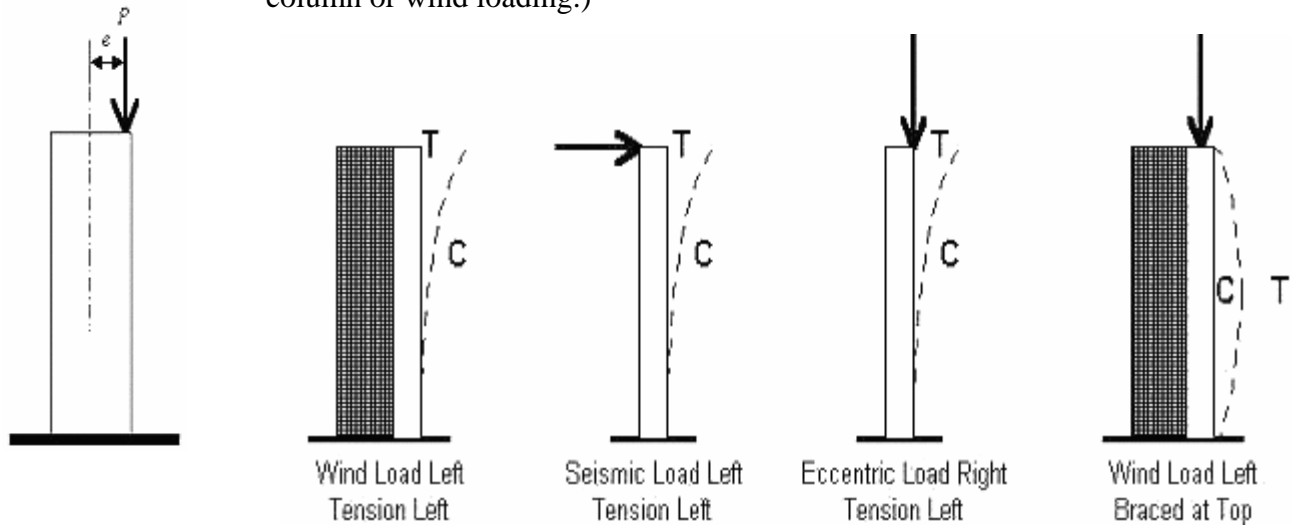


Column Design – Centric & Eccentric Loading

Loading Location

Centric loading: The load is applied at the centroid of the cross section. The limiting allowable stress is determined from strength (P/A) or buckling.

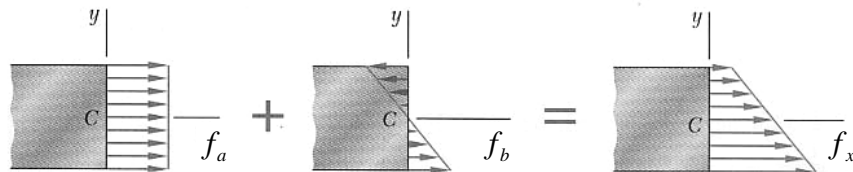
Eccentric loading: The load is offset from the centroid of the cross section because of how the beam load comes into the column. This offset introduces bending along with axial stress. (This can also happen with continuous beams across a column or wind loading.)



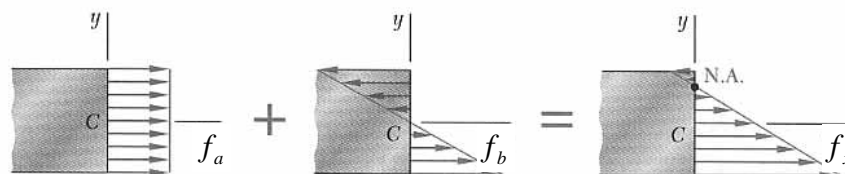
Eccentric Loading

The eccentricity causes bending stresses by a moment of value $P \times e$. Within the elastic range (linear stresses) we can *superposition* or add up the normal and bending stresses:

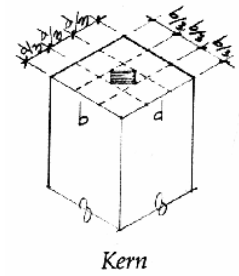
$$f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}$$



The resulting stress distribution is still *linear*. And the n.a. *moves* (if there is one).



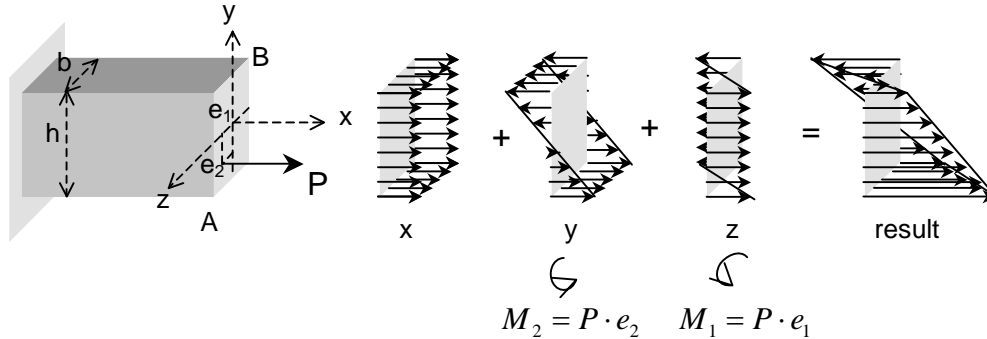
The value of e (or location of P) that causes the stress at an edge to become zero is at the edge of the **kern**. As long as P stays within the kern, there will *not* be any tension stress.



