APPLIED ARCHITECTURAL STRUCTURES:
STRUCTURAL ANALYSIS AND SYSTEMS
ARCH 631
DR. ANNE NICHOLS
FALL 2013

lecture
four

design methods & beams


Forum, Pompeii

Allowable Stress Design

- historical method
- a.k.a.
  working stress,
  stress design
- stresses stay in ELASTIC range

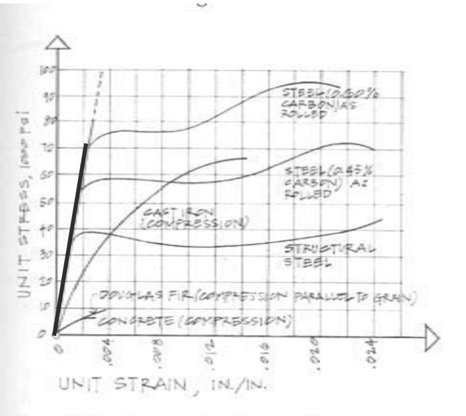

Figure 5.20 Stress-strain diagram for various materials.

Allowable Stress Design

- codes
  - wood
    - National Design Specification
    - Manual of Timber Construction (glulam)
  - masonry
    - Masonry Specification Joint Code
  - steel
    - Steel Joist Institute
    - American Institute of Steel Construction


www.thfrc.gov

Limit State Design

- stresses go to limit (strain outside elastic range)
- loads may be factored
- resistance or capacity reduced by a factor
- based on material behavior
- “state of the art”

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Limit State Design

- codes
  - – wood
    - National Design Specification
  - – masonry
    - Masonry Specification Joint Code
  - – concrete
    - American Concrete Institute
    - Precast & Prestressed Concrete
  - – steel
    - American Institute of Steel Construction

Reinforced Concrete Design

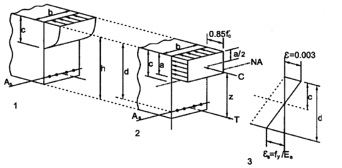
- ultimate strength design
- ϕ factor applied to capacity
  - – different for flexure, shear, bearing....
- factors applied to loads (ASCE 7)
  - – may be different for combinations

$$U = 1.2D + 1.6L$$
$$U = 1.2D + 1.0W + 1.0L$$
$$\vdots$$

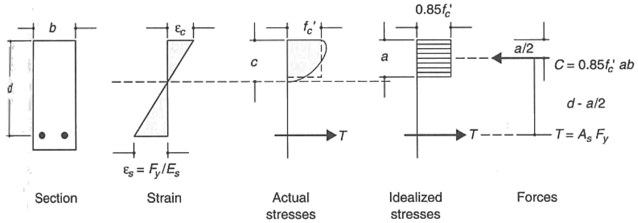
  - – <u>can</u> use alternate values & factors (older codes)

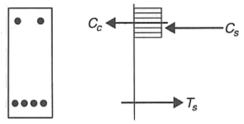
Reinforced Concrete Design

- want steel to yield first
  - – ductile failure
  - – underreinforced
- find flexure capacity or resistance from
  - – ultimate stresses in steel
  - – "uniform stress block" in concrete

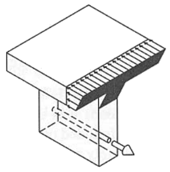


Reinforced Concrete Design





Beams with compression steel.



Reinforced concrete T beam. The shape of the section causes the stresses in the top flange to be lower than stresses in the web of the member. Concrete is used at the top and steel where the section is assumed to be cracked.

Steel Design

- load and resistance factor design
- like concrete, but capacity related to material

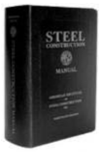
$$R_u \le \phi R_n$$

load factors
load types

nominal strength

resistance factor

- R_u combinations, ex:
  - 1.4D
  - 1.2D + 1.6L
- compression $\phi_c = 0.85$
- capacity $P_n = A_g F_{cr}$



Plastic Design

- bending & beams
- all of material sees ultimate stress
- refers primarily to steel behavior
- statically indeterminate systems

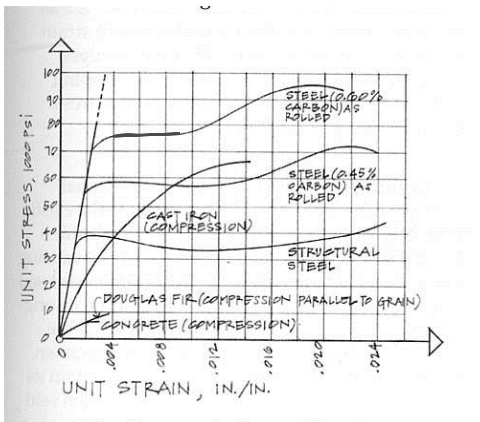


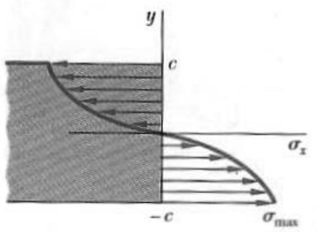
Figure 5.20 Stress-strain diagram for various materials.

