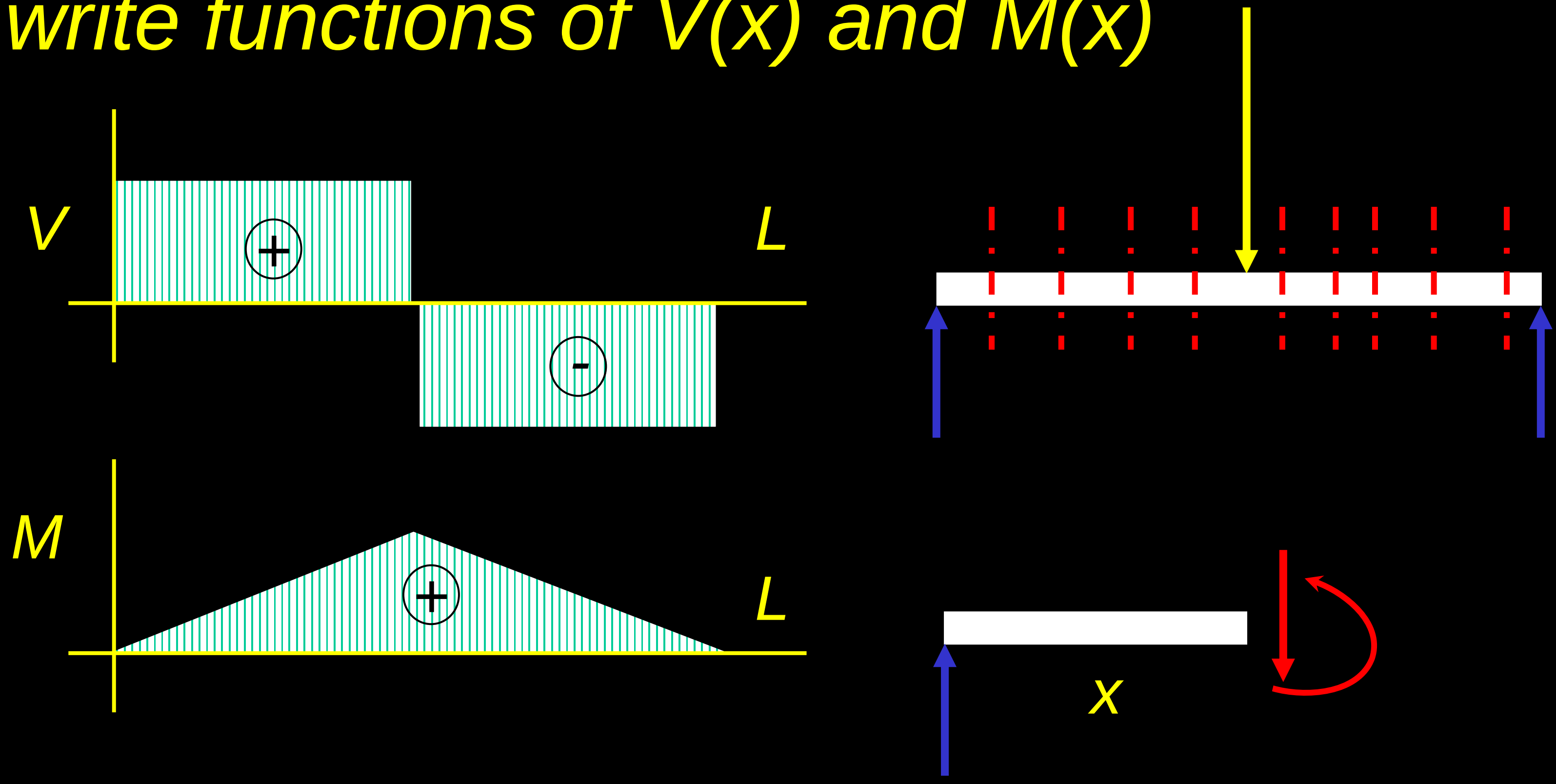
- *along the beam length, plot V, plot M*





