ARCH 631

# APPLIED <br> ARCHITECTURAL STRUCTURES 

## COURSE NOTE SET <br> Fall 2012



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# ARCH 631. Applied Architectural Structures 

Instructor: Prof. Anne B. Nichols Office Hours: 9-10 am and 2-3 pm MW<br>A413 Langford<br>(979) 845-6540 anichols @tamu.edu<br>11-12 pm TR<br>(and by appointment $M-R$ )

Prerequisites: ARCH $331 \&$ ARCH 431 (or equivalent hours and content)
Course Description: Structural analysis of building structural systems: components, frames, shapes; selection and economics of structural systems; survey of current structural design codes; supervision practices in structural construction.
Goals: ARCH 631 encompasses structural analysis of building structural systems: components, frames, and shapes. Also covered is the selection and economics of structural systems; survey of current structural design codes; supervision practices in structural construction. Case studies and writing exercises will be utilized. The course follows the content areas of the ARE 4.0 section of Structural Systems section: General Structures is the application of general structural principles to building design and construction considering the code requirements, implication of alternate systems, materials and construction details along with site and environmental characteristics. Wind Forces, Seismic Forces \& Lateral Forces is the application of lateral force principles to the design and construction of buildings to resist lateral, wind and seismic forces considering the code requirements, implication of alternative systems, materials and constructions details along with site and environmental characteristics.

Objective: To synthesize knowledge of components, systems and framing with environmental loads (particularly hazard) and design codes and standards.
Text: $\quad$ Structures, $6^{\text {th }}$ ed., Daniel L. Schodek and Martin Bechthold, (2007) Pearson - Prentice Hall, ISBN 0-13-178939-2

Recommended Texts:
A Structures Primer, Kaufman, (2010) Prentice Hall, ISBN 978-0-13-230256-3
Understanding Structures, Moore, (1999) McGraw-Hill, ISBN 9780070432536
The Structural Basis of Architecture, Sandaker, et.al, (2011) Routledge, ISBN 9780415415477

References: AIA Publications
Adoptable codes (ICBO, SBCCI, BOCA, CABO)
International Building Code, International Residential Code
Structural Design Codes (ACI, PCI, AISC, MSJC, etc.)
Material and Professional Standard Documents (ANSI, ASCE, ASTM, ASHRAE)
Timetable: 9:35-10:50 am Lecture T,R (section 600)
Grading: Assignments 20\%
Mid-term Exams 40\%
Team Project 20\%
Final Exam 20\%
Letter Grades (Approximate):
$\frac{\frac{90-100 \ldots \ldots . \mathrm{A}}{80-89 \ldots \ldots \ldots \mathrm{~B}}}{\frac{70-79 \ldots \ldots \ldots \mathrm{C}}{60-69 \ldots \ldots . \mathrm{D}}} \frac{0-59 \ldots \ldots \ldots . \mathrm{F}}{}$

Policy: 1) Attendance: Necessary. Required.* And subject to University Policy. See Part I Section 7 in Texas A\&M University Student Rules: http://student-rules.tamu.edu/ Absences related to illness or injury must be documented according to http://shs.tamu.edu/attendance.htm including the Explanatory Statement for Absence from class for 3 days or less. Doctor visits not related to immediate illness or injury are not excused absences.
2) Lecture: The lecture slides should be viewed prior to class. Class-will also require problem solving with the lecture examples, assignments, and case studies. The lecture slide handouts are available on the class web page (see \#3) and Vista (see \#9). Use of electronic devices during lecture is prohibited.
3) Notes: The notes and related handouts are available on the class web page at http://faculty.arch.tamu.edu/anichols/631frame.html, or on Vista (see \#8). A bound set can be purchased from the Notes-n-Quotes at 701 W . University, directly across from the Mitchell Physics Building in the Northgate Neighborhood.
4) Assignments: Due as stated on the assignment statements. Only one assignment without University excuse may be turned in for credit no later than one week after the due date and before final exams begin. All other assignments and projects will receive no credit if late without a recognized excuse or after final exams have begun.
5) Team Project: A term project to be completed in teams is due the last week of class. Presentations of the projects will be made during class periods.
6) Mid-term Exams: Mid-term exams will be given in lecture at any time during the period. Make-up exams without an excuse will not be given.
7) Teaching Assistant:

9) Vista: Vista is a web course tool for posting, reading messages and replying as well as recording scores and is accessed with your neo account. This will be used to post questions and responses by class members and the instructor, for posting scores and for email. It can be accessed at http://elearning.tamu.edu/
10) Final Exam: The final exam will be comprehensive and is officially scheduled for 12:30-2:30 PM Friday, December 7.
11) Other Resources: The Student Learning Center provides tutoring in math and physics. See their schedule at http://slc.tamu.edu/tutoring.shtml The Student Counseling Center has programs for study and learning (PASS), and tutoring services. See the resources at http://scs.tamu.edu/
12) Aggie Honor Code: "An Aggie does not lie, cheat, or steal or tolerate those who do."

The University policy will be strictly enforced. See Part I Section 20 in Texas A\&M University Student Rules: http://student-rules.tamu.edu/ Plagiarism (deliberate misrepresentation of someone else's work as your own) will be treated strictly according to University policy as outlined by the Office of the Aggie Honor System: http:/www.tamu.edu/aggiehonor/
13) The American with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring accommodation, please contact the Department for

* Except for death in the family, medical or deans' excuse, and natural disasters.

Student Life, Services for Students with Disabilities, in Cain Hall or call 845-1637. Also contact Prof. Nichols at the beginning of the semester.
14) Grievances: For grievances other than those listed in Part III in Texas A\&M University Student Rules: http://student-rules.tamu.edu/ the instructor must be the first point of contact.

## Learning Objectives:

1) The student will be able to read a text or article about structural technology, identify the key concepts and related equations, and properly apply the concepts and equations to appropriate structural problems (relevance). The student will also be able to define the answers to key questions in the reading material. The student will be able to evaluate their own skills, or lack thereof, with respect to reading and comprehension of structural concepts, clarity of written communication, reasonable determination of precision in numerical data, and accuracy of computations.
2) The student will be able to read a problem statement, interpret the structural wording in order to identify the concepts and select equations necessary to solve the problem presented (significance). The student will be able to identify common steps in solving structural problems regardless of the differences in the structural configuration and loads, and apply these steps in a clear and structured fashion (logic). The student will be able to draw representational structural models and diagrams, and express information provided by the figures in equation form. The student will compare the computational results in a design problem to the requirements and properly decide if the requirements have been met. The student will take the corrective action to meet the requirements.
3) The student will create structural models with a computer application based on the concepts of the behavior and loading of the structural member or assemblage. The student will be able to interpret the modeling results and relate the results to the solution obtained by manual calculations.
4) The student will be able to articulate the physical phenomena, behavior and design criteria which influence structural space and form. (depth) The student will be able to identify the structural purpose, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies. (breadth) The student will be able to identify the configuration, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies with respect to materials (e.g. reinforced concrete beams or frames). The student will draw upon existing organizational and communication skills to clearly present concepts and personal interpretation of structural knowledge in writing assignments and examinations (clarity, precision, accuracy, relevance, depth, breadth, logic, significance).
5) The student will interact and participate in group settings to facilitate peer-learning and teaching. In addition, the student will be able to evaluate the comprehension of concepts, clarity of communication of these concepts or calculations, and the precision and accuracy of the data used in the computations in the work of their peers. The student will participate in the classification and identification of structural components and assemblages and purposes with a case study chosen by a group in order to show synthesis of structural knowledge including modeling and analysis.

Tentative Schedule (subject to change at any time throughout the semester)
Lecture
Text Topic
Articles/Problems

| 1. | Structures: An Overview <br> Introduction to Structural Analysis and Design | Read*: Ch. 1 <br> Solve: Assignment 1 (start) |
| :---: | :---: | :---: |
| 2. | Review of Statics and Mechanics | Read: Ch. 2; note sets 2.1 \& 2.2 Reference: Appendices 1-5 |
| 3. | Overview of Building Codes | Read: Ch. 3; note sets 3.1 \& 3.2 <br> Reference: note sets 3.3, 3.4 \& 3.5 |
| 4. | Overview of Design Philosophies and Beams | Read: § 6.1-6.4.1 \& § 8.1-8.3 <br> Reference: Appendices 6-9; note set 4.2 <br> Due: Assignment 1 over material from lectures 1-2 |
| 5. | Trusses \& Columns | Read: Ch. 4 \& § 7.1-7.4.2 <br> Reference: note set 5.1 |
| 6. | Funicular Structures: Cables \& Arches | Read: Ch. 5 <br> Due: Assignment 2 over material from lectures 3-4 |
| 7. | Rigid Frames: Analysis \& Design | Read: Ch. 9; note set 7.1 <br> Reference: note set 7.2 <br> Due: CPR 1 Text over material from lecture 4 |
| 8. | Plates and Grids | Read: Ch. 10 \& § 8.4; note set 8.1 <br> Due: Assignment 3 over material from lectures 5-6 \& CPR 1 Reviews |
| 9. |  | Mid-term Exam |
| 10. | Reinforced Concrete Construction | Read: § 15.3, 6.4.4-6.4.7, 7.4.5 \& 8.4.6, Appendix 12; note set 10.1 |
| 11. | CASE STUDY - Reinforced Concrete | Read: note set 11 |
| 12. | CASE STUDY - Reinforced Concrete | Read: note set 11 <br> Due: Assignment 4 over material from lecture 7 |
| 13. | Membrane, Net, and Shell Structures | Read: Ch. 11 \& 12; note set 13.1 |
| 14. | Structural Planning \& Design Issues | Read: Ch. 13; note set 14 <br> Due: Assignment 5 over material from lectures 7-8 |

*Note: Material in the Class Note Set not specifically mentioned above are provided as references or aids.

| Lectur | e Text Topic | Articles/Problems |
| :---: | :---: | :---: |
| 15. | Design for Lateral Loads Wind and Flood | Read: § 14.1; note set 15.1 <br> Re-read: § 1.3.1, 1.3.2, 3.3.3 <br> Due: CPR 2 Text over material from lecture 10 |
| 16. | Design for Lateral Loads Seismic | Read: § 14.2; note sets $16.1,16.2 \& 16.3$ <br> Re-read: § 3.3.4 <br> Due: Assignment 6 over material from lectures 10-12 \& CPR 2 Reviews |
| 17. | Structural Connections: Wood and Steel | Read: § 16.1-16.3; note set 17.1 |
| 18. |  | Mid-term Exam |
| 19. | Wood Construction | Read: § 15.2, 6.4.2, \& 7.4.3; note set 19.1 |
| 20. | CASE STUDY - Wood | Read: note set 20 <br> Due: Assignment 7 over material from lectures 13-15 |
| 21. | Steel Construction | Read: § 15.4, 6.4.3 \& 7.4.4; note set 21.1 <br> Due: CPR 3 Text over material from lectures 15 and 17 |
| 22. | CASE STUDY - Steel | Read: note set 22 <br> Due: Assignment 8 over material from lectures 15-17 \& CPR 3 reviews |
| 23. | Masonry Construction | Read: note set 23.1 |
| 24. | Foundations and Retaining Walls | Read: §15.5; note sets $24.1 \& 24.2$ <br> Due: Assignment 9 over material from lectures 19-22 |
| 25. |  | Mid-term Exam |
| Thanksgiving Break |  |  |
| 26. Project Presentations |  |  |
| 27. Project Presentations |  |  |
| 28. | Construction \& Inspection Review | Reference: note set 28.1 <br> Due: Assignment 10 over material from lectures 23-24 \& Project Report |
|  | Final Exam Period | Exam |

*Note: Materials in the Class Note Set not specifically mentioned above are provided as references or aids.

|  | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 5 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | 19 | 20 | 21 | 22 | 23 | 24 <br> last day to register | 25 |
|  | $\begin{array}{\|l\|} \hline 26 \\ \text { freshm } \\ \text { convoc } \end{array}$ | $27$ <br> classes begin | 28 Lect 1 | 29 | 30 Lect 2 | $31$ <br> last day to add | 1 |
|  | 2 | 3 | 4 Lect 3 | 5 | $6 \quad$ Lect 4 \#1 due academic convocation | $7$ | 8 |
|  | 9 | 10 | 11 Lect 5 | 12 | 13 Lect 6 $\# 2$ due | 14 | 15 |
|  | 16 | 17 | 18 Lect 7 <br> CPR 1 text due  | 19 | $\begin{array}{\|l\|} \hline 20 \\ \# 3 \& \text { CPR } 1 \text { rev. } \end{array}$ | $\begin{gathered} \hline 21 \\ \text { due } \end{gathered}$ | 22 |
|  | 23 | 24 | 25 Lect 9 Exam 1 | 26 | 27 Lect 10 | 28 | 29 |
|  | 30 | 1 | 2 Lect 11 | 3 | \#4 due | 5 | 6 |
| $\begin{aligned} & \text { 荘 } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | 7 | 8 | 9 Lect 13 | 10 | $11 \quad$ Lect 14 $\# 5$ due | 12 | 13 |
|  | 14 | $15$ <br> (mid-term grades due) | 16 Lect 15 <br> CPR 2 text due  | 17 | $\begin{array}{\|l\|l\|} \hline 18 & \text { Lect } 16 \\ \# 6 \end{array} \text { \& CPR } 2 \text { rev. } .$ | $\begin{gathered} 19 \\ \text { due } \end{gathered}$ | 20 |
|  | 21 | 22 college classes canceled for Symposium | 23 Lect 17 | 24 |  | 26 | 27 |
|  | 28 | 29 | 30 Lect 19 | 31 | $1 \quad$ Lect 20 $\# 7$ due | last day to Q-drop | 3 |
|  | 4 | 5 | 6 <br> CPR 3 text 21 <br>  | 7 | $\begin{array}{\|lc\|} \hline 8 & \text { Lect } 22 \\ \# 8 & \text { \& } \\ \text { CPR } & \text { rev. } \end{array}$ | $\begin{aligned} & 9 \\ & \text { due } \end{aligned}$ | 10 |
|  | 11 | 12 | 13 Lect 23 | 14 | $15 \quad$ Lect 24 $\# 9$ due | 16 | 17 |
|  | $\begin{array}{\|l\|} \hline 18 \\ \text { Bonfin } \\ \text { Remen } \\ \hline \end{array}$ | $19$ | $\begin{array}{\|l\|} \hline 20 \\ \text { Exam } 3 \end{array}$ | 21 | 22 Thanksgivin | $123$ <br> g Holiday | 24 |
|  | 25 | 26 | 27 Lect 26 <br> presentations  | 28 | 29 Lect 27 <br> presentations  | 30 | 1 |
|  | 2 | 3 (dead day) Friday classes | 4 $\# 10$, project 28 \& ev | $5$ | $\begin{aligned} & \hline 6 \\ & \text { Days } \end{aligned}$ | 7Final exams <br> $12: 30-2: 30$ am <br> 631 <br> 63 | 8 |
|  | 9 | 10 | $11{ }^{\text {a }}$ | 12 | 13 | 14 <br> Commencement (and Saturday) | 15 |
|  | 16 | $17$ <br> Grades due | 18 | 19 | 20 | 21 | 22 |
|  | 23 | 24 | 25 | $26$ <br> Winter Holiday | 27 | 28 | 29 |

## ARCH 631. Student Understandings

1) I understand that there are intellectual standards in this course and that I am responsible for monitoring my own learning.
2) I understand that the class will focus on application and synthesis, not on lecture.
3) I understand that I am responsible for preparing for lecture with the assigned reading by internalizing key concepts, recognizing key questions, and evaluating what makes sense and what doesn't make sense to me. $\qquad$
4) I understand that I will be held regularly responsible for assessing my own work using criteria and standards discussed in class.
5) I understand that if at any time in the semester I feel unsure about my "grade", I may request an assessment from the instructor. $\qquad$
6) I understand that there are 13 practice assignments, one due every week during the bulk of the semester. $\qquad$
7) I understand that there are group projects and I will be responsible to take an active part in advancing the work of the group. $\qquad$
8) I understand that I will occasionally be required to assess the work of my classmates in an objective manor using the same criteria and standards used to assess my own work. $\qquad$
9) I understand that there are 3 graded exams distributed throughout the semester.
10) I understand that there is a final exam in the course. $\qquad$
11) I understand that I must do a Learning Evaluation in which I make my "case" for receiving a particular grade using criteria provided in class and citing evidence from my work across the semester. $\qquad$
12) I understand that the work of the course requires consistent classroom attendance and active participation. $\qquad$
13) I understand that I will regularly be required to demonstrate that I have prepared for lecture. $\qquad$
14) I understand that the class will not be graded on a curve. I understand that it is theoretically possible for the whole class to get an A or an F $\qquad$
15) I understand the basis of the final grade as outlined in the syllabus.
16) I understand that I will uphold academic integrity and abide by the Aggie Honor Code. $\qquad$

NAME (sign and print) $\qquad$

DATE $\qquad$


Academic integrity is defined differently from culture to culture. In some cultures, sharing homework assignments, providing friends with answers to test questions, and allowing someone to copy one's papers is considered "helping". In the US, each of these actions is considered a violation of academic integrity, and violators are punished severely. At Texas A \& M University, the least severe punishment is a suspension for a stated period of time (possibly a semester or longer), which could cause a student to have to reapply for continuation of their academic program. Following the suspension, an academic department could refuse to readmit the individual. Furthermore, it usually means a loss of legal non-immigrant status as student. Once this legal status is lost, an international student is no longer eligible for employment in the US and must apply for reinstatement to legal status by the Immigration and Naturalization Services. But, applying does not guarantee that reinstatement to legal status will occur.

While most international students adapt very well to the US academic system, it is important to understand the US expectations for students in relation to academic integrity and to avoid problems that could be caused by lack of cultural understanding about these issues. Indeed, many international students are simply unaware of the US expectations and university rules about this important issue and would never violate them on purpose. While this lack of understanding can have serious consequences, there are a variety of resources to assist you in learning what the rules and expectations are.

For example, the most common problem of international students in relation to academic integrity concerns giving appropriate credit to others when using someone else's ideas in a written paper, such as a dissertation or theses. This is called plagiarism. The majority of this handout focuses on this problem and how to avoid it. However, your best source of information about appropriate writing styles and formats for research papers, theses, or dissertations is the faculty member teaching the class or advising you in your research project. You should also ask your professor if it is appropriate to work with another student on homework or other class projects.

Students who are accused of scholastic dishonesty have rights within the university regulations that govern the process. For more information about the process, the punishments, and your rights and responsibilities as a student, refer to the Academic Integrity brochure published by the Department of Student Life or to the Texas A \& M University Student Rules and Regulations, available in your academic department, at International Student Services, and at http://student-rules.tamu.edu.

Plagiarism - Plagiarism occurs when someone else's ideas are described or words are used in a written document, but that individual is not given appropriate or correct credit for those ideas or words. The effect that is created is that the writer of the document appears to be taking credit for ideas or words that are not his or her own. In essence, it becomes a theft of ideas. This is a very serious offense in a university, which is an institution designed to explore ideas and create new knowledge.

Fabrication of Results/Data - It is never appropriate to create false data to include in a written paper or to leave data out of a study to make the results appear more significant. The academic community condemns this above all other forms of scholastic dishonesty.

Reference Citation - There are appropriate ways to give credit for someone else's ideas in a written paper that you are writing. This is called making a reference citation. Each academic discipline is different in its expectations for correctly paraphrasing and documenting sources. For example, some disciplines use footnotes; some use endnotes. Some use names in the body of the text; some use numbers. Each uses its own stylistic format from a scientific journal or professional association. Therefore, it is vital to ask your academic advisor for help in learning what is appropriate for your academic major or a specific class. There are some basic examples listed on this page to point out some of these differences.

Literature Review - This part of a thesis or dissertation is a review of the current state of the discipline about a chosen research topic. It is never the writer's own ideas, and it is never copied from a previously written dissertation or thesis of a research group member. It is a compilation of the ideas of others, so each idea that comes from another source must be quoted or paraphrased and appropriately given credit through a reference citation.

## How to Avoid Plagiarism When Writing a Paper

1. Quotation Marks -- Never copy the ideas of someone else word for word without using quotation marks. In most disciplines it is not acceptable to use too many quotations in a written paper. In general, the reason to use quotations is that the original writer has used such impressive wording that it must be read as it was originally written or it will lose its impact or importance. In most cases, such quotations are not longer than a few sentences. Always check with your academic advisor if you have questions about what is appropriate for your field.
2. Use of Tables, Charts, and Figures - Sometimes, you will want to use a chart or some type of diagram in your paper that was originally created by someone else. Generally, this is all right to do if you provide credit to the original author. However, here are circumstances where it is not appropriate. For example, if a paper is being published, it may be necessary to obtain permission in advance from the author before the table or chart can be used in your published paper. If you are not sure about whether you should use a table or figure in your papers, ask your professor.
3. Paraphrasing - This occurs when you read something from an author that you want to include, but you summarize the original author's ideas and write them in your own words. Even using paraphrasing, it is still necessary to provide a reference citation naming the original author, to let the reader know that this idea is not your original idea. If you use too many of the original author's wording or phrasing in your paraphrase, you may still be guilty of plagiarism. If you are not sure that your paraphrasing is acceptable, ask the professor for whom you are writing the paper. Below are some examples of this.

## Paraphrasing Sample 1 :

## Original Text

"The association of lipids with proteins not only solubilizes lipids but also aids in their transport into cells. Triacylglycerols are transported to tissues either in chylomicrons or in VLDL."

Note the use of quotation marks and the reference number " 1 " at the end of the quotation to indicate the reference citation.

## Reference Citation:

1. Mathews, C. K., \& van Holde, K. E. (1990) Biochemistry, pp. 576, Benjamin/Cummings Press, Menlo Park, CA.

This reference citation would appear, numbered, in a list at the end of the paper. Using the reference citation provides proper credit to the original author. Remember that citation formats are discipline specific. Ask your professor what is appropriate for your papers in class or your research group.

## Bad Paraphrase of Original Text:

Lipids associate with proteins not only to solubilize themselves in blood but also to help their transport into cells. Triacylglycerols are carried to tissues by chylomicrons or VLDL particles. ${ }^{1}$

This paraphrase is worded too much like the original quotation. Even though the phrasing is not exactly the same as the original, this pharphrasing could still be considered plagiarism.

## Better Paraphrase of Original Text:

Plasma lipoproteins are composed of lipid and protein domains and are responsible for delivering the water-insoluble lipids to cells. For example, chylomicrons or VLDL particles are involved in the delivery of triacylglycerols. ${ }^{1}$

Now, the idea of the original author is more obviously stated in the words of the writer. Notice that both paraphrases still use the reference number " 1 " at the end of the idea to indicate that the original idea came from a source other than the writer of this paper.

## Paraphrasing Sample 2 :

Original Text
"Skills are dimensions of the ability to behave effectively in situations of action. Skill is a hybrid term that refers both to a property of concrete behavior and to a property of theories of action" (Argyris \& Schon, 1974, p. 12).

Note the use of quotation marks and inclusion of names of the authors, the date of the original publication, and the page number where the quotation appeared in the original text. Some disciplines use this type of reference citation format instead of numbers as used in sample 1.

Reference Citation:
Argyris, C. and Schon, D.A. (1974). Theory in practice: Increasing professional effectiveness. San Francisco, California: Jossey-Bass Publishers.

Note that this reference citation is different than that in Sample 1. In this discipline, the references are also listed at the end of the paper, but they are listed alphabetically by the author's last name. To make the last name easy to see, the first line of the citation sometimes hangs over the following lines as this example shows. Again, always ask your professor what is most appropriate for your papers in class or your research group.

## Bad Paraphrase of Original Text:

In active situations, skills are aspects of one's ability to act effectively. Referring to both to definite action as well as a theory of action, skill is a hybrid term (Argyris \& Schon, 1974).

This paraphrase is worded too much like the original quotation. Even though the phrasing is not exactly the same as the original, this pharphrasing could still be considered plagiarism.

Better Paraphrase of Original Text:
The term "skill" can refer both to physical actions that allow one to perform effectively in a specific situation as well as to mental concepts that comprise one's theories of action (Argyris \& Schon, 1974).

Now, the idea of the original author is more obviously stated in the words of the writer. Notice that both paraphrases still use reference citations to indicate that the original idea came from a source other than the writer of this paper.

