APPLIED ARCHITECTURAL STRUCTURES:
STRUCTURAL ANALYSIS AND SYSTEMS

ARCH 631

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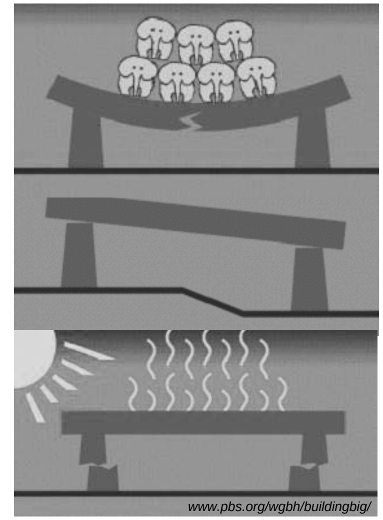
FALL 2013

lecture
two

structural analysis (statics & mechanics)

Structural Requirements

- serviceability
  - strength
  - deflections
- efficiency
  - economy of materials
- construction
- cost
- other



Structure Requirements

- strength & equilibrium
  - safety
  - stresses not greater than strength
  - adequate foundation

Figure 1.16 Equilibrium and Stability?—*sculpture by Richard Byer. Photo by author.*

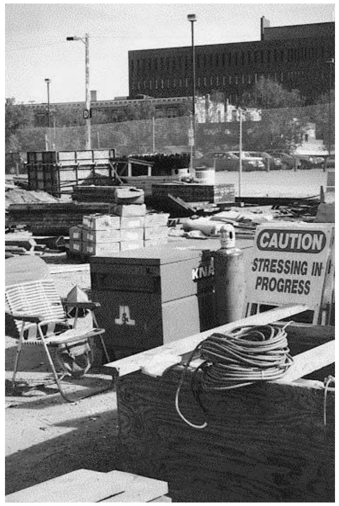
Structure Requirements

- stability & stiffness
  - stability of components
  - minimum deflection and vibration
  - adequate foundation

Figure 1.15 Stability and the strength of a structure—the collapse of a portion of the UW Husky stadium during construction (1987) due to a lack of adequate bracing to ensure stability. Photo by author.

Structure Requirements

- *economy and construction*
  - *minimum material*
  - *standard sized members*
  - *simple connections and details*
  - *maintenance*
  - *fabrication/ erection*



Relation to Architecture

"The geometry and arrangement of the load-bearing members, the use of materials, and the crafting of joints all represent opportunities for buildings to express themselves. The best buildings are not designed by architects who after resolving the formal and spatial issues, simply ask the structural engineer to make sure it doesn't fall down." - Onouy & Kane

Structural Loads - STATIC

- *dead load*
  - *static, fixed, includes material weights, fixed equipment*
- *live load*
  - *transient and moving loads (including occupants)*
- *snow load*

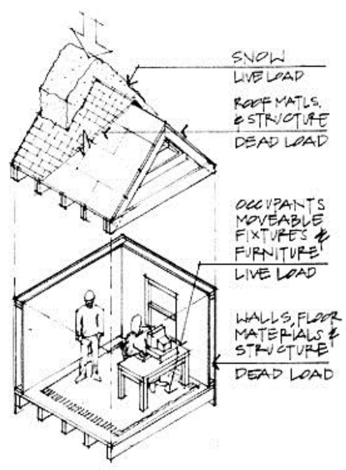


Figure 1.12 Typical building loads.

Structural Loads – STATIC & DYNAMIC

- *wind loads*
  - *dynamic, wind pressures treated as lateral static loads on walls, pressure or suction*
  - *pressure determined from wind velocity, q_h*
  - *dynamic effects include motion from buffeting or "vortex shedding"*

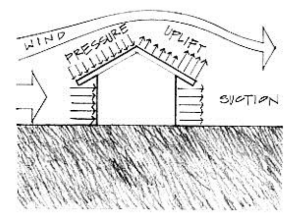


Figure 1.13 Wind loads on a structure.

$$F_W = C_d q_h A$$

Structural Loads - DYNAMIC

- *earthquake loads*
  - *seismic, movement of ground (3D)*
  - *building mass responds*
  - *static models often used, V is static shear*
- *impact loads*
  - *rapid, energy loads*