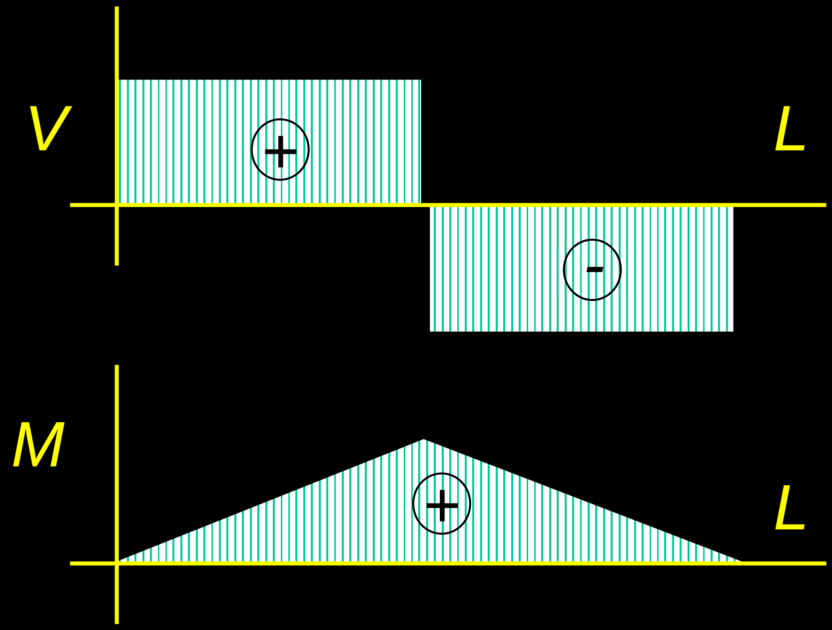
Mathematical Method

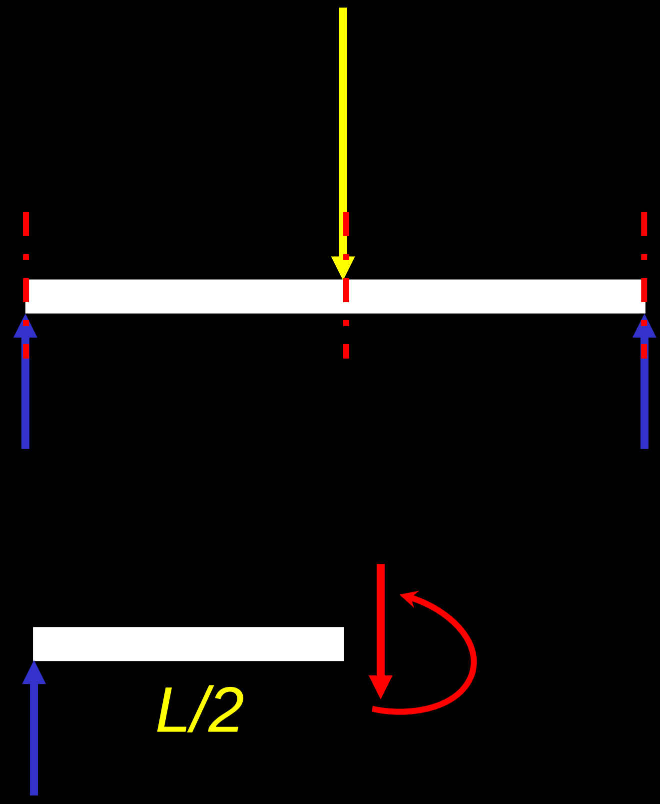
- cut sections with x as width
- write functions of V(x) and M(x)