## List of Symbol Definitions

$a \quad$ long dimension for a section subjected to torsion (in, mm); acceleration ( $\mathrm{ft} / \mathrm{sec}^{2}, \mathrm{~m} / \mathrm{sec}^{2}$ );
acceleration due to gravity, $32.17 \mathrm{ft} / \mathrm{sec}^{2}, 9.81 \mathrm{~m} / \mathrm{sec}^{2}$ (also see g)
unit area ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ );
distance used in beam formulas ( $\mathrm{ft}, \mathrm{m}$ );
depth of the effective compression block in a concrete beam (in, mm);
equivalent square column size in spread footing design (in, ft, mm, m)
$\boldsymbol{A}$ area bounded by the centerline of a thin walled section subjected to torsion ( $\mathrm{in}^{2}, \mathrm{~mm}^{2}$ )
A area, often cross-sectional ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{b} \quad$ nominal cross section bolt area ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{e} \quad$ net effective area, equal to the total area ignoring any holes and modified by the lag factor, $U$, ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ ) (see $A_{\text {net }}$ )
$A_{g} \quad$ gross area, equal to the total area ignoring any holes ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{g v} \quad$ gross area in shear, equal to the total area ignoring any holes ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{\text {net }} \quad$ net effective area, equal to the gross area subtracting any holes $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right)\left(\right.$ see $\left.A_{e}\right)$
$A_{n t} \quad$ net area in shear of a bolted connection subject to shear rupture $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right.$ )
$A_{n v} \quad$ net area in tension of a bolted connection subject to shear rupture ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ ); net shear area for a masonry member ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{p} \quad$ bearing area $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right)$
$A_{\text {throat }}$ area across the throat of a weld ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{s} \quad$ area of steel reinforcement in concrete beam design $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right.$ )
$A_{s^{\prime}} \quad$ area of compression steel reinforcement in concrete beam design ( $\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}$ )
$A_{v} \quad$ area of concrete shear stirrup reinforcement $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right)$; seismic coefficient for acceleration
$A_{\text {web }} \quad$ web area in a steel beam equal to the depth x web thickness $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right.$ )
$A_{1} \quad$ area of column in spread footing design ( $\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right)$
$A_{2} \quad$ projected bearing area of column load in spread footing design $\left(\left(\mathrm{in}^{2}, \mathrm{ft}^{2}, \mathrm{~mm}^{2}, \mathrm{~m}^{2}\right)\right.$
ASD Allowable Stress Design
$b \quad$ width, often cross-sectional (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ );
narrow dimension for a section subjected to torsion (in, mm);
number of truss members (also see n);
rectangular column dimension in concrete footing design (in, mm, m); distance used in beam formulas ( $\mathrm{ft}, \mathrm{m}$ )
$b_{E} \quad$ effective width of the flange of a concrete T beam cross section (in, mm)
$b_{f} \quad$ width of the flange of a steel or concrete T beam cross section (in, mm)
$b_{o} \quad$ perimeter length for two-way shear in concrete footing design (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$b_{w} \quad$ width of the stem of a concrete T beam cross section (in, mm)
$B \quad$ spread footing dimension in concrete design ( $\mathrm{ft}, \mathrm{m}$ );
dimension of a steel base plate (in, mm, m)
$B_{s} \quad$ width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design ( $\mathrm{ft}, \mathrm{m}$ )
$B_{l} \quad$ factor for determining $M_{u}$ for combined bending and compression
$c \quad$ distance from the neutral axis to the top or bottom edge of a beam (in, mm, m); rectangular column dimension in concrete footing design (in, mm, m)
$c_{1} \quad$ coefficient for shear stress for a rectangular bar in torsion
$c_{2} \quad$ coefficient for shear twist for a rectangular bar in torsion
$C L, \&$ center line
C compression label;
compression force (lb, kips, N, kN);
dimension of a steel base plate for concrete footing design (in, mm, m);
seismic design coefficient dependent on the building period of vibration;
constant for moment calculation of plates with respect to boundary conditions;
coefficient for eccentrically loaded bolt groups
$C_{a} \quad$ constant for moment calculation of plates with respect to boundary conditions
$C_{b} \quad$ modification factor for LRFD steel beam design;
constant for moment calculation of plates with respect to boundary conditions
$C_{d} \quad$ pressure coefficient for wind force calculation
$C_{D} \quad$ load duration factor for wood design
$C_{F} \quad$ size factor for wood design
$C_{f u} \quad$ flat use factor for wood design
$C_{H} \quad$ shear stress factor for wood design
$C_{i} \quad$ incising factor for wood design
$C_{L} \quad$ beam stability factor for wood design
$C_{m} \quad$ modification factor for combined stress in steel design
$C_{M} \quad$ wet service factor for wood design
$C_{p} \quad$ column stability factor for wood design
$C_{r} \quad$ repetitive member factor for wood design
$C_{s} \quad$ seismic design coefficient based on soil, response and acceleration
$C_{v} \quad$ web shear coefficient for steel design
$C_{V} \quad$ glulam volume factor for wood design
$C_{t} \quad$ temperature factor for wood design;
seismic coefficient based on structural system and number of stories to determine building period
$d \quad$ diameter of a circle (in, mm, m);
depth, often cross-sectional (in, mm, m);
perpendicular distance from a force to a point in a moment calculation (in, mm, m) ;
effective depth from the top of a reinforced concrete beam to the centroid of the steel (in, mm);
effective depth from the top of a reinforced masonry member to the centroid of the steel (in, mm); critical cross section dimension of a rectangular timber column cross section related to the profile (axis) for buckling (in, mm, m);
symbol in calculus to represent a very small change (like the greek letters for d , see $\delta \& \Delta$ )
$d^{\prime} \quad$ effective depth from the top of a reinforced concrete beam to the centroid of the compression steel (in, mm)
$d_{b} \quad$ depth of a steel wide flange section (in, mm);
bar diameter of concrete reinforcement (in, mm)
$d_{f} \quad$ depth of a steel column flange (wide flange section) (in, mm)
$d_{x} \quad$ difference in the $x$ direction between an area centroid and the centroid of the composite shape (in, mm)
$d_{y} \quad$ difference in the $y$ direction between an area centroid and the centroid of the composite shape (in, mm)
$D \quad$ diameter of a circle (in, mm, m);
dead load for LRFD design
DL dead load
$e \quad$ dimensional change to determine strain (in, mm) (see sor $\varepsilon$ ); eccentric distance of application of a force ( P ) from the centroid of a cross section (in, mm)
$E \quad$ modulus of elasticity ( psi ; ksi, $\mathrm{kPa}, \mathrm{MPa}, \mathrm{GPa}$ );
earthquake load for LRFD design
$E_{c} \quad$ modulus of elasticity of concrete ( $\mathrm{psi} ; \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}, \mathrm{GPa}$ )
$E_{s} \quad$ modulus of elasticity of steel ( $\mathrm{psi} ; \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}, \mathrm{GPa}$ )
$f \quad$ symbol for stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ );
symbol for function with respect to some variable, ie. $f(\mathrm{t})$
$f_{a} \quad$ calculated axial stress (psi, ksi, kPa, MPa)
$f_{b} \quad$ calculated bending stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{c} \quad$ calculated compressive stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{c}^{\prime} \quad$ concrete design compressive stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{c r} \quad$ calculated column stress based on the critical column load $\mathrm{P}_{\mathrm{cr}}(\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa})$
$f_{m} \quad$ calculated compressive stress in masonry ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{m}^{\prime} \quad$ masonry design compressive stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{n} \quad$ natural frequency of a suspended cable $\left(\mathrm{sec}^{-1}, \mathrm{~Hz}\right)$
$f_{p} \quad$ calculated bearing stress $(\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa})$
$f_{r} \quad$ calculated radial stress for a glulam timber ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{s} \quad$ calculated steel stress for reinforced masonry ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{t} \quad$ calculated tensile stress (psi, ksi, kPa, MPa)
$f_{v} \quad$ calculated shearing stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$f_{x} \quad$ combined stress in the direction of the major axis of a column (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$f_{y} \quad$ yield stress (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F \quad$ force (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
capacity of a nail in shear ( $\mathrm{lb}, \mathrm{kip}, \mathrm{N}, \mathrm{kN}$ );
hydraulic fluid load for LRFD design
$F_{a} \quad$ allowable axial stress $(\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa})$
$F_{b} \quad$ allowable bending stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$F_{b}^{\prime \prime} \quad$ allowable bending stress for combined stress for wood design (psi, ksi, kPa, MPa)
$F_{c} \quad$ allowable compressive stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
critical unfactored compressive stress for LRFD steel design
$F_{c r} \quad$ flexural buckling (column) stress in ASD and LRFD (psi, ksi, kPa, MPa)
$F_{c \perp} \quad$ allowable compressive stress perpendicular to the wood grain (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{\text {connector }} \quad$ resistance capacity of a connector (lb, kips, $\mathrm{N}, \mathrm{kN}$ )
$F_{c E}^{\prime} \quad$ intermediate compressive stress for ASD wood column design dependant on material (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{c r} \quad$ critical column stress due to buckling (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{c}^{\prime} \quad$ allowable compressive stress for ASD wood column design (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F^{*}{ }_{c} \quad$ intermediate compressive stress for ASD wood column design dependant on load duration (psi, ksi, kPa, MPa)
$F_{e} \quad$ elastic critical buckling stress is steel design
$F_{\text {EXX }}$ yield strength of weld material (psi, ksi, kPa, MPa)
$F_{\text {horizontal-resist }}$ resultant frictional force resisting sliding in a footing or retaining wall (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$F_{n} \quad$ nominal stress (psi, ksi, kPa, MPa)
$F_{n v} \quad$ nominal shear stress (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{n t} \quad$ nominal tensile stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$F_{p} \quad$ allowable bearing stress parallel to the wood grain (psi, $\mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$F_{r} \quad$ allowable radial stress for a curved glulam (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{\text {sliding }}$ resultant force causing sliding in a footing or retaining wall (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$F_{t} \quad$ allowable tensile stress (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{v} \quad$ allowable shear stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ );
allowable shear stress in a welded connection (psi, ksi, kPa, MPa)
$F_{v m} \quad$ allowable shear stress in the reinforced masonry (psi, ksi, kPa, MPa)
$F_{v s} \quad$ allowable shear stress in the reinforcement for masonry (psi, ksi, $\mathrm{kPa}, \mathrm{MPa}$ )
$F_{x} \quad$ force component in the x coordinate direction (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$F_{y} \quad$ force component in the y coordinate direction (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
yield stress ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$F_{y w} \quad$ yield stress in the web of a steel wide flange section ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
$F_{u} \quad$ ultimate stress a material can sustain prior to failure ( $\mathrm{psi}, \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ )
F.S. factor of safety (also see SF)
$g \quad$ acceleration due to gravity, $32.17 \mathrm{ft} / \mathrm{sec}^{2}, 9.81 \mathrm{~m} / \mathrm{sec}^{2}$ (also see a)
gage spacing of staggered bolt holes (in, mm)
$G \quad$ shear modulus ( $\mathrm{psi} ; \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}, \mathrm{GPa}$ );
gigaPascals ( $10^{9} \mathrm{~Pa}$ or $1 \mathrm{kN} / \mathrm{mm}^{2}$ );
relative stiffness of columns to beams in a rigid connection (see $\Psi$ )
$h$ depth, often cross-sectional (in, ft, mm, m);
sag of a cable structure ( $\mathrm{ft}, \mathrm{m}$ );
height (in, ft, mm, m);
effective height of a wall or column, $\left(\operatorname{see} \ell_{e}\right)$
$h_{c} \quad$ height of the web in a wide flange section (in, $\left.\mathrm{ft}, \mathrm{mm}, \mathrm{m}\right)$ (also see $t_{w}$ )
$h_{f} \quad$ depth of a flange in a T section (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ );
height of a concrete spread footing (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$h_{n} \quad$ building height for determination of period for seismic design
$H$ hydraulic soil load for LRFD design;
height of retaining wall (ft, m)
$H_{A} \quad$ horizontal load from active soil or water pressure ( $\mathrm{lb}, \mathrm{k}, \mathrm{N}, \mathrm{kN}$ )
I moment of inertia ( $\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}$ );
seismic importance factor based on building occupancy
$\bar{I} \quad$ moment of inertia about the centroid $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$\bar{I}_{T} \quad$ moment of inertia about the centroid of a composite shape $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right.$ ) (also see $\hat{I}$ )
$\hat{I} \quad$ moment of inertia about the centroid of a composite shape (in ${ }^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}$ ) (also see $I_{c}$ )
$I_{c} \quad$ moment of inertia about the centroid of a composite shape $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$I_{\text {min }} \quad$ minimum moment of inertia of $\mathrm{I}_{\mathrm{x}}$ and $\mathrm{I}_{\mathrm{y}}\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$I_{n e t} \quad$ moment of inertia of plate area excluding bolt holes $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}\right.$ )
$I_{o} \quad$ moment of inertia about the centroid $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$I_{\text {transformed }}$ moment of inertia of a multi-material section transformed to one material (in ${ }^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}$ )
$I_{x} \quad$ moment of inertia with respect to an x -axis $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right.$ )
$I_{y} \quad$ moment of inertia with respect to a y-axis $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$j \quad$ number of connections in a truss (also see $n$ ); multiplier by effective depth of concrete or masonry section for moment arm, jd (see $d$ )
$J, J_{o} \quad$ polar moment of inertia $\left(\mathrm{in}^{4}, \mathrm{~mm}^{4}, \mathrm{~m}^{4}\right)$
$k \quad$ kips ( 1000 lb );
shape factor for plastic design of steel beams, $\mathrm{M}_{\mathrm{p}} / \mathrm{M}_{\mathrm{y}}$;
effective length factor for columns (also $K$ );
distance from outer face of flange to the web toe of fillet of a wide flange section (in, mm); spring constant (lb/in, N/mm);
multiplier by effective depth of masonry section for neutral axis, kd
kg kilograms
$k N \quad$ kiloNewtons $\left(10^{3} \mathrm{~N}\right)$
$k P a \quad$ kiloPascals $\left(10^{3} \mathrm{~Pa}\right)$
$K \quad$ effective length factor with respect to column end conditions (also $k$ );
masonry mortar strength designation
$K_{A} \quad$ empirically derived coefficient based on soil properties
$K_{c E} \quad$ material factor for wood column design
$\ell \quad$ length (in, ft, mm, m);
cable span ( $\mathrm{ft}, \mathrm{m}$ )
$\ell_{d} \quad$ development length of concrete reinforcement (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$\ell_{d c} \quad$ development length of compression reinforcement in concrete footing design (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$l_{d h}$ development length for hooks (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$\ell_{e} \quad$ effective length that can buckle for wood column design (in, ft, mm, m)
$\ell_{n} \quad$ effective clear span for concrete one-way slab design (ft, m)
$l b \quad$ pound force
$L \quad$ length (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ );
live load for LRFD design;
spread footing dimension in concrete design ( $\mathrm{ft}, \mathrm{m}$ )
$L_{b} \quad$ unbraced length of a steel beam in LRFD design ( $\mathrm{ft}, \mathrm{m}$ )
$L_{c} \quad$ clear distance between the edge of a bolt hole and the edge of the next hole or edge of the connected steel plate in the direction of the load (in, mm)
$L_{d} \quad$ development length of reinforcement in concrete ( $\mathrm{ft}, \mathrm{m}$ )
$L_{e} \quad$ effective length that can buckle for column design ( $\mathrm{ft}, \mathrm{m}$ )
$L_{m} \quad$ projected length for bending in concrete footing design ( $\mathrm{ft}, \mathrm{m}$ )
$L_{p} \quad$ maximum unbraced length of a steel beam in LRFD design for full plastic flexural strength (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$L_{r} \quad$ roof live load in LRFD design;
maximum unbraced length of a steel beam in LRFD design for inelastic lateral-torsional buckling (in, ft, mm, m)

$L L \quad$ live load
LRFD Load and Resistance Factor Design
$m \quad$ mass (lb-mass, $\mathrm{g}, \mathrm{kg}$ );
meters;
moment per unit width ( $\mathrm{lb}-\mathrm{ft} / \mathrm{ft}, \mathrm{kN}-\mathrm{m} / \mathrm{m}$ );
edge dimension in a steel base plate (in, mm)
$\mathrm{mm} \quad$ millimeters
$M$ moment of a force or couple (lb-ft, kip-ft, N-m, kN-m);
bending moment (lb-ft, kip-ft, N-m, kN-m);
masonry mortar strength designation
$M_{a} \quad$ required bending moment in steel ASD beam design (unified) (lb-ft, kip- $\mathrm{ft}, \mathrm{N}-\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M_{A} \quad$ moment value at quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N $\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M_{B} \quad$ moment value at half point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, $\mathrm{kN}-\mathrm{m}$ );
nominal moment capacity of a reinforced concrete beam at the balanced steel ratio ( $\rho_{b}$ ) for limiting strains in both concrete and steel (lb-ft, kip-ft, N-m, kN-m)
$M_{C} \quad$ moment value at three quarter point of unbraced beam length for LRFD beam design (lb-ft, kip$\mathrm{ft}, \mathrm{N}-\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M_{m} \quad$ moment capacity of a reinforced masonry beam (lb-ft, kip-ft, N-m, kN-m)
$M_{n} \quad$ nominal moment capacity of a reinforced concrete beam based on steel yielding and concrete design strength (lb-ft, kip-ft, $\mathrm{N}-\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M_{\text {overturning }}$ resulting moment from all forces on a footing or retaining wall causing overturning (lb-ft, kip-ft, N-m, kN-m)
$M_{p} \quad$ internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N $\mathrm{m}, \mathrm{kN}-\mathrm{m}$ ) (also see $M_{u l t}$ )
$M_{\text {resist }}$ resulting moment from all forces on a footing or retaining wall resisting overturning (lb-ft, kip$\mathrm{ft}, \mathrm{N}-\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M_{u} \quad$ factored moment calculated in concrete design from load factors (lb-ft, kip-ft, N-m, kN-m)
$M_{u l t} \quad$ internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N$\mathrm{m}, \mathrm{kN}-\mathrm{m}$ ) (also see $M_{p}$ )
$M_{y} \quad$ internal bending moment when the extreme fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)
$M_{1} \quad$ smaller end moment used to calculate $\mathrm{C}_{\mathrm{m}}$ for combined stresses in a beam-column (lb-ft, kip-ft, N-m, kN-m)
$M_{2}$ larger end moment used to calculate $\mathrm{C}_{\mathrm{m}}$ for combined stresses in a beam-column (lb-ft, kip- ft , $\mathrm{N}-\mathrm{m}, \mathrm{kN}-\mathrm{m}$ )
$M P a$ megaPascals $\left(10^{6} \mathrm{~Pa}\right.$ or $\left.1 \mathrm{~N} / \mathrm{mm}^{2}\right)$
$n \quad$ number of truss joints or members, nails or bolts;
modulus of elasticity transformation coefficient for steel to concrete;
edge dimension in a steel base plate (in, mm)
n.a. neutral axis (axis connecting beam cross-section centroids)
$n$, equivalent edge dimension in a steel base plate for design (in, mm)
$N \quad$ Newtons (kg-m/sec ${ }^{2}$ );
bearing-type connection with bolt threads included in shear plane;
normal load (lb, kip, N, kN);
bearing length on a wide flange steel section (in, mm);
dimension of a steel base plate (in, mm, m);
masonry mortar strength designation
$N_{\phi} \quad$ meridional in-plane internal force per unit length in a shell (lb/ft, $\mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ )
$N_{\theta} \quad$ hoop in-plane internal force per unit length in a shell ( $\mathrm{lb} / \mathrm{ft}, \mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ )
o.c. on-center
$O$ point of origin;
masonry mortar strength designation
$p \quad$ pitch of nail spacing (in, mm) (also see $s)$;
pressure (lb/in ${ }^{2}, \mathrm{lb} / \mathrm{ft}^{2}$, kip/in ${ }^{2}$, kip/ $\mathrm{ft}^{2}, \mathrm{~Pa}, \mathrm{MPa}$ ):
unit weight of soil for determining active lateral pressure $\left(\mathrm{lb} / \mathrm{ft}^{3}, \mathrm{kN} / \mathrm{m}^{3}\right)$
$p_{A} \quad$ active soil pressure $\left(\mathrm{lb} / \mathrm{ft}^{3}, \mathrm{kN} / \mathrm{m}^{3}\right)$
$p_{r} \quad$ internal pressure $\left(\mathrm{lb} / \mathrm{in}^{2}, \mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kip} / \mathrm{in}^{2}, \mathrm{kip} / \mathrm{ft}^{2}, \mathrm{~Pa}, \mathrm{MPa}\right)$
$P \quad$ force, concentrated (point) load (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{a} \quad$ required axial force in ASD steel design (unified) (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{c} \quad$ available axial strength for steel unified design (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{c r} \quad$ critical (failure) load in column calculations (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{e 1} \quad$ Euler buckling strength in steel unified design (lb, kip, N, kN)
$P_{n} \quad$ maximum column load capacity in LRFD steel and concrete design (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
nominal axial load for a tensile member or connection in LRFD steel (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{o} \quad$ maximum axial force with no concurrent bending moment in a reinforced concrete column (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{r} \quad$ required axial force in steel unified design (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P_{u}$ factored column load calculated from load factors in LRFD steel and concrete design (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
factored axial load for a tensile member or connection in LRFD steel (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$P a \quad$ Pascals (N/m²)
$q \quad$ shear flow (lb/in, kips/ft, N/m, kN/m) );
soil bearing pressure ( $\mathrm{lb} / \mathrm{ft}^{2}$, kips/ $\mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{~Pa}, \mathrm{MPa}$ )
$q_{\text {allowed }}$ allowable soil bearing pressure $\left(\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kips} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{~Pa}, \mathrm{MPa}\right)$
$q_{h} \quad$ static wind velocity pressure for wind force calculation ( $\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kips} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}, \mathrm{Pa}, \mathrm{MPa}$ )
$q_{\text {net }} \quad$ net allowed soil bearing pressure ( $\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kips} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}, \mathrm{Pa}, \mathrm{MPa}$ )
$q_{u} \quad$ factored soil bearing pressure in concrete design from load factors $\left(\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kips} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}, \mathrm{Pa}\right.$, MPa)
$Q \quad$ first moment area used in shearing stress calculations $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}\right.$ )
$Q_{\text {connected }}$ first moment area used in shear calculations for built-up beams $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}\right)$
$Q_{x} \quad$ first moment area about an x axis (using y distances) ( $\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}$ )
$Q_{y} \quad$ first moment area about an y axis (using x distances) (in ${ }^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}$ )
$r$ radius of a circle or arc (in, mm, m);
radius of gyration (in, mm, m)
$r_{o} \quad$ polar radius of gyration (in, $\mathrm{mm}, \mathrm{m}$ )
$r_{x} \quad$ radius of gyration with respect to an x -axis (in, mm, m)
$r_{y} \quad$ radius of gyration with respect to a y-axis (in, mm, m)
$R \quad$ force, reaction or resultant (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
radius of curvature of a beam or radius of a shell ( $\mathrm{ft}, \mathrm{m}$ );
rainwater or ice load for LRFD design;
seismic response modification based on structural type;
calculated reduction in live load limited to $60 \%$ (in percent);
generic load quantity (force, shear, moment, etc.) for LRFD design
$R_{a} \quad$ required strength (ASD-unified) (also see $V_{a}, M_{a}$ )
$R_{n} \quad$ concrete beam design ratio $=\mathrm{M}_{\mathrm{u}} / \mathrm{bd}^{2}\left(\mathrm{lb} / \mathrm{in}^{2}, \mathrm{MPa}\right)$
nominal value for LRFD design to be multiplied by $\phi$ (also see $P_{n}, M_{n}$ ) nominal value for ASD design to be divided by the safety factor $\Omega$
$R_{u} \quad$ design value for LRFD design based on load factors (also see $P_{u}, M_{u}$ )
$R_{w} \quad$ seismic response modification based on structural type
$R_{x} \quad$ reaction or resultant component in the x coordinate direction (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$R_{y} \quad$ reaction or resultant component in the y coordinate direction (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$s \quad$ strain (=change in length divided by length) ( no units);
displacement with respect to time ( $\mathrm{ft}, \mathrm{m}$ );
length of a segment of a thin walled section (in, mm);
pitch of nail spacing (in, mm) (also see p);
longitudinal center-to-center spacing of any two consecutive holes (in, mm);
spacing of stirrups in reinforced concrete beams (in, mm)
s.w. self-weight
$S \quad$ section modulus ( $\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}$ );
snow load for LRFD design;
allowable strength of a weld for a given size ( $\mathrm{lb} / \mathrm{in}, \mathrm{kips} / \mathrm{in}, \mathrm{N} / \mathrm{mm}, \mathrm{kN} / \mathrm{m}$ )
seismic soil profile;
masonry mortar strength designation
$S_{\text {net }} \quad$ section modulus of plate area excluding bolt holes (in ${ }^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}$ )
$S_{\text {required }}$ section modulus required to not exceed allowable bending stress ( $\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}$ )
$S_{x} \quad$ section modulus with respect to the x-centroidal axis $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}\right)$
$S_{y} \quad$ section modulus with respect to the y-centroidal axis $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}, \mathrm{~m}^{3}\right.$ )
$S C$ slip critical bolted connection
SF safety factor (also see F.S.)
S4S surface-four-sided
$t \quad$ thickness (in, mm, m);
time ( $\mathrm{sec}, \mathrm{hrs}$ )
$t_{f} \quad$ thickness of the flange of a steel beam cross section (in, mm, m)
$t_{w} \quad$ thickness of the web of a steel beam cross section (in, mm, m)
$T$ tension label;
tensile force (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
torque (lb-ft, kip-ft, N-m, kN-m);
throat size of a weld (in, mm);
effect of thermal load for LRFD design;
seismic building period (sec);
depth in web of wide flange section from fillet to fillet (in, mm)
$U \quad$ shear lag factor for steel tension member design (see $A_{e}$ and $A_{\text {net }}$ )
$U_{b s} \quad$ reduction coefficient for block shear rupture
$v \quad$ velocity ( $\mathrm{ft} / \mathrm{sec}, \mathrm{m} / \mathrm{sec}, \mathrm{mi} / \mathrm{h}$ );
shear force per unit length ( $\mathrm{lb} / \mathrm{ft}, \mathrm{k} / \mathrm{ft}, \mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ ) (see q)
$V \quad$ shearing force ( $\mathrm{lb}, \mathrm{kip}, \mathrm{N}, \mathrm{kN}$ );
seismic base shear force (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{a} \quad$ required shear in steel ASD design (unified) (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{c} \quad$ shear force capacity in concrete (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{n} \quad$ nominal shear force capacity for concrete design (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{s} \quad$ shear force capacity in steel (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{u} \quad$ factored shear calculated in concrete design from load factors (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{u l} \quad$ factored one-way shear calculated in concrete footing design from load factors (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$V_{u 2}$ factored two-way shear calculated in concrete footing design from load factors (lb, kip, $\mathrm{N}, \mathrm{kN}$ )
$w \quad$ load per unit length on a beam (lb/ft, kip/ft, $\mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ );
load per unit area on a surface ( $\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kip} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{kN} / \mathrm{m}^{2}$ ) (see w');
width dimension (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$w_{c} \quad$ weight of reinforced concrete per unit volume $\left(\mathrm{lb} / \mathrm{ft}^{3}, \mathrm{~N} / \mathrm{m}^{3}\right)$
$w_{u} \quad$ factored load per unit length on a beam from load factors (lb/ft, kip/ft, $\mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ );
factored load per unit area on a surface from load factors ( $\mathrm{lb} / \mathrm{ft}^{2}$, $\mathrm{kip} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{kN} / \mathrm{m}^{2}$ )
$w^{\prime} \quad$ load per unit area on a surface $\left(\mathrm{lb} / \mathrm{ft}^{2}, \mathrm{kip} / \mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{kN} / \mathrm{m}^{2}\right)$ (see $w$ );
$W \quad$ weight (lb, kip, $\mathrm{N}, \mathrm{kN}$ );
total load from a uniform distribution (lb, kip, N, kN);
wind load for LRFD design;
seismic building weight ( lb , kip, $\mathrm{N}, \mathrm{kN}$ );
wide flange shape designation (i.e. W $21 \times 68$ )
$x \quad$ a distance in the x direction (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ )
$\bar{x} \quad$ the distance in the x direction from a reference axis to the centroid of a shape (in, mm)
$\hat{x}$ the distance in the x direction from a reference axis to the centroid of a composite shape (in, mm )
$X \quad$ bearing-type connection with bolt threads excluded from shear plane;
design constant for steel base plate design based on concrete bearing capacity
$y \quad$ a distance in the $y$ direction (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ );
distance from the neutral axis to the $y$-level of a beam cross section (in, mm)
$\bar{y} \quad$ the distance in the $y$ direction from a reference axis to the centroid of a shape (in, mm)
$\bar{y}_{T} \quad$ the distance in the y direction from a reference axis to the centroid of a composite shape (in, mm ) (also see $\hat{y}$ )
$\hat{y}$ the distance in the $y$ direction from a reference axis to the centroid of a composite shape (in, mm ) (also see $\bar{y}_{T}$ )
$z \quad$ the distance from a unit area to a reference axis (in, $\mathrm{ft}, \mathrm{mm}, \mathrm{m}$ ) (also see $d_{x}$ and $d_{y}$ )
$Z \quad$ plastic section modulus of a steel beam $\left(\mathrm{in}^{3}, \mathrm{~mm}^{3}\right)$;
seismic geographic factor based on zone
, symbol for feet
" symbol for inches
\# symbol for pounds
$=$ symbol for equal to
$\approx \quad$ symbol for approximately equal to
$\propto \quad$ symbol for proportional to
$\leq \quad$ symbol for less than or equal to
」 symbol for integration
$\alpha \quad$ coefficient of thermal expansion $\left(/{ }^{\circ} \mathrm{C}, /{ }^{\circ} \mathrm{F}\right)$;
angle, in a math equation (degrees, radians)
$\beta \quad$ angle, in a math equation (degrees, radians)
$\beta_{c} \quad$ ratio of long side to short side of the column in concrete footing design
$\beta_{1} \quad$ coefficient for determining stress block height, $a$, based on concrete strength, $f_{c}^{\prime}$
$\delta \quad$ elongation (in, mm) (also see e)
$\delta_{P} \quad$ elongation due to axial load (in, mm)
$\delta_{s} \quad$ shear deformation (in, mm)
$\delta_{T} \quad$ elongation due to change in temperature (in, mm)
$\Delta$ beam deflection (in, mm);
story drift (in, mm);
an increment
$\Delta_{L L} \quad$ beam deflection due to live load (in, mm)
$\Delta_{\text {max }} \quad$ maximum calculated beam deflection (in, mm)
$\Delta_{T L} \quad$ beam deflection due to total load (in, mm)
$\Delta T \quad$ change in temperature $\left({ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{F}\right)$
$\varepsilon \quad$ strain (no units)
$\varepsilon_{t} \quad$ thermal strain (no units)
$\phi \quad$ diameter symbol;
angle of twist (degrees, radians);
resistance factor in LRFD steel design and reinforced concrete design; angle defining the shell cutoff (degrees, radians)
$\kappa \quad$ limit of timber slenderness for intermediate length columns (no units)
$\lambda \quad$ design constant for steel base plate design
$\mu \quad$ Poisson's ratio (also see v);
coefficient of static friction
$v \quad$ Poisson's ratio (also see $\mu$ )
$\gamma \quad$ specific gravity of a material ( $\mathrm{lb} / \mathrm{in}^{3}, \mathrm{lb} / \mathrm{ft}^{3}, \mathrm{~N} / \mathrm{m}^{3}, \mathrm{kN} / \mathrm{m}^{3}$ ); angle, in a math equation (degrees, radians);
shearing strain (no units);
load factor in LRFD design
$\gamma_{D}$ dead load factor in LRFD steel design
$\gamma_{L} \quad$ live load factor in LRFD steel design
$\theta \quad$ angle, in a trig equation, ex. $\sin \theta$ (degrees, radians);
slope of the deflection of a beam at a point (degrees, radians)
$\pi \quad$ pi $\left(180^{\circ}\right)$
$\rho \quad$ radial distance (in, mm);
radius of curvature in beam deflection relationships (ft, m);
reinforcement ratio in concrete beam design $=\mathrm{A}_{s} / \mathrm{bd}$ (or possibly $\mathrm{A}_{s} / \mathrm{bt}, \mathrm{A}_{\mathrm{s}} / \mathrm{bh}$ ) (no units)
$\rho_{b} \quad$ balanced reinforcement ratio in concrete beam design
$\rho_{g} \quad$ reinforcement ratio in concrete column design $=\mathrm{A}_{\mathrm{st}} / \mathrm{A}_{\mathrm{g}}$
$\rho_{\max }$ maximum reinforcement ratio allowed in concrete beam design for ductile behavior
$\sigma \quad$ engineering symbol for normal stress (axial or bending)
$\tau \quad$ engineering symbol for shearing stress
$v_{c} \quad$ shearing stress capacity in concrete design ( $\mathrm{psi} ; \mathrm{ksi}, \mathrm{kPa}, \mathrm{MPa}$ );
$\omega \quad$ load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m) (see w);
load per unit area ( $\mathrm{lb} / \mathrm{ft}^{2}$, kips/ $\mathrm{ft}^{2}, \mathrm{~N} / \mathrm{m}^{2}, \mathrm{~Pa}, \mathrm{MPa}$ )
$\omega^{\prime} \quad$ load per unit volume ( $\mathrm{lb} / \mathrm{ft}, \mathrm{kip} / \mathrm{ft}, \mathrm{N} / \mathrm{m}, \mathrm{kN} / \mathrm{m}$ ) (see $\gamma$ )
$\Sigma$ summation symbol
$\Omega \quad$ safety factor for ASD of steel (unified)
$\Psi \quad$ relative stiffness of columns to beams in a rigid connection (see $G$ )

## Structural Glossary

Allowable strength: Nominal strength divided by the safety factor.
Allowable stress: Allowable strength divided by the appropriate section property, such as section modulus or cross section area.