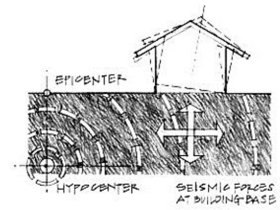
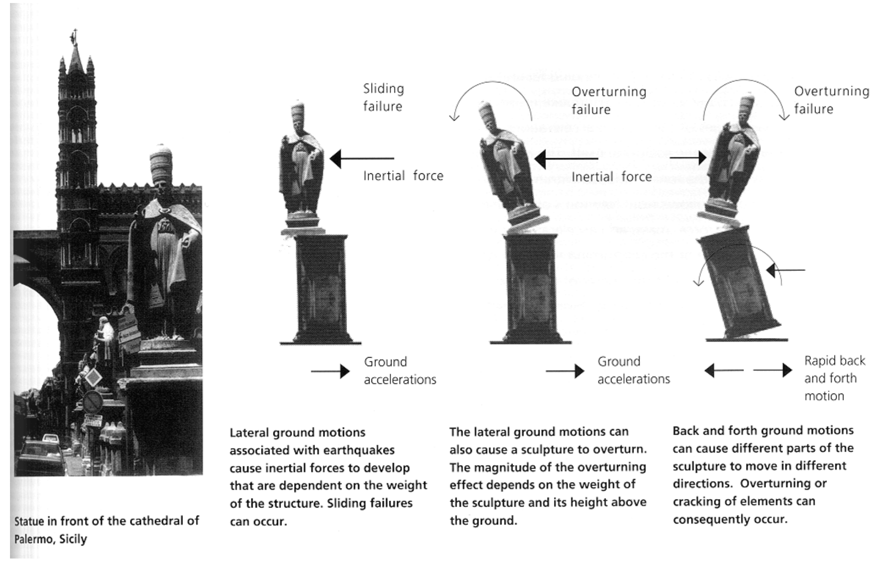


Figure 1.14 Earthquake loads on a structure.

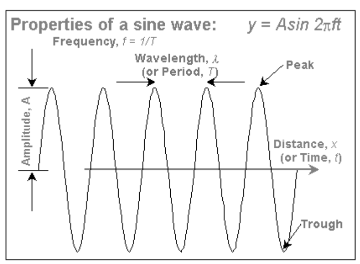
$$V = \frac{ZICW}{R_W}$$

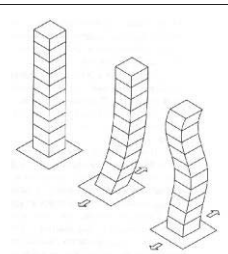
Dynamic Response



Dynamic Response

- *period of vibration or frequency*
  - *wave*
  - *sway/time period*
- *damping*
  - *reduction in sway*
- *resonance*
  - *amplification of sway*





Statics & Mechanics Review

- *how loads affect our structures*
  - *statics: things don't move*
    - *forces*
    - *supports & connections*
    - *equilibrium*
  - *mechanics: things can change shape*
    - *stress & strain*
    - *deflections*
    - *buckling*

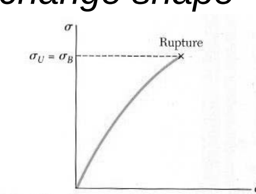


Fig. 2.11 Stress-strain diagram for a typical brittle material.

Structural Math

- quantify environmental loads
  – how big is it?
- evaluate geometry and angles
  – where is it?
  – what is the scale?
  – what is the size in a particular direction?
- quantify what happens in the structure
  – how big are the internal forces?
  – how big should the beam be?