Method 1: Equilibrium

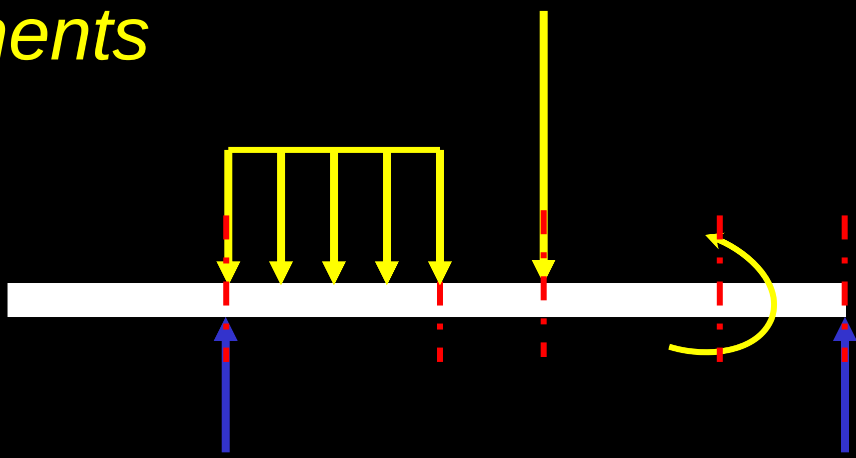
- cut sections at important places
- plot V & M





Method 1: Equilibrium

- *important places*
  - *supports*
  - *concentrated loads*
  - *start and end of distributed loads*
  - *concentrated moments*
- *free ends*
  - *zero forces*



Method 2: Semigraphical

- *by knowing*
  - *area under loading curve = change in V*
  - *area under shear curve = change in M*
  - *concentrated forces cause "jump" in V*
  - *concentrated moments cause "jump" in M*

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx \qquad M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Method 2

- *relationships*

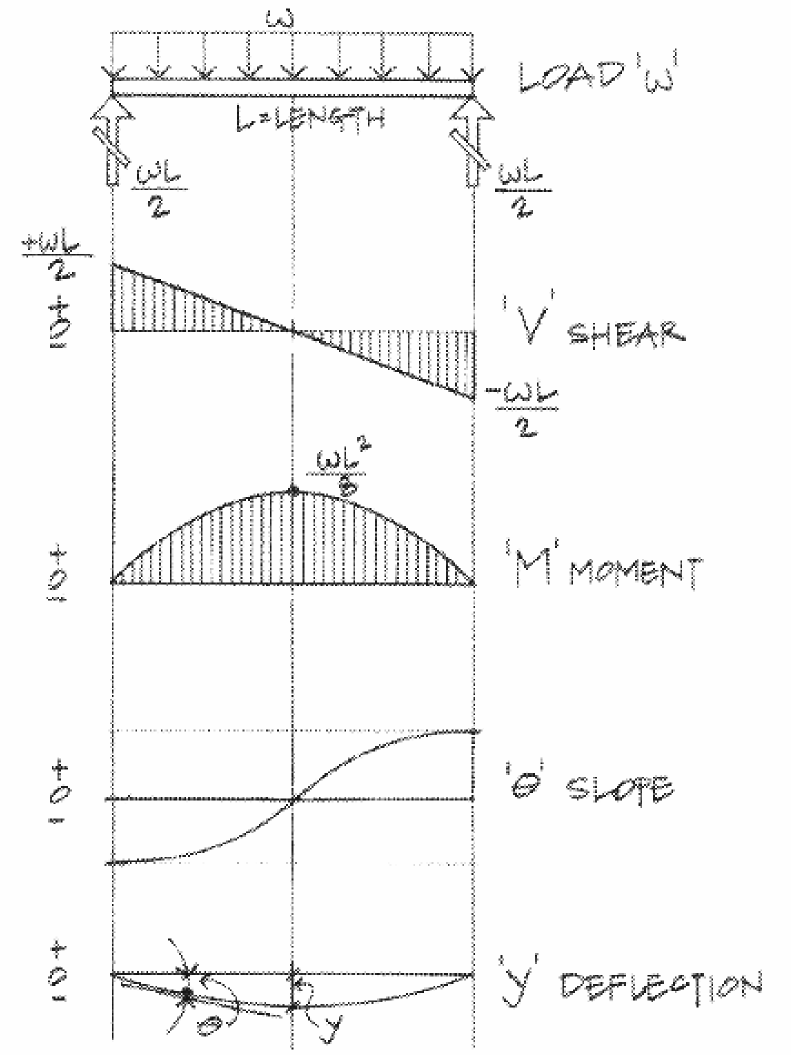
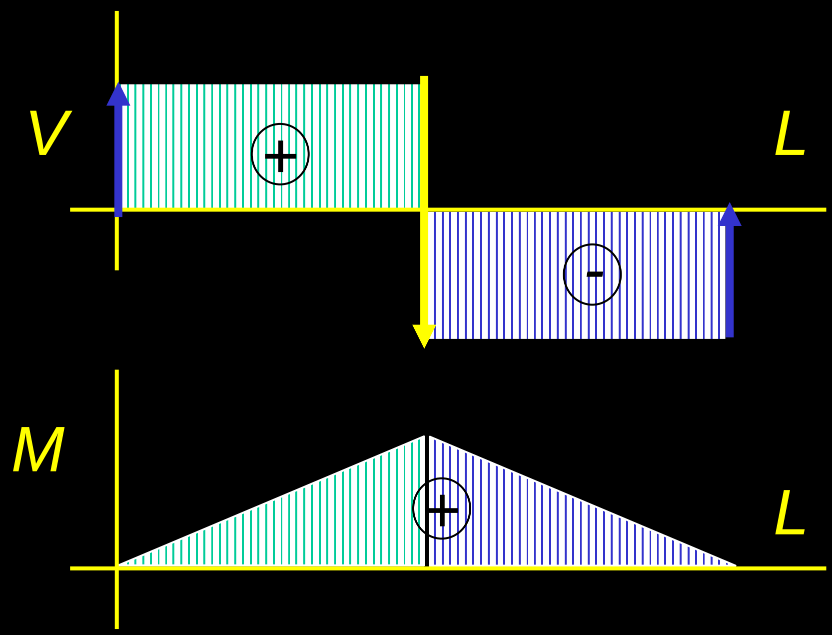