Applicable building code: Building code und which the structure is designed.
ASD (Allowable Strength Design): Method of proportioning structural components such that the allowable strength equals or exceeds the required strength of the component under the action of the ASD load combinations.

ASD load combination: Load combination in the applicable building code intended for allowable strength design (allowable stress design).
ASTM standards: The American Society of Testing and Materials specifies standards for performance and testing of construction materials.
Axial force: A force that is acting along the longitudinal axis of a structural member.
Base shear: A lateral (wind or seismic) force acting at the base of a structure.
Beam: Structural member that has the primary function of resisting bending moments.
Beam-column: Structural member that resists both axial force and bending moment.
Bearing (local compressive yielding): Limit state of local compressive yielding due to the action of a member bearing against another member or surface.

Bending moment: A force rotating about a point; causes bending in beams, etc.
Block shear rupture: In a connection, limit state of tension fracture along one path and shear yielding or shear fracture along another path.
Bracing: Diagonal members that are used to stiffen a structure, by utilizing the inherent in-plane stiffness of a triangular framework.

Braced frame: An essentially vertical truss system that provides resistance to lateral forces and provides stability for the structural system.
Buckling: Limit state of sudden change in the geometry of a structure or any of its elements under a critical loading condition.
Buckling strength: Nominal strength for buckling or instability limit states.
Built-up member, cross-section, section, shape: Member, cross-section, section or shape fabricated from elements that are nailed, welded, glued or bolted together.

Camber: Curvature fabricated into a beam or truss so as to compensate for deflection induced by loads.

Cantilevers: Structural elements or systems that are supported only at one end.
Cement: The generic name for cementitious (binder) materials used in concrete, which is a commonly used building material.

Center of gravity: The location of resultant gravity forces on an object or objects.
Centroid: The center of mass of a shape or object.

Chord member: Primary member that extends, usually horizontally, through a truss connection.
Cold-rolled steel structural member: Shape manufactured by roll forming cold-or hot- rolled coils or sheets without manifest addition of heat such as would be required for hot forming.

Collector: An element that transfers load from a diaphragm to a resisting element.
Column: Structural member that has the primary function of resisting axial force.
Component (of vector): One of several vectors combined to a resultant vector.
Composite: Condition in which steel and concrete elements and members work as a unit in the distribution of internal forces.

Composite materials: Those consisting of a combination of two of more distinct materials, together yielding superior characteristics to those of the individual constituents.

Compression: A force that tends to shorten or crush a member or material.
Concentrated force: A force acting on a single point.
Concentrated load: An external concentrated force (also known as a point load).
Concrete: Material composed mainly of cement, crushed rock or gravel, sand and water.
Concrete crushing: Limit state of compressive failure in concrete having reached the ultimate strain.

Connection: A connection joins members to transfer forces or moments from one to the other.
Cope: Cutout made in a structural member to remove a flange and conform to the shape of an intersecting member.

Couple: A couple is a system of two equal forces of opposite direction offset by a distance. A couple causes a moment whose magnitude equals the magnitude of the force times the offset distance.

Cover plate: Plate welded or bolted to the flange of a member to increase cross-sectional area, section modulus or moment of inertia.

Creep: Plastic deformation that proceeds with time.
Damping: Reduces vibration amplitude, like amplitude seismic vibration.
Dead load: The weight of a structure or anything permanently attached to it.
Deflection: Deflection is the vertical moment under gravity load of beams for example, while lateral movement under wind of seismic load is called drift.

Deformation: A change of the shape of an object or material.
Design load: Applied load determined in accordance with either LRFD load combinations or ASD load combinations, whichever is applicable.
Design strength: Resistance factor multiplied by the nominal strength, $\varnothing \mathrm{R} n$.
Design stress range: Magnitude of change in stress due to the repeated application and removal of service live loads. For locations subject to stress reversal it is the algebraic difference of the peak stresses.
Design stress: Design strength divided by the appropriate section property, such as section modulus or cross section area.

Determinate structure: A structure with the number of reactions equal to the number of static equations (3).
Diagonal Bracing: Inclined structural member carrying primarily axial force in a braced fame.
Diaphragm plate: Plate possessing in-plane shear stiffness and strength, used to transfer forces to the supporting elements.

Diaphragm: Roof, floor or other membrane or bracing system that transfers in-plane forces to the lateral force resisting system.

Displacement: May be a translation, a rotation, or a combination of both.
Distributed load: An external force which acts over a length or an area.
Double curvature: Deformed shape of a beam with one or more inflection points within the span.
Double-concentrated forces: Two equal and opposite forces that form a couple on the same side of the loaded member.

Drift: Lateral deflection of structure due to lateral wind or seismic load.
Ductibility: The capacity of a material to deform without breaking; it is measured as the ratio of total strain at failure, divided by the strain at the elastic limit.
Durability: Ability of a material, element or structure to perform its intended function for its required life without the need for replacement or significant repair, but subject to normal maintenance.

Dynamic equilibrium: Equilibrium of a moving object without change of motion.
Dynamic load: Cyclic load, such as gusty wind or seismic loads.
Effective length factor, $K$ : Ratio between the effective length and the unbraced length of the member.

Effective length: Length of an otherwise identical column with the same strength when analyzed with pinned end conditions.

Effective net area: Net area modified to account for the effect of shear lag.
Effective section modulus: Section modulus reduced to account for buckling of slender compression elements.

Effective width: Reduced width of a plate or slab with an assumed uniform stress distribution which produces the same effect on the behavior of a structural member as the actual plate or slab with its nonuniform stress distribution.

Elastic: A material or structure is elastic if it returns to its original geometry upon unloading.
Elastic/plastic: Materials that have both an elastic zone and a plastic zone (i.e. steel).
Elastic limit: The point of a stress/strain graph beyond which deformation of a material is plastic, i.e. remains permanently deformed.

Elastic modulus: The linear slope value relating material stress to strain.
End-bearing pile: A pile supported on firm soil or rock.
Energy: The work to move a body a distance; energy is the product of forces times distance.
Epicenter: The point on the Earth's surface above the hypocenter where an earthquake originates.

Equilibrium: An object is in equilibrium if the resultant of all forces acting on it has zero magnitude.

External force: A force acting on an object; external forces are also called applied forces.
Factored load: Product of a load factor and the nominal load.
Fatigue: Limit state of crack initiation and growth resulting from repeated application of live loads, usually by reversing the loading direction.
Fillet weld: Weld of generally triangular cross section made between intersecting surfaces of elements.

Fitted bearing stiffener: Stiffener used at a support or concentrated load that fits tightly against one or both flanges of a beam so as to transmit load through bearing.
Fixed connection: A connection that resists axial and shear forces and bending moments.
Flexure: Bending deformation (of increasing curvature).
Flexural buckling: Buckling mode in which a compression member deflects laterally without twist or change in cross-sectional shape.

Flexural-torsional buckling: Buckling mode in which a compression member bends and twists simultaneously without change in cross-sectional shape.
Force: Resultant of distribution of stress over a prescribed area, or an action that tends to change the shape of an object, move an object, or change the motion of an object.

Foundations: There are two basic types: 'shallow,' which includes pad footing, strip footings and rafts and 'deep' i.e. piles. The choice is a function of the strength and stiffness of the underlying strata and the load to be carried, the aim being to limit differential settlement on the structure and more importantly the finishes.

Fully restrained moment connection: Connection capable of transferring moment with negligible rotation between connected members.

Funicular: The shape of a chain or string suspended form two points under any load.
Gravity: An attractive force between objects; each object accelerates at the attractive force divided by its mass.

Groove weld: Weld in a groove between connection elements.
Gusset plate: Plate element connecting truss members of a strut or brace to a beam or column.
Hertz: Cycles per second.
Horizontal diaphragm: A floor or roof deck to resist lateral load.
Horizontal shear: Force at the interface between steel and concrete surfaces in a composite beam.
Indeterminate structure: A structure with more unknown reactions than static equations (3).
Inelastic: Inelastic (plastic) strain implies permanent deformation.
Inertia: Tendency of objects at rest to remain at rest and objects in motion to remain in motion.
In-plane instability: Limit state of a beam-column bent about its major axis while lateral buckling or lateral-torsional buckling is prevented by lateral bracing.

Instability: Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry produces large displacements.
Internal force: The force within an object that resists external forces, also called resisting force.
Joint: Area where two or more ends, surfaces, or edges are attached. Categorized by type of fastener or weld used and method of force transfer.

Joist: A repetitive light beam.
K-connection: Connection in which forces in branch members or connecting elements transverse to the main member are primarily equilibrated by forces in other branch members or connecting elements on the same side of the main member.

Kern: The core of a post or other compression member which limits eccentric stresses being tensile.

Lacing: Plate, angle or other steel shape, in a lattice configuration, that connects two steel shapes together.

Lap joint: Joint between two overlapping connection elements in parallel planes.
Lateral bracing: Diagonal bracing, shear walls or equivalent means for providing in-plane lateral stability.

Lateral load resisting system: Structural system designed to resist lateral loads and provide stability for the structure as a whole.

Lateral load: Load, such as that produced by wind or earthquake effects, acting in a lateral direction.

Lateral-torsional buckling: Buckling mode of a flexural member involving deflection normal to the plane of bending occurring simultaneously with twist about the shear center of the crosssection.

Length effects: Consideration of the reduction in strength of a member based on its unbraced length.

Limit state: Condition in which a structure or component becomes unfit for service and is judged either to be no longer useful for its intended function (serviceability limit state) or to have reached its ultimate load-carrying capacity (strength limit state).
Linear: A structural or material behavior is linear if its deformation is directly proportional to the loading.
Line of action: The line of action defines the location and incline of a vector.
Linear elastic: A force-displacement relationship which is both linear and elastic.
Live load: Any load not permanently attached to the structure.
Load: Force or other action that results from the weight of building materials, occupants and their possessions, environmental effects, differential movement, or restrained dimensional changes.
Load effect: Forces, stresses and deformations produced in a structural component by the applied loads.

Load factor: Factor that accounts for deviations of the nominal load from the actual load, for uncertainties in the analysis that transforms the load into a load effect and for the probability that more than one extreme load will occur simultaneously.
Local bending: Limit state of large deformation of a flange under a concentrated tensile force.
Local buckling: Limit state of buckling of a compression element within a cross section.
Local crippling: Limit state of local failure of web plate in the immediate vicinity of a concentrated load or reaction.

Local yielding: Yielding that occurs in a local area of an element.
LRFD (Load and Resistance Factor Design): Method of proportioning structural components such that the design strength equals or exceeds the required strength of the component under the action of the LRFD load combinations.

LRFD load combination: Load combination in the applicable building code intended for strength design (load and resistance factor design).
Main member: Chord member or column to which branch members or other connecting elements are attached.

Mass: Mass is the property of an object to resist acceleration.
Magnitude: a scalar value of physical units, such as force or displacement.
Modulus of elasticity: The proportional constant relating stress/strain of material in the linear elastic range: calculated as stress divided by strain. The modulus of elasticity is the slope of the stress-strain graph, usually denoted as E, also as Young's Modulus Y, or E-Modulus.
Moment: A force causing rotation without translation; defined as force times lever arm.
Moment of inertia: Moment of inertia is the capacity of an object to resist bending or buckling, defined as the sum of all parts of the object times the square of their distance from the centroid.

Moment connection: Connection that transmits bending moment between connected members.
Moment frame: Framing system that provides resistance to lateral loads and provides stability to the structural system, primarily by shear and flexure of the framing members and their connections.

Net area: Gross area reduced to account for removed material.
Nominal dimension: Designated or theoretical dimension, as in the tables of section properties.
Nominal load: Magnitude of the load specified by the applicable building code.
Nominal strength: Strength of a structure or component (without the resistance factor or safety factor applied) to resist load effects, as determined in accordance with this Specification.

Normal stress: Stress acting parallel to the axis of an object in compression and tension; normal stress is usually simply called stress.

Out-of-plane buckling: Limit state of a beam-column bent about its major axis while lateral buckling or lateral-torsional buckling is not prevented by lateral bracing.

Overlap connection: Connection in which intersecting branch members overlap.
Overturn: Topping, or tipping over under lateral load.

Permanent load: Load in which variations over time are rare or of small magnitude. All other loads are variable loads.

Pin connection: A pin connection transfers axial and shear forces but no bending moment.
Pin support: A pin support resists axial and shear forces but no bending moment.
Pitch: Longitudinal center-to-center spacing of fasteners. Center-to-center spacing bolt threads along axis of bolt.

Plastic: Material may be elastic or plastic. Plastic deformation of a structure or material under load remains after the load is removed.

Plastic analysis: Structural analysis based on the assumption of rigid-plastic behavior, in other words, that equilibrium is satisfied throughout the structure and the stress is at or below the yield stress.
Plastic hinge: Yielded zone that forms in a structural member when the plastic moment is attained. The member is assumed to rotate further as if hinged, except that such rotation is restrained by the plastic moment.
Plastic moment: Theoretical resisting moment developed within a fully yielded cross section.
Plastic stress distribution method: Method for determining the stresses in a composite member assuming that the steel section and the concrete in the cross section are fully plastic.

Plate girder: Built-up beam.
Plug weld: Weld made in a circular hole in one element of a joint fusing that element to another element.

Post-buckling strength: Load or force that can be carried by an element, member, or frame after initial buckling has occurred.

Pressure: Similar to stress, the force intensity at a point, except that pressure is acting on the surface of an object rather than within the object.

Prying action: Amplification of the tension force in a bolt caused by leverage between the point of applied load, the bolt and the reaction of the connected elements.

Punching load: Component of branch member force perpendicular to a chord.
$P-\delta$ effect: Effect of loads acting on the deflected shape of a member between joints or nodes.
$P-\Delta$ effect: Effect of loads acting on the displaced location of joints or nodes in a structure. In tiered building structures, this is the effect of loads acting on the laterally displaced location of floors and roofs.

Radius of gyration: A mathematical property, determining the stability of a cross section, defined as: $\mathrm{r}=\sqrt{I / A}$, where $\mathrm{I}=$ moment of inertia and $\mathrm{A}=$ cross section area.

Reaction: The response of a structure to resist applied load.
Required strength: Forces, stresses and deformations acting on the structural component, determined by either structural analysis, for the LRFD or ASD load combinations, as appropriate, or as specified by the Specification or Standard.
Resilience: The property of structures to absorb energy without permanent deformation of fracture.

Resistance factor $\phi$ : Factor that accounts for unavoidable deviations of the nominal strength from the actual strength and for the manner and consequences of failure.

Resultant: The resultant of a system of forces is a single force or moment whose magnitude, direction, and location make it statically equivalent to the system of forces.
Retaining wall: Wall used to hold back soil or other materials.
Roller support: In two dimensions, a roller support restrains one translation degree of freedom.
Rupture strength: In a connection, strength limited by tension or shear rupture.
Safety factor: Factor that accounts for deviations of the actual strength from the nominal strength, deviations of the actual load from the nominal load, uncertainties in the analysis that transforms the load into a load effect, and for the manner and consequence of failure.
Scalar: A mathematical entity with a numeric value but no direction (in contrast to a vector).
Section modulus: The property of a cross section defined by its shape and orientation; section modulus is denoted S , and $\mathrm{S}=\mathrm{I} / \mathrm{c}$, where $\mathrm{I}=$ moment of inertia about the centroid and c is the distance from the centroid to the edge of the section,
Service load combination: Load combination under which serviceability limit states are evaluated.

Service load: Load under which serviceability limit states are evaluated.
Serviceability limit state: Limiting condition affecting the ability of a structure to preserve its appearance, maintainability, durability or the comfort of its occupants or function of machinery, under normal usage.

Shear: A sliding force, pushing and pulling in opposite directions.
Shear buckling: Buckling mode in which a plate element, such as the web of a beam, deforms under pure shear applied in the plane of the plate.

Shear connector: Headed stud, cannel, plate or other shape welded to a steel member and embedded in concrete of a composite member to transmit shear forces at the interface between the two materials.

Shear connector strength: Limit state of reaching the strength of a shear connector, as governed by the connector bearing against the concrete in the slab or by the tensile strength of the connector.

Shear modulus: The ratio of shear stress divided by the shear strain in a linear elastic material.
Shear rupture: Limit state of rupture (fracture) due to shear.
Shear strain: Strain measuring the intensity of racking in a material. Shear strain is measured as the change in angle of a small square part of a material.

Shear stress: Stress acting parallel to an imaginary plane cut through an object.
Shear wall: Wall that provides resistance to lateral loads in the plane of the wall and provides stability for the structural system.

Shear yielding: Yielding that occurs due to shear.
Shear yielding (punching): In a connection, limit state based on out-of-plane shear strength of the chord wall to which branch members are attached.

Slip: In a bolted connection, limit state of relative motion of connected parts prior to the attainment of the available strength of the connection.
Slip-critical connection: Bolted connection designed to resist movement by friction on the faying surface of the connection under the clamping forces of the bolts.
Slot weld: Weld made in an elongated hole fusing an element to another element.
Splice: Connection between two structural elements joined at their ends to forma single, longer element.

Stability: Condition reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry does not produce large displacements.
Static equilibrium: Equilibrium of an object at rest.
Stiffness: The capacity of a material to resist deformation.
Stiffened element: Flat compression element with adjoining out-of-plane elements along both edges parallel to the direction of loading.
Stiffener: Structural element, usually an angle or plate, attached to a member to distribute load, transfer shear or prevent buckling.
Stiffness: Resistance to deformation of a member or structure, measured by the ratio of the applied force (or moment) to the corresponding displacement (or rotation).

Strain: Change of length along an axis, calculated as $\varepsilon=\Delta \mathrm{L} / \mathrm{L}$, where L is the original length and $\Delta \mathrm{L}$ is the change of length.

Strength: The capacity of a material to resist breaking.
Strength design: A design method based on factored load and ultimate strength for concrete design.

Strength limit state: Limiting condition affecting the safety of the structure, in which the ultimate load-carrying capacity is reached.

Stress: Force per unit area caused by axial force, moment, shear or torsion.
Stress concentration: Localized stress considerably higher than average (even in uniformly loaded cross sections of uniform thickness) due to abrupt changes in geometry or localized loading.

Stress resultant: A system of forces which is statically equivalent to a stress distribution over an area.

Stress: The internal reaction to an applied force, measured in force per unit area.
Structure: Composition of elements that define form and resist applied loads.
Structural Aluminum: Elements manufactured of aluminum for structural purposes, generally $50 \%$ larger than comparable steel elements due to the lower modulus of elasticity.

Structural Steel: Elements manufactured of steel with properties designated by ASTM standards, including A36, A992 \& A572.
Strong axis: Major principal centroidal axis of a cross section.
Structural analysis: Determination of load effects on members and connections based on principles of structural mechanics.

Structural component: Member, connector, connecting element or assemblage.
Structural system: An assemblage of load-carrying components that are joined together to provide interaction or interdependence.
T-connection: Connection in which the branch member or connecting element is perpendicular to the main member and in which forces transverse to the main member are primarily equilibrated by shear in the main member.

Tensile rupture: Limit state of rupture (fracture) due to tension.
Tensile strength (of material): Maximum tensile stress that a material is capable of sustaining as defined by ASTM.

Tensile strength (of member): Maximum tension force that a member is capable of sustaining.
Tensile yielding: Yielding that occurs due to tension.
Tension: A force that tends to elongate or enlarge an object.
Tension and shear rupture: In a bolt, limit state of rupture (fracture) due to simultaneous tension and shear force.

Tie plate: Plate element used to join parallel components of a built-up column, girder or strut rigidly connected to the parallel components and designed to transmit shear between them.

Torsion: A twisting moment.
Torsional bracing: Bracing resisting twist of a beam or column.
Torsional buckling: Buckling mode in which a compression member twists about its shear center axis.

Torsional yielding: Yielding that occurs due to torsion.
Translation: Motion of an object along a straight line path without rotation.
Transverse reinforcement: Steel reinforcement in the form of closed ties or welded wire fabric providing confinement for the concrete surrounding the steel shape core in an encased concrete composite column.
Transverse stiffener: Web stiffener oriented perpendicular to the flanges, attached to the web.
Truss: A linear support system consisting of triangular panels usually with pin joints.
Ultimate strength: The utmost strength reached by a material before breaking.
Unbraced length: Distance between braced points of a member, measured between the centers of gravity of the bracing members.

Uneven load distribution: In a connection, condition in which the load is not distributed through the cross section of connected elements in a manner that can be readily determined.

Unframed end: The end of a member not restrained against rotation by stiffeners of connection elements.

Unstiffened elements: Flat compression element with an adjoining out-of-plane element along one edge parallel to the direction of loading.
Uplift: Upward force, usually wind uplift.
Variable load: Load not classified as permanent load.

Vector: A mathematical entity having a magnitude, line of action, and a direction in space.
Vertical bracing system: System of shear walls, braced frames or both, extending through one or more floors of a building.

Vertical diaphragm: A wall to resist lateral load.
Vibration: The cyclic motion of an object.
Wall: A vertical element to resist load and define space; shear walls also resist lateral loads.
Weak axis: Minor principal centroidal axis of a cross section.
Web buckling: Limit state of lateral instability of a web.
Web compression buckling: Limit state of out-of-plane compression buckling of the web due to a concentrated compression force.
Web sideway buckling: Limit state of lateral buckling of the tension flange opposite the location of a concentrated compression force.

Weld metal: Portion of a fusion weld that has been completely melted during welding. Weld metal has elements of filler metal and base metal melted in the weld thermal cycle.
Working stress: The same as allowable stress.
Yield moment: In a member subjected to bending, the moment at which the extreme outer fiber first attains the yield stress.
Yield point: First stress in a material at which an increase in strain occurs without an increase in stress as defined by ASTM.

Yield strength: Stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain as defined by ASTM.
Yield strain: The strain of a material which occurs at the level of yield stress.
Yield stress: Generic term to denote either yield point or yield strength, as appropriate for the material.

Yielding: Limit state of inelastic deformation that occurs after the yield stress is reached.
Yielding (plastic moment): Yielding throughout the cross section of a member as the bending moment reaches the plastic moment.
Yielding (yield moment): Yielding at the extreme fiber on the cross section of a member when the bending moment reached the yield moment.