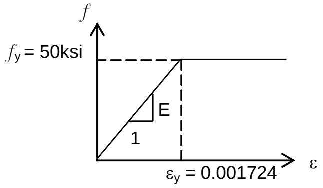
Elastic vs. Plastic Behavior

- Hooke's law valid

$$f = E\varepsilon$$

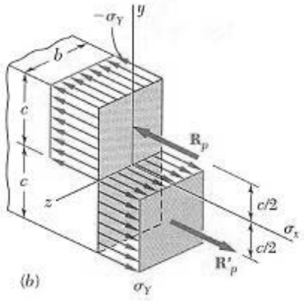
- yield point is end of elastic range for a ductile material
- continued strain with no more load





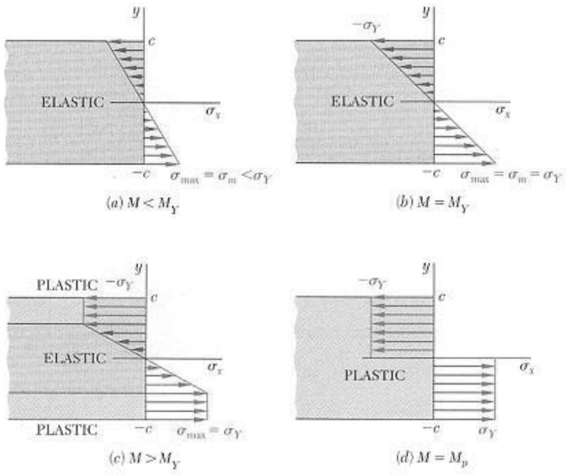
Internal Moments - ALL at yield

- all parts reach yield
- plastic hinge forms
- ultimate moment
- $A_{tension} = A_{compression}$



$$M_{ult} \text{ or } M_p = bc^2 f_y = \frac{3}{2} M_y$$

Plastic Hinge Development

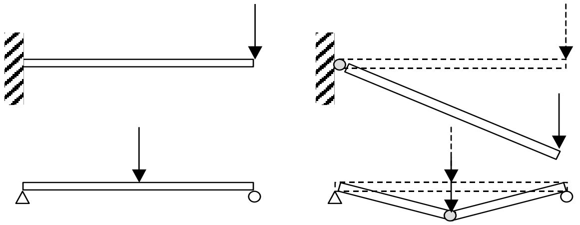


(a) $M < M_Y$ (b) $M = M_Y$

(c) $M > M_Y$ (d) $M = M_p$

Plastic Hinge Examples

- stability can be effected



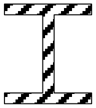
Plastic Section Modulus

- shape factor, k

$$k = {M_p}\!\big/\!{M_y}$$

= 3/2 for a rectangle

≈ 1.1 for an *I*



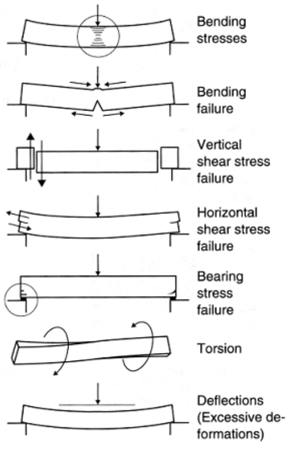
$$k = {Z}\!\big/\!{S}$$

- plastic modulus, Z

$$Z = \frac{M_p}{f_y}$$

Beams

- transverse loading
- sees:
  - – bending
  - – shear
  - – deflection
  - – torsion
  - – bearing
- cross section shape



Transverse Loadings

Beams

- maximum stress distribution
- principal stresses
  - resultant of shear and bending stress

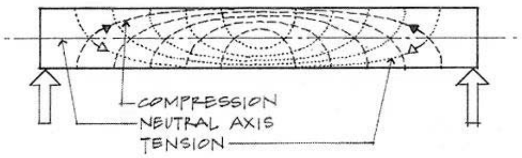


Figure 5.34 Stress trajectories in a beam (flexure).