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

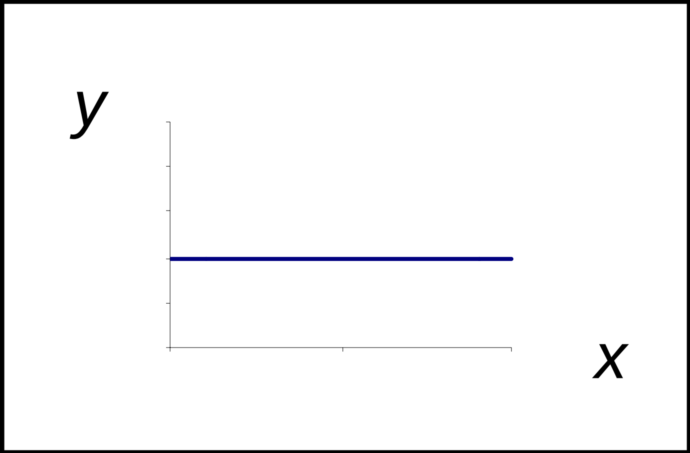
Method 2: Semigraphical

- M_{max} occurs where $V = 0$ (calculus)

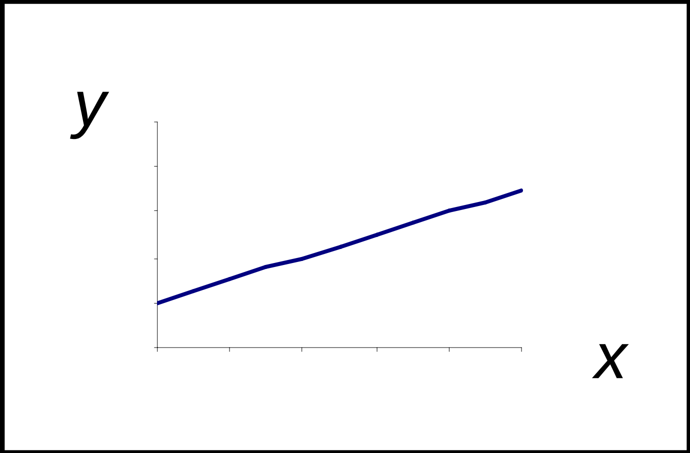


Curve Relationships

- *integration of functions*
- *line with 0 slope, integrates to sloped*



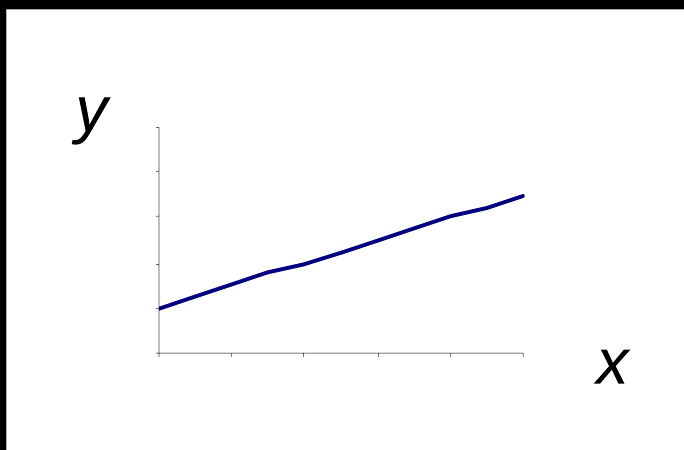
⇒



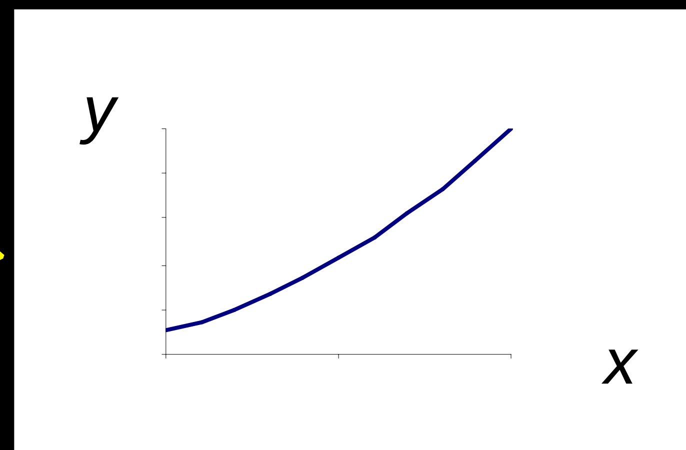
- *ex: load to shear, shear to moment*

Curve Relationships

- line with slope, integrates to parabola