References:
AISC, Specifications for Structural Steel Buildings, $13^{\text {th }}$ ed. (2005)
Jacqueline Glass, Encyclopaedia of Architectural Technology, Wiley, Cornwall (2002)

## Statics Primer

\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{Notation:} <br>
\hline $a$ \& = name for acceleration \& \multirow[t]{2}{*}{$Q_{y}$} \& \multirow[t]{2}{*}{$=$ first moment area about an $y$ axis (using x distances)} <br>
\hline \multirow[t]{2}{*}{A} \& $=$ area (net $=$ with holes, bearing $=$ in \& \& <br>
\hline \& contact, etc...) \& $R$ \& $=$ name for resultant vectors <br>
\hline \multirow[t]{3}{*}{(C)
$d$} \& $=$ shorthand for compression \& $R_{x}$ \& $=$ resultant component in the x <br>
\hline \& $=$ perpendicular distance to a force \& \& direction <br>
\hline \& from a point \& $R_{y}$ \& $=$ resultant component in the y <br>
\hline \multirow[t]{3}{*}{$d_{x}$

$d$} \& $=$ difference in the x direction \& \& direction <br>

\hline \& between an area centroid ( $\bar{x}$ ) and the centroid of the composite shape \& tail \& $$
\begin{aligned}
& =\text { start of a vector (without } \\
& \text { arrowhead) }
\end{aligned}
$$ <br>

\hline \& ( $\hat{\mathrm{x}}$ ) \& tip \& $=$ direction end of a vector (with <br>

\hline \multirow[t]{3}{*}{$d_{y}$} \& $=$ difference in the $y$ direction between an area centroid ( $\bar{y}$ ) and \& (T) \& | arrowhead) |
| :--- |
| $=$ shorthand for tension | <br>

\hline \& the centroid of the composite shap \& V \& = internal shear force <br>
\hline \& ( y ) \& $w$ \& $=$ name for distributed lo <br>
\hline F \& $=$ name for force vectors, as is $A, B$, $C, T$ and $P$ \& \multicolumn{2}{|l|}{$w_{s(e l f)} w(t)=$ name for distributed load from self weight of member} <br>
\hline $F_{x}$ \& $=$ force component in the x direction \& W \& = name for force due to weight <br>
\hline $F_{y}$ \& $=$ force component in the y direction \& $\stackrel{x}{\bar{x}}$ \& $=\mathrm{x}$ axis direction or algebra var <br>
\hline $g$ \& $=$ acceleration due to gravity \& $\bar{x}$ \& he distance in the x directio <br>
\hline $h$ \& $=$ name for height \& \& a reference axis to the centroid of a <br>
\hline $\bar{I}$ \& $=$ moment of inertia about the centroid \& $y$ \& $=\begin{aligned} & \text { shape } \\ & = \\ & \text { axis direction or algebra variable }\end{aligned}$ <br>
\hline $I_{x}$

$I$ \& $=$ moment of inertia with respect to an x -axis \& $\bar{y}$ \& $=$ the distance in the y direction from a reference axis to the centroid of a shape <br>
\hline $I_{y}$ \& $=$ moment of inertia with respect to a $y$-axis \& $\alpha$ \& $=$ angle, in math <br>
\hline L \& $=$ beam span length \& $\beta$ \& = angle, in math <br>
\hline m \& $=$ name for mass \& $\gamma$ \& = angle, in math <br>
\hline M \& $=$ moment due to a force or internal \& $\mu$ \& $=$ coefficient of static friction <br>
\hline \& bending moment \& $\theta$ \& $=$ angle, in a trig equation, ex. $\sin \theta$, <br>
\hline $N$ \& = name for normal force to a surface \& \& that is measured between the x axis <br>
\hline $p$ \& $=$ pressure \& \& and tail of a vector <br>
\hline $Q_{x}$ \& $=$ first moment area about an x axis (using y distances) \& $\Sigma$ \& $=$ summation symbol <br>
\hline
\end{tabular}

## Newton's Laws of Motion

Newton's laws govern the behavior of physical bodies, whether at rest or moving:

- First Law. A particle originally at rest, or moving in a straight line with constant velocity, will remain in this state provided the particle is not subjected to an unbalanced force.
- Second Law. A particle of mass $\boldsymbol{m}$ acted upon by an unbalanced force experiences an acceleration that has the same direction as the force and a magnitude that is directly proportional to the force. This is expressed mathematically as: $\bar{F}=m \bar{a}$,
where $\mathbf{F}$ and $\boldsymbol{a}$ are vector (directional) quantities, and $\boldsymbol{m}$ is a scalar quantity.
- Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear.


## Units

Units are necessary to define quantities. Standards exist to relate quantities in a convention system, such as the International System of Units (SI) or the U.S. Customary system.

| Units | Mass | Length | Time | Force |
| :---: | :---: | :---: | :---: | :---: |
| SI | kg | m | s | $N=\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^{2}}$ |
| Absolute English | lb | ft | S | $\text { Poundal }=\frac{l b \cdot f t}{s^{2}}$ |
| Technical English | $\text { slug }=\frac{l b_{f} \cdot s^{2}}{f t}$ | ft | S | $\mathrm{lb}_{\text {force }}$ |
| Engineering English | lb | ft | S | $\mathrm{lb}_{\text {force }}$ |
| $l b_{\text {force }}=l b_{(\text {mass })} \times 32.17 \mathrm{ft} / \mathrm{s}^{2}$ |  |  |  |  |
| gravitational constant | $g_{c}=32.17 \mathrm{ft} / \mathrm{s}^{2}$ | (English) |  | $F=m g$ |
|  | $g_{c}=9.81 \mathrm{~m} / \mathrm{s}^{2}$ | (SI) |  |  |
| conversions (pg. vii) | $\begin{aligned} & 1 \mathrm{in}=25.4 \mathrm{~mm} \\ & 1 \mathrm{lb}=4.448 \mathrm{~N} \end{aligned}$ |  |  |  |

## Conversions

Conversion of a quantity from a category within a unit system to a more useful category or to another unit system is very common. Tables of conversion can be found in most physics, statics and design texts.

## Numerical Accuracy

Depends on 1) accuracy of data you are given
2) accuracy of the calculations performed

The solution CANNOT be more accurate than the less accurate of \#1 and \#2 above!
DEFINITIONS: precision the number of significant digits
accuracy the
possible error
Relative error measures the degree of accuracy:

$$
\frac{\text { relative error }}{\text { measurement }} \times 100=\text { degree of accuracy }
$$

For engineering problems, accuracy rarely is less than $0.2 \%$.

## Math for Structures

1. Parallel lines never intersect.
2. Two lines are perpendicular (or normal) when they intersect at a right angle $=90^{\circ}$.
3. Intersecting (or concurrent) lines cross or meet at a point.
4. If two lines cross, the opposite angles are identical:
5. If a line crosses two parallel lines, the intersection angles with the same orientation are identical:
6. If the sides of two angles are parallel and intersect in the same fashion, the angles are identical.
7. If the sides of two angles are parallel, but intersect in the opposite fashion, the angles are supplementary: $\alpha+\beta=180^{\circ}$.
8. If the sides of two angles are perpendicular and intersect in the same fashion, the angles are identical.
9. If the sides of two angles are perpendicular, but intersect in the opposite fashion, the angles are supplementary: $\alpha+\beta=180^{\circ}$.
10. If the side of two angles bisects a right angle, the angles are complimentary: $\alpha+\gamma=90^{\circ}$.
11. If a right angle bisects a straight line, the remaining angles are complimentary: $\alpha+\gamma=$ $90^{\circ}$.
12. The sum of the interior angles of a triangle $=180^{\circ}$.
13. For a right triangle, that has one angle of $90^{\circ}$, the sum of the other angles $=90^{\circ}$.
14. For a right triangle, the sum of the squares of the sides equals the square of the

$$
A B^{2}+A C^{2}=C B^{2}
$$

hypotenuse:
15. Similar triangles have identical angles in the same orientation. Their sides are related by:

Case 1:

Case 2:


$$
\frac{A B}{A D}=\frac{A C}{A E}=\frac{B C}{D E}
$$

$$
C^{\prime} \int_{B^{\prime}} \frac{A B}{A^{\prime} B^{\prime}}=\frac{A C}{A^{\prime} C^{\prime}}=\frac{B C}{B^{\prime} C^{\prime}}
$$

16. For right triangles:

$$
\begin{aligned}
& \sin =\frac{\text { opposite side }}{\text { hypotenuse }}=\sin \alpha=\frac{A B}{C B} \\
& \cos =\frac{\text { adjacent side }}{\text { hypotenuse }}=\cos \alpha=\frac{A C}{C B} \\
& \tan =\frac{\text { opposite side }}{\text { adjacent side }}=\tan \alpha=\frac{A B}{A C}
\end{aligned}
$$



## (SOHCAHTOA)

17. If an angle is greater than $180^{\circ}$ and less than $360^{\circ}$, $\sin$ will be less than 0 .

If an angle is greater than $90^{\circ}$ and less than $270^{\circ}$, cos will be less than 0 .
If an angle is greater than $90^{\circ}$ and less than $180^{\circ}$, tan will be less than 0.
If an angle is greater than $270^{\circ}$ and less than $360^{\circ}$, tan will be less than 0.
18. LAW of SINES (any triangle)

$$
\frac{\sin \alpha}{A}=\frac{\sin \beta}{B}=\frac{\sin \gamma}{C}
$$

19. LAW of COSINES (any triangle)


$$
A^{2}=B^{2}+C^{2}-2 B C \cos \alpha
$$

20. Surfaces or areas have dimensions of width and length and units of length squared (ex. $i n^{2}$ or inches x inches).
21. Solids or volumes have dimension of width, length and height or thickness and units of length cubed (ex. $\mathrm{m}^{3}$ or mx m x m )
22. Force is defined as mass times acceleration. So a weight due to a mass is accelerated upon by gravity: $\quad \mathrm{F}=\mathrm{m} \cdot \mathrm{g} \quad \mathrm{g}=9.81 \mathrm{~m} / \mathrm{sec}^{2}=32.17 \mathrm{ft} / \mathrm{sec}^{2}$
23. Weight can be determined by volume if the unit weight or density is known: $\mathrm{W}=\mathrm{V} \cdot \gamma$ where $\cdot \mathrm{V}$ is in units of length ${ }^{3}$ and $\gamma$ is in units of force/unit volume
24. Algebra: If

$$
\mathrm{a} \cdot \mathrm{~b}=\mathrm{c} \cdot \mathrm{~d} \quad \text { then it can be rewritten: }
$$

$$
\begin{array}{ll}
a \cdot b+k=c \cdot d+k & \text { add a constant } \\
c \cdot d=a \cdot b & \text { switch sides } \\
a=\frac{c \cdot d}{b} & \text { divide both sides by } b
\end{array}
$$

$$
\frac{a}{c}=\frac{d}{b} \quad \text { divide both sides by } b \cdot c
$$

25. Cartesian Coordinate System

26. Solving equations with one unknown:
$1^{\text {st }}$ order polynomial:

$$
\begin{array}{ll}
2 x-1=0 \cdots & 2 x=1 \cdots \\
a x+b=0 \cdots & x=\frac{-b}{a}
\end{array}
$$

$$
x=\frac{1}{2}
$$

$2^{\text {nd }}$ order polynomial

$$
\begin{array}{llc}
a x^{2}+b x+c=0 \cdots & x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} & \begin{array}{c}
\text { two answers } \\
\text { (radical cannot be } \\
\text { negative) }
\end{array} \\
x^{2}-1=0 \cdots & x=\frac{-0 \pm \sqrt{0^{2}-4(-1)}}{2 \cdot 1} \cdots & x= \pm 1
\end{array}
$$

27. Solving 2 linear equations simultaneously:

One equation consisting only of variables can be rearranged and then substituted into the second equation:
$\begin{array}{lll}\text { ex: } & 5 x-3 y=0 & \text { add 3y to both sides to rearrange } \\ 4 x-y=2 & \text { divide both sides by } 5 & 5 x=3 y \\ & \text { substitute } x \text { into the other equation } & x=\frac{3}{5} y \\ & \left.\begin{array}{l}\text { add like terms } \\ 5\end{array} y\right)-y=2 \\ & \text { simplify } & \frac{7}{5} y=2 \\ & & y=\frac{10}{7}=1.43 \\ \end{array}$
Equations can be added and factored to eliminate one variable:
ex:

$$
\begin{array}{ll}
2 x+3 y=8 \\
4 x-y=2 & \text { multiply both sides by } 3 \\
& \text { and add } \\
& \text { simplify } \\
& \text { put } x=1 \text { in an equation for } \mathrm{y} \\
& \text { simplify }
\end{array}
$$

$$
\begin{aligned}
& 2 x+3 y=8 \\
& 12 x-3 y=6 \\
& \hline 14 x+0=14 \\
& x=1 \\
& 2 \cdot 1+3 y=8 \\
& 3 y=6 \\
& y=2
\end{aligned}
$$

28. Derivatives of polynomials

$$
\begin{array}{ll}
y=\text { constant } & \frac{d y}{d x}=0 \\
y=x & \frac{d y}{d x}=1 \\
y=a x & \frac{d y}{d x}=a \\
y=x^{2} & \frac{d y}{d x}=2 x \\
y=x^{3} & \frac{d y}{d x}=3 x^{2}
\end{array}
$$

29. The minimum and maximum of a function can be found by setting the derivative $=0$ and solving for the unknown variable.
30. Calculators (and software) process equations by an "order of operations", which typically means they process functions like exponentials and square roots before simpler functions such as + or - . BE SURE to specify with parenthesis what order you want, or you'll get the wrong answers. It is also important to have degrees set in your calculator for trig functions.

For instance, Excel uses - for sign (like -1) first, then will process exponents and square roots, times and divide, followed by plus and minus. If you type $4 \times 10^{\wedge} 2$ and really mean $(4 \times 10)^{\wedge} 2$ you will get an answer of 400 instead of 1600 .

## General Procedure for Analysis

1. $\left.\begin{array}{c}\text { Inputs } \\ \text { Outputs } \\ \text { "ritical Path" }\end{array} \quad \square \quad \begin{array}{l}\underline{\text { GIVEN: }} \\ \frac{\text { FIND: }}{\text { SOLUTION }}\end{array}\right\}$ on graph paper
2. Draw simple diagram of body/bodies \& forces acting on it/them.
3. Choose a reference system for the forces.
4. Identify key geometry and constraints.
5. Write the basic equations for force components.
6. Count the equations \& unknowns.
7. SOLVE
8. "Feel" the validity of the answer. (Use common sense. Check units...)

Example: Two forces, $\mathrm{A} \& \mathrm{~B}$, act on a particle. What is the resultant?

1. GIVEN: Two forces on a particle and a diagram with size and orientation

FIND: The "resultant" of the two forces
SOLUTION:

2. Draw what you know (the diagram, any other numbers in the problem statement that could be put on the drawing....)
3. Choose a reference system. What would be the easiest? Cartesian, radian?
4. Key geometry: the location of the particle as the origin of all the forces

Key constraints: the particle is "free" in space
5. Write equations:

$$
\begin{aligned}
& \text { size of } A^{2}+\text { size of } B^{2}=\text { size of resultant } \\
& \sin \alpha=\frac{\text { size of } B}{\text { size of } A+B}
\end{aligned}
$$

6. Count: Unknowns: 2, magnitude and direction $\leq$ Equations: $2 \therefore$ can solve
7. Solve: graphically or with equations
8. "Feel": Is the result bigger than A and bigger than B? Is it in the right direction? (like A \& B)

## Forces

Forces are vectors, which means they have a direction, size and point or line of application. External forces can be moved along the line of action by the law of transmissibility. Internal forces are within a material or at a connection between elements.

Force systems can be classified as concurrent, collinear, coplanar, coplanarparallel, or space.

Because they are vector quantities, they cannot be simply added. They must be graphically added or analytically added by resolving forces into components using trigonometry and summed.
$\theta$ is: between $x \& F$
$\mathrm{~F}_{\mathrm{x}}=\mathrm{F} \cdot \cos \theta$
$\mathrm{F}_{\mathrm{y}}=\mathrm{F} \cdot \sin \theta$
$\mathrm{F}=\sqrt{F_{x}^{2}+F_{y}^{2}}$
$\tan \theta=\frac{F_{y}}{F_{x}}$


$$
R_{x}=\sum F_{x}, R_{y}=\sum F_{y} \quad \text { and } R=\sqrt{R_{x}^{2}+R_{y}^{2}} \quad \tan \theta=\frac{R_{y}}{R_{x}}
$$

## Types of Forces

Forces can be classified as concentrated at a point or distributed over a length or area.
Uniformly distributed loads are quite common and have units of $\mathrm{lb} / \mathrm{ft}$ or $\mathrm{N} / \mathrm{m}$. The total load is commonly wanted from the distribution, and can be determined based on an "area" calculation with the load value as the "height".

Equivalent force systems are the reorganization of the loads in a system so there is a equivalent force put at the same location that would cause the same translation and rotation (see Moments).


To determine a distributed load due to hydrostatic pressure, the height of the water, $h$, is multiplied by the material density, $\gamma\left(62.4 \mathrm{lb} / \mathrm{ft}^{3}\right): p=h \gamma$.

To determine a weight of a beam member per length, the cross section area, $A$, is multiplied by the material density, $\gamma\left(\right.$ ex. concrete $\left.=150 \mathrm{lb} / \mathrm{ft}^{3}\right): w_{\text {s.w. }}=A \gamma$. (Care must be taken with units.)

## Friction

Friction is a resulting force from the contact of two materials and a normal force. It can be static or kinematic. Static friction is defined as the product of the normal force, $N$, with the coefficient of static friction, $\mu$, which is a constant dependant upon the materials in contact: $F=\mu N$


## Moments

Moments are the tendency of forces to cause rotation and are vector quantities with rotational direction. Most physics texts define positive rotation as counter clockwise. With the sign convention, moments can be added.

Moments are defined as the product of the force magnitude, $F$, with the perpendicular distance from the point of interest to the line of action of the force, $d_{\perp}: \quad M=F \cdot d_{\perp}$


Moment couples can be identified with forces of equal size in opposite direction that are parallel. The equations is $M=F \cdot d_{\perp}$ where F is the size of one of the forces.

## Support Conditions

Reaction forces and moments occur at supports for structural elements. The force component directions and moments are determined by the motion that is resisted, for example no rotation will mean a reaction moment. Supports are commonly modeled as these general types, with the drawing symbols of triangles, circles and ground:

Structural Analysis, $4^{\text {th }}$ ed., R.C. Hibbeler

Table 2-1 Supports for Coplanar Structures


## Equilibrium

Equilibrium is the state when all the external forces acting on a rigid body form a system of forces equivalent to zero. There will be no rotation or translation. The forces are referred to as balanced.

$$
R_{x}=\sum F_{x}=0 \quad R_{y}=\sum F_{y}=0 \quad \text { AND } \quad \sum M=0
$$

Equilibrium for a point already satisfies the sum of moments equal to zero because a force acting through a point will have zero moment from a zero perpendicular distance. This is a very useful concept to apply when summing moments for a rigid body. If the point summed about has unknown forces acting through it, that force variable will not appear in the equilibrium equation as an unknown quantity, allowing for much easier algebra.

## Free Body Diagrams

1. Determine the free body of interest. (What body is in equilibrium?)
2. Detach the body from the ground and all other bodies ("free" $i t$ ).
3. Indicate all external forces which include:

- action on the free body by the supports \& connections
- action on the free body by other bodies
- the weigh effect (=force) of the free body itself (force due to gravity)

4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting on the body not by the body.
5. Dimensions/angles should be included for moment computations and force computations.
6. Indicate the unknown angles, distances, forces or moments, such as those reactions or constraining forces where the body is supported or connected.

- The line of action of any unknown should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc...) If the sense isn't obvious, assume a sense. When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the direction is opposite the assumed direction. DON'T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.

With the 3 equations of equilibrium, there can be no more than 3 unknowns for statics. If there are, and the structure is stable, it means that it is statically indeterminate and other methods must be used to solve the unknowns. When it is not stable, it is improperly constrained and may still look like it has 3 unknowns. It will prove to be unsolvable.

## Conditions for Equilibrium of a Rigid Body

1. Two-force body: Equilibrium of a body subjected to two forces on two points requires that those forces be equal and opposite and act in the same line of action.


(A)

(B)

(C)
2. Three-force body: Equilibrium of a body subjected to three forces on three points requires that the line of action of the forces be concurrent (intersect) or parallel AND that the resultant equal zero.

(A) -no

(B)

(C)

## Geometric Properties

Area is an important quantity to be calculated in order to know material quantities and to find geometric properties for beam and column cross sections. Charts are available for common mathematical relationships.

Centroid For a uniform material, the geometric center of the area is the centroid or center of gravity. It can be determined with calculus. $\quad \bar{x}=\frac{\sum(x \Delta A)}{\boldsymbol{A}} \quad \bar{y}=\frac{\sum(y \Delta A)}{\boldsymbol{A}}$

First Moment Area The product of an area with respect to a distance about an axis is called the first moment area, $Q$. The quantity is useful for shear stress calculations and to determine the moment of inertia.

$$
\mathrm{Q}_{\mathrm{x}}=\int \mathrm{ydA}=\overline{\mathrm{y}} \mathrm{~A} \quad \mathrm{Q}_{\mathrm{y}}=\int \mathrm{xdA}=\overline{\mathrm{x}} \mathrm{~A}
$$

Geometric Properties of Areas

| Rectangle |  | $\begin{aligned} \bar{I}_{x^{\prime}} & =\frac{1}{12} b h^{3} \\ \bar{I}_{y^{\prime}} & =\frac{1}{12} b^{3} h \\ I_{x} & =\frac{1}{3} b h^{3} \\ I_{y} & =\frac{1}{3} b^{3} h \\ J_{C} & =\frac{1}{12} b h\left(b^{2}+h^{2}\right) \end{aligned}$ | Area $=b h$ $\bar{x}=\mathrm{b} / 2$ <br> $\bar{y}=\mathrm{h} / 2$ |
| :---: | :---: | :---: | :---: |
| Triangle |  | $\begin{aligned} & \bar{I}_{x^{\prime}}=\frac{1}{36} b h^{3} \\ & I_{x}=\frac{1}{12} b h^{3} \\ & \bar{I}_{y^{\prime}}=\frac{1}{36} b^{3} h \end{aligned}$ | $\begin{aligned} & \text { Area }=b h / 2 \\ & \bar{x}=b / 3 \\ & \bar{y}=h / 3 \end{aligned}$ |
| Circle |  | $\begin{aligned} \bar{I}_{x} & =\bar{I}_{y}=\frac{1}{4} \pi r^{4} \\ J_{o} & =\frac{1}{2} \pi r^{4} \end{aligned}$ | $\begin{aligned} & \text { Area }=\pi r^{2}=\pi d^{2} / 4 \\ & \bar{x}=0 \\ & \bar{y}=0 \end{aligned}$ |
| Semicircle |  | $\begin{aligned} & \bar{I}_{x}=0.1098 r^{4} \\ & \bar{I}_{y}=\pi r^{4} / 8 \end{aligned}$ | $\begin{aligned} & \text { Area }=\pi r^{2} / 2=\pi d^{2} / 8 \\ & \bar{x}=0 \quad \bar{y}=4 r / 3 \pi \end{aligned}$ |
| Quarter circle |  | $\begin{aligned} & \bar{I}_{x}=0.0549 \mathrm{r}^{4} \\ & \bar{I}_{y}=0.0549 \mathrm{r}^{4} \end{aligned}$ | $\begin{aligned} & \text { Area }=\pi r^{2} / 4=\pi d^{2} / 16 \\ & \bar{x}=4 r / 3 \pi \\ & \bar{y}=4 r / 3 \pi \end{aligned}$ |
| Ellipse |  | $\begin{aligned} & \bar{I}_{x}=\frac{1}{4} \pi a b^{3} \\ & \bar{I}_{y}=\frac{1}{4} \pi a^{3} b \\ & J_{O}=\frac{1}{4} \pi a b\left(a^{2}+b^{2}\right) \end{aligned}$ | Area $=\pi a b$ $\begin{aligned} & \bar{x}=0 \\ & \bar{y}=0 \end{aligned}$ |
| Semiparabolic area <br> Parabolic area |  | $\begin{aligned} & \bar{I}_{x}=16 \mathrm{ah}^{3} / 175 \\ & \bar{I}_{y}=4 \mathrm{a}^{3 h} / 15 \end{aligned}$ | $\begin{aligned} & \text { Area }=4 a \mathrm{~h} / 3 \\ & \bar{x}=0 \quad \bar{y}=3 h / 5 \end{aligned}$ |
| Parabolic spandrel |  | $\begin{aligned} & \bar{I}_{x}=37 \mathrm{ah}^{3} / 2100 \\ & \bar{I}_{y}=\mathrm{a}^{3 \mathrm{~h}} / 80 \end{aligned}$ | $\begin{aligned} & \text { Area }=a h / 3 \\ & \bar{x}=3 a / 4 \quad \bar{y}=3 h / 10 \end{aligned}$ |

Moment of Inertia The moment of inertia is the second area moment of an area, and is found using calculus. For a composite shape, the moment of inertia can be found using the parallel axis theorem: $\quad I_{x}=\bar{I}_{x}+A d_{y}{ }^{2} \quad I_{y}=\bar{I}_{y}+A d_{x}{ }^{2}$

The theorem states that the sum of the centroid of each composite shape about an axis (subscript axes) can be added but must be added to the second moment area of the shape by the distance between parallel axes (opposite axes direction).

## Internal Forces

If a body is in equilibrium, it holds that any section of that body is in equilibrium. Two forcebodies will have internal forces that are in line with the body (end points), while three-force bodies will see an internal force that will not be axial, in addition to an internal moment called a bending moment. An axial force that is pulling the body from both ends is referred to as a tensile force, $\longrightarrow$ and a force pushing on the body at both ends is referred to as a compressive force $\longrightarrow \square$.

## Cable Analysis

Cables can only see tensile forces. If cables are straight, they are two-force bodies and the geometry of the cable determines the direction of the force.

If cables drape (are funcular) by having distributed or gravity loads, the internal vertical force component changes, while the internal horizontal force component does not.

## Truss Analysis

Truss members are assembled such that the pins connecting them are the only location of forces (internal and external). This loading assumption relies on there being no bending in the members, and all truss members are then two-force bodies.

Equilibrium of the joints will only need to satisfy the x force components summing to 0 and the y force components summing to zero. The member forces will have direction in the geometry of the member. Assuming the unknown forces in tension is represented by drawing arrows "away" from the joint. When compression forces are known, they must be drawn "in" to the point.

Equilibrium of the section will only be possible if the section cut is through three or less members exposing three or less unknown forces. This method relies on the sum of moment equilibrium equation. The member forces are in the direction of the members, and the line of action of those forces runs through the member location in order to find the perpendicular distance. It is helpful to find points of intersection of unknown forces to sum moments.

## Pinned Frame. Arch and Compound Beam Analysis

Connecting or "internal" pins, mean a frame is made up of multiple bodies, just like a truss. But unlike a truss, the member will not all be two-force bodies, so there may be three equations of equilibrium required for each member in an assembly, in addition to the three equations of equilibrium for the entire structure. The force reactions on one side of the pin are equal and opposite those to the other side, so there are only two unknown component forces per pin.


## Beam Analysis

Statically determinate beams have a limited number of support arrangements for a limit of three unknown reactions. The cantilever condition has a reaction moment.

simply supported (most common)

overhang

cantilever

The internal forces and moment are particularly important for design. The axial force (commonly equal zero) is labeled $P$, while the transverse force is called shear, $V$, and the internal moment is called bending moment, $M$.

The sign convention for positive shear is a downward force on a left section cut (or upward force on a right section cut).

The sign convention for positive bending moment corresponds to a downward deflection (most common or positive curvature.) That is a counter clockwise moment on a right section cut and a clockwise moment of a left section cut.


## Shear and Bending Moment Diagrams

Diagrams of the internal shear at every location along the beam and of the internal bending moment are extremely useful to locate maximum quantities to design the beams for. There are two primary methods to construct them. The equilibrium method relies on section cuts over distances and writes expressions based on the variable of distance. These functions are plotted as lines or curves. The semi-graphical method relies on the calculus relationship between the "load" curve (or load diagram), shear curve, and bending moment curve. If the area under a curve is known, the result in the next plot is a change by the amount of the area.

The location of the maximum bending moment corresponds to the location of zero shear.
On the deflected shape of a beam, the point where the shape changes from smile up to frown is called the inflection point. The bending moment value at this point is zero.


## Semigraphical Method Proceedure:

1. Find all support forces.

## $V$ diagram:

2. At free ends and at simply supported ends, the shear will have a zero value.
3. At the left support, the shear will equal the reaction force.

4. The shear will not change in $x$ until there is another load, where the shear is reduced if the load is negative. If there is a distributed load, the change in shear is the area under the loading.
5. At the right support, the reaction is treated just like the loads of step 4.
6. At the free end, the shear should go to zero.

