

## ARCH 631. Topic 2 Reading Notes

- Traditional engineering topics separate out into:
  - mechanics (forces and motion), covering equilibrium (state of balance)
    - statics – specific to equilibrium
    - dynamics – specific to bodies in motion
  - strength of materials, addressing relationship between applied forces and internal effects, such as deformation
- Chapter only provides an overview, and refers to detailed texts for more info
- Mobile example has external forces from the weights, internal forces or reactions within the structure and is in equilibrium defined by the sum of forces equal to zero as well as the sum of moments ( $F \times d$ ) equal to zero.
- Equilibrium or free-body diagrams are analytical drawings with the force system on the geometry used to determine reaction values.
- Knowing reactions and internal forces, stresses (unit force) and strains (deformation) can be determined
- Force has magnitude and sense of direction and is a vector, typically represented by a line (arrow) with scaled length, with line of action.
- Can be transferred to any location on that line when acting on a rigid body
- A scalar quantity has magnitude and no directions
- Net result of several vector forces (resultant) can be found by vector addition methods:
  - parallelogram law – find the diagonal (graphical)
  - “tip-to-tail” – find the final path (graphical)
  - summation of components (analytical)
- The two forces with equivalent resultant to a force are called components – typically in Cartesian coordinates ( $x$  &  $y$ )
- With 90 degree angle between components, trig functions from a right triangle can be used to define the components, the hypotenuse is the magnitude of the resultant, and the angle can be found from sin, cos or tan
- Statically equivalent systems have forces that result in the same motion – translation and rotation – which means resultant along the same line of action
- Concurrent force systems act through the same point; non-concurrent don't
- Moment of a force is the tendency to cause the body to rotate, with size defined by  $M$  times perpendicular distance ( $r$  or lever arm)
- Moment from a distributed load is found by locating the equivalent total force  $wL$
- Moments due to multiple forces can be added
- Moment due to parallel forces with same size in opposite direction is called a moment couple and is independent of the reference point selected as moment center

- Equilibrium exists when a force system causes no translation or rotation; resultant is zero and forces are concurrent on a point; sum of x component forces = 0, sum of y component forces = 0, sum of z component forces = 0, sum of moments = 0
- Equilibrant is a force of opposite direction and equal size to a resultant that is applied to put a force system into equilibrium
- Positive forces are to the right and up, negative forces are to left and down for equilibrium
- Positive moment is a tendency to rotate counter clockwise (by text), other way is negative (for equilibrium)
- Two-force members (or bodies) can only be in equilibrium if the forces are collinear and the line of action is through the two points the forces are at
- Three-force members (those with **exactly** three forces by this text) will have concurrent forces (acting through a common point)
- Applied forces act on a structure
- Reactive forces are generated by the action of one body on another usually at connections or supports
- Reactive forces will maintain equilibrium and must be included in free-body diagrams
- Support conditions determined the type of reactive forces present:
  - pinned connections – have force reaction in any direction
  - roller connection – has force reaction only perpendicular to the face of the support
  - fixed connection – has force reaction in any direction and moment reaction
- To ensure stability there must be sufficient restraint from the support to satisfy all three equilibrium equations
- Statically indeterminate structures have more restraint and can't be solved using equilibrium equations because there are too many unknown reactive forces/moments
- Principle of Superposition: with multiple loadings, the reactions can be found from each individual load and added together to get an equivalent reaction
- Overturning results when the moment (typically from a force) is larger than a resisting moment (commonly from the force due to weight) and the body rotates about a point
- Fixed-end connections (the type on cantilever beams) fully restrain a body and have a resisting moment reaction; multiple fixed-ends make a structure statically indeterminate
- Cable support have reactive forces along the direction of the cable and are called tensile (because the cable can't be compressed) and are internal forces
- Internal forces are moments are typically in tension, compression, shear, or bending; are rarely constant throughout a structure
- Axial forces are along the axis of a member and are tensile or compressive
- Shear and bending moment develop in a member subjected to transverse loading (perpendicular to the axis) and are internal resisting forces and moment to keep the section of body in equilibrium
- Shear and moment diagrams are plots of the value of the internal shear and moment across all sections; where the diagram peaks are critical locations (members are sized for these values)