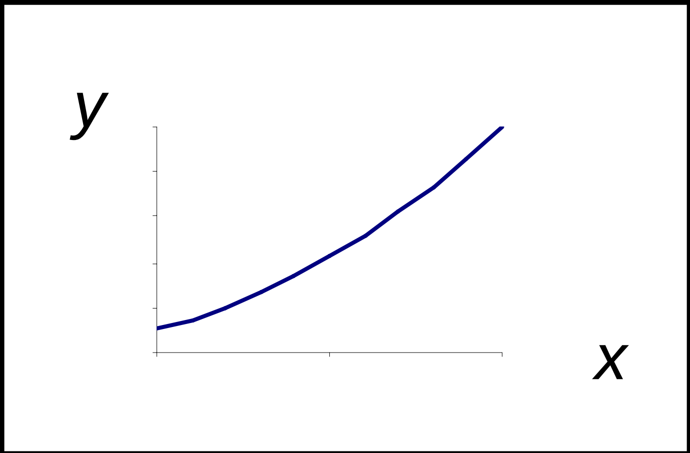
$\Rightarrow$



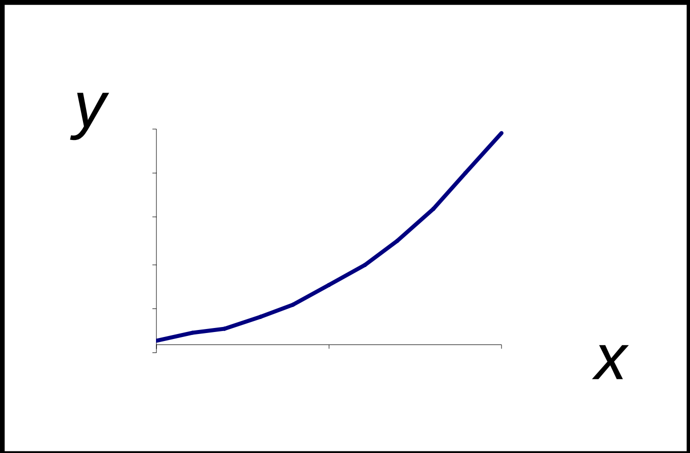
- ex: load to shear, shear to moment

Curve Relationships

- *parabola, integrates to 3rd order curve*



$\Rightarrow$



- *ex: load to shear, shear to moment*

Basic Procedure

1. Find reaction forces & moments

   Plot axes, underneath beam load diagram

V:

2. Starting at left
3. Shear is 0 at free ends
4. Shear jumps with concentrated load
5. Shear changes with area under load