## $M$ diagram:

7. At free ends and at simply supported ends, the moment will have a zero value.
8. At the left support, the moment will equal the reaction moment (if there is one).
9. The moment will not change in $x$ until there is another load or applied moment, where the moment is reduced if the applied moment is negative. If there is a value for shear on the V diagram, the change in moment is the area under the shear diagram.
For a triangle in the shear diagram, the width will equal the height $\div$ w!
10. At the right support, the moment reaction is treated just like the moments of step 9.
11. At the free end, the moment should go to zero.


## Indeterminate Structures

Structures with more unknowns than equations of equilibrium are statically indeterminate. The number of excess equations is the degree to which they are indeterminate. Other methods must be used to generate the additional equation. These structures will usually have three-force bodies, and possibly rigid connections which mean internal axial, shear and bending moment at the members and at the joints. Bending moment and shear diagrams can be constructed.


Example 1 Determine the resultant vector analytically with the component method.

## Example Problem 2.9 (Figure 2.29)

This is the same problem as Example Problem 2.2, which was solved earlier using the graphical methods.

$$
\begin{aligned}
-A_{x} & =-A \cos 30^{\circ}=-(400 \mathrm{lb} .)(0.866)=-346.4 \mathrm{lb} \\
-A_{y} & =-A \sin 30^{\circ}=-(400 \mathrm{lb} .)(0.50)=-200 \mathrm{lb} \\
+B_{x} & =+B \cos 45^{\circ}=+(600 \mathrm{lb} .)(0.707)=+424.2 \mathrm{lb} \\
-B_{y} & =-B \sin 45^{\circ}=-(600 \mathrm{lb} .)(0.707)=-424.2 \mathrm{lb}
\end{aligned}
$$


(a)

$$
R_{x}=\sum F_{x}=-A_{x}+B_{x}
$$

$$
=-346.4 \mathrm{lb} .+424.2 \mathrm{lb} .=+77.8 \mathrm{lb}
$$

$$
R_{y}=\sum F_{y}=-A_{y}-B_{y}
$$

$$
=-200 \mathrm{lb} .-424.2 \mathrm{lb} .=-624.2 \mathrm{lb} .
$$



$$
R=\sqrt{\left(R_{x}\right)^{2}+\left(R_{y}\right)^{2}}=\sqrt{(+77.8)^{2}+(-624.2)^{2}}=629 \mathrm{lb} .
$$

$$
\tan \theta=\left(\frac{R_{y}}{R_{x}}\right) \quad \theta=\tan ^{-1}\left(\frac{624.2}{77.8}\right)=82.9^{\circ}
$$



## Example 2

## Example Problem 2.13 (Figure 2.35)



A 1-foot-wide slice of a 4 -foot-thick concrete gravity dam weighs 10,000 pounds and the equivalent force due to water pressure behind the dam is equal to 1200 pounds. The stability of the dam against overturning is evaluated about the "toe" at $A$.
Determine the resultant moment at $A$ due to the two forces shown. Is the dam stable?

$$
\begin{aligned}
M_{A} & =-\left(F_{w}\right) \times(6 \mathrm{ft} .)+(W) \times(2 \mathrm{ft} .) \\
M_{A} & =-(1200 \mathrm{lb} .)(6 \mathrm{ft} .)+(10,000 \mathrm{lb} .)(2 \mathrm{ft} .) \\
& =+12,800 \mathrm{lb} . \mathrm{ft} .
\end{aligned}
$$

Yes, because the ground will stop the rotation.

## Example 3



Figure 2.40 (a) Three forces on a vertical pole. (b) Forces resolved into x and y components.

## Example Problem 2.17

A 8-meter vertical pole is used to support three cable forces as shown in Figure 2.40a. Determine the moment at the base of the pole at $A$.

## Solution (Figure 2.40b):

Resolve forces $F_{1}$ and $F_{2}$ into their respective $x$ and $y$ components.

$$
\begin{aligned}
& F_{1 x}=F_{1} \sin 30^{\circ}=(1800 \mathrm{~N})(0.5)=900 \mathrm{~N} \\
& F_{1 y}=F_{1} \cos 30^{\circ}=(1800 \mathrm{~N})(0.866)=1560 \mathrm{~N} \\
& F_{2 x}=\frac{3}{5} F_{2}=\frac{3}{5}(900 \mathrm{~N})=540 \mathrm{~N} \\
& F_{2 y}=\frac{4}{5} F_{2}=\frac{4}{5}(900 \mathrm{~N})=720 \mathrm{~N}
\end{aligned}
$$

The moment at the base of the pole at $A$ is the algebraic sum of the moments due to force $F_{3}$ and the component forces of $F_{1}$ and $F_{2}$.

$$
\begin{aligned}
M_{A}= & +\left(F_{1 x}\right)(8 \mathrm{~m})-\left(F_{2 x}\right)(8 \mathrm{~m})-\left(F_{3}\right)(8 \mathrm{~m}) \\
M_{A}= & +(900 \mathrm{~N})(8 \mathrm{~m})-(540 \mathrm{~N})(8 \mathrm{~m}) \\
& -(360 \mathrm{~N})(8 \mathrm{~m}) \\
M_{A}= & +(7200 \mathrm{~N}-\mathrm{m})-(4320 \mathrm{~N}-\mathrm{m}) \\
& -(2880 \mathrm{~N}-\mathrm{m})=0
\end{aligned}
$$

A zero resultant moment at $A$ means that there is no tendency for the pole to rotate about the base for this particular combination of forces. Also, note that the vertical components of forces $F_{1}$ and $F_{2}$ did not appear in the moment equation because neither had a moment arm.

Forces that intersect the reference point have no moment arms and will cause no tendency for rotation about the point.

## Example 4

## Example Problem 4.1 (Method of Joints)

An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. Determine the support reactions at eachend, then Using the method of joints, solve for all member forces. Summarize the results of all member forces on -a FBD (this diagram is referred to as a force summation diagram).


$$
\begin{gathered}
\sum F_{y}=-\left(\frac{B A}{\sqrt{2}}\right)+2400 \mathrm{lb} .=0 \\
B A=+2400 \sqrt{2} \mathrm{lb} .=+3390 \mathrm{lb}
\end{gathered}
$$

$$
\sum F_{x}=\left(+\frac{B A}{\sqrt{2}}\right)-A F=0 ; \quad A F=\left(+\frac{2400 \sqrt{2} \mathrm{lb} .}{\sqrt{2}}\right)=+2400 \mathrm{lb} . \text { (tension) }
$$



$$
\begin{aligned}
& \sum F_{x}=(-3600 \mathrm{lb} .)+(2400 \mathrm{lb} .)+\left(\frac{D B}{\sqrt{2}}\right)=0 \\
&D B=(1200 \sqrt{2} \mathrm{lb} .)=+1696 \mathrm{lb} . \text { (tension }) \\
& \sum F_{y}=+D B_{y}-C D=0 \quad C D=\frac{D B}{\sqrt{2}}=\frac{1200 \sqrt{2} \mathrm{lb} .}{\sqrt{2}}=1200 \mathrm{lb} . \text { (compression) }
\end{aligned}
$$



$$
\begin{aligned}
& \sum F=+E C_{x}-B C_{x}=0 \\
& B C_{x}=\frac{2 B C}{\sqrt{5}} \text { and } E C_{x}=\frac{(2 \times 4025 \mathrm{lb} .)}{\sqrt{5}}=3600 \mathrm{lb} . \\
& \sum F_{y}=\left(+\frac{4025 \mathrm{lb} .}{\sqrt{5}}\right)-1200 \mathrm{lb} .+1200 \mathrm{lb} .-\left(\frac{4025 \mathrm{lb} .}{\sqrt{5}}\right)=0 \\
& 0=0 ; \quad \text { checks }
\end{aligned}
$$

## Example 5

Figure 3.15 shows a radial three-hinged arch, so named because the shape of the two-member structure is an arc of a circle with a $42-\mathrm{ft}$ radius that is pinned at its two external supports with a third pin connecting the two members at the crown of the arch. Such frames are commonly used to form circular dome and barrel arch buildings and, as in this case, arch bridges.


FIGURE 3.15
This bridge structure consists of four arches spaced 18 ft apart, with each supporting a roadway deck having a uniform dead (including allowance for the arch self-weight) plus averaged live load of 2,000 plf. As shown, this horizontal load is delivered to the arch through vertical columns spaced 8 ft apart, each delivering the same vertical load to the supporting arch. In this instance, or whenever four or more uniformly spaced equal concentrated loads act on a structural element, it is reasonable to assume the element is uniformly loaded.

We want to know the external reaction components at supports $A$ and $B$. Since there are four support reactions-two per hinge-we cannot simply determine them by application of the three equilibrium equations to the entire $80-\mathrm{ft}$ structure. By taking it apart at pin $C$, however, we see that we have a total of six unknowns (two per pin) and three equations of equilibrium for each of the two separated members-six equations and six unknowns. Note that the two components of the force in hinge $C$ must be assumed to be equal and opposite on the left and right members.

By summing moments at $A$ and $B$, respectively, we get the following two equations with the two unknown components of force in pin $C$ :

$$
\begin{array}{r}
80,000(20)-C_{11}(30)-C_{1}(40)=0 \\
-80,000(20)+C_{11}(30)-C_{1}(40)=0
\end{array}
$$

From these, $C_{11}=53,300 \mathrm{lb}$ and $C_{\mathrm{V}}=0$. Summing vertical forces on each arch element shows us that $A_{\mathrm{Y}}=B_{\mathrm{Y}}=80,000 \mathrm{lb}$, and summation of horizontal forces on both members indicates that the outward kick of the arch members, called the horizontal thrust, is

$$
A_{11}=B_{11}=C_{11}=53,300 \mathrm{lb}
$$

Thus, the force with which the foundation reacts to support the arch bridge is given as

$$
F=\sqrt{\left(80,000^{2}+53,300^{2}\right)}=96,100 \mathrm{lb}
$$

This force makes an angle with a vertical axis of

$$
z=\arctan \left(\frac{53,300}{80,000}\right)=33.7^{\circ}
$$

Actually, we could have made quick work of determining the arch reaction components by applying the simple arch equations discussed in the last chapter. Since it is uniformly loaded, the vertical component of the reaction at $A$ would be $V=w L / 2=2000(80) / 2=80,000 \mathrm{lb}$. The horizontal component would be $H=w L^{2} / 8 s=2000\left(80^{2}\right) /(8 \times 30)=53,300 \mathrm{lb}$.

## Example 6

## Example Problem 8.5 (Semi-Graphical Method)

A cantilever beam supports a uniform load of $\omega=2 \mathrm{kN} / \mathrm{m}$ over its entire span, plus a concentrated load of 10 kN at 0.75 m from the free end. Construct the $V$ and $M$ diagrams (Figure 8.29).


SOLUTION:
Determine the reactions:
$\sum F_{x}=R_{B x}=0 \quad \mathrm{R}_{\mathrm{Bx}}=0 \mathrm{kN}$
$\sum F_{y}=-10 k N-(2 \mathrm{kN} / m)(3 m)+R_{B y}=0 \quad \mathrm{R}_{\mathrm{By}}=16 \mathrm{kN}$
$\sum M_{B}=(10 k N)(2.25 m)+(6 k N)(1.5 m)+M_{R B}=0 \quad \mathrm{M}_{\mathrm{RB}}=-31.5^{\mathrm{kN}-\mathrm{m}}$


Draw the load diagram with the distributed load as given with the reactions.
Shear Diagram:
Label the load areas and calculate:
Area I $=(-2 \mathrm{kN} / \mathrm{m})(0.75 \mathrm{~m})=-1.5 \mathrm{kN}$
Area II $=(-2 \mathrm{kN} / \mathrm{m})(2.25 \mathrm{~m})=-4.5 \mathrm{kN}$
$V_{A}=0$
$\mathrm{V}_{\mathrm{C}}=\mathrm{V}_{\mathrm{A}}+$ Area $\mathrm{I}=0-1.5 \mathrm{kN}=-1.5 \mathrm{kN}$ and
$V_{C}=V_{C}+$ force at $C=-1.5 \mathrm{kN}-10 \mathrm{kN}=-11.5 \mathrm{kN}$
$\mathrm{V}_{\mathrm{B}}=\mathrm{V}_{\mathrm{C}}+$ Area $\mathrm{II}=-11.5 \mathrm{kN}-4.5 \mathrm{kN}=-16 \mathrm{kN}$ and
$V_{B}=V_{B}+$ force at $B=-16 \mathrm{kN}+16 \mathrm{kN}=0 \mathrm{kN}$

## Bending Moment Diagram:

Label the load areas and calculate:

$$
\begin{array}{ll}
\text { Area III }=(-1.5 \mathrm{kN})(0.75 \mathrm{~m}) / 2=-0.5625 \mathrm{kN}-\mathrm{m} & \\
\text { Area IV }=(-11.5 \mathrm{kN})(2.25 \mathrm{~m})=-25.875 \mathrm{kN}-\mathrm{m} & \\
\text { Area } \mathrm{V}=(-16-11.5 \mathrm{kN})(2.25 \mathrm{~m}) / 2=-5.0625 \mathrm{kN}-\mathrm{m} & \\
& \\
\mathrm{M}_{\mathrm{A}}=0 & \\
\mathrm{M}_{\mathrm{C}}=\mathrm{M}_{\mathrm{A}}+\text { Area III }=0-0.5625 \mathrm{kN}-\mathrm{m}=-0.5625 \mathrm{kN}-\mathrm{m} & \\
\mathrm{M}_{\mathrm{B}}=\mathrm{M}_{\mathrm{C}}+\text { Area IV }+ \text { Area } \mathrm{V}=-0.5625 \mathrm{kN}-\mathrm{m}-25.875 \mathrm{kN}-\mathrm{m}-5.0625 \mathrm{kN}-\mathrm{m}= \\
& =-31.5 \mathrm{kN}-\mathrm{m} \text { and } \\
\mathrm{M}_{\mathrm{B}}=\mathrm{M}_{\mathrm{B}}+\text { moment at } B=-31.5 \mathrm{kN}-\mathrm{m}+31.5 \mathrm{kN}-\mathrm{m}=0 \mathrm{kN}-\mathrm{m} &
\end{array}
$$



## Mechanics of Materials Primer

## Notation:

| $\begin{aligned} A= & \text { area }(\text { net }=\text { with holes, bearing }=\text { in } \\ & \text { contact, etc...) } \end{aligned}$ | $Q_{\text {connected }}=$ first moment area about a neutral axis for the connected part |
| :---: | :---: |
| $\begin{aligned} b= & \text { total width of material at a } \\ & \text { horizontal section } \end{aligned}$ | $=$ radius of gyration or radius of a hole |
| $d \quad=$ diameter of a hole | $S \quad=$ section modulus |
| $D=$ symbol for diameter | = thickness of a hole or member |
| $E \quad=\begin{aligned} & \text { modulus of elasticity or Young's } \\ & \text { modulus }\end{aligned}$ | $\begin{array}{ll} T & =\text { name for axial moment or torque } \\ V & =\text { internal shear force } \end{array}$ |
| $f \quad=$ symbol for stress | $y=$ vertical distance |
| $f_{\text {allowable }}=$ allowable stress | $\alpha=$ coefficient of thermal expansion for |
| $\begin{aligned} f_{\text {critical }}= & \text { critical buckling stress in column } \\ & \text { calculations from } P_{\text {critical }} \end{aligned}$ | a material $\delta=$ elongation or length change |
| $f_{v} \quad=$ shear stress | $\delta_{T}=$ elongation due or length change |
| $f_{p} \quad=$ bearing stress (see P ) | due to temperature |
| $F_{\text {allowed }}=$ allowable stress (used by codes) | = strain |
| $F_{\text {connector }}=\text { shear force capacity per }$ connector | $\begin{array}{ll} \varepsilon_{T} & =\text { thermal strain (no units) } \\ \phi & =\text { angle of twist } \end{array}$ |
| $I \quad=\quad \begin{aligned} & \text { moment of inertia with respect to } \\ & \text { neutral axis bending }\end{aligned}$ | = shear strain |
| $J=$ polar moment of inertia | $=\mathrm{pi}$ (3.1415 radians or 180 |
| $K=$ effective length factor for columns | $\theta \quad=$ angle of principle stress |
| $L \quad=$ length | = slope of the beam deflection curve |
| $L_{e} \quad=$ effective length that can buckle for column design, as is $\ell_{e}, L_{\text {effective }}$ | $\begin{array}{ll} \rho & =\text { name for radial distance } \\ \sigma & =\text { engineering symbol for normal } \\ \text { stress } \end{array}$ |
| $M=$ internal bending moment, as is $M^{\prime}$ <br> $n \quad=$ number of connectors across a joint | $\tau \quad=$ engineering symbol for shearing |
| $p \quad=$ pitch of connector spacing | displacement due to |
| $P \quad=$ name for axial force vector, as is $P^{\prime}$ | = displaceme |
| $\begin{aligned} P_{\text {crit }}= & \text { critical buckling load in column } \\ & \text { calculations, as is } P_{\text {critical }}, P_{c r} \end{aligned}$ | $\begin{array}{ll} \Delta T & =\text { change in temperature } \\ \int & =\text { symbol for integration } \end{array}$ |
| $Q \quad=\begin{aligned} & \text { first moment area about a neutral } \\ & \text { axis } \end{aligned}$ |  |

Mechanics of Materials is a basic engineering science that deals with the relation between externally applied load and its effect on deformable bodies. The main purpose of Mechanics of Materials is to answer the question of which requirements have to be met to assure STRENGTH, RIGIDITY, AND STABILITY of engineering structures.

## Normal Stress

Stress that acts along an axis of a member; can be internal or external; can be compressive or tensile.

$$
f=\sigma=\frac{P}{A_{\text {net }}} \quad \text { Strength condition: } f=\frac{P}{A_{\text {net }}}<f_{\text {allowable }} \text { or } F_{\text {allowed }}
$$

## Shear Stress (non beam)

Stress that acts perpendicular to an axis or length of a member, or parallel to the cross section is called shear stress.

Shear stress cannot be assumed to be uniform, so we refer to average shearing stress.

$$
f_{v}=\tau=\frac{P}{A_{\text {net }}} \quad \text { Strength condition: } f_{v}=\frac{P}{A_{\text {net }}}<\tau_{\text {allowable }} \text { or } F_{\text {allowed }}
$$

## Bearing Stress

A compressive normal stress acting between two bodies.

$$
f_{p}=\frac{P}{A_{\text {bearing }}}
$$

## Torsional Stress

A shear stress caused by torsion (moment around the axis).

$$
f_{v}=\frac{T \rho}{J}
$$

## Bolt Shear Stress

Single shear - forces cause only one shear "drop" across the bolt. $f=\frac{P}{1 A_{\text {bolt }}}$
Double shear - forces cause two shear changes across the bolt. $f=\frac{P}{2 A}$ bolt

Bearing of a bolt on a bolt hole - The bearing surface can be represented by projecting the cross section of the bolt hole on a plane (into a rectangle).

$$
f_{p}=\frac{P}{A}=\frac{P}{t d}
$$

## Bending Stress

A normal stress caused by bending; can be compressive or tensile. The stress at the neutral surface or neutral axis, which is the plane at the centroid of the cross section is zero.

$$
f_{b}=\frac{M y}{I}=\frac{M}{S}
$$



## Beam Shear Stress

$f_{v-a v e}=0$ on the beam's surface. Even if Q is a maximum at $\mathrm{y}=0$, we don't know that the thickness is a minimum there.

$$
f_{v}=\frac{V}{\Delta A}=\frac{V}{b \cdot \Delta x} \quad f_{v-a v e}=\frac{V Q}{I b}
$$

## Rectangular Sections


$f_{v-\max }$ occurs at the neutral axis: $\quad f_{v}=\frac{V Q}{I b}=\frac{3 V}{2 A}$

## Webs of Beams

In steel W or S sections the thickness varies from the flange to the web. We neglect the shear stress in the flanges and consider the shear stress in the web to be constant:

$$
f_{v-\max }=\frac{3 V}{2 A} \approx \frac{V}{A_{w e b}}
$$

## Connectors in Bending

Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices. The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p .

$$
\frac{V_{\text {longitudial }}}{p}=\frac{V Q}{I} \quad n F_{\text {connector }} \geq \frac{V Q_{\text {connectedarea }}}{I} \cdot p
$$

where
$\mathrm{p}=$ pitch length
$\mathrm{n}=$ number of connectors connecting the connected area to the rest of the cross section
$\mathrm{F}=$ force capacity in one connector
$\mathrm{Q}_{\text {connected area }}=\mathrm{A}_{\text {connected area }} \times \mathrm{y}_{\text {connected area }}$
$\mathrm{y}_{\text {connected area }}=$ distance from the centroid of the connected area to the neutral axis

## Normal Strain

In an axially loaded member, normal strain, $\varepsilon$ is the change in the length, $\delta$ with respect to the original length, L .

$$
\varepsilon=\frac{\delta}{L}
$$

## Shearing Strain

In a member loaded with shear forces, shear strain, $\gamma$ is the change in the sheared side, $\delta_{\mathrm{s}}$ with respect to the original height, L. For small angles: $\tan \phi \cong \phi$.

$$
\gamma=\frac{\delta_{s}}{L}=\tan \phi \cong \phi
$$


(a)

(b)

In a member subjected to twisting, the shearing strain is a measure of the angle of twist with respect to the length and distance from the center, $\rho$ :

$$
\gamma=\frac{\rho \phi}{L}
$$

## Stress vs. Strain

Behavior of materials can be measured by recording deformation with respect to the size of the load. For members with constant cross section area, we can plot stress vs. strain.

BRITTLE MATERIALS - ceramics, glass, stone, cast iron; show abrupt fracture at small strains.

DUCTILE MATERIALS - plastics, steel; show a yield point and large strains (considered plastic) and "necking" (give warning of failure)

SEMI-BRITTLE MATERIALS - concrete;

show no real yield point, small strains, but have some "strain-hardening".

## Linear-Elastic Behavior

In the straight portion of the stress-strain diagram, the materials are elastic, which means if they are loaded and unloaded no permanent deformation occurs.

## True Stress \& Engineering Stress

True stress takes into account that the area of the cross section changes with loading.
Engineering stress uses the original area of the cross section.

## Hooke's Law - Modulus of Elasticity

In the linear-elastic range, the slope of the stress-strain diagram is constant, and has a value of E, called Modulus of Elasticity or Young's Modulus.

$$
f=E \cdot \varepsilon
$$



Isotropic Materials - have the same E with any direction of loading.
Anisotropic Materials - have different E's with the direction of loading.

## Orthotropic Materials - have directionally based E's

Table D-1 Elastic moduli of selected materials

| Material | Modulus of clasticity E |  | Shear modulus $G$ |  | Poisson's ratio $\nu$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $10^{61} \mathrm{psi}$ | GPa | $10^{6} \mathrm{psi}$ | GPa |  |
| Aluminum | 10 | 70 | 3.8 | 26 | 0.33 |
| Aluminum alloys | 10-12 | 70-80 | 3.8-4.4 | 26-30 | 0.33 |
| 2014-T6 | 10.6 | 73 | 4 | 28 | 0.33 |
| 6061-T6 | 10 | 70 | 3.8 | 26 | 0.33 |
| 7075-T6 | 10.4 | 72 | 3.9 | 27 | 0.33 |
| Brick (compression) | 1.5-3.5 | 10-24 |  |  |  |
| Cast iron | 12-25 | 80-170 | 4.5-10 | 31-69 | 0.2-0.3 |
| Gray cast iron | 14 | 97 | 5.6 | 39 | 0.25 |
| Concrete (compression) | 2.6-4.4 | 18-30 |  |  | 0.1-0.2 |
| Copper | 17 | 115 | 6.2 | 43 | 0.35 |
| Copper alloys | 14-18 | 96-120 | 5.2-6.8 | 36-47 | 0.33-0.35 |
| Brass | 14-16 | 96-110 | 5.2-6 | 36-41 | 0.34 |
| 80\% Cu, $20 \% \mathrm{Zn}$ | 15 | 100 | 5.5 | 38 | 0.33 |
| Naval brass | 15 | 100 | 5.5 | 38 | 0.33 |
| Bronze | 14-17 | 96-120 | 5.2-6.3 | 36-44 | 0.34 |
| Manganese bronze | 15 | 100 | 5.6 | 39 | 0.35 |
| Glass | 7-12 | 50-80 | 2.9-5 | 20-33 | 0.20-0.27 |
| Magnesium | 5.8 | 40 | 2.2 | 15 | 0.34 |
| Nickel | 30 | 210 | 11.4 | 80 | 0.31 |
| Nylon | 0.3-0.4 | 2-3 |  | $0.0003-0.0014$ | 0.4 |
| Rubber | 0.0001-0.0006 | 0.001-0.004 | 0.00004-0.0002 | 0.0003-0.0014 | 0.44-0.50 |
| Steel | 28-32 | 190-220 | 10.8-12.3 | 75-85 | 0.28-0.30 |
| Stone (compression) |  |  |  |  |  |
| Granite | 6-10 | 40-70 |  |  | 0.2-0.3 |
| Marble | 7-14 | 50-100 |  |  | 0.2-0.3 |
| Titanium | 16 | 110 | 5.8 | 40 | 0.33 |
| Titanium alloys | 15-18 | 100-124 | 5.6-6.8 | 39-47 | 0.33 |
| Tungsten | 52 | 360 | 22 | 150 | 0.2 |
| Wood (bending) $10-11$ |  |  |  |  |  |
| Ash | 1.5-1.6 | 10-11 |  |  |  |
| Oak | 1.6-1.8 | 11-12 |  |  |  |
| Southern pine | 1.6-2 | 11-14 |  |  |  |
| Wrought iron | 28 | 190 | 10.9 | 75 | 0.3 |

## Plastic Behavior \& Fatigue

Permanent deformations happen outside the linear-elastic range and are called plastic deformations. Fatigue is damage caused by reversal of loading.

- The proportional limit (at the end of the elastic range) is the greatest stress valid using Hooke's law.
- The elastic limit is the maximum stress that can be applied before permanent deformation would appear upon unloading.

- The yield point (at the yield stress) is where a ductile material continues to elongate without an increase of load. (May not be well defined on the stress-strain plot.)
- The ultimate strength is the largest stress a material will see before rupturing, also called the tensile strength.
- The rupture strength is the stress at the point of rupture or failure. It may not coincide with the ultimate strength in ductile materials. In brittle materials, it will be the same as the ultimate strength.
- The fatigue strength is the stress at failure when a member is subjected to reverse cycles of stress (up \& down or compression \& tension). This can happen at much lower values than the ultimate strength of a material.
- Toughness of a material is how much work (a combination of stress and strain) us used for fracture. It is the area under the stress-strain curve.

Concrete does not respond well to tension and is tested in compression. The strength at crushing is called the compression strength.

Materials that have time dependent elongations when loaded are said to have creep. Concrete and wood creep. Concrete also has the property of shrinking over time.

