Architectural Structures I: Statics and Strength of Materials

ENDS 231

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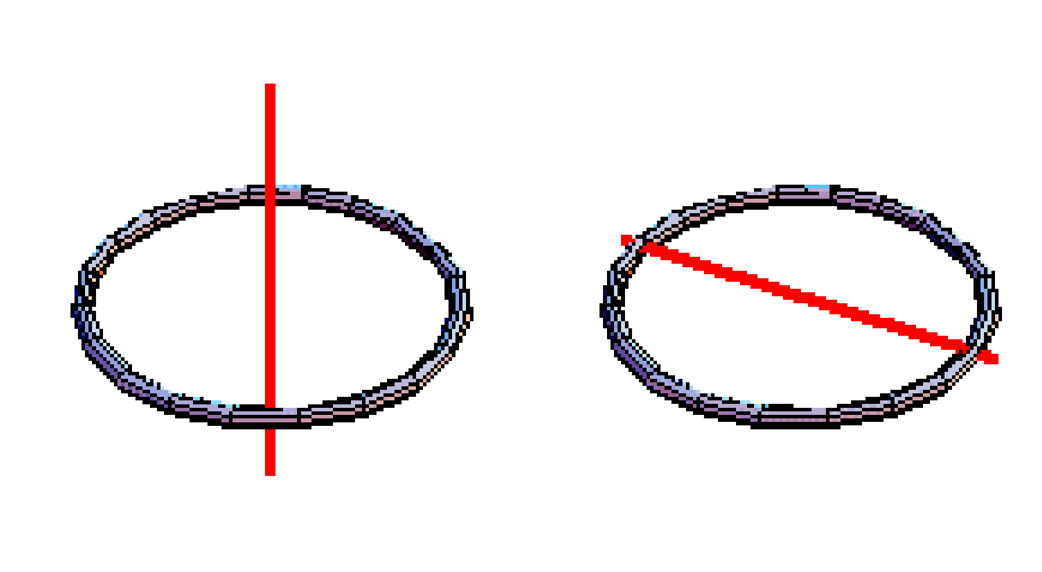
Summer 2006