Physical Math

- physics takes observable phenomena and relates the measurement with rules: <u>mathematical relationships</u>
- need
  – reference frame 
  – measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light
  – calculations & geometry

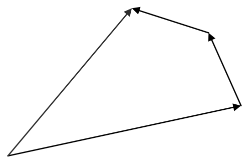
Units

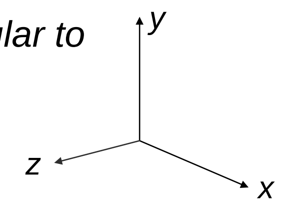
- measures
  – US customary & SI

Units	US	SI
Length	in, ft, mi	mm, cm, m
Volume	gallon	liter
Mass	lb mass	g, kg
Force	lb force	N, kN
Temperature	F	C

Vectors

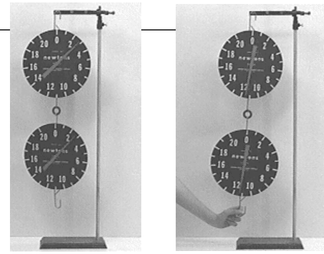
- scalars – any quantity
- vectors - quantities with direction
  – like displacements
  – summation results in the "straight line path" from start to end
  – <u>normal</u> vector is perpendicular to something



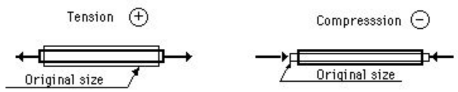


Forces & Reactions

- *Newton's 3rd law:*
  - *for every force of action there is an equal and opposite reaction along the same line*



- *external forces act on bodies*
  - *can cause moments*
- *internal forces are*
  - *in bodies*
  - *between bodies (connections)*



Force Components

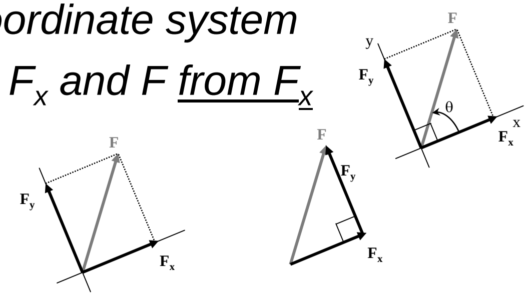
- *convenient to resolve into 2 vectors*
- *at right angles*
- *in a "nice" coordinate system*
- *θ is between F_x and F from F_x*

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

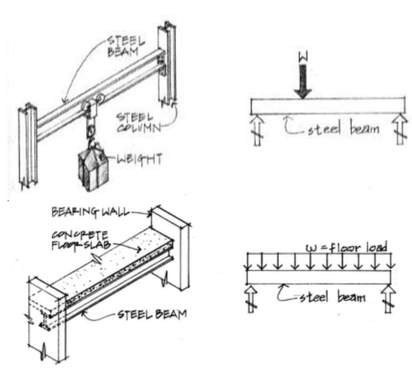


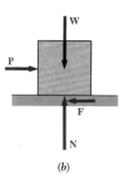
Load Types

- *weight (F = ma)*

$$W = \gamma t A$$

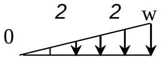
- *concentrated*
- *distributed*
  - *uniform*
  - *linear*
- *friction*
  - *F= μN*





$\cdot x = W$ w

$\frac{w \cdot x}{2} = \frac{W}{2}$ w w



x

W

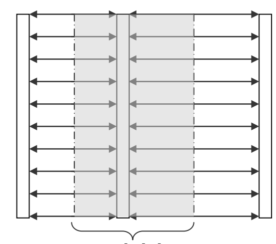
W/2

x/2 x/2 2x/3 x/3 x/2 x/6 x/3

Load Tracing

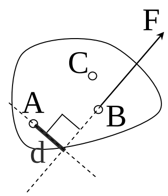
- *tributary load*
  - *think of water flow*
  - *"concentrates" load of area into center*

$$w = \left(\frac{load}{area}\right) \times (tributary\ width)$$



Moments

- defined by magnitude and direction
- units: N·m, k·ft
- direction:
  - + ccw (right hand rule)
  - - cw
- value found from F and ⊥ distance