## Poisson's Ratio

For an isometric material that is homogeneous, the properties are the same for the cross section:

$$
\varepsilon_{y}=\varepsilon_{z}
$$

There exists a linear relationship while in the linear-elastic range of the material between longitudinal strain and lateral strain:

$$
\mu=-\frac{\text { lateral strain }}{\text { axial strain }}=-\frac{\varepsilon_{y}}{\varepsilon_{x}}=-\frac{\varepsilon_{z}}{\varepsilon_{x}} \quad \varepsilon_{y}=\varepsilon_{z}=-\frac{\mu f_{x}}{E}
$$

Positive strain results from an increase in length with respect to overall length. Negative strain results from a decrease in length with respect to overall length.
$\mu$ is the Poisson's ratio and has a value between 0 and $1 / 2$, depending on the material

## Relation of Stress to Strain

$f=\frac{P}{A} ; \varepsilon=\frac{\delta}{L}$ and $E=\frac{f}{\varepsilon}$ so $E=\frac{P / A}{\delta / L}$ which rearranges to: $\quad \delta=\frac{P L}{A E}$

## Stress Concentrations

In some sudden changes of cross section, the stress concentration changes (and is why we used average normal stress). Examples are sharp notches, or holes or corners.

## Plane of Maximum Stress

When both normal stress and shear stress occur in a structural member, the maximum stresses can occur at some other planes (angle of $\theta$ ).


Maximum Normal Stress happens at $\theta=0^{\circ}$ AND
Maximum Shearing Stress happens at $\theta=45^{\circ}$ with only normal stress in the $x$ direction.

## Thermal Strains

Physical restraints limit deformations to be the same, or sum to zero, or be proportional with respect to the rotation of a rigid body.
We know axial stress relates to axial strain: $\quad \delta=\frac{P L}{A E} \quad$ which relates $\delta$ to P
Deformations can be caused by the material reacting to a change in energy with temperature. In general (there are some exceptions):

- Solid materials can contract with a decrease in temperature.
- Solid materials can expand with an increase in temperature.

The change in length per unit temperature change is the coefficient of thermal expansion, $\alpha$. It has units of $/{ }^{\circ} \mathrm{F}$ or $/{ }^{\circ} \mathrm{C}$ and the deformation is related by:

$$
\delta_{T}=\alpha(\Delta T) L
$$

## Coefficient of Thermal Expansion

| Material | Coefficients $(\alpha)$ <br> $\left[\right.$ in. $\left./ \mathrm{in} . /^{\circ} \mathrm{F}\right]$ | Coefficients $(\alpha)$ <br> $\left[\mathrm{mm} / \mathrm{mm} /{ }^{\circ} \mathrm{C}\right]$ |
| :--- | :---: | :---: |
| Wood | $3.0 \times 10^{-6}$ | $5.4 \times 10^{-6}$ |
| Glass | $4.4 \times 10^{-6}$ | $8.0 \times 10^{-6}$ |
| Concrete | $5.5 \times 10^{-6}$ | $9.9 \times 10^{-6}$ |
| Cast Iron | $5.9 \times 10^{-6}$ | $10.6 \times 10^{-6}$ |
| Steel | $6.5 \times 10^{-6}$ | $11.7 \times 10^{-6}$ |
| Wrought Iron | $6.7 \times 10^{-6}$ | $12.0 \times 10^{-6}$ |
| Copper | $9.3 \times 10^{-6}$ | $16.8 \times 10^{-6}$ |
| Bronze | $10.1 \times 10^{-6}$ | $18.1 \times 10^{-6}$ |
| Brass | $10.4 \times 10^{-6}$ | $18.8 \times 10^{-6}$ |
| Aluminum | $12.8 \times 10^{-6}$ | $23.1 \times 10^{-6}$ |


(a)

(b) Thermal Strain: $\quad \varepsilon_{T}=\alpha \Delta T$

There is no stress associated with the length change with free movement, BUT if there are restraints, thermal deformations or strains can cause internal forces and stresses.

## How A Restrained Bar Feels with Thermal Strain

1. Bar pushes on supports because the material needs to expand with an increase in temperature.
2. Supports push back.
3. Bar is restrained, can't move and the reaction causes internal stress.


(b)

## Superposition Method

If we want to solve a statically indeterminate problem that has extra support forces:

- We can remove a support or supports that makes the problem look statically determinate
- Replace it with a reaction and treat it like it is an applied force
- Impose geometry restrictions that the support imposes


## Beam Deflections

$$
\theta=\text { slope }=\frac{1}{E I} \int M(x) d x
$$

If the bending moment changes, $\mathrm{M}(\mathrm{x})$ across a beam of constant material and cross section then the curvature will change:

The slope of the n.a. of a beam, $\theta$, will be tangent to the radius of curvature, R :
The equation for deflection, y , along a beam is:

$$
y=\Delta=\frac{1}{E I} \int \theta d x=\frac{1}{E I} \iint M(x) d x
$$

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well.

Elastic curve equations can be superpositioned ONLY if the stresses are in the elastic range.

## Column Buckling

Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations. A column loaded centrically can experience unstable equilibrium, called buckling, because of how tall and slender they are. This instability is sudden and not good.

Buckling can occur in sheets (like my "memory metal" cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of $\mathrm{P} \cdot \Delta$ ) as a function of the end conditions.

Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):

$$
P_{\text {critical }}=\frac{\pi^{2} E I_{\text {min }}}{(L)^{2}} \quad \text { or } \quad P_{c r}=\frac{\pi^{2} E I}{\left(L_{e}\right)^{2}}=\frac{\pi^{2} E A}{\left(L_{e} / r\right)^{2}}
$$


and the critical stress (if less than the normal stress) is:

$$
f_{\text {critical }}=\frac{P_{\text {critical }}}{A}=\frac{\pi^{2} E A r^{2}}{A\left(L_{e}\right)^{2}}=\frac{\pi^{2} E}{\left(L_{e} / r\right)^{2}}
$$

where $\mathrm{I}=\mathrm{Ar}^{2}$ and $L_{e} / r$ is called the slenderness ratio. The smallest I of the section will govern.
Radius of gyration is a relationship between I and A. It is useful for comparing columns of different shape cross section shape.

$$
r_{x}=\sqrt{\frac{I_{x}}{A}} \quad r_{y}=\sqrt{\frac{I_{y}}{A}}
$$

## Yield Stress and Buckling Stress

The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

## Effective Length and Bracing

Depending on the end support conditions for a column, the effective length can be found from the deflected shape
 (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, K.

$$
L_{e}=K \cdot L
$$

| Buckled shape of column shown by dashed line |  |  |  | (d) | (e) | (f) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Theoretical $K$ value | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Recommended design values when ideal conditions are approximated | 0.65 | 0.80 | 1.0 | 1.2 | 2.10 | 2.0 |
| End conditions code | Rotation fixed, Translation fixed <br> Rotation free, Translation fixed $\square$ Rotation fixed, Translation free <br> i Rotation free, Translation free | Rotation fixed, Translation fixed <br> Rotation free, Translation fixed <br> Rotation fixed, Translation free <br> Rotation free, Translation free |  |  |  |  |



## Example 1

## Example Problem 6.8 (Figures 6.18 to 6.20 )

A pipe storage rack is used for storing pipe in a shop. The support rack beam is fastened to the main floor beam using steel straps $1^{\prime \prime} 2^{\prime \prime} \times 2^{\prime \prime}$ in dimension. Round bolts are used to fasten the strap to the floor beam in single shear. (a) If the weight of the pipes impose a maximum tension load of 10,000 pounds in each strap, determine the tension stress developed in the steel strap. (b) Also, what diameter bolt is necessary to fasten the strap to the floor beam if the allowable shear stress for the bolts equals $F_{v}=15,000^{\mathrm{lb}} / \mathrm{in.}^{2}$ ?

## Solution:

a. The tensile stress developed in the steel strap (Figure 6.19) can be determined using the direct stress formula.
$f_{t}=\frac{P}{A}=\frac{10,000 \mathrm{lb} .}{\left(1 / 2^{\prime \prime} \times 2^{\prime \prime}\right)}=10,000^{\mathrm{lb}} / \mathrm{in} .^{2}$
In mild steel (A36), the maximum permissible tensile stress (allowable) is equal to

$$
\underset{\text { (allowable) }}{F_{t}}=22,000 \mathrm{psi}
$$

Therefore, the strap size is adequate to support the tensile load safely.
b. To determine the size bolt necessary to carry the load safely in single shear, the design form of the equation must be used.

$$
\begin{aligned}
f_{v}=\frac{P}{A} ; \quad A & =\frac{P}{F_{v}}=\frac{10,000 \mathrm{lb} .}{15,000^{\mathrm{lb}} / \mathrm{in} .^{2}}=0.67 \mathrm{in} .^{2} \\
A=\frac{\pi D^{2}}{4} ; \quad D^{2} & =\frac{4 \times A}{\pi}=\frac{4 \times 0.67 \mathrm{in} .^{2}}{3.14} \\
& =0.854 \mathrm{in} .^{2}
\end{aligned}
$$

$D=0.92$ in.; Use: $1^{\prime \prime} \phi$ bolt.


Figure 6.18 Pipe storage rack.


Figure 6.19 Section.


Figure 6.20 Bolt in single shear.

## Example 2

8.11 A built-up plywood box beam with $2 \times 4 \mathrm{~S} 4 \mathrm{~S}$ top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 \#/ft. along the 26 -foot span. Assume the nails have a shear capacity of 80\# each.

## Solution:

Construct the shear ( $V$ ) diagram to obtain the critical shear condition and its location
Note that the condition of shear is critical at the supports, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing $P$ varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.

$$
\begin{aligned}
& f_{v}=\frac{V Q}{I b} \\
& I_{x}=\frac{\left(4.5^{\prime \prime}\right)\left(18^{\prime \prime}\right)^{3}}{12}-\frac{\left(3.5^{\prime \prime}\right)\left(15^{\prime \prime}\right)^{3}}{12}=1,202.6 \text { in. }^{4} \\
& \varepsilon=\frac{\delta}{2} \\
& f_{v-\max }=\frac{(2,600 \#)\left(83.3 \text { in. } .^{3}\right)}{\left(1,202.6 \text { in. }^{4}\right)\left(1 / 2^{\prime \prime}+1 / 2^{\prime \prime}\right)}=180.2 \mathrm{psi}
\end{aligned}
$$

Assume:

$$
\begin{aligned}
& F= \text { Capacity of two nails (one each side) at the } \\
& \text { flange; representing two shear surfaces }
\end{aligned}
$$

(n) $F \geq f_{v} \times b \times p=\frac{V Q}{I b} \times b p$

$\therefore(\mathrm{n}) F \geq p \times \frac{V Q}{I} ; \quad p \leq \frac{(\mathrm{n}) F I}{V Q}$


At the maximum shear location (support) where $V=2,600$ \#

$$
Q=A \bar{y}=\left(5.25 \mathrm{in}^{2} .^{2}\right)\left(8.25^{\prime \prime}\right)=43.3 \mathrm{in} .^{3}
$$


where:

$$
A_{v}=\text { shear area }
$$

Shear force $=f_{v} \times A_{v}$

$$
p \leq \frac{(2 \text { nails } \times 80 \# / \text { nail })\left(1,202.6 \text { in. }^{4}\right)}{(2,600 \#)\left(43.3 \text { in. }{ }^{3}\right)}=1.71^{\prime \prime}
$$

## Example 3

### 6.4 THERMAL EFFECTS

Most structural materials expand in volume when subjected to heat and contract when cooled. Whenever a design prevents the change in length of a member subjected to temperature variation, internal stresses develop. Sometimes these thermal stresses may be sufficiently high to exceed the elastic limit and cause serious damage. Free, unrestrained members experience no stress changes with temperature changes, but dimensional change results. For example, it is common practice to provide expansion joints between sidewalk pavements to allow movement during hot summer days. Prevention of expansion on a hot day would undoubtedly result in severe buckling of the pavement.
The dimensional change due to temperature changes is usually described in terms of the change in a linear dimension. The change in length of a structural member, $\Delta L$, is directly proportional to both the temperature change $(\Delta T)$ and the original length of the member $L$. Thermal sensitivity, called the coefficient of linear expansion ( $\alpha$ ), has been determined for all engineering materials (see Table 6.3). Careful measurements have shown that the ratio of strain $\varepsilon$ to temperature change $\Delta T$ is a constant:

$$
\alpha=\frac{\text { strain }}{\text { temp. change }}=\frac{\varepsilon}{\Delta T}=\frac{\delta / L}{\Delta T}
$$

Solving this equation for the deformation:
where:

$$
\delta=\alpha L \Delta T
$$

where:

$$
\begin{aligned}
\alpha & =\text { coefficient of thermal expansion or contraction } \\
L & =\text { original length of the member (inches or } \mathrm{mm} \text { ) } \\
\Delta T & =\text { change in temperature }\left({ }^{\circ} \mathrm{F} \text { or }{ }^{\circ} \mathrm{C}\right. \text { ) } \\
\delta & =\text { total change in length (in. or } \mathrm{mm} \text { ) }
\end{aligned}
$$

Of perhaps even greater importance in engineering design are the stresses developed by restraining the free expansion and contraction of members subjected to temperature variations. To calculate these temperature stresses, it is useful to determine first the free expansion or contraction of the member involved and, second, the force and unit stress developed in forcing the member to attain its original length. The problem from this point on is exactly the same as those solved in the earlier portions of this chapter dealing with axial stresses, strains, and deformations. The amount of stress developed by restoring a bar to its original length $L$ is:

$$
\begin{aligned}
f & =\varepsilon E=\frac{\delta}{L} E=\frac{\alpha L \Delta T E}{L}=\alpha \Delta T E \\
\therefore f & =\alpha \Delta T E
\end{aligned}
$$



Figure 6.57 Steel rail subjected to thermal change.

## Example Problem 6.21 (Figure 6.57)

A 60 ' length of steel rail is laid on a day when the temperature is $40^{\circ} \mathrm{F}$. In order to prevent the rail from developing any internal stresses due to a thermal increase of $60^{\circ} \mathrm{F}$, what is the amount of deformation that needs to be accommodated with respect to the slotted connection at the rail $\operatorname{end}(\mathrm{s}) ? E_{s t}=29 \times 10^{3} \mathrm{ksi}$.

## Solution:

Steel has a coefficient of expansion $\alpha=6.5 \times 10^{-6} /{ }^{\circ} \mathrm{F}$ (see Table 6.3).
Using the deformation equation due to thermal change:

$$
\begin{aligned}
\delta & =\alpha L \Delta T=\left(6.5 \times 10^{-6} /{ }^{\circ} \mathrm{F}\right)\left(60^{\prime} \times 12^{\mathrm{in} .} / \mathrm{ft}\right)\left(60^{\circ} \mathrm{F}\right) \\
& =0.28^{\prime \prime}
\end{aligned}
$$

This amount of deformation ( $0.28^{\prime \prime}$ ) for a 60 '-long rail section may not seem large but if there are no provisions made to allow movement during thermal changes, large internal stress may result. If the rail section in this example has a cross-sectional area of $A=10.5$ in. ${ }^{2}$, determine the amount of internal compressive stress that can result if the rail is restrained from moving.

$$
\begin{aligned}
f & =\alpha \Delta T E=\left(6.5 \times 10^{-6} /{ }^{\circ} \mathrm{F}\right)\left(60^{\circ} \mathrm{F}\right)\left(29 \times 10^{\left.3 \mathrm{k} / \mathrm{in.}^{2}\right)}\right. \\
& =11.31 \mathrm{ksi}
\end{aligned}
$$

(a very large internal stress which can potentially cause the rail to buckle)

## Example 4

A short concrete column measuring 12 in . square is reinforced with four \#8 bars $\left(A_{s}=4 \times 0.79\right.$ in. $^{2}=3.14$ in. $\left.^{2}\right)$ and supports an axial load of 250 k . Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$$
\begin{aligned}
& E_{c}=3 \times 10^{6} \mathrm{psi} \text { and } \\
& E_{s}=29 \times 10^{6} \mathrm{psi}
\end{aligned}
$$

## Solution:

From equilibrium:


$$
\begin{aligned}
& {\left[\Sigma F_{y}=0\right]-250 \mathrm{k}+f_{s} A_{\mathrm{s}}+f_{c} A_{c}=0} \\
& A_{s}=3.14 \mathrm{in} .^{2} \\
& A_{c}=\left(12^{\prime \prime} \times 12^{\prime \prime}\right)-3.14 \mathrm{in}^{2} \cong 141 \mathrm{in} .^{2} \\
& 3.14 f_{s}+141 f_{c}=250 \mathrm{k}
\end{aligned}
$$

From the deformation relationship:

$$
\begin{aligned}
& \delta_{s}=\delta_{c} ; L_{s}=L_{c} \\
& \therefore \frac{\delta_{s}}{L}=\frac{\delta_{c}}{L}
\end{aligned}
$$

and

$$
\varepsilon_{s}=\varepsilon_{c}
$$

Since

$$
E=\frac{f}{\varepsilon}
$$

and

$$
\begin{aligned}
& \frac{f_{s}}{E_{s}}=\frac{f_{c}}{E_{c}} \\
& f_{s}=f_{c} \frac{E_{s}}{E_{c}}=\frac{29 \times 10^{3}\left(f_{c}\right)}{3 \times 10^{3}}=9.67 f_{c}
\end{aligned}
$$

Substituting into the equilibrium equation:

$$
\begin{aligned}
& 3.14\left(9.76 f_{c}\right)+141 f_{c}=250 \\
& 30.4 f_{c}+141 f_{c}=250 \\
& 171.4 f_{c}=250 \\
& f_{c}=1.46 \mathrm{ksi} \\
& \therefore f_{s}=9.67(1.46) \mathrm{ksi} \\
& f_{s}=14.1 \mathrm{ksi}
\end{aligned}
$$

## Example 5

Determine the deflection in the steel beam if it is a W15 x 88. $\mathrm{E}=30 \times 10^{3} \mathrm{ksi}$.


Deflection Check:

$$
\begin{aligned}
\Delta_{\text {actual }} & =\frac{P L^{3}}{48 E I}+\frac{5 \omega L^{4}}{384 E I} \\
\Delta_{\text {actual }} & =\frac{(20 \mathrm{k})\left(28^{\prime}\right)^{3}(1728)}{48\left(30 \times 10^{3}\right)(890)} \\
& +\frac{5(1.06 \mathrm{k} / \mathrm{ft} .)\left(28^{\prime}\right)^{4}(1728)}{384\left(30 \times 10^{3}\right)(890)} \\
\Delta_{\text {actual }} & =0.59^{\prime \prime}+0.55^{\prime \prime}=1.14^{\prime \prime}
\end{aligned}
$$

## Example 6

Example Problem 10.6 (Figures 10.28 to 10.30 )
A W8 $\times 40$ steel column supports trusses framed into its web, which serve to fix the weak axis and light beams that attach to the flange, simulating a pin connection about the strong axis. If the base connection is assumed as a pin, determine the critical buckling load the column is capable of supporting.

## Solution:

$$
\begin{aligned}
\text { W } 8 \times 40 ; \quad(A & =11.7 \mathrm{in}^{2} .^{2}, r_{x}=3.53 "{ }^{\prime}, I_{x}=146 \mathrm{in} .^{4}, \\
r_{y} & \left.=2.044^{\prime}, I_{y}=49.1 \mathrm{in} .^{4}\right)
\end{aligned}
$$

The first step is to determine the critical axis for buckling (i.e., which one has the larger $K L / r$ ).

## Weak Axis:

$$
\begin{aligned}
L_{e} & =K L=0.7\left(34^{\prime}\right)=23.8^{\prime} \\
\frac{K L}{r_{y}} & =\frac{23.8^{\prime} \times 12^{\mathrm{in} . / \mathrm{ft}}}{2.04^{\prime \prime}}=140
\end{aligned}
$$

Strong Axis:

$$
\begin{aligned}
& L_{e}=L ; \quad K=1.0 ; \quad K L=37^{\prime} \\
& \frac{K L}{r_{x}}=\frac{\left(37^{\prime} \times 12^{\mathrm{in} .} / \mathrm{ft}\right)}{3.53^{\prime}}=125.8
\end{aligned}
$$

The weak axis for this column is critical since

$$
\begin{aligned}
\frac{K L}{r_{y}} & >\frac{K L}{r_{x}} \\
P_{\text {cr. }} & =\frac{\pi^{2} E I_{y}}{(K L)^{2}}=\frac{(3.14)^{2}\left(29 \times 10^{3} \mathrm{ksi}\right)\left(49.1 \mathrm{in} .^{4}\right)}{\left(23.8^{1} \times 12^{\mathrm{in} . / \mathrm{ft}}\right)^{2}} \\
& =172.1 \mathrm{k} \\
f_{\text {critical }} & =\frac{P_{\text {crit. }}}{A}=\frac{172.1 \mathrm{k}}{11.7 \mathrm{in.}^{2}}=14.7 \mathrm{ksi}
\end{aligned}
$$



Figure 10.28 Truss/column framing.


Figure 10.30 (a) Weak axis. (b) Strong axis.

## Conversion Factors and Units of Measurement

Simplified Engineering for Architects and Builders, $10^{\text {th }}$ ed., Ambrose \& Tripeny, 2006

TABLE 3 Factors for Conversion of Units

| To Convert from |  |  |  |
| :--- | :--- | :--- | :--- |
| U.S. Units to SI |  | To Convert from |  |
| Units, Multiply by: | U.S. Unit |  | SI Units to U.S. |
| 25.4 | in. | SI Unit | Units, Multiply by: |

TABLE 2 Units of Measurement: SI System

| Name of Unit | Abbreviation | Use in Building Design |
| :---: | :---: | :---: |
| Length |  |  |
| Meter <br> Millimeter | $\begin{aligned} & \mathrm{m} \\ & \mathrm{~mm} \end{aligned}$ | Large dimensions, building plans, beam spans Small dimensions, size of member cross sections |
| Area |  |  |
| Square meters <br> Square millimeters | $\begin{aligned} & \mathrm{m}^{2} \\ & \mathrm{~mm}^{2} \end{aligned}$ | Large areas <br> Small areas, properties of member cross sections |
| Volume |  |  |
| Cubic meters Cubic millimeters | $\begin{aligned} & \mathrm{m}^{3} \\ & \mathrm{~mm}^{3} \end{aligned}$ | Large volumes Small volumes |
| Mass |  |  |
| Kilogram <br> Kilograms per cubic meter | kg $\mathrm{kg} / \mathrm{m}^{3}$ | Mass of material (equivalent to weight in U.S. units) Density (unit weight) |
| Force, Load |  |  |
| Newton Kilonewton | $\begin{aligned} & \mathrm{N} \\ & \mathrm{kN} \end{aligned}$ | Force or load on structure 1000 Newtons |
| Stress |  |  |
| Pascal <br> Kilopascal <br> Megapascal <br> Gigapascal | Pa <br> kPa <br> MPa <br> GPa | Stress or pressure ( 1 pascal $=1 \mathrm{~N} / \mathrm{m}^{2}$ ) <br> 1000 pascals <br> 1,000,000 pascals <br> $1,000,000,000$ pascals |
| Temperature |  |  |
| Degree Celsius | ${ }^{\circ} \mathrm{C}$ | Temperature |

TABLE 1 Units of Measurement: U.S. System

| Name of Unit | Abbreviation | Use in Building Design |
| :--- | :--- | :--- |
| Length |  |  |
| Foot | ft | Large dimensions, building plans, beam spans <br> Inch |
|  | in. | Small dimensions, size of member cross <br> sections |
|  |  |  |
| Area |  |  |
| Square feet |  |  |
| Square inches |  |  |
| Volume |  |  |

## Design and Technology in Architecture, Revised Ed., David Guise, 1991, Van Nostrand Reinhold, NY.

## Chapter 7 <br> BUILDING CODES AND ZONING ORDINANCES



In a complex society, regulation is one of the facts of life. The buildings in which people live, work, and play are subject to many controls. Local and regional government agencies have been established to protect the public and the environment from dangerous and undesirable conditions that sometimes occur when manmade structures are erected, and the result is innumerable building codes and zoning ordinances. In the United States, these rules and regulations can vary from community to community, but all are based on fundamental construction methods necessary to protect public safety and welfare.

## BUILDING CODES

Building codes not only affect the selection of the materials that an architect uses to build a structure; they can influence the size and shape of the building as well. For example, depending upon how fire-resistant the selected construction materials are, the codes will permit different maximum areas per floor and different total numbers of floors for the building.

The impact of the relationship between building materials and the size of a building can be most easily demonstrated by an example. Assume that an architect is planning to design a resort. He wishes to use exposed, laminated-wood beams and other wood construction in order to create a rustic atmosphere. Chart 1 , reproduced from the National Building Code, known as BOCA, lists hotels under the use-group R-1. When $\mathrm{R}-1$ is intersected with construction type 4 (heavy timber), the chart shows that the maximum height permitted for a structure of this type is 4 stories or 50 feet, whichever is greater. The chart also indicates that no more than 14,400 square feet of space is permitted on each floor. If the client's needs can be accommodated within these height and area limitations, then all is fine. If not, a different type of framing system will have to be considered-one that permits either more height or more area.

Chart 2 provides specific information regarding the amount of fire protection that is required for each particular part of a building. The information differs based on the variations in construction types listed in Chart 1. The building code can also be referred to for further explanation, and indeed the chart often refers the reader to the code. For example, under type 4 construction, Chart 2 shows that bearing walls require a two-hour rating (see 1 under Structure Element).

Park Avenue, New York, New York.

## CHART 1

Table 501
height and area limitations of builoings
Height limitations of buildings (shown in upper figure as stories and feet above grade). and area limitations of one or two story buildings facing on one street or public space not less than 30

MP - Mot permilted feet wide (shown in lower figure as area in square feet per floor). See Note a.

