# **Equilibrium of a Particle**

• EQUILIBRIUM is the state where the resultant of the forces on a particle is zero.

ex: 2 forces of same size, opposite direction

ex: 4 forces, polygon rule shows that it closes

- Analytically:  $R_x = \sum F_x = 0$   $R_y = \sum F_y = 0$  (scalar addition)  $M = \sum M = 0$  (always true when the forces run through the point)
- NEWTON'S FIRST LAW: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

#### Collinear Force System

- All forces act along the same line. Only one equilibrium equation is needed:  $\sum F_{(in-line)} = 0$
- Equivalently:  $R_x = \sum F_x =$ \_\_\_\_\_ and  $R_y = \sum F_y =$ \_\_\_\_\_ We know that \_\_\_\_\_ has to equal 0 for no rotation.

#### Concurrent Force System

• All forces act through the same point. Only two equilibrium equations are needed:

 $R_x = \sum F_x =$  \_\_\_\_\_ and  $R_y = \sum F_y =$  \_\_\_\_\_ We know that \_\_\_\_\_ has to equal 0 for no rotation.

- FREE BODY DIAGRAM (aka FBD): Sketch of a significant isolated particle of a body or structure showing all the forces acting on it. Forces can be from
  - externally applied forces
  - weight of the rigid body
  - reaction forces or constraints
  - externally applied moments
  - moment reactions or constraints
  - forces developed within a section member

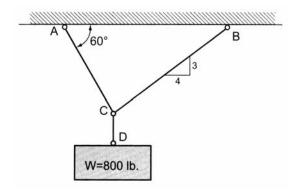
- How to solve when there are more than three forces on a free body:
  - 1. Resolve all forces into x and y components using known and unknown forces and angles. (Tables are helpful.)
  - 2. Determine if any unknown forces are related to other forces and write an equation.
  - *3.* Write the two equilibrium equations (in x and y).
  - 4. Solve the equations simultaneously when there are the same number of equations as unknown quantities. (see math handout)
- Common problems have unknowns of: 1)

2)

### Example 1 (pg 49)

#### Example Problem 3.1: Equilibrium of a Particle

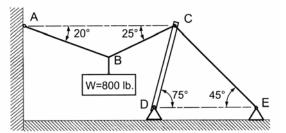
Two cables, shown in Figure 3.8, are used to support a weight W = 800 lb., suspended at concurrent point *C*. Determine the tension developed in cables *CA* and *CB* for the system to be in equilibrium. Solve this problem analytically and check the answer graphically.



# Example 2 (pg 56)

### Example Problem 3.5

A compound cable system supports a weight W = 800 lb. at point B, as shown in Figure 3.18. Cable BA is attached to a wall support at A and concurrent point C is supported by a compression strut DC. Determine all of the cable forces and the compression in strut DC.



• CABLES: have the same tension all along the length if they are not cut. The force *magnitude* is the same everywhere in the cable *even if it changes angles*. Cables CANNOT be in \_\_\_\_\_\_. (They flex instead.)

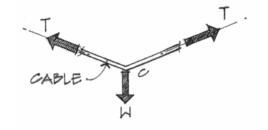
# • CABLE STRUCTURES:

*High-strength steel* is the most common material used for cable structures because it has a high strength to weight ratio.

Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

Cables with a single load have a \_\_\_\_\_\_ force system. It will only be in equilibrium if the cable is

The forces anywhere in a *straight segment* can be resolved into x and y components of  $T_x = T \cos \theta$  and  $T_y = T \sin \theta$ .



The shape of a cable having a *uniform distributed load* is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h(Lx - x^2) / L^2$$

where y is the vertical distance from the straight line from cable start to end

h is the vertical sag (maximum y)

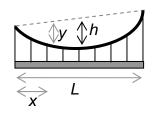
x is the distance from one end to the

location of y

L is the horizontal span.

 $L_{total} = L(1 + \frac{8}{3}h^2/L^2 - \frac{32}{5}h^4/L^4)$ 

where  $L_{total}$  is the total length of parabolic cable h and L are defined above.



# Example 3 (pg 55)

### Example Problem 3.4

Determine the maximum weight *W* that can be safely supported by the cable system shown in Figure 3.15 if cables *AB* and *BC* have a breaking strength of 500 pounds each.