Beams

- deflections

$$\frac{d^2y}{dx^2} = curvature = \frac{M(x)}{EI}$$

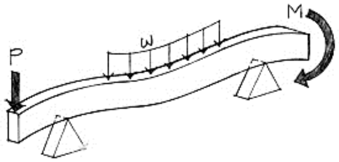
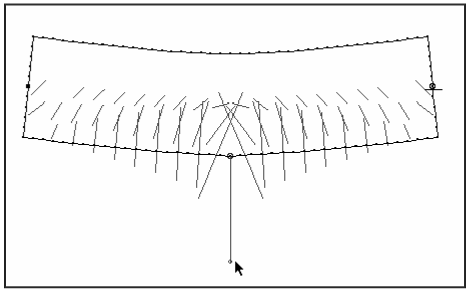


Figure 5.4 Bending (flexural) loads on a beam.



U.Washington – ENGR 220

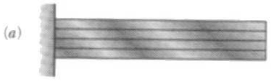
Beams

- design:
  - bending stress not exceeding allowable or limit stress

$$F_{all} \geq f_b = \frac{Mc}{I} \qquad S_{req'd} \geq \frac{M}{F_{all}}$$

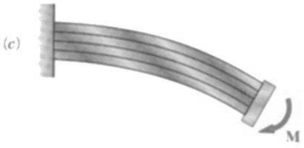
Beams

- bending stresses dominate



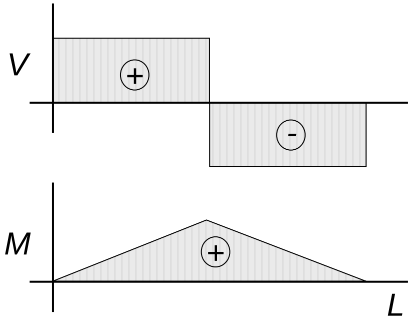
- shear stresses exist horizontally with shear

- no shear stresses with pure bending



Beams

- V & M drawings help determine M_{max}



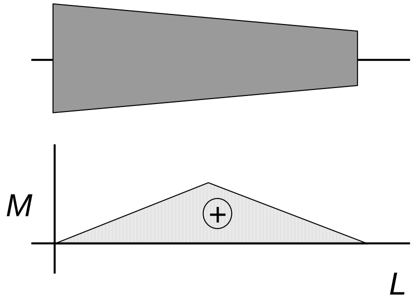
$$V = \Sigma(-w)dx$$

$$M = \Sigma(V)dx$$

  – inflection points where slope of M=0

Beams

- prismatic (constant cross section)
  – maximum stress ⇔ maximum moment
- non-prismatic
  – S varies



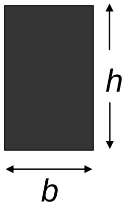
Beam Design

1. Know F_{all} for the material or f_u for LRFD

2. Draw V & M, finding M_{max}

3. Calculate $S_{req'd}$

4. Determine section size



$$S = \frac{bh^2}{6}$$

Beam Design

4*. Include self weight for M_{max}
  – and repeat 3 & 4 if necessary

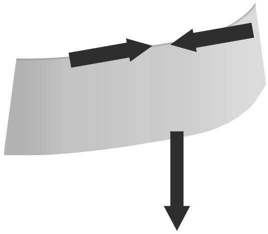
5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper

Beam Design

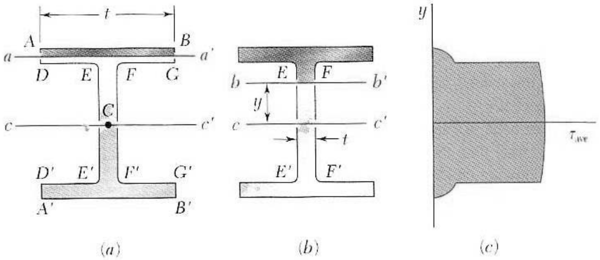
5. Consider lateral stability (cont)
   - lateral buckling caused by compressive forces at top couples with insufficient rigidity
   - can occur at low stress levels
   - stiffen or brace



Beam Design

6. Evaluate shear stresses - horizontal
   - W and rectangles

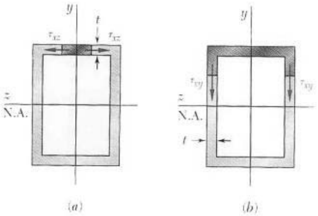
$$\tau_{\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