ML - Mot IImited

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{5}{*}{Use Group

Note a} \& \multicolumn{10}{|c|}{Type of construction} <br>
\hline \& \multicolumn{5}{|c|}{Noncombustible} \& \multicolumn{3}{|l|}{Noncombustible/Combustible} \& \multicolumn{2}{|r|}{Combustible} <br>

\hline \& \multicolumn{2}{|r|}{\multirow[t]{2}{*}{| Type 1 |
| :--- |
| Protected Note b |}} \& \multicolumn{3}{|c|}{Type 2} \& \multicolumn{2}{|r|}{Type 3} \& Type 4 \& \multicolumn{2}{|r|}{Type 5} <br>

\hline \& \& \& \multicolumn{2}{|c|}{Protected} \& Unprotected \& Protected \& Unprotected \& Heavy timber \& Protected \& Unprotected <br>
\hline \& 1A \& 18 \& 2A \& 28 \& 2 C \& 3A \& 38 \& 4 \& 5A \& 58 <br>

\hline A-I Assembly, theaters \& NL. \& NL \& $$
\begin{aligned}
& \hline \text { 5St. } 65 \\
& 19,950 \\
& \hline
\end{aligned}
$$ \& \[

$$
\begin{gathered}
3 \mathrm{St} .40^{\circ} \\
13,125 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\hline \text { St. } 30 \\
8.400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 3 \text { St. } 4 \sigma \\
& 11,550
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
8.400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 3 \text { St. } 40 \\
& 12,600
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
8 . S 25
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
1 \mathrm{St} .20 \\
4.200
\end{array}
$$
\] <br>

\hline A-2 Assembly, nightclubs and similar uses \& NL \& $$
\begin{aligned}
& \mathrm{NL} \\
& 7.200
\end{aligned}
$$ \& \[

$$
\begin{gathered}
\text { 3St. } 40^{\circ} \\
5.700
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
3,750
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 1 \text { St. } 20^{\circ} \\
& 2.400
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St.} 3 \mathrm{O} \\
3,300
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{St} .20 \\
2.400
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
3.600
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{St} .20 \\
2.550
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{St} .20 \\
1,200
\end{gathered}
$$
\] <br>

\hline Lecture halls, recr terminals, restaurants other than night clubs \& NL \& NL. \& $$
\begin{gathered}
5 \text { St. } 65 \\
19.950
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& 3 \mathrm{St} .40 \\
& 13,125
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \text { St. } 30 \\
8,400
\end{array}
$$

\] \& \[

$$
\begin{gathered}
3 \text { St } 40 \\
11,550
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \text { St. } 30 \\
8,400
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .40^{\circ} \\
12,600
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{St} .20 \\
8,925
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 1 \text { St. } 20 \\
& 4.200 \\
& \hline
\end{aligned}
$$
\] <br>

\hline A- Assembly, churches Noted \& NL. \& NL \& $$
\begin{gathered}
\text { 5St. } 65 \\
34.200
\end{gathered}
$$ \& \[

$$
\begin{gathered}
3 \mathrm{St} .40^{\circ} \\
22.500
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \hline \text { 2 St:30 } \\
& 14,400
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
\text { 3St. } 40^{\circ} \\
19,800
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\text { 2 St. } 30 \\
14.400
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 3 \text { St. } 40^{\prime} \\
& 21.600
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 1 \mathrm{St.} 20 \\
& 15.300
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
1 \text { St. } 200 \\
7.200
\end{array}
$$
\] <br>

\hline 8 Business \& NL \& NL \& $$
\begin{gathered}
7 \text { St. } 85 \\
34.200 \\
\hline
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& \hline \text { 5St. } 65 \\
& 22.500
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St} .4 \sigma \\
& 14.400
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 4 \mathrm{St} .50 \\
& 19.800
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \text { 3 St. } 40 \\
& 14.400
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 5 \mathrm{St} .65 \\
& 21.600 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St} .40^{\circ} \\
& 15.300
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \text { St. } 30 \\
7.200 \\
\hline
\end{array}
$$
\] <br>

\hline E Educational Note c,d \& NL \& NL \& $$
\begin{gathered}
\text { 5St. } 65 \\
34,200
\end{gathered}
$$ \& \[

$$
\begin{gathered}
3 \mathrm{St} .40^{\circ} \\
22.500
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\text { 2St. } 30 \\
14,400
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 36 \mathrm{t} .40^{\prime} \\
& 19,800
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .3 \mathrm{y} \\
14,400
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St.} 40^{\circ} \\
& 21.600
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& \text { 1 St. } 20 \\
& 15.300
\end{aligned}
$$
\]

Note e \& $$
\begin{array}{r}
1 \text { St. } 20 \\
7.200 \\
\text { Note e }
\end{array}
$$ <br>

\hline F-1 Factory and industrial.

moderate $\quad$ Note i \& NL. \& NL \& \[
$$
\begin{aligned}
& 6 \text { St. } 75 \\
& 22.800 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
4 \text { St. } 50^{\prime} \\
15.000 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\text { 2 SL. } 30 \\
9.600
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St.} .40^{\circ} \\
13.200
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\text { 2 St. } 30 \\
9.600 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \hline \text { 4St. } 50^{\circ} \\
& 14.400 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 2 \mathrm{SLL} 30 \\
& 10,200
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{St} .20 \\
4.800
\end{gathered}
$$
\] <br>

\hline F-2 Factory and industrial.

low $\quad$ Note i \& NL. \& NL . \& \[
$$
\begin{array}{r}
7 \mathrm{St} .85 \\
34,200 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& \text { 5St. } 65 \\
& 22.500
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \text { St } 40 \\
& 14.400
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
4 \mathrm{St} .50 \\
19,800 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .40 \\
14,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \hline \text { 5St. } 66^{\prime \prime} \\
& 21,600
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St} .40^{\circ} \\
& 15,300 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \text { St. } 30 \\
7.200
\end{array}
$$
\] <br>

\hline H High hazard Note I \& $$
\begin{aligned}
& \hline 5 \mathrm{St} .65 \\
& 16.800 \\
& \hline
\end{aligned}
$$ \& \[

$$
\begin{gathered}
\hline 3 \mathrm{St} .40^{\circ} \\
14.400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\hline 3 \mathrm{St} .40^{\prime} \\
11.400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
25 \mathrm{St} .30 \\
7.500 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
4,800 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
2 \mathrm{St.} 30 \\
6.600 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{gathered}
1 \mathrm{SLL} 20 \\
4,800
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .300 \\
7,200
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
1 \text { SL. } 20 \\
5,100
\end{array}
$$
\] \& NP <br>

\hline -1 Institutional, residential care \& NL. \& NL \& $$
\begin{gathered}
\hline \text { 9St. } 100 \\
19.950 \\
\hline
\end{gathered}
$$ \& \[

$$
\begin{gathered}
4 \mathrm{St} .50 \\
13,125 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .40 \\
8,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \hline \text { 4St.50 } \\
& 11.550 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
\hline 3 \mathrm{St} .40^{\gamma} \\
8,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \text { 4St.50 } \\
& 12,600 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
3 \mathrm{St.40} \\
8,925 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .35 \\
4.200 \\
\hline
\end{gathered}
$$
\] <br>

\hline 1-2 Institutional, incapacitated \& NL. \& $$
\begin{gathered}
8 \mathrm{St} .90 \\
21.600 \\
\hline
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& \text { 4St. 50 } \\
& 17,100 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 2 \mathrm{St} .30 \\
& 11.250 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
7,200 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
1 \mathrm{St.} 20 \\
9.900 \\
\hline
\end{array}
$$

\] \& NP \& \[

$$
\begin{aligned}
& \text { 1st. } 20 \\
& 10.800
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
15 \mathrm{St.20} \\
7.650 \\
\hline
\end{array}
$$
\] \& NP <br>

\hline 1.3 Institutional, restrained \& NL. \& $$
\begin{aligned}
& 6 \text { St. } 75^{\prime} \\
& 18.000
\end{aligned}
$$ \& \[

$$
\begin{aligned}
& \hline \text { 4St. } 50 \\
& 14.250
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
9.375
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
6,000
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St.} 3 \gamma^{\prime} \\
8,250
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
6,000
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
2 \text { St. } 30^{\circ} \\
9,000
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
1 \text { St. } 20 \\
6.375
\end{gathered}
$$
\] \& NP <br>

\hline M Mercantile \& NL. \& NL \& $$
\begin{gathered}
6 \mathrm{St.75} \\
22.800 \\
\hline
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& \text { 4St. } 50 \\
& 15.000 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \mathrm{St} .30 \\
9.600 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St.} 40^{\prime} \\
& 13,200 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St.} 30 \\
9.600
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \text { 4St. } 50 \\
& \text { 14.400 }
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \mathrm{St.} 30 \\
10.200 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{array}{r}
1 \text { SL. } 20 \\
4,800
\end{array}
$$
\] <br>

\hline R. 1 Residential, hotels \& NL \& NL. \& $$
\begin{gathered}
9 \text { St. } 100^{\prime} \\
22.800
\end{gathered}
$$ \& \[

$$
\begin{gathered}
4 \text { St. } 50 \\
15,000
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .40^{\circ} \\
9,600
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 4 \mathrm{St} .50 \\
& 13.200
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .4 \sigma \\
9.600
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \text { 4St. } 50^{\circ} \\
& 14,400
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& \text { 3St. } 40^{\prime} \\
& 10.200
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .35 \\
4.800
\end{gathered}
$$
\] <br>

\hline R-2 Residential, multiple-family \& NL \& NL. \& $$
\begin{gathered}
9 \text { St. } 100 \\
22,800
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& 4 \text { St. } 50 \\
& 15.000 \\
& \text { Note } 0
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St.} 40^{\circ} \\
9.600
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
4 \mathrm{SLL} 5 \mathrm{O} \\
13,200 \\
\text { Note } 9 \\
\hline
\end{array}
$$

\] \& \[

$$
\begin{gathered}
3 \text { St. } 4 \sigma \\
9,600
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 4 \mathrm{St} .5 \sigma^{\prime} \\
& 14.400
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St}, 4 \sigma^{3} \\
& 10,200
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{SL} .35 \\
4.800
\end{gathered}
$$
\] <br>

\hline R.3 Residential, one-and two-family \& NL. \& NL \& $$
\begin{aligned}
& 4 \mathrm{St} .50 \\
& 22.800 \\
& \hline
\end{aligned}
$$ \& \[

$$
\begin{aligned}
& \text { 4St.50 } \\
& 15,000 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
3 \text { SL } 40 \\
9,600 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\text { 4St. } 50 \\
13,200 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{StL} .40^{\circ} \\
9.600
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \text { 4St.5V } \\
& 14,400
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St}, 40 \\
& 10,200 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{array}{r}
2 \mathrm{St} .35 \\
4.800 \\
\hline
\end{array}
$$
\] <br>

\hline 8.1 Storage, moderate \& NL . \& NL. \& $$
\begin{gathered}
\hline 5 \mathrm{St} .65 \\
19.950 \\
\hline
\end{gathered}
$$ \& \[

$$
\begin{aligned}
& 4 \text { St. } 50 \\
& 13.125 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
8,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 3 \mathrm{St} .40 \\
& 11.550 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \mathrm{St} .30 \\
8,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 4 \mathrm{St} .50 \\
& 12.600 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
2 \text { St. } 30 \\
8,925 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& 1 \text { St. } 20^{\circ} \\
& 4.200
\end{aligned}
$$
\] <br>

\hline 8-2 Storage. low Note h \& NL \& NL \& $$
\begin{array}{r}
7 \text { St. } 85 \\
34.200 \\
\hline
\end{array}
$$ \& \[

$$
\begin{aligned}
& \hline 5 \text { St. } 65 \\
& 2.500 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& \hline 3 \mathrm{St} .40^{\circ} \\
& 14,400 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{aligned}
& \text { 4St. } 5 \mathrm{y} \\
& 19,800 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St} .4 \sigma \\
14,400 \\
\hline
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& \hline \text { 5St.65 } \\
& 21.600 \\
& \hline
\end{aligned}
$$

\] \& \[

$$
\begin{gathered}
3 \mathrm{St.} 4 \gamma \\
15.300
\end{gathered}
$$

\] \& \[

$$
\begin{array}{r}
2 \mathrm{SL.} 3 \mathrm{O} \\
7,200
\end{array}
$$
\] <br>

\hline U Utility, miscellaneous \& NL. \& NL \& \& \& \& \& - \& \& \& <br>
\hline
\end{tabular}

Note a. See the following sections for general exceptions to Table 501:
Section 501.4 Allowable area reduction for multistory buildings.
Section 502.2 Allowable area increase due to street frontage.
Section 502.3 Allowable area increase due to automatic sprinkler system installation.
Section 503.1 Allowable height increase due to automatic sprinkler system installation.
Section 504.0 Unlimited area one-story buildings.
Note b. Buildings of Type 1 construction permitted to be of unlimited tabular heights and areas are not subject to special requirements that allow increased heights
and areas for other types of construction (see Section 501.5 )
Mote c. For tabular area increase in buildings of Use Group E. see Section 502.4.
Mote d. For height exceptions for auditoriums in buildings of Use Groups A-4 and E, see Section 503.2 .
Mote a. For height exceptions for day care centers of Type 5 construction, see Section 503.3 .
Motet. For exceptions to height and area limitations for buildings of Use Group H. see Article 6 governing the specific use. For other special fireresistive requirements
governing specific uses, see Section 904.0.
Mote g . For exceptions to height of buildings for Use Group R-2 of Types 2B and 3A construction, see Section 904.2
Note h. For height and area exceptions for open parking structures, see Section 607.0 .
Note I. For exceptions to height and area limitations for special industrial uses, see Section 501.1.1
Note |. 1 foot $=304.8 \mathrm{~mm} ; 1$ square foot $=0.093 \mathrm{~m}^{2}$.

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## CHART 2

Table 401
FIRERESISTANCE RATINGS OF STRUCTURE ELEMENTS (IM HOURS)

|  | Structure element <br> Note a |  |  | Type of construction Section 401.0 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | Nonco | ustible |  |  | combustible/C | mbustible |  | bustible |
|  |  |  |  |  | $02.0$ |  | $\begin{array}{r} \text { TyI } \\ \text { Sectio } \end{array}$ | ${ }_{403.0}^{2}$ |  | $\begin{aligned} & \text { e } 3 \\ & 1404.0 \end{aligned}$ | Type 4 Section 405.0 | Sect | $\begin{aligned} & e 5 \\ & 1406.0 \end{aligned}$ |
|  |  |  |  |  |  |  |  | Unprotected | Protected | Unprotected | Heavy timber Note C | Protected | Unprotected |
|  |  |  |  | 1 A | 18 | 2 A | 26 | 2 C | 3 A | 3 B | 4 | 5A | 58 |
| 1 | Exterior walls | Loadbe | earing | $\stackrel{4}{4}$ | 3 | 2 | $\begin{aligned} & 1 \\ & \text { Not les } \end{aligned}$ | 0 <br> than the rating | $\begin{gathered} 2 \\ 15 e d ~ o n ~ f i r e ~ \end{gathered}$ | $\frac{2}{\text { paration distan }}$ | $\stackrel{2}{\text { (see Section }}$ | 1 | $0 \longrightarrow$ |
|  |  | Nonloa | adbearing |  |  |  |  | than the rating | sed on fir | paration dista | (see Section 9 |  |  |
| 2 | Fire walls and party (Section 907.0) |  |  | $\stackrel{4}{4}$ | 3 | 2 | 2 | $\mathbf{- N o t ~ l e s s ~}_{2}^{2}$ | $\frac{2}{2}$ | $\frac{2}{\text { equired by Tabi }}$ | $907.1^{2}$ | 2 | 2 |
|  |  |  | Fire enclosure of of exits (Sections 817.11, 909.0 and Note b) | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | Fire separation assemblies (Section 909.0 ) |  | Shafts (other than exits) and elevator hoistways (Sections 909.0 915.0 and Note b) | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 |
|  |  |  | Mixed use separation (Section 313.0) |  |  |  |  | ance rating co | esponding |  | by Table 313.1 |  |  |
|  |  |  | Other separation assemblies (Note i |  | 1 |  |  | $1$ | 1 | 1 | 1 | 1 | 1 |
| 4 Fire partitions (Section 910.0) |  |  | Exit access corridors (Notes t. g) | Note d |  |  |  |  | 1 | 1 | 1 | 1 | 1 |
|  |  |  | Tenant spaces separations (Note f) |  | 1 | 1 | $\mathrm{d}^{1}$ | $\xrightarrow{ }$ | 1 | 0 | 1 | 1 | 0 |
| 5 | $\begin{aligned} & \text { Dwelling unit separations } \\ & \text { (Sections } 910.0 .913 .0 \text { and Notes } I \text { and i) } \end{aligned}$ |  |  |  |  |  |  |  | 1 | 1 | 1 | 1 | 1 |
| 6 | Smoke barriers (Section 911.0 and Note g) |  |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | Other nonbearing partitions |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 |
| 8 Interior bearing walls, bearing partitions, columns, girders. russes (other than roof trusses) and framing (Section 912.0) |  |  | Supporting more than one floor | 4 | 3 | 2 | 1 | 0 | 1 | 0 | $\begin{gathered} \text { see } \\ \text { Sec. } 405.0 \end{gathered}$ | 1 | 0 |
|  |  |  | Supporting one floor only or a roof only | 3 | 2 | 11/2 | 1 | 0 | 1 | 0 | $\begin{gathered} \text { see } \\ \text { Sec. } 405.0 \end{gathered}$ | 1 | 0 |
| $9 \begin{gathered}\text { Structural members supporting wall } \\ \text { (Section } 912.0 \text { and Note g) }\end{gathered}$ |  |  |  | - $\begin{aligned} & \text { a }\end{aligned}$ |  |  |  |  |  |  |  | 1 | 0 |
| 10 | $\begin{aligned} & 0 \text { Floor construction including beams } \\ & \text { (Section } 913.0 \text { and Note h) } \end{aligned}$ |  |  |  |  |  |  |  |  |  |  | 1 | 0 |
| 11 | Roof construction, including beams, trusses and framing. arches and roof deck (Section 914.0 and Notes e. i) |  | $15{ }^{\prime}$ or less in height to lowest member | 2 | 11/2 | 1 | $\begin{gathered} 1 \\ - \text { Note } \end{gathered}$ | $0$ | 1 | 0 | see Sec. 405.0 Note C | 1 | 0 |
|  |  |  | More than $15^{\prime}$ but less than $20^{\prime}$ in height to lowest member | 1 |  | ${ }^{1} \mathrm{No}$ |  | 0 | 0 | 0 | see Sec. 405.0 | 1 | 0 |
|  |  |  | $20^{\prime}$ or more in height to lowest member |  |  | $-{ }^{0} \text { Note }$ | 0 | 0 | 0 | 0 | see Sec. 405.0 | 0 | 0 |

Note a. For fireresistance rating requirements for structural members and assemblies which support other fireresistance rated members or assemblies, see Section
912.1.

Note b. For reductions in the required fireresistance rating of exit and shaft enclosures, see Sections 817.11 and 915.3 .
Note c. For substitution of other structural materials for timber in Type 4 construction, see Section 1703.1.1.
Note d. Fireretardant-treated wood permitted, see Sections 904.3 and 1702.4
Note e. For permitted uses of heavy timber in roof construction in buildings of Types 1 and 2 construction, see Section 914.4.
Note i. For reductions in required fireresistance ratings of exit access corridors, tenant separations and dwelling unit separations, see Section 810.4 and 810.4 .1 .
Note g . For exceptions to the required fireresistance rating of construction supporting exit access corridor walls, tenant separation walls in covered mall buildings,
and smoke barriers, see Sections 911.4 and 912.2 .
Note h. For buildings having habitable or occupiable stories or basements below grade, see Section 807.3.1.
Note i. Not less than the rating required by code.
Note 1. For Use Group R-3, see Section 309.4
Mete k. 1 foot $=304.8 \mathrm{~mm}$.

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Upon further investigation, to determine the required fire rating for columns supporting more than one floor (intersect line 8 under Structure Element with Type 4 construction), Chart 2 refers the reader to the building code because the information is too detailed to be included in a chart. The text of the code under that section states that columns supporting floor loads cannot be less than 8 inches by 8 inches in cross-section or less than 6 inches by 8 inches when supporting roofs.

Building codes are concerned with innumerable items and often run to hundreds and even thousands of pages. Every section of the code obviously has some impact on a building, but not every section has a major impact of a form-giving nature. The list that follows notes code items that often affect a building's overall design. The list is selective; individual architects might include different items or omit some of the items on this list.

1. Total permitted area as a function of construction materials.
2. Total permitted height as a function of construction materials.
3. Number and location of required stairs and exits.
4. Required amount of natural and/or artificial light.
5. Required amount of natural and/or artificial ventilation.
6. Required number and types of plumbing fixtures (for washrooms).
7. Pipe spaces required for plumbing and stormdrainage systems.
8. Heating equipment.
9. Air-conditioning equipment.
10. Elevator machine rooms and shafts.
11. Electric-equipment spaces and shafts.
12. Fire-protection systems.
13. Fire-extinguishing equipment.
14. Total building size as a function of building use.

## ZONING ORDINANCES

While building codes tend to tell an architect how a structure can be built, zoning regulations tell him or her where the structure can be built and how bulky it can be. They define the areas of a community in which
buildings intended for certain specific uses can be constructed. For example, manufacturing is often allowed only in a particular area, which is usually some distance from residential areas. Zoning ordinances can also limit the overall bulk of buildings and the percentage of the ground they can cover. In addition, they may mandate such things as how many parking spaces must be provided; the amount of open space; yards and plaza sizes; and, in major cities, the type of vertical setbacks that are required.

Among the major items covered by most zoning ordinances are the following:

1. Building use permitted in each area of the community.
2. Lot-area regulations.
3. Yard-size regulations.
4. Building height and setback requirements.
5. Distances between buildings.
6. Parking and truck-dock requirements.
7. Ratio of floor area to total building size.
8. Ratio of open space on the ground to the maximum height of the structure.

## RELATIONSHIP BETWEEN CODES AND ZONING ORDINANCES

Apparent jurisdictional overlap may occur between zoning ordinances and building codes. For example, they could conflict over minimum side-yard requirements. A zoning ordinance may specify the size of a yard or set back from a property line that must be provided for a particular type of building, while the building code may establish a minimum yard dimension that is required in order to provide adequate light and ventilation for a window facing onto a yard. Often these requirements are not the same, and since both requirements must be met, the stricter of the two prevails.

Another type of conflict can occur when a building code does not limit a building's height provided proper fire rated materials are used in its construction, but the town zoning ordinance states that no building can be more than, for example, eight stories high. Or the reverse situation might apply: that is, a town zoning ordinance might permit an eight-story hotel while the building code specifies that hotels may not be more than four stories high if they are of heavy-timber con-
struction. In either case, a solution must be found that satisfies all requirements, and the decisions that result from such conflicts inevitably influence the design of the building.

Apart from building codes and zoning ordinances, the requirements of special-interest agencies can also affect a building. For example, the Board of Health sets up rules for restaurants and hospitals, and the Department of Labor has requirements to protect workers such as mandating guardrails or window ledges to protect window washers. Many of these types of rules will affect the design of buildings. The list of special-interest requirements is enormous, but fortunately their effect on the design of a building is relatively minor, especially when compared with the requirements of the building codes and zoning ordinances. Occasionally, though, a special-interest agency regulation does influence the design of a building.

## EXIT REQUIREMENTS

One of the decisive form-givers in any major building is the location of its required means of egress. This is a separate problem from the location of decorative or ceremonial stairs, which codes refer to as "convenient" or "ornamental" stairs.

Although building codes go into minute detail describing exit requirements and the way in which exiting enclosures must be constructed, the five points that follow have a major impact on the overall building design.

1. Use of the building, for example, as an office, store, or school.
2. Total number of people in the building as a determinant of the required number of separate exits.
3. Limitations on the maximum travel distance permitted to reach an exit enclosure.
4. Provision for a choice of paths to an exit, and a choice of exits in case one exit is blocked.
5. Provision that exits must lead the occupants to a safe area.
Items 1,2 , and 5 are an automatic spellout of the codes. Items 3 and 4 require proper proportioning and shaping of spaces by the architect in order to comply with
a specific maximum travel distance and a specific maximum dead-end corridor length. This proportioning can
have a dramatic effect on the overall shape of the build-
ing. For example, most codes will not permit any dead
end corridors in a hospital; therefore, the stairs must be located at the ends of the building.


Elevators can make a major design contribution or be part of a nondescript core, but in either case they may not count as a means of exiting as they could fail during an emergency. Hyatt Regency Hotel, Atlanta, Georgia.
APPROXIMATE AREAS of CODE INFLUENCE

Figure 1. Geographical Influence of Model Codes

# SEI/ASCE 7-10: <br> Minimum Design Loads for Buildings and Other Structures 

Chapter 1
GENERAL

### 1.1 SCOPE

This standard provides minimum load requirements for the design of buildings and other structures that are subject to building code requirements. Loads and appropriate load combinations, which have been developed to be used together, are set forth for strength design and allowable stress design. For design strengths and allowable stress limits, design specifications for conventional structural materials used in buildings and modifications contained in this standard shall be followed.

### 1.2 DEFINITIONS AND NOTATIONS

### 1.2.1 Definitions

The following definitions apply to the provisions of the entire standard.

ALLOWABLE STRESS DESIGN: A method of proportioning structural members such that elastically computed stresses produced in the members by nominal loads do not exceed specified allowable stresses (also called "working stress design").

AUTHORITY HAVING JURISDICTION: The organization, political subdivision, office, or individual charged with the responsibility of administering and enforcing the provisions of this standard.

BUILDINGS: Structures, usually enclosed by walls and a roof, constructed to provide support or shelter for an intended occupancy.

DESIGN STRENGTH: The product of the nominal strength and a resistance factor.

ESSENTIAL FACILITIES: Buildings and other structures that are intended to remain operational in the event of extreme environmental loading from flood, wind, snow, or earthquakes.

FACTORED LOAD: The product of the nominal load and a load factor.

HIGHLY TOXIC SUBSTANCE: As defined in 29 CFR 1910.1200 Appendix A with Amendments as of February 1, 2000.

IMPORTANCE FACTOR: A factor that accounts for the degree of risk to human life, health, and welfare associated with damage to property or loss of use or functionality.

LIMIT STATE: A condition beyond which a structure or member becomes unfit for service and is
judged either to be no longer useful for its intended function (serviceability limit state) or to be unsafe (strength limit state).

LOAD EFFECTS: Forces and deformations produced in structural members by the applied loads.

LOAD FACTOR: A factor that accounts for deviations of the actual load from the nominal load, for uncertainties in the analysis that transforms the load into a load effect, and for the probability that more than one extreme load will occur simultaneously.

LOADS: Forces or other actions that result from the weight of all building materials, occupants and their possessions, environmental effects, differential movement, and restrained dimensional changes. Permanent loads are those loads in which variations over time are rare or of small magnitude. All other loads are variable loads (see also "nominal loads").

NOMINAL LOADS: The magnitudes of the loads specified in this standard for dead, live, soil, wind, snow, rain, flood, and earthquake.

NOMINAL STRENGTH: The capacity of a structure or member to resist the effects of loads, as determined by computations using specified material strengths and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions.

OCCUPANCY: The purpose for which a building or other structure, or part thereof, is used or intended to be used.

OTHER STRUCTURES: Structures, other than buildings, for which loads are specified in this standard.

P-DELTA EFFECT: The second order effect on shears and moments of frame members induced by axial loads on a laterally displaced building frame.

RESISTANCE FACTOR: A factor that accounts for deviations of the actual strength from the nominal strength and the manner and consequences of failure (also called "strength reduction factor").

RISK CATEGORY: A categorization of buildings and other structures for determination of flood, wind, snow, ice, and earthquake loads based on the risk associated with unacceptable performance. See Table 1.5-1.

STRENGTH DESIGN: A method of proportioning structural members such that the computed forces produced in the members by the factored loads do not

## CHAPTER 1 GENERAL

## Table 1.5-1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

Use or Occupancy of Buildings and Structures
Buildings and other structures that represent a low risk to human life in the event of failure
All buildings and other structures except those listed in Risk Categories I, III, and IV
Buildings and other structures, the failure of which could pose a substantial risk to human life.
Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial
economic impact and/or mass disruption of day-to-day civilian life in the event of failure.
Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that
manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous
chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity
exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat
to the public if released.
Buildings and other structures designated as essential facilities.
Buildings and other structures, the failure of which could pose a substantial hazard to the community.
Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store,
use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing
sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by
the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to
the public if released. ${ }^{\text {a }}$
Buildings and other structures required to maintain the functionality of other Risk Category IV structures.
IV
arildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category
if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5 .2 that a
release of the substances is commensurate with the risk associated with that Risk Category.
exceed the member design strength (also called "load and resistance factor design").

TEMPORARY FACILITIES: Buildings or other structures that are to be in service for a limited time and have a limited exposure period for environmental loadings.

TOXIC SUBSTANCE: As defined in 29 CFR 1910.1200 Appendix A with Amendments as of February 1, 2000.

### 1.1.2 Symbols and Notations

$\boldsymbol{F}_{x}$ A minimum design lateral force applied to level $x$ of the structure and used for purposes of evaluating structural integrity in accordance with Section 1.4.2.
$W_{x}$ The portion of the total dead load of the structure, $D$, located or assigned to Level $x$.
D Dead load.
L Live load.
$L_{r} \quad$ Roof live load.
$N$ Notional load used to evaluate conformance with minimum structural integrity criteria.
$\boldsymbol{R} \quad$ Rain load.
S Snow load.