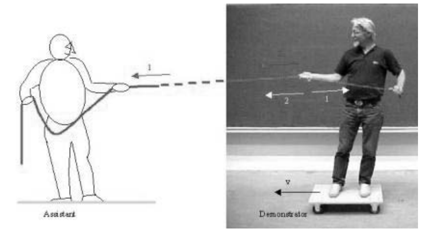
$$M = F \cdot d$$

- d also called “lever” or “moment” arm

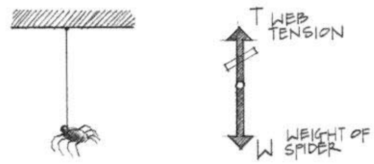
Equilibrium

- analytically

$$R_x = \sum F_x = 0$$
$$R_y = \sum F_y = 0$$
$$M = \sum M = 0$$

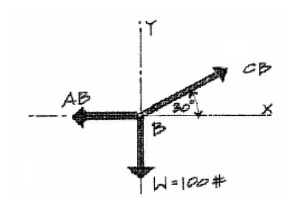


- free body diagrams

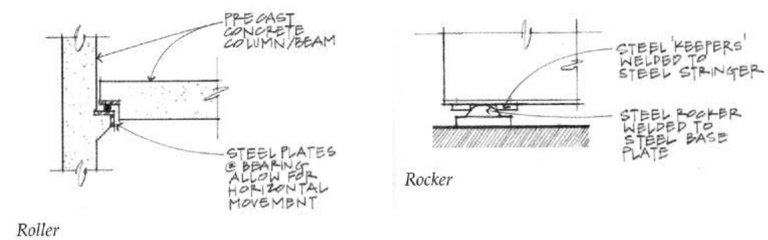


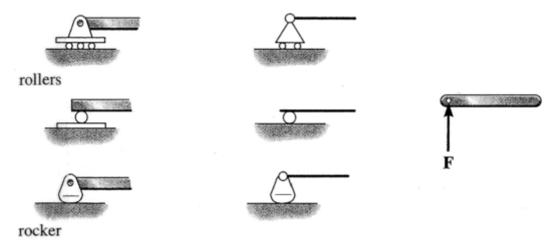
Free Body Diagram

- FBD (sketch)
- tool to see all forces on a body or a point including
  - – external forces
  - – weights
  - – force reactions
  - – external moments
  - – moment reactions
  - – internal forces



Supports and Connections





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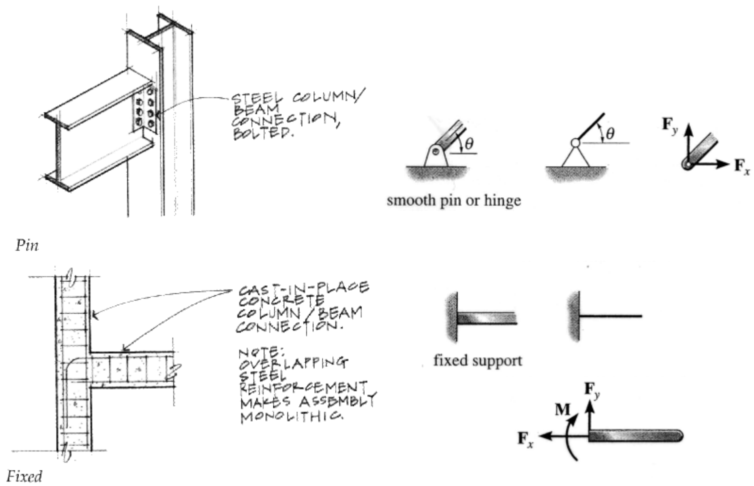
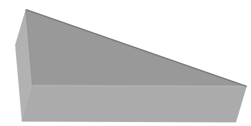


Centroid

- "average" x & y of an area
- for a volume of constant thickness
  - $\Delta W = \gamma t \Delta A$ where γ is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum (x \Delta A)}{\boldsymbol{A}}$$

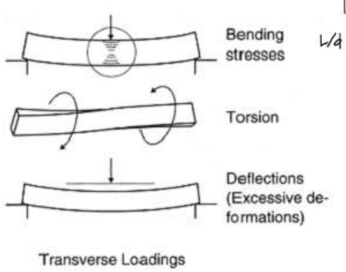
$$\bar{y} = \frac{\sum (y \Delta A)}{\boldsymbol{A}}$$

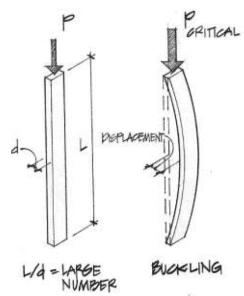


Moments of Inertia

- 2nd moment area
  - math concept
  - area x (distance)^2
- need for behavior of
  - beams
  - columns