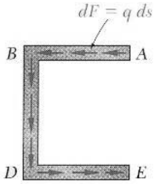


Beam Design

6. Evaluate shear stresses (cont)
   - thin walled – open or closed

$$\tau_{ave} = \frac{VQ}{Ib} \qquad q = \frac{VQ}{I}$$



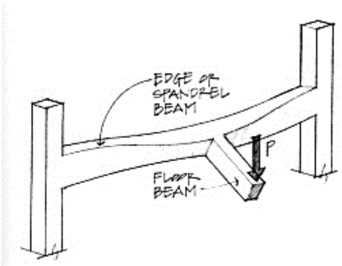


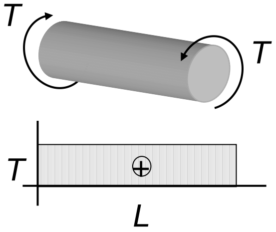
Beam Design

7. Provide adequate bearing area at supports

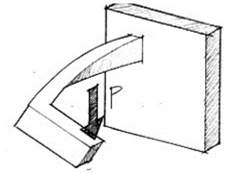
$$f_p = \frac{P}{A}$$

8. Evaluate torsion
   - cross section

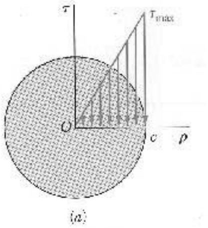




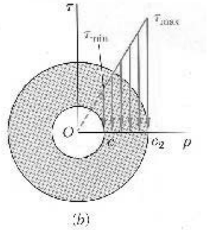
$$\tau = \frac{T\rho}{J}$$



Beam Design



(a)



(b)

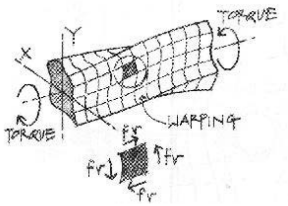
8. Torsion (cont)
   - round-ish

$$\tau = \frac{T\rho}{J}$$

$$J = \tfrac{1}{2}\pi c^4$$

   - rectangular

$$\tau_{max} = \frac{T}{c_1 ab^2}$$



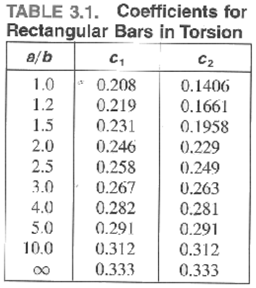


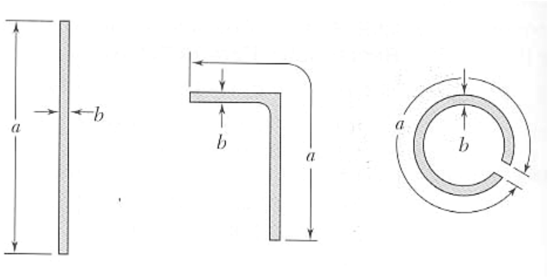
TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Beam Design

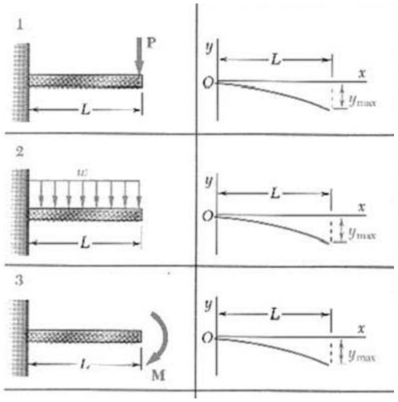
8. Torsion (cont)
   - open long sections

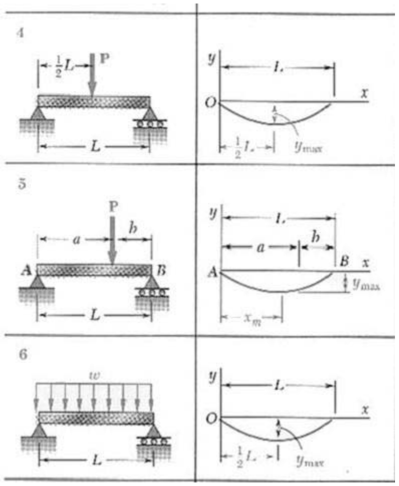
$$\tau_{max} = \frac{T}{\frac{1}{3}ab^2}$$