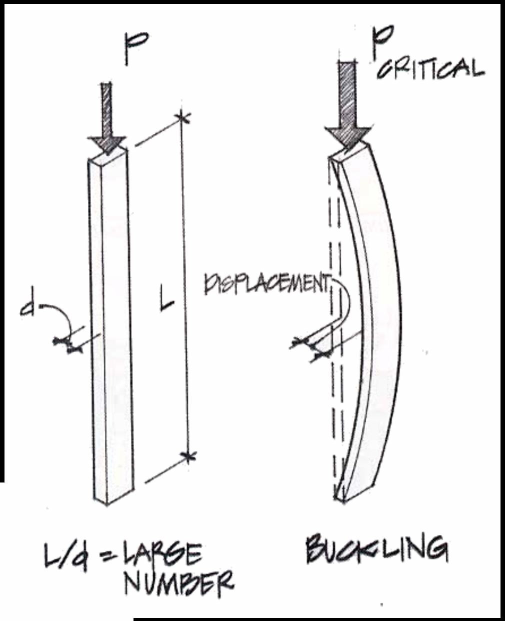
lecture
eleven

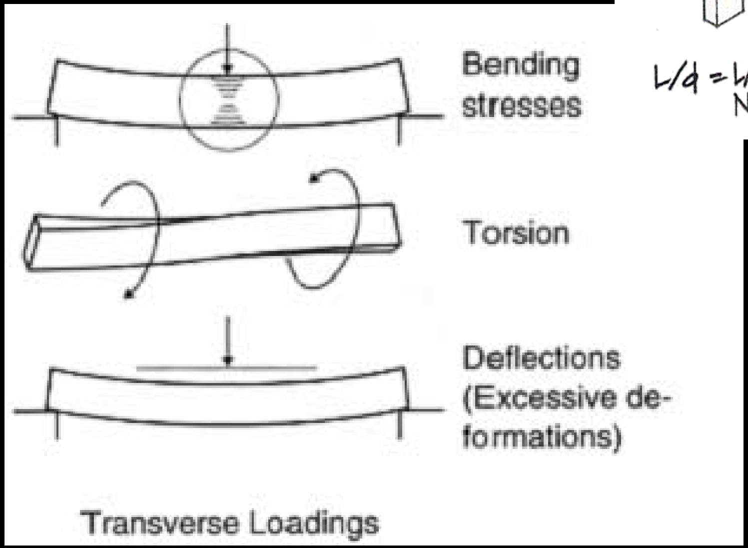


moment of inertia of an area

Moments of Inertia

- 2^{nd} moment area
  – math concept
  – area x $(distance)^2$
- need for behavior of
  – beams
  – columns