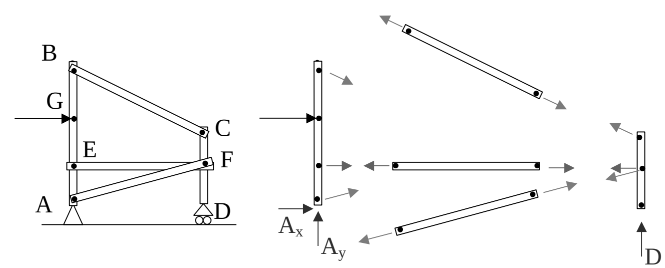
$$I_x = I_{cx} + A d_y^{\,2}$$





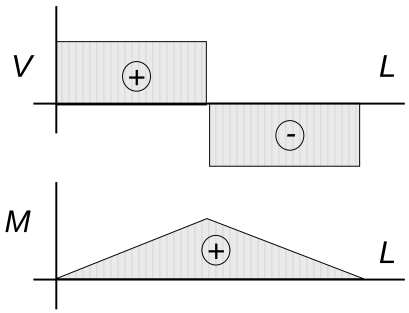
Internal and Pin Forces

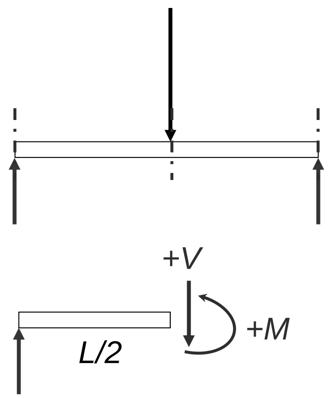
- 3 equations per three-force body
- two-force body forces in line
- 2 reactions per pin + support forces



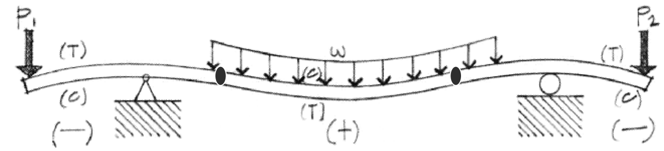
Internal Beam V & M (+P)

- *maximums needed for design*
- M_{max} *at V = 0*





Deflected Shape



- *positive bending moment*
  - *tension in bottom, compression in top*
- *negative bending moment*
  - *tension in top, compression in bottom*
- *zero bending moment*
  - *inflection point*

Stress

- *stress is a term for the intensity of a force, like a pressure*
- *internal or applied*
- *force per unit area*

$$stress = \frac{P}{A}$$

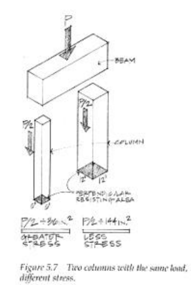
Stress Types

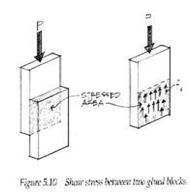
- *normal stress is normal to the cross section*

$$f_{t\ or\ c} = \frac{P}{A}$$

- *shear stress parallel to a surface*

$$f_v = \frac{P}{A} = \frac{P}{td}$$





Stress Types

- <u>bearing</u> stress on a surface by contact in compression

$$f_p = \frac{P}{A} = \frac{P}{td}$$

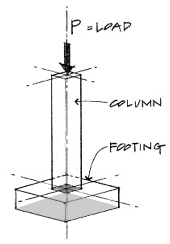
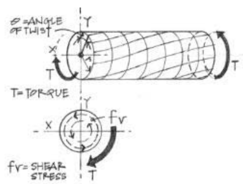


Figure 5.3 Centric loads.

- <u>torsional</u> stress by shear from twisting

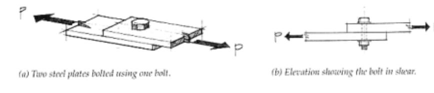
$$f_v = \frac{T\rho}{J}$$



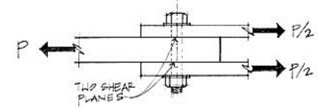
Bolt Stresses

- <u>single</u> shear

$$f_v = \frac{P}{A} = \frac{P}{\pi \, d^2/4}$$

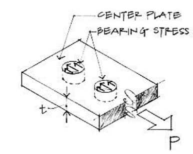


- <u>double</u> shear



$$f_v = \frac{P}{2A} = \frac{P/2}{A} = \frac{P/2}{\pi \, d^2/4}$$

- <u>bearing</u> $$f_p = \frac{P}{A_{projected}} = \frac{P}{td}$$



Bearing stress on plate.