Beam Design

9. Evaluate deflections
   - y_{max} & location





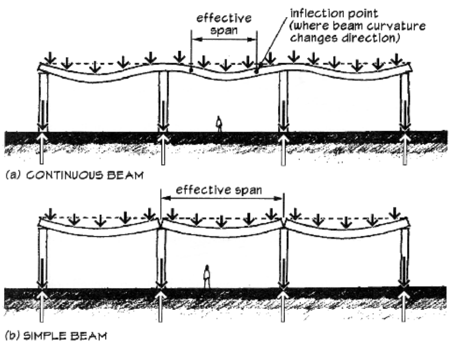
Deflection Limits

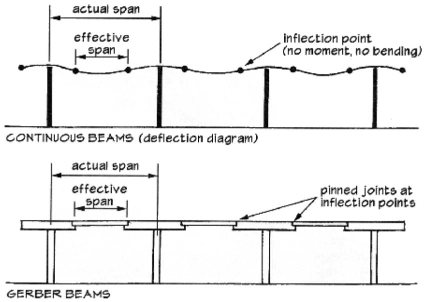
- based on service condition, severity

Use	LL only	DL+LL
Roof beams:		
Industrial	L/180	L/120
Commercial		
plaster ceiling	L/240	L/180
no plaster	L/360	L/240
Floor beams:		
Ordinary Usage	L/360	L/240
Roof or floor (damageable elements)		L/480

Continuous Beams

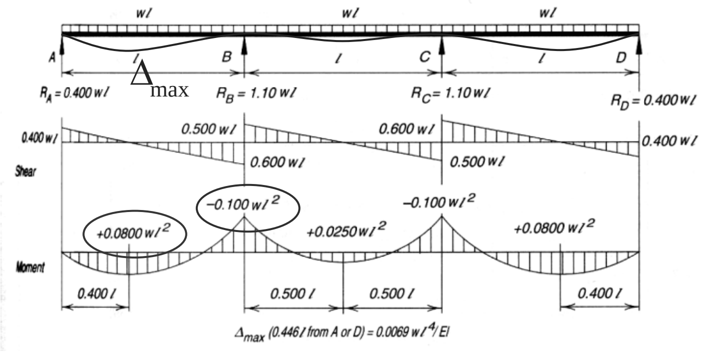
- statically indeterminate
- reduced moments than simple beam