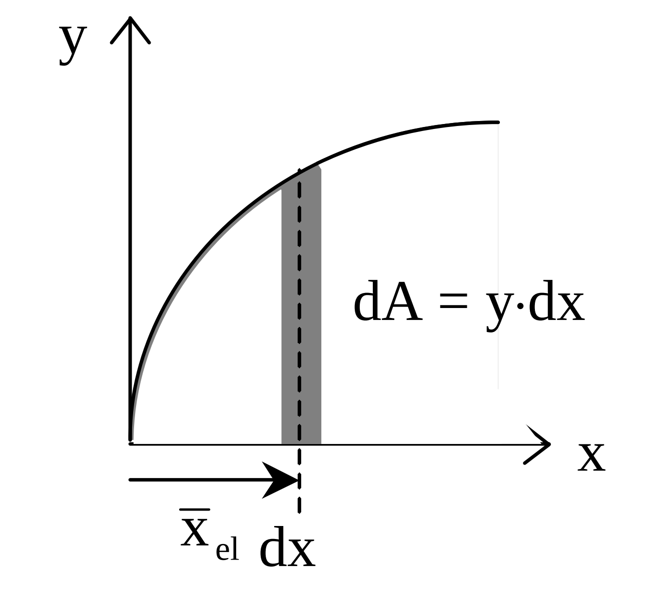


Moment of Inertia

- *about any reference axis*
- *can be negative*

$$I_y = \int x^2 dA$$

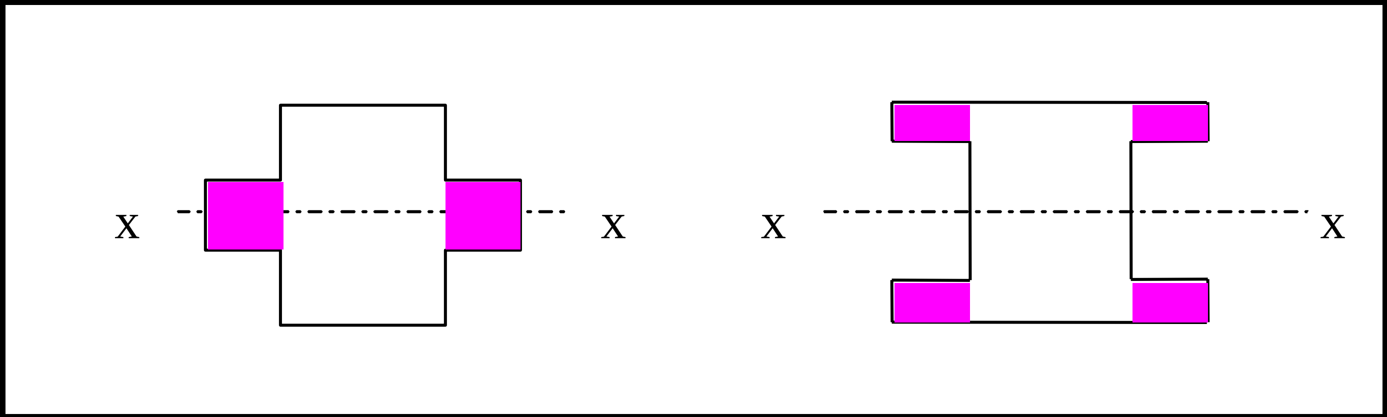
$$I_x = \int y^2 dA$$



- *resistance to bending and buckling*

Moment of Inertia

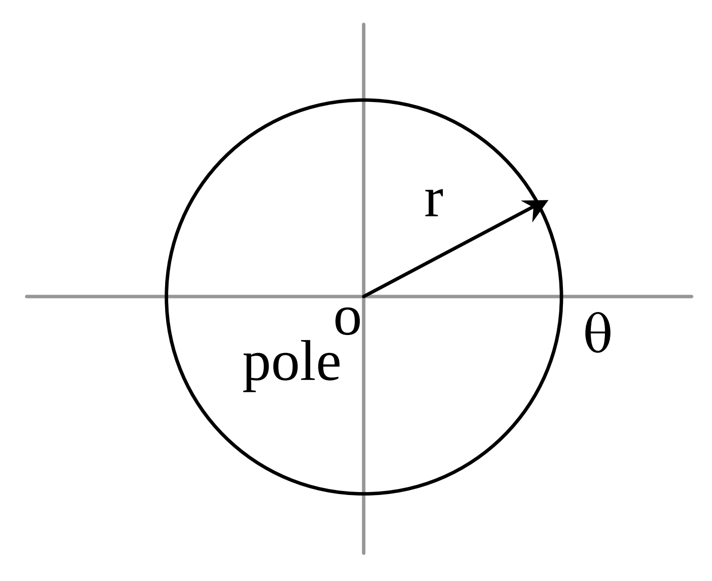
- *larger area away for same distance*
  - *larger I*



Polar Moment of Inertia

- *for round-ish shapes*
- *uses polar coordinates (r and θ)*
- *resistance to twisting*