Bending Stresses

- tension and compressive stress caused by bending

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

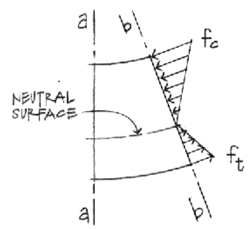
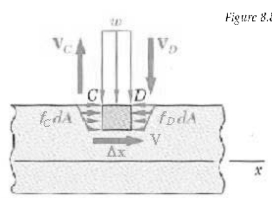


Figure 8.8 Bending stresses on section b-b.

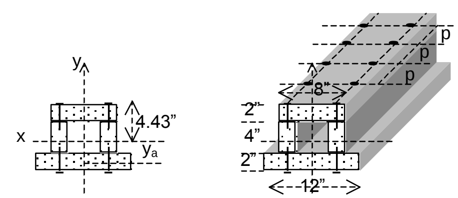
- <u>shear</u> stress from bending

$$f_{v-ave} = \frac{VQ}{Ib}$$



Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices

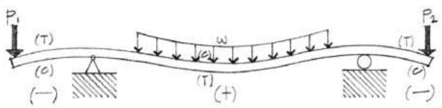


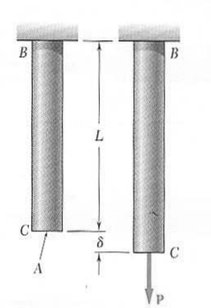
$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$

Strain

- *materials deform*
- *axially loaded materials change length*
- *bending materials deflect*

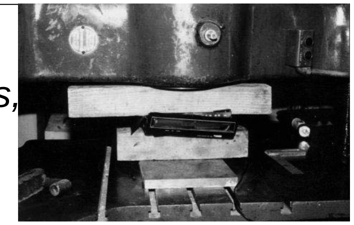




- *STRAIN:*
  - *change in length over length*

$$strain = \frac{\Delta L}{L}$$

Problem Solving



1. *STATICS:*

   equilibrium of external forces, internal forces, <u>stresses</u>

2. *GEOMETRY:*

   cross section properties, deformations and conditions of geometric fit, <u>strains</u>

3. *MATERIAL PROPERTIES:*

   <u>stress-strain relationship</u> for each material obtained from testing

Stress to Strain

- *important to us in f-ε diagrams:*
  - *straight section*
  - *LINEAR-ELASTIC*

$$f = E \cdot \varepsilon$$

  - *recovers shape (no permanent deformation)*

$$\delta = \frac{PL}{AE}$$

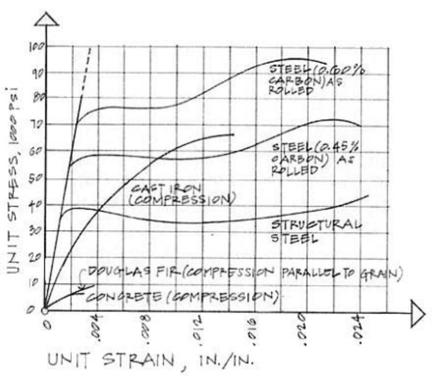


Figure 5.20 Stress-strain diagram for various materials.

Behavior Types

- *brittle*

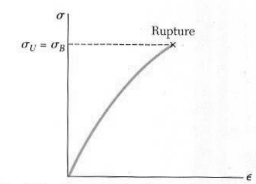


Fig. 2.11 Stress-strain diagram for a typical brittle material.

- *semi-brittle*

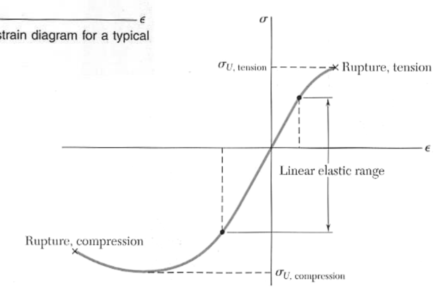


Fig. 2.14 Stress-strain diagram for concrete.