If there is bending in two directions (**bi-axial bending**), there will be one more bending stress added to the total:

$$f_x = f_a + f_{bx} + f_{by} = \frac{P}{A} + \frac{M_1 y}{I_z} + \frac{M_2 z}{I_y}$$

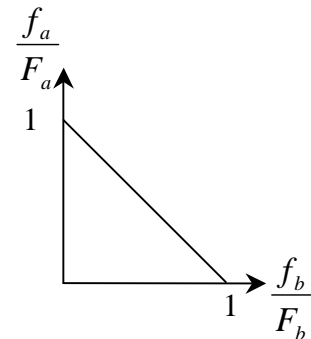
With P , M_1 , and M_2 :



Eccentric Loading Design

Because there are combined stresses, we can't just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the **interaction** diagram. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress = 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).

The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These type of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the **P-Δ (P-delta)** effect.



Limit Criteria Methods

- 1) $\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$ interaction formula (bending in one direction)
- 2) $\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$ interaction formula (biaxial bending)
- 3) $\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_b} \leq 1.0$ interaction formula (P-Δ effect)

Steel - ASD:

The modification factors are included in the form:
$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1.0$$

where:

$1 - \frac{f_a}{F'_e}$ = magnification factor accounting for P- Δ

$F'_e = \frac{12\pi^2 E}{23(KL/r)^2}$ = allowable buckling stress

C_m = modification factor accounting for end conditions,
 = $0.6 - 0.4 (M_1/M_2)$ where M_1 and M_2 are the end moments and $M_1 < M_2$.
 M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature
 = 0.85, members in frames subject to joint translation (sidesway)

Wood: - NDS:

The modification factors are included in the form:
$$\left[\frac{f_c}{F'_c}\right]^2 + \frac{f_{bx}}{F'_{bx} \left[1 - f_c / F_{cEx}\right]} \leq 1.0$$

where:

$1 - \frac{f_c}{F_{cEx}}$ = magnification factor accounting for P- Δ

F'_{bx} = allowable bending stress

Steel – LRFD:

The modification factors are included in the form for two conditions.

$$\text{For } \frac{P_u}{\phi_c P_n} \geq 0.2: \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

$$\text{For } \frac{P_u}{\phi_c P_n} < 0.2: \quad \frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

where:

$\phi_c = 0.85$ for compression

$\phi_b = 0.90$ for flexure

Column Design Methodology

In order to design an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
 - buckling
 - axial stress
 - combined stress
2. See if we can find values for r or A or $S (=I/c_{max})$
3. Pick a trial section based on if we think r or A is going to govern the section size.
4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.
5. Did the section pass the stress test?
 - If not, do you *increase* r or A or S ?
 - If so, is the difference really big so that you could *decrease* r or A or S to make it more efficient (economical)?
6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Example 1

Example 12.10.1

Investigate the acceptability of a W16×67 used as a column under the loading shown in Fig. 12.10.1. Steel is A992.

$F_b = 27.8$ ksi.

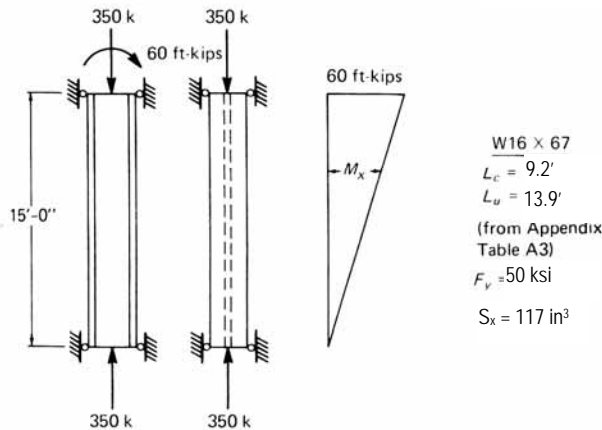


Fig. 12.10.1 Beam-column for Example 12.10.1.