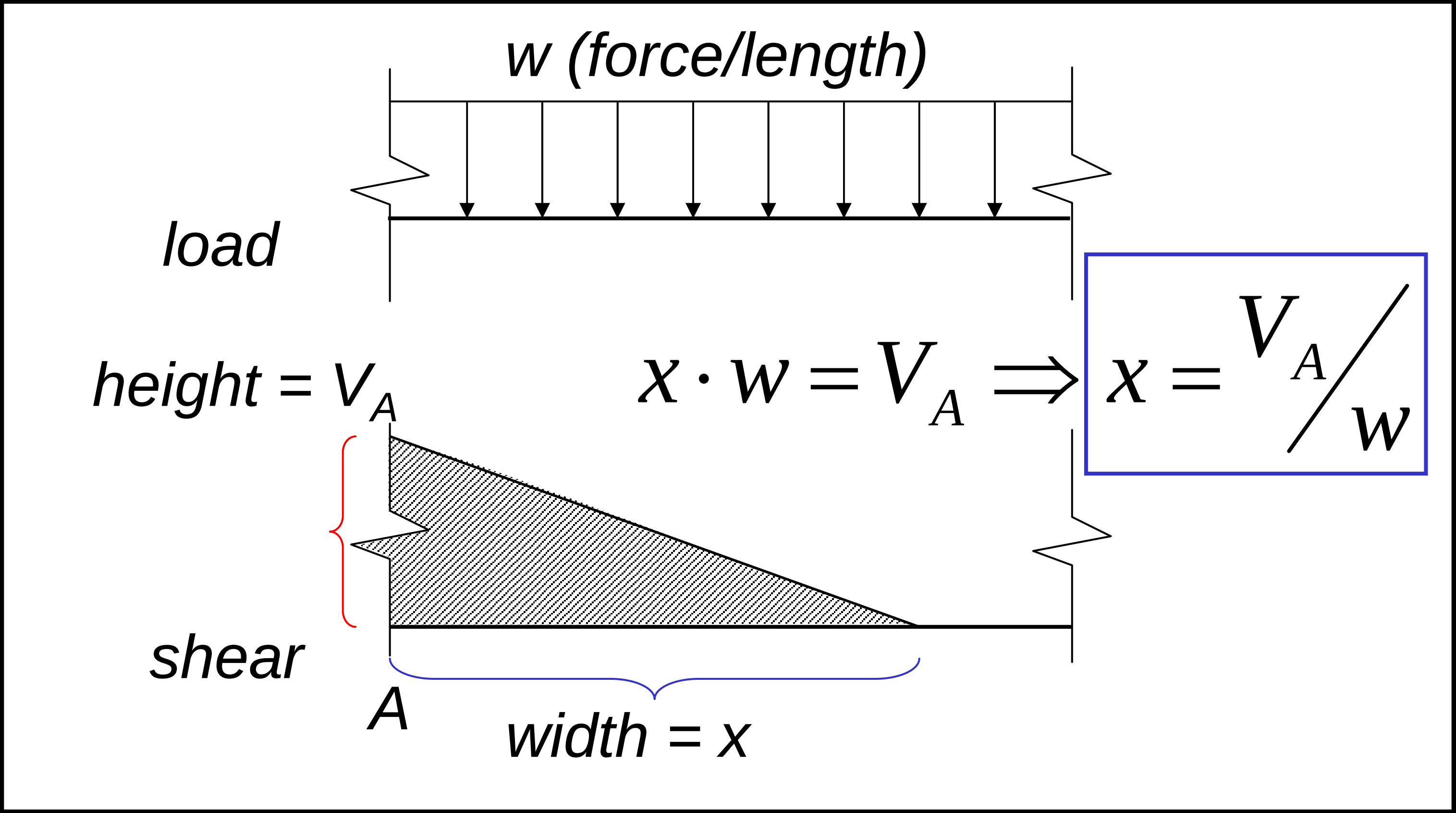
Basic Procedure

M:

6. *Starting at left*
7. *Moment is 0 at free ends*
8. *Moment jumps with moment*
9. *Moment changes with area under V*