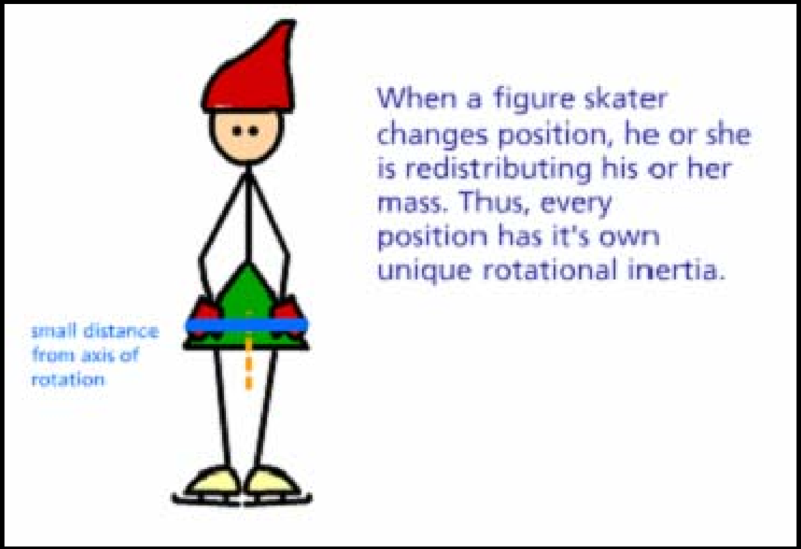
$$J_o = \int r^2 dA$$

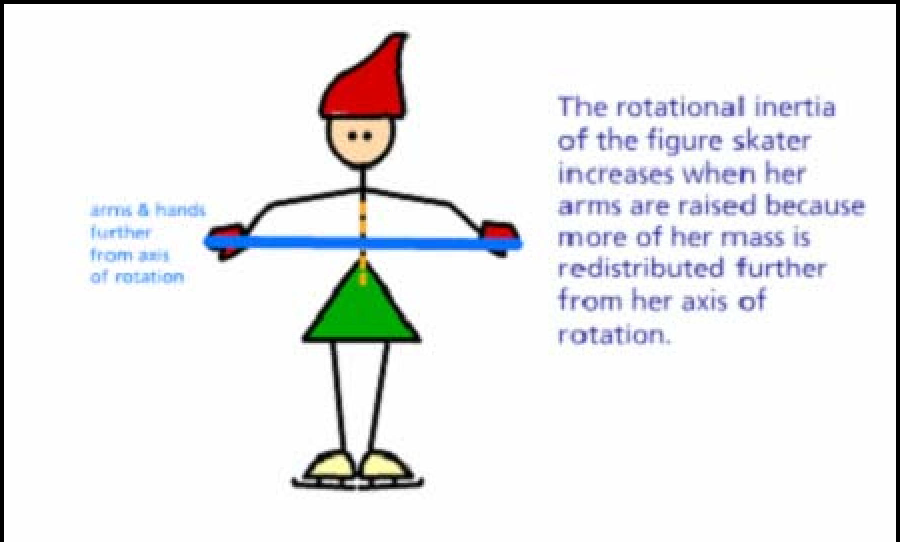


Radius of Gyration

- *measure of inertia with respect to area*

$$r_x = \sqrt{\frac{I_x}{A}}$$





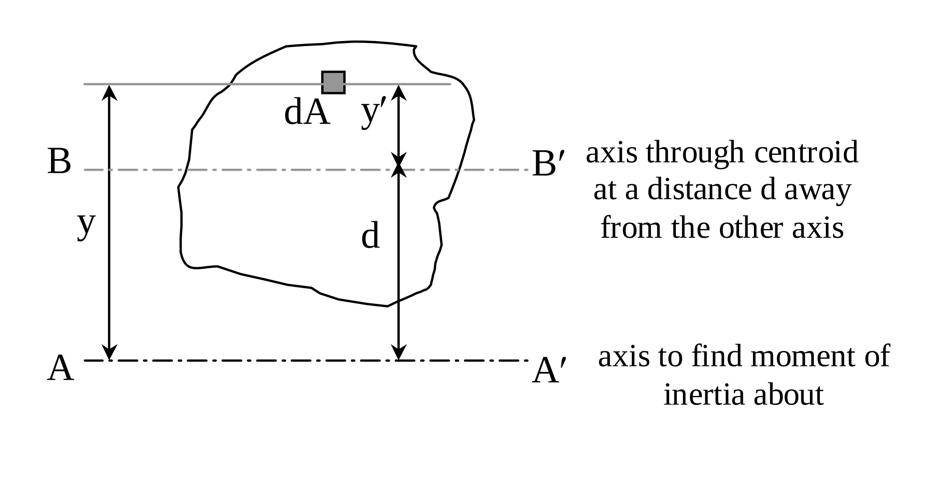
Parallel Axis Theorem

- can find composite I once composite centroid is known (basic shapes)

$$I_x = I_{cx} + Ad_y^2$$
$$= \bar{I}_x + Ad_y^2$$

$$I = \sum \bar{I} + \sum Ad^2$$

$$\boxed{\bar{I} = I - Ad^2}$$



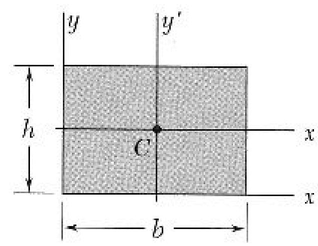
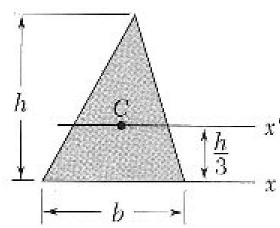
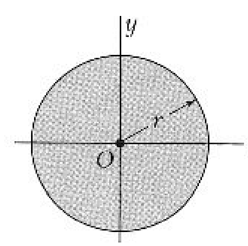
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with A, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$'s, d's, and Ad^2's
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$'s and Ad^2's

$$(d_x = \hat{x} - \bar{x})$$
$$(d_y = \hat{y} - \bar{y})$$

Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - x, y
  - x', y'
  - C

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$