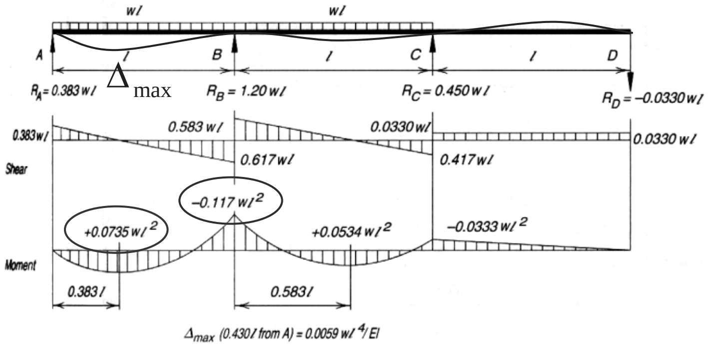
Continuous Beams

- loading pattern affects
  - moments & deflection



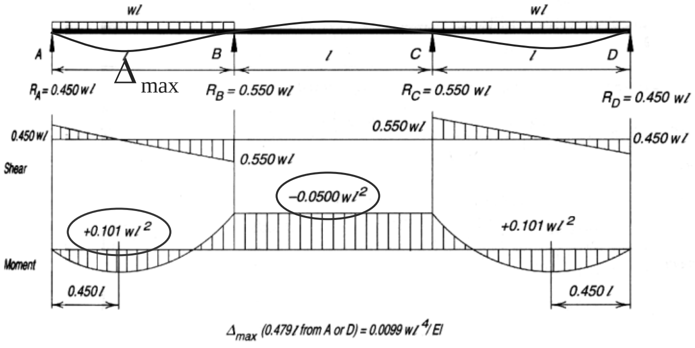
Continuous Beams

- unload end span



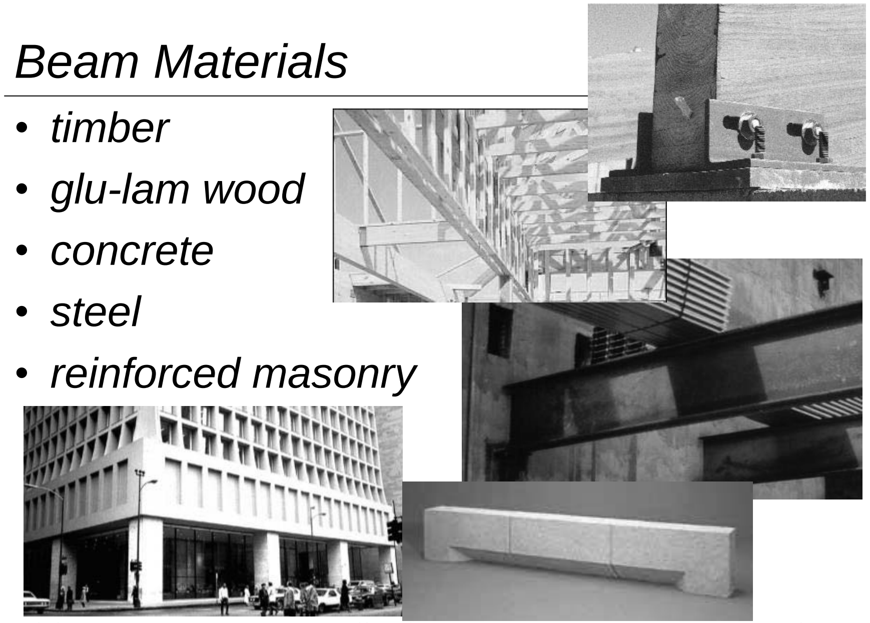
Continuous Beams

- unload middle span



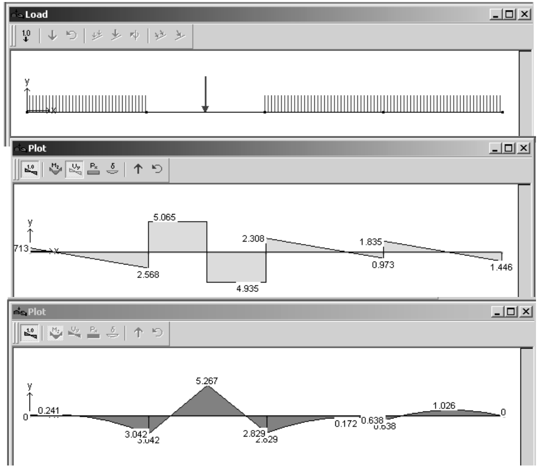
Beam Materials

- timber
- glu-lam wood
- concrete
- steel
- reinforced masonry



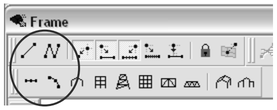
Tools – Multiframe

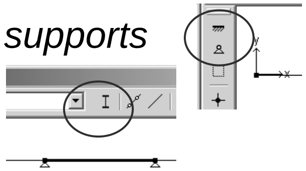
- in computer lab

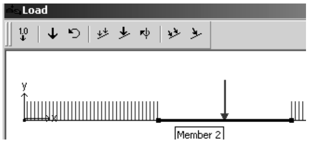


Tools – Multiframe

- frame window
  - define beam members
  - select points, assign supports
  - select members, assign <u>section</u>
- load window
  - select point or member, add point or distributed loads

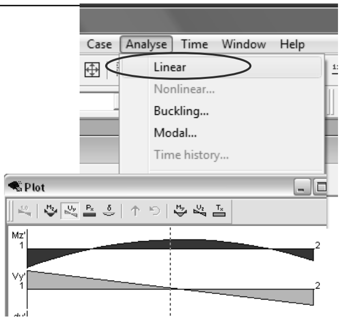


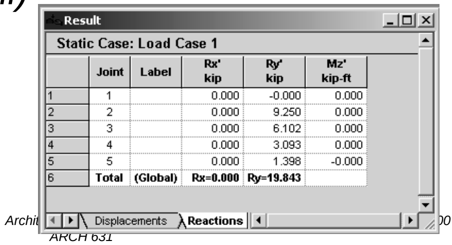




Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear
- plot
  - choose options
  - double click (all)
- results
  - choose options




Result

Static Case: Load Case 1

	Joint	Label	Rx' kip	Ry' kip	Mz' kip-ft
1	1		0.000	-0.000	0.000
2	2		0.000	9.250	0.000
3	3		0.000	6.102	0.000
4	4		0.000	3.093	0.000
5	5		0.000	1.398	-0.000
6	Total	(Global)	Rx=0.000	Ry=19.843	