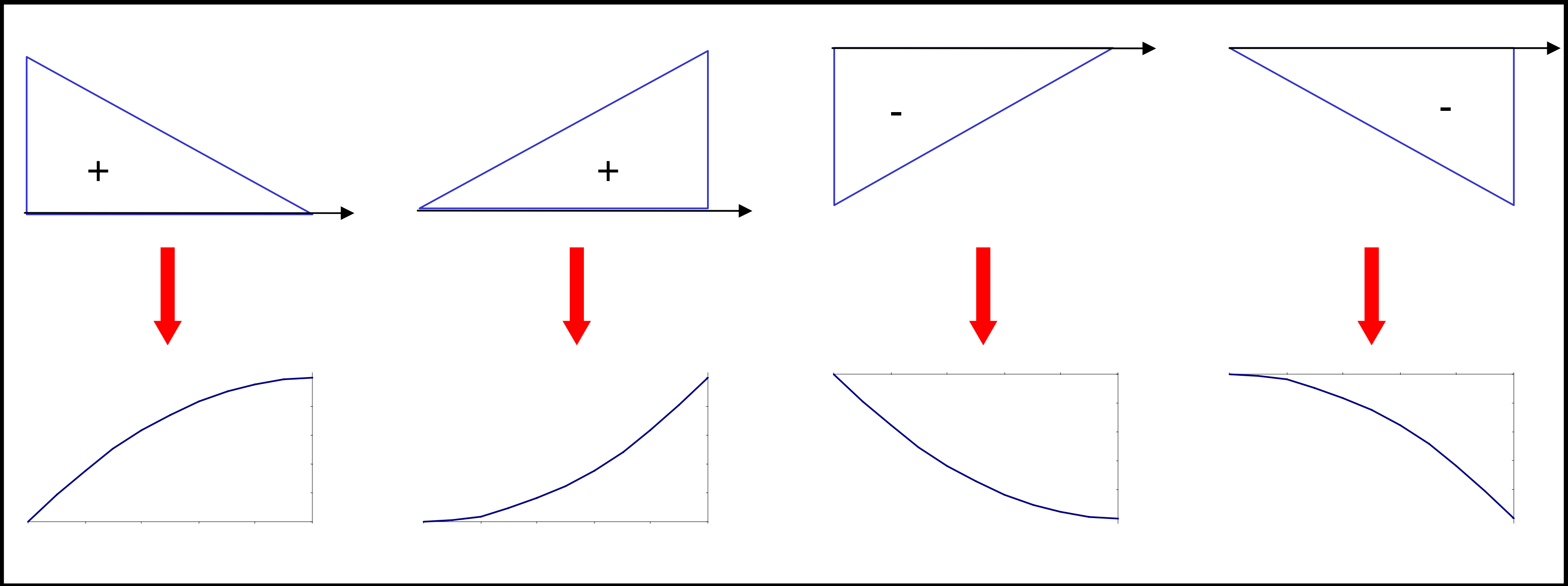
Triangle Geometry

- slope of V is w (-w:1)



Parabolic Shapes

- *cases*



up fast, then slow

up slow, then fast

down fast, then slow

down slow, then fast