SOLUTION

(a) Column effect.

$$\frac{KL}{r_y} = \frac{15(12)}{2.46} = 73$$

$$F_a = 20.38 \text{ ksi}$$

$$f_a = \frac{P}{A} = \frac{350}{19.7} = 17.8 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{17.8}{20.38} = 0.873$$

(b) bending

$$C_m = 0.6 - 0.4(M_1/M_2) = 0.60$$

$$f_b = \frac{60(12)}{117} = 6.15 \text{ ksi}$$

$$\frac{C_m f_b}{F_b} = \frac{0.6(6.15)}{27.8} = 0.13$$

(c) Moment magnification.

$$\frac{KL}{r_x} = \frac{15(12)}{6.96} = 25.9; \quad F'_e = \frac{12\pi^2(30000)}{23(25.9)^2} = 230 \text{ ksi}$$

where the x -axis is the axis of bending.

$$\frac{1}{1 - f_a/F'_e} = \frac{1.0}{1 - 17.8/230} = 1.084$$

(d) Check of AISC Formulas:

For stability, Formula (1.6-1a),

$$\frac{f_a}{F_a} + \frac{C_m f_b}{F_b} \left(\frac{1.0}{1 - f_a/F'_e} \right) =$$

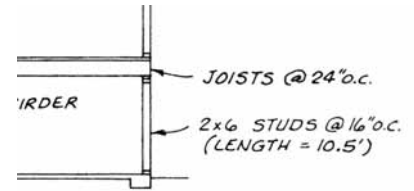
$$0.873 + 0.13(1.084) = 1.01 \approx 1.0$$

The W16×67 is acceptable for the given loading.

Example 2

EXAMPLE 7.16 Combined Bending and Compression in a Stud Wall

Check the 2 × 6 stud in the first-floor bearing wall in the building shown in Fig. 7.20a. Consider the given vertical loads and lateral forces. Lumber is No. 2 DF-L. $MC \leq 19$ percent and normal temperatures apply. Allowable stresses are to be in accordance with the NDS. $F'_b = 2152$ psi $F_c = 1350$ psi



$$A = 8.25 \text{ in}^2$$

$$S_x^* = 7.56 \text{ in}^3$$

COLUMN CAPACITY:

Sheathing provides lateral support about the weak axis of the stud. Therefore, check column buckling about the x axis only ($L = 10.5$ ft and $d_x = 5.5$ in.):

$$\left(\frac{l_e}{d}\right)_y = 0 \quad \text{because of sheathing}$$

$$\left(\frac{l_e}{d}\right)_{\max} = \left(\frac{l_e}{d}\right)_x = \frac{10.5 \text{ ft} \times 12 \text{ in./ft}}{5.5 \text{ in.}} = 22.9$$

$$E = 1,600,000 \text{ psi}$$

For visually graded sawn lumber:

$$K_{cE} = 0.3$$

$$c = 0.8$$

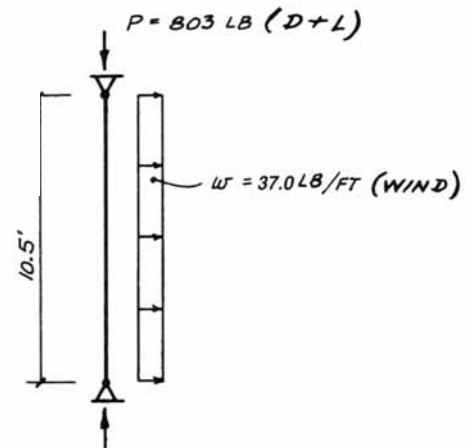
$$F_{cE} = \frac{K_{cE} E'}{(l_e/d)^2} = \frac{0.3(1,600,000)}{(22.9)^2} = 915 \text{ psi}$$

$$F_c^* = F_c(C_D) \quad C_D = 1.6 \text{ from wind loading}$$

$$= 1350(1.6) = 2376 \text{ psi}$$

$$\frac{F_{cE}}{F_c^*} = \frac{915}{2376} = 0.385 \quad C_p = 0.36$$

$$F'_c = F_c(C_D)(C_p) = 2376(0.36) = 855 \text{ psi}$$



Load Case 2: Gravity Loads + Lateral Forces

BENDING:

Wind governs over seismic. Force to one stud:

$$\text{Wind} = 27.8 \text{ psf}$$

$$w = 27.8 \text{ psf} \times \frac{16 \text{ in}}{12 \text{ in/ft}} = 37.0 \text{ lb/ft}$$

$$M = \frac{wL^2}{8} = \frac{37.0(10.5)^2}{8} = 510 \text{ ft-lb} = 6115 \text{ in.-lb}$$

AXIAL:

$$f_b = \frac{M}{S} = \frac{6115}{7.56} = 809 \text{ psi} \quad F'_b = 2152 \text{ psi}$$

$$D + W: f_c = \frac{P}{A} = \frac{378}{8.25} = 46 \text{ psi}$$

COMBINED STRESS:

The simplified interaction formula from Example 7.13 (Sec. 7.12) applies:

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{bx}}{F'_{bx}(1 - f_c/F_{cEx})} \leq 1.0$$

$$F_{cEx} = F_{cE} = 915 \text{ psi}$$

D + W:

In this load combination, D produces the axial stress f_c and W results in the bending stress f_{bx} .

$$\left(\frac{f_c}{F'_c}\right)^2 + \left(\frac{1}{1 - f_c/F_{cEx}}\right) \frac{f_{bx}}{F'_{bx}} =$$

$$\left(\frac{46}{855}\right)^2 + \left(\frac{1}{1 - 46/915}\right) \frac{809}{2152} = 0.399 < 1.0$$

2 × 6 No. 2 DF-L exterior bearing wall OK