- Positive bending moment is associated with tension on the bottom and compression on the top
- Negative bending moment is associated with compression on the bottom and tension on the top
- Method to construct diagrams involves cutting a “section” and for the left-hand section, the sum of all the forces “down” will be resisted by an upward (positive) shear while the sum of the moments about the cut will be resisted by a counterclockwise (positive) bending moment (associated with an upward curving-concave deformation)
- *Positive moment values can be plotted on the “up” or compressive side, or plotted on the “down” or tension side which corresponds to the deflected shape*
- Principle of Superposition applies to shear and bending moment diagrams: with multiple loadings, diagram values can be found from each individual load and added together (for the same location) to get an equivalent diagram
- Concentrated loads (without distributed loads) result in constant shears across sections of the diagram while the moment diagrams vary linearly over those sections (sloped)
- Uniformly distributed loads result in sloped shear diagrams and the moment diagrams are parabolic curves
- Maximum (or minimum) bending moment occurs where shear is zero
- Deflected shape can be sketched knowing the curving direction associated with bending moments and that the curvature changes when the bending moment is zero (called point of inflection)
- Change in shear is defined as the change in load (integral sign) between sections, change in bending moment is defined as the change in shear (integral sign) between sections
- Stress is force intensity per unit area and can be tensile, compressive, shear, or bending
- Tension or compression forces result in normal or axial stress that is uniform across a section and defined by force divided by area (represented by  $f$ )
- Prestressing is a technique to pre-compress a system so that more loads can be applied before tensile stresses occur under load or to reduce the cross section size (reduced stresses); common for beams
- The capacity of a material to resist stresses up to a failure or other limit stress (like yielding) is commonly referred to as strength
- Allowable stress is a limiting stress for design that takes the limit stress and divides by a factor of safety (bigger than 1.0)
- Mechanical properties of materials refer to the stress and strain behavior of various materials and constant or critical values for those materials
- Elastic behavior for a material is a proportional increase in strain with increase of stress – unloading means the member returns to a state of zero strain (original length)
- Plastic behavior for a material is an increase in strain without a significant increase in stress – unloading will result in permanent deformation (strain)
- Steel has both an elastic range and plastic range; concrete doesn't
- Strain is defined as change in size or shape with respect to the original size or shape (dimensionless)
- Hooke's law defines the relationship of stress divided by strain as a constant = modulus of elasticity (E) in the elastic range (linear or constant slope in the stress vs. strain plot found experimentally)

- Each material has a unique value for modulus of elasticity (material property or constant)
- End of elastic range is termed the proportional limit (which is not always obvious or existent)
- When there is a strain in one direction, there will be a negative strain in an orthogonal direction; the fraction of the strains is called the Poisson's ratio (also a material constant)
- For steel in the plastic range more stress can be resisted (strain hardening) up to an ultimate strength after which the stress drops off and rupture occurs; the cross section drastically reduces as well and is called "necking"
- Ductile materials have plastic ranges (like steel); characteristic load-deformation curves have sloped section, then fairly flat section
- Brittle materials have elastic ranges and rupture with little evident deformation (like glass); characteristic load-deformation curves have sloped sections
- Ductile behavior is preferred for structural design because the plastic range holds some "reserve" strength (although deforms permanently)
- Allowable strength design is conservative because the limit is well below the proportional limit
- If the rate of loading (how fast the load is added) increases, many materials start behaving more like brittle materials
- If the temperature decreases, ductile materials can also behave like brittle materials (the Liberty ships of world war II cracked due to brittle behavior in cold water)
- Creep is defined as deformation under load due to duration (constant stress level). Concrete and timber exhibit this behavior.
- Fatigue is a failure due to repeated loading under reverse cycles (tension to compression or bending back and forth) and is unrelated to the stress-strain behavior.
- Flaws, cracks (or sharp geometries) are locations for high stresses referred to as concentrated stresses; cracks can propagate rapidly in brittle materials
- Deformations in the elastic range can be related by the modulus of Elasticity (Hooke's law) and the definition of strain =  $fL/E$