Plastic Behavior

- ductile

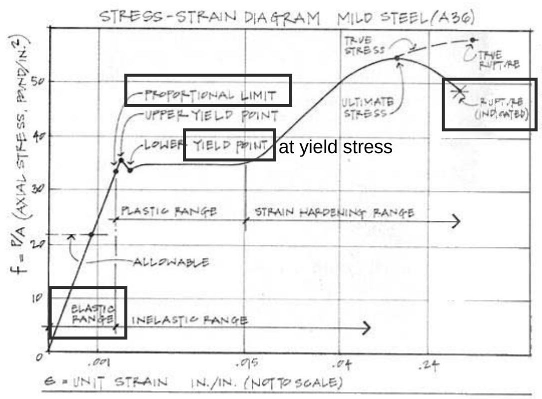
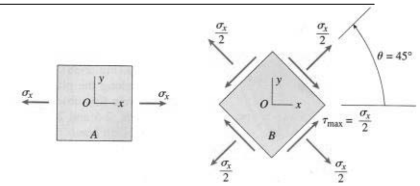


Figure 5.22 Stress-strain diagram for mild steel (A36) with key points highlighted.

Maximum Stresses

- *if we need to know where max f and f_v happen:*



$$\theta = 0^\circ \rightarrow \cos\theta = 1 \qquad f_{\max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos\theta = \sin\theta = \sqrt{0.5}$$

$$f_{v-\max} = \frac{P}{2A_o} = \frac{f_{\max}}{2}$$

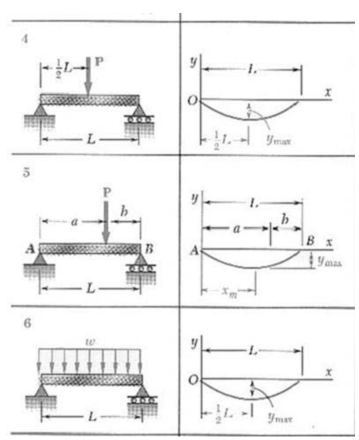
Thermal Deformation

- α - *the rate of strain per degree*
- *UNITS :* $/{}^\circ F$, $/{}^\circ C$
- *length change:* $\delta_T = \alpha(\Delta T)L$
- *thermal strain:* $\varepsilon_T = \alpha(\Delta T)$
  - *<u>no stress</u> when movement allowed*

Beam Deflections

- *curvature, R*

$$\frac{1}{R} = \frac{M}{EI} \qquad curvature = \frac{M(x)}{EI}$$



$$\theta = slope = \int \frac{M(x)}{EI} dx$$

$$\Delta = deflection = \int\int \frac{M(x)}{EI} dx$$

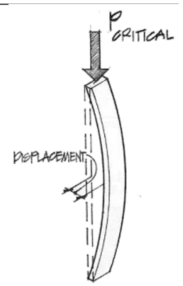
$$y_{\max}(x) = \Delta_{actual} \leq \Delta_{allowable}$$

Column Stability

- *short columns*

$$f_{critical} = \frac{P_{actual}}{A} < F_a$$

- *slenderness ratio = L_e/r (L/d)*
- *radius of gyration =* $r = \sqrt{\frac{I}{A}}$

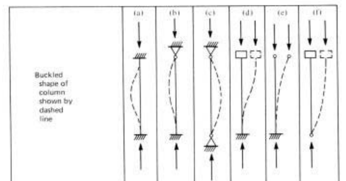


weak axis

$$f_{critical} = \frac{P_{critical}}{A} = \frac{\pi^2 EAr^2}{A(L_e)^2} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2} \qquad P_{critical} = \frac{\pi^2 EA}{\left(\frac{L_e}{r}\right)^2}$$

Column Stresses

- *when a column gets stubby, F_y will limit the load*
- *real world has loads with eccentricity*
- *end conditions* $L_e = K \cdot L$



	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design values when ideal conditions are approximated	0.65	0.80	1.0	1.2	2.10	2.0

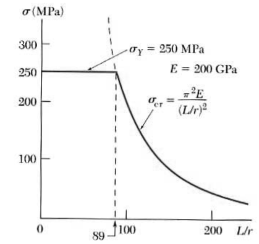


Fig. 10.9