### 1.3 BASIC REQUIREMENTS

### 1.3.1 Strength and Stiffness

Buildings and other structures, and all parts thereof, shall be designed and constructed with adequate strength and stiffness to provide structural stability, protect nonstructural components and systems from unacceptable damage, and meet the serviceability requirements of Section 1.3.2.

Acceptable strength shall be demonstrated using one or more of the following procedures:
a. the Strength Procedures of Section 1.3.1.1,
b. the Allowable Stress Procedures of Section 1.3.1.2, or
c. subject to the approval of the authority having jurisdiction for individual projects, the Performance-Based Procedures of Section 1.3.1.3.

## MINIMUM DESIGN LOADS

It shall be permitted to use alternative procedures for different parts of a structure and for different load combinations, subject to the limitations of Chapter 2. Where resistance to extraordinary events is considered, the procedures of Section 2.5 shall be used.

### 1.3.1.1 Strength Procedures

Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.3 of this Standard without exceeding the applicable strength limit states for the materials of construction.

### 1.3.1.2 Allowable Stress Procedures

Structural and nonstructural components and their connections shall have adequate strength to resist the applicable load combinations of Section 2.4 of this Standard without exceeding the applicable allowable stresses for the materials of construction.

### 1.3.1.3 Performance-Based Procedures

Structural and nonstructural components and their connections shall be demonstrated by analysis or by a combination of analysis and testing to provide a reliability not less than that expected for similar components designed in accordance with the Strength Procedures of Section 1.3.1.1 when subject to the influence of dead, live, environmental, and other loads. Consideration shall be given to uncertainties in loading and resistance.
1.3.1.3.1 Analysis Analysis shall employ rational methods based on accepted principles of engineering mechanics and shall consider all significant sources of deformation and resistance. Assumptions of stiffness, strength, damping, and other properties of components and connections incorporated in the analysis shall be based on approved test data or referenced Standards.
1.3.1.3.2 Testing Testing used to substantiate the performance capability of structural and nonstructural components and their connections under load shall accurately represent the materials, configuration, construction, loading intensity, and boundary conditions anticipated in the structure. Where an approved industry standard or practice that governs the testing of similar components exists, the test program and determination of design values from the test program shall be in accordance with those industry standards and practices. Where such standards or practices do not exist, specimens shall be constructed to a scale similar to that of the intended application unless it can
be demonstrated that scale effects are not significant to the indicated performance. Evaluation of test results shall be made on the basis of the values obtained from not less than 3 tests, provided that the deviation of any value obtained from any single test does not vary from the average value for all tests by more than $15 \%$. If such deviaton from the average value for any test exceeds $15 \%$, then additional tests shall be performed until the deviation of any test from the average value does not exceed $15 \%$ or a minimum of 6 tests have been performed. No test shall be eliminated unless a rationale for its exclusion is given. Test reports shall document the location, the time and date of the test, the characteristics of the tested specimen, the laboratory facilities, the test configuration, the applied loading and deformation under load, and the occurrence of any damage sustained by the specimen, together with the loading and deformation at which such damage occurred.
1.3.1.3.3 Documentation The procedures used to demonstrate compliance with this section and the results of analysis and testing shall be documented in one or more reports submitted to the authority having jurisdiction and to an independent peer review.
1.3.1.3.4 Peer Review The procedures and results of analysis, testing, and calculation used to demonstrate compliance with the requirements of this section shall be subject to an independent peer review approved by the authority having jurisdiction. The peer review shall comprise one or more persons having the necessary expertise and knowledge to evaluate compliance, including knowledge of the expected performance, the structural and component behavior, the particular loads considered, structural analysis of the type performed, the materials of construction, and laboratory testing of elements and components to determine structural resistance and performance characteristics. The review shall include the assumptions, criteria, procedures, calculations, analytical models, test setup, test data, final drawings, and reports. Upon satisfactory completion, the peer review shall submit a letter to the authority having jurisdiction indicating the scope of their review and their findings.

### 1.3.2 Serviceability

Structural systems, and members thereof, shall be designed to have adequate stiffness to limit deflections, lateral drift, vibration, or any other deformations that adversely affect the intended use and performance of buildings and other structures.

## Chapter 2 <br> COMBINATIONS OF LOADS

### 2.1 GENERAL

Buildings and other structures shall be designed using the provisions of either Section 2.3 or 2.4. Where elements of a structure are designed by a particular material standard or specification, they shall be designed exclusively by either Section 2.3 or 2.4 .

### 2.2 SYMBOLS

$A_{k}=$ load or load effect arising from extra ordinary event $A$
$D=$ dead load
$D_{i}=$ weight of ice
$E=$ earthquake load
$F=$ load due to fluids with well-defined pressures and maximum heights
$F_{a}=$ flood load
$H=$ load due to lateral earth pressure, ground water pressure, or pressure of bulk materials
$L=$ live load
$L_{r}=$ roof live load
$R=$ rain load
$S$ = snow load
$T=$ self-straining load
$W=$ wind load
$W_{i}=$ wind-on-ice determined in accordance with Chapter 10

### 2.3 COMBINING FACTORED LOADS USING STRENGTH DESIGN

### 2.3.1 Applicability

The load combinations and load factors given in Section 2.3.2 shall be used only in those cases in which they are specifically authorized by the applicable material design standard.

### 2.3.2 Basic Combinations

Structures, components, and foundations shall be designed so that their design strength equals or exceeds the effects of the factored loads in the following combinations:

1. 1.4 D
2. $1.2 D+1.6 L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
3. $1.2 D+1.6\left(L_{r}\right.$ or $S$ or $\left.R\right)+(L$ or $0.5 W)$
4. $1.2 D+1.0 W+L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
5. $1.2 D+1.0 E+L+0.2 S$
6. $0.9 D+1.0 W$
7. $0.9 D+1.0 E$

## EXCEPTIONS:

1. The load factor on $L$ in combinations 3,4 , and 5 is permitted to equal 0.5 for all occupancies in which $L_{o}$ in Table 4-1 is less than or equal to 100 psf , with the exception of garages or areas occupied as places of public assembly.
2. In combinations 2,4 , and 5 , the companion load $S$ shall be taken as either the flat roof snow load $\left(p_{f}\right)$ or the sloped roof snow load $\left(p_{s}\right)$.
Where fluid loads $F$ are present, they shall be included with the same load factor as dead load $D$ in combinations 1 through 5 and 7.

Where load $H$ are present, they shall be included as follows:

1. where the effect of $H$ adds to the primary variable load effect, include $H$ with a load factor of 1.6 ;
2. where the effect of $H$ resists the primary variable load effect, include $H$ with a load factor of 0.9 where the load is permanent or a load factor of 0 for all other conditions.
Effects of one or more loads not acting shall be investigated. The most unfavorable effects from both wind and earthquake loads shall be investigated, where appropriate, but they need not be considered to act simultaneously. Refer to Section 12.4 for specific definition of the earthquake load effect $E .{ }^{1}$

Each relevant strength limit state shall be investigated.

### 2.3.3 Load Combinations Including Flood Load

When a structure is located in a flood zone (Section 5.3.1), the following load combinations shall be considered in addition to the basic combinations in Section 2.3.2:

1. In V-Zones or Coastal A-Zones, 1.0 W in combinations 4 and 6 shall be replaced by $1.0 W+2.0 F_{a}$.
2. In noncoastal A-Zones, 1.0W in combinations 4 and 6 shall be replaced by $0.5 W+1.0 F_{a}$.
[^0]
### 2.3.4. Load Combinations Including Atmospheric Ice Loads

When a structure is subjected to atmospheric ice and wind-on-ice loads, the following load combinations shall be considered:

1. $0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$ in combination 2 shall be replaced by $0.2 D_{i}+0.5 S$.
2. $1.0 W+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$ in combination 4 shall be replaced by $D_{i}+W_{i}+0.5 S$.
3. 1.0 W in combination 6 shall be replaced by $D_{i}+W_{i}$.

### 2.3.5 Load Combinations Including Self-Straining Loads

Where applicable, the structural effects of load $T$ shall be considered in combination with other loads. The load factor on load $T$ shall be established considering the uncertainty associated with the likely magnitude of the load, the probability that the maximum effect of $T$ will occur simultaneously with other applied loadings, and the potential adverse consequences if the effect of $T$ is greater than assumed. The load factor on $T$ shall not have a value less than 1.0.

### 2.3.6 Load Combinations for Nonspecified Loads

Where approved by the Authority Having Jurisdiction, the Responsible Design Professional is permitted to determine the combined load effect for strength design using a method that is consistent with the method on which the load combination requirements in Section 2.3.2 are based. Such a method must be probability-based and must be accompanied by documentation regarding the analysis and collection of supporting data that is acceptable to the Authority Having Jurisdiction.

### 2.4 COMBINING NOMINAL LOADS USING ALLOWABLE STRESS DESIGN

### 2.4.1 Basic Combinations

Loads listed herein shall be considered to act in the following combinations; whichever produces the most unfavorable effect in the building, foundation, or structural member being considered. Effects of one or more loads not acting shall be considered.
$\begin{array}{ll}\text { 1. } & D \\ \text { 2. } & D+L \\ \text { 3. } & D+\left(L_{r} \text { or } S \text { or } R\right)\end{array}$
4. $D+0.75 L+0.75\left(L_{r}\right.$ or $S$ or $\left.R\right)$
5. $D+(0.6 W$ or $0.7 E)$

6a. $D+0.75 L+0.75(0.6 W)+0.75\left(L_{r}\right.$ or $S$ or $\left.R\right)$
6b. $D+0.75 L+0.75(0.7 E)+0.75 S$
7. $0.6 D+0.6 W$
8. $0.6 D+0.7 E$

## EXCEPTIONS:

1. In combinations 4 and 6 , the companion load $S$ shall be taken as either the flat roof snow load $\left(p_{f}\right)$ or the sloped roof snow load $\left(p_{s}\right)$.
2. For nonbuilding structures, in which the wind load is determined from force coefficients, $C_{f}$, identified in Figures 29.5-1, 29.5-2 and 29.5-3 and the projected area contributing wind force to a foundation element exceeds 1,000 square feet on either a vertical or a horizontal plane, it shall be permitted to replace $W$ with $0.9 W$ in combination 7 for design of the foundation, excluding anchorage of the structure to the foundation.
3. It shall be permitted to replace 0.6 D with 0.9 D in combination 8 for the design of Special Reinforced Masonry Shear Walls, where the walls satisfy the requirement of Section 14.4.2.
Where fluid loads $F$ are present, they shall be included in combinations 1 through 6 and 8 with the same factor as that used for dead load $D$.

Where load $H$ is present, it shall be included as follows:

1. where the effect of $H$ adds to the primary variable load effect, include $H$ with a load factor of 1.0 ;
2. where the effect of $H$ resists the primary variable load effect, include $H$ with a load factor of 0.6 where the load is permanent or a load factor of 0 for all other conditions.

The most unfavorable effects from both wind and earthquake loads shall be considered, where appropriate, but they need not be assumed to act simultaneously. Refer to Section 1.4 and 12.4 for the specific definition of the earthquake load effect $E .{ }^{2}$

Increases in allowable stress shall not be used with the loads or load combinations given in this standard unless it can be demonstrated that such an increase is justified by structural behavior caused by rate or duration of load.

[^1]
## MINIMUM DESIGN LOADS

### 2.4.2 Load Combinations Including Flood Load

When a structure is located in a flood zone, the following load combinations shall be considered in addition to the basic combinations in Section 2.4.1:

1. In V-Zones or Coastal A-Zones (Section 5.3.1), $1.5 F_{a}$ shall be added to other loads in combinations 5,6 , and 7 , and $E$ shall be set equal to zero in 5 and 6.
2. In non-coastal A-Zones, $0.75 F_{a}$ shall be added to combinations 5,6 , and 7 , and $E$ shall be set equal to zero in 5 and 6 .

### 2.4.3 Load Combinations Including Atmospheric Ice Loads

When a structure is subjected to atmospheric ice and wind-on-ice loads, the following load combinations shall be considered:

1. $0.7 D_{i}$ shall be added to combination 2 .
2. ( $L_{r}$ or $S$ or $R$ ) in combination 3 shall be replaced by $0.7 D_{i}+0.7 W_{i}+S$.
3. 0.6 W in combination 7 shall be replaced by $0.7 D_{i}+$ $0.7 W_{i}$.

### 2.4.4 Load Combinations Including Self-Straining Loads

Where applicable, the structural effects of load $T$ shall be considered in combination with other loads. Where the maximum effect of load $T$ is unlikely to occur simultaneously with the maximum effects of other variable loads, it shall be permitted to reduce the magnitude of $T$ considered in combination with these other loads. The fraction of $T$ considered in combination with other loads shall not be less than 0.75 .

### 2.5 LOAD COMBINATIONS FOR EXTRAORDINARY EVENTS

### 2.5.1 Applicability

Where required by the owner or applicable code, strength and stability shall be checked to ensure that structures are capable of withstanding the effects of extraordinary (i.e., low-probability) events, such as fires, explosions, and vehicular impact without disproportionate collapse.

### 2.5.2 Load Combinations

### 2.5.2.1 Capacity

For checking the capacity of a structure or structural element to withstand the effect of an extraordinary event, the following gravity load combination shall be considered:

$$
\begin{equation*}
(0.9 \text { or } 1.2) D+A_{k}+0.5 L+0.2 S \tag{2.5-1}
\end{equation*}
$$

in which $A_{k}=$ the load or load effect resulting from extraordinary event $A$.

### 2.5.2.2 Residual Capacity

For checking the residual load-carrying capacity of a structure or structural element following the occurrence of a damaging event, selected load-bearing elements identified by the Responsible Design Professional shall be notionally removed, and the capacity of the damaged structure shall be evaluated using the following gravity load combination:

$$
\begin{equation*}
(0.9 \text { or } 1.2) D+0.5 L+0.2\left(L_{r} \text { or } S \text { or } R\right) \tag{2.5-2}
\end{equation*}
$$

### 2.5.3 Stability Requirements

Stability shall be provided for the structure as a whole and for each of its elements. Any method that considers the influence of second-order effects is permitted.

## AISC Manual of Load and Resistance Factor Design, $3^{\text {rd }}$ ed.

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## Structural Load Requirements <br> International Building Code (2012)

TABLE 1607.1


## Live Loads \& Allowed Reductions

1607.10 Reduction in uniform live loads. Except for uniform live loads at roofs, all other minimum uniformly distributed live loads, $L_{\mathrm{o}}$, in Table 1607.1 are permitted to be reduced in accordance with Section 1607.10.1 or 1607.10.2. Uniform live loads at roofs are permitted to be reduced in | accordance with Section 1607.12.2.
1607.10.1 Basic uniform live load reduction. Subject to 【 the limitations of Sections 1607.10.1.1 through 1607.10.1.3 and Table 1607.1, members for which a value of $K_{u} A_{7}$ is 400 square feet $\left(37.16 \mathrm{~m}^{2}\right)$ or more are permitted to be designed for a reduced uniformly distributed live load, $L$, in accordance with the following equation:

$$
L=L_{o}\left(0.25+\frac{15}{\sqrt{K_{L L} A_{T}}}\right)
$$

(Equation 16-23)

For SI: $\quad L=L_{\rho}\left(0.25+\frac{4.57}{\sqrt{K_{L L} A_{T}}}\right)$
where:
$L=$ Reduced design live load per square foot $\left(\mathrm{m}^{2}\right)$ of area supported by the member.
$L_{\mathrm{o}}=$ Unreduced design live load per square foot $\left(\mathrm{m}^{2}\right)$ of area supported by the member (see Table 1607.1).
$K_{L L}=$ Live load element factor (see Table 1607.10.1).
$A_{T}=$ Tributary area, in square feet $\left(\mathrm{m}^{2}\right)$.
$L$ shall not be less than $0.50 L_{6}$ for members supporting one floor and $L$ shall not be less than $0.40 L_{0}$ for members supporting two or more floors.

TABLE 1607.10.1
LIVE LOAD ELEMENT FACTOR, $\boldsymbol{K}_{\text {LL }}$

| ELEMENT | $\boldsymbol{K}_{\mathbf{u}}$ |
| :--- | :---: |
| Interior columns | 4 |
| Exterior columns without cantilever slabs | 4 |
| Edge columns with cantilever slabs | 3 |
| Comer columns with cantilever slabs | 2 |
| Edge beams without cantilever slabs | 2 |
| Interior beams | 2 |
| All other members not identified above including: <br> Edge beams with cantilever slabs <br> Cantilever beams <br> One-way slabs <br> Two-way slabs <br> Members without provisions for continuous shear <br> transfer normal to their span |  |

1607.10.1.1 One-way slabs. The tributary area, $A_{T}$, for use in Equation 16-23 for one-way slabs shall not exceed an area defined by the slab span times a width normal to the span of 1.5 times the slab span.
1607.10.1.2 Heavy live loads. Live loads that exceed $100 \mathrm{psf}\left(4.79 \mathrm{kN} / \mathrm{m}^{2}\right)$ shall not be reduced.

## Exceptions:

1. The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than $L$ as calculated in Section 1607.10.1.
2. For uses other than storage, where approved, additional live load reductions shall be permitted where shown by the registered design professional that a rational approach has been used and that such reductions are warranted.
1607.10.1.3 Passenger vehicle garages. The live loads shall not be reduced in passenger vehicle garages.

Exception: The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than $L$ as calculated in Section 1607.10.1.
1607.10.2 Alternative uniform live load reduction. As an alternative to Section 1607.10 .1 and subject to the limitations of Table 1607.1, uniformly distributed live loads are permitted to be reduced in accordance with the following provisions. Such reductions shall apply to slab systems, beams, girders, columns, piers, walls and foundations.

1. A reduction shall not be permitted where the live load exceeds $100 \mathrm{psf}\left(4.79 \mathrm{kN} / \mathrm{m}^{2}\right)$ except that the design live load for members supporting two or more floors is permitted to be reduced by a maximum of 20 percent.

Exception: For uses other than storage, where approved, additional live load reductions shall be permitted where shown by the registered design professional that a rational approach has been used and that such reductions are warranted.
2. A reduction shall not be permitted in passenger vehicle parking garages except that the live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent.
3. For live loads not exceeding $100 \mathrm{psf}\left(4.79 \mathrm{kN} / \mathrm{m}^{2}\right)$, the design live load for any structural member supporting 150 square feet $\left(13.94 \mathrm{~m}^{2}\right)$ or more is permitted to be reduced in accordance with Equation 16-24.
4. For one-way slabs, the area, $A$, for use in Equation $16-24$ shall not exceed the product of the slab span and a width normal to the span of 0.5 times the slab span.
$R=0.08(A-150) \quad$ (Equation 16-24)
For SI: $R=0.861(A-13.94)$
Such reduction shall not exceed the smallest of:

1. 40 percent for horizontal members;
2. 60 percent for vertical members; or
3. $R$ as determined by the following equation.
$R=23.1\left(1+D / L_{o}\right)$
(Equation 16-25)
where:

$$
\begin{aligned}
& A=\text { Area of floor supported by the member, square } \\
& \text { feet ( } \mathrm{m}^{2} \text { ). } \\
& D=\text { Dead load per square foot }\left(\mathrm{m}^{2}\right) \text { of area } \\
& \text { supported. } \\
& L_{\mathrm{o}}=\text { Unreduced live load per square foot }\left(\mathrm{m}^{2}\right) \text { of } \\
& \text { area supported. } \\
& R=\text { Reduction in percent. }
\end{aligned}
$$

1607.11 Distribution of floor loads. Where uniform floor live loads are involved in the design of structural members arranged so as to create continuity, the minimum applied loads shall be the full dead loads on all spans in combination with the floor live loads on spans selected to produce the greatest load effect at each location under consideration. Floor live loads are permitted to be reduced in accordance with Section 1607.10.

## Minimum Roof Loads

1607.12 Roof loads. The structural supports of roofs and marquees shall be designed to resist wind and, where applicable, snow and earthquake loads, in addition to the dead load of construction and the appropriate live loads as prescribed in this section, or as set forth in Table 1607.1. The live loads acting on a sloping surface shall be assumed to act vertically on the horizontal projection of that surface.
1607.12.1 Distribution of roof loads. Where uniform roof live loads are reduced to less than $20 \mathrm{psf}\left(0.96 \mathrm{kN} / \mathrm{m}^{2}\right)$ in accordance with Section 1607.12.2.1 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the most unfavorable load effect. See Section 1607.12.2 for reductions in minimum roof live loads and Section 7.5 of ASCE 7 for partial snow loading.
1607.12.2 General. The minimum uniformly distributed live loads of roofs and marquees, $L_{0}$, in Table 1607.1 are permitted to be reduced in accordance with Section 1607.12.2.1.
1607.12.2 1 Ordinary roofs, awnings and canopies. Ordinary flat, pitched and curved roofs, and awnings and canopies other than of fabric construction supported by a skeleton structure, are permitted to be designed for a reduced uniformly distributed roof live load, $L_{r}$, as specified in the following equations or other controlling combinations of loads as specified in Section 1605, whichever produces the greater load effect.

In structures such as greenhouses, where special scaffolding is used as a work surface for workers and materials during maintenance and repair operations, a lower roof load than specified in the following equations shall not be used unless approved by the building official. Such structures shall be designed for a minimum roof live load of $12 \mathrm{psf}\left(0.58 \mathrm{kN} / \mathrm{m}^{2}\right)$.
$L_{r}=L_{o} R_{1} R_{2}$
(Equation 16-26)
where: $12 \leq L_{\mathrm{r}} \leq 20$
For SI: $L_{\mathrm{r}}=L_{0} R_{1} R_{2}$
where: $0.58 \leq L_{r} \leq 0.96$
$L_{o}=$ Unreduced roof live load per square foot $\left(\mathrm{m}^{2}\right)$ of horizontal projection supported by the member (see Table 1607.1).
$L_{r}=$ Reduced roof live load per square foot $\left(\mathrm{m}^{2}\right)$ of horizontal projection supported by the member.

The reduction factors $R_{1}$ and $R_{2}$ shall be determined as follows:

$$
\begin{aligned}
& R_{I}=1 \text { for } A_{1} \leq 200 \text { square feet }\left(18.58 \mathrm{~m}^{2}\right) \\
& (\text { Equation 16-27) } \\
& R_{I}=1.2-0.001 A_{t} \text { for } 200 \text { square feet } \\
& \quad<A_{t}<600 \text { square feet } \quad \text { (Equation 16-28) } \\
& \text { For SI: } 1.2-0.011 A_{t} \text { for } 18.58 \text { square meters }<A_{t}< \\
& 55.74 \text { square meters } \\
& R_{I}=0.6 \text { for } A_{t} \geq 600 \text { square feet }\left(55.74 \mathrm{~m}^{2}\right)
\end{aligned}
$$

(Equation 16-29)
where:
$A_{t}=$ Tributary area (span length multiplied by effective width) in square feet ( $\mathrm{m}^{2}$ ) supported by the nember, and
$R_{2}=1$ for $F \leq 4$
(Equation 16-30)
$R_{2}=1.2-0.05 F$ for $4<F<12$ (Equation 16-31)
$R_{2}=0.6$ for $F \geq 12$
(Equation 16-32)
where:
$F=$ For a sloped roof, the number of inches of rise per foot (for SI: $F=0.12 \times$ slope, with slope expressed as a percentage), or for an arch or dome, the rise-to-span ratio multiplied by 32 .
1607.12.3 Occupiable roofs. Areas of roofs that are occupiable, such as roof gardens, or for assembly or other similar purposes, and marquees are permitted to have their uniformly distributed live loads reduced in accordance with Section 1607.10.
1607.12.3.1 Landscaped roofs. The uniform design live load in unoccupied landscaped areas on roofs shall be $20 \mathrm{psf}\left(0.958 \mathrm{kN} / \mathrm{m}^{2}\right)$. The weight of all landscaping materials shall be considered as dead load and shall be computed on the basis of saturation of the soil.
1607.12.4 Awnings and canopies. Awnings and canopies shall be designed for uniform live loads as required in Table 1607.1 as well as for snow loads and wind loads as specified in Sections 1608 and 1609.

Minimum Snow Loads


## Documentation of Loads

## SECTION 1603 CONSTRUCTION DOCUMENTS

1603.1 General. Construction documents shall show the size, section and relative locations of structural members with floor levels, column centers and offsets dimensioned. The design loads and other information pertinent to the structural design required by Sections 1603.1.1 through 1603.1.9 shall be indicated on the construction documents.

Exception: Construction documents for buildings constructed in accordance with the conventional light-frame construction provisions of Section 2308 shall indicate the following structural design information:

1. Floor and roof live loads.
2. Ground snow load, $P_{g}$.
3. Ultimate design wind speed, $V_{u l t}$ ( 3 -second gust), miles per hour ( mph ) $(\mathrm{km} / \mathrm{hr}$ ) and nominal design wind speed, $V_{\text {asd }}$, as determined in accordance with Section 1609.3.1 and wind exposure.
4. Seismic design category and site class.
5. Flood design data, if located in flood hazard areas established in Section 1612.3.
6. Design load-bearing values of soils.
1603.1.1 Floor live load. The uniformly distributed, concentrated and impact floor live load used in the design shall be indicated for floor areas. Use of live load reduction in accordance with Section 1607.10 shall be indicated for each type of live load used in the design.
1603.1.2 Roof live load. The roof live load used in the design shall be indicated for roof areas (Section 1607.12).
1 1603.1.3 Roof snow load data. The ground snow load, $P_{g}$, shall be indicated. In areas where the ground snow load, $P_{g}$, exceeds 10 pounds per square foot ( psf ) $\left(0.479 \mathrm{kN} / \mathrm{m}^{2}\right)$, the following additional information shall also be provided, regardless of whether snow loads govern the design of the roof:
7. Flat-roof snow load, $P_{f}$
8. Snow exposure factor, $C_{e}$.
9. Snow load importance factor, $I$.
10. Thermal factor, $C_{r}$.
1603.1.4 Wind design data. The following information related to wind loads shall be shown, regardless of whether wind loads govern the design of the lateral forceresisting system of the structure:
11. Ultimate design wind speed, $V_{\text {ult }}$ (3-second gust), miles per hour ( $\mathrm{km} / \mathrm{hr}$ ) and nominal design wind speed, $V_{a s d}$, as determined in accordance with Section 1609.3.1.
12. Risk category.
13. Wind exposure. Where more than one wind exposure is utilized, the wind exposure and applicable wind direction shall be indicated.
14. The applicable internal pressure coefficient.
15. Components and cladding. The design wind pressures in terms of $\mathrm{psf}\left(\mathrm{kN} / \mathrm{m}^{2}\right)$ to be used for the design of exterior component and cladding materials not specifically designed by the registered design professional.
1603.1.5 Earthquake design data. The following information related to seismic loads shall be shown, regardless of whether seismic loads govern the design of the lateral force-resisting system of the structure:

## 1. Risk category.

2. Seismic importance factor, $I_{c}$.
3. Mapped spectral response acceleration parameters, $S_{s}$ and $S_{l}$.
4. Site class.
5. Design spectral response acceleration parameters, $S_{D S}$ and $S_{D I}$.
6. Seismic design category.
7. Basic seismic force-resisting system(s).
8. Design base shear(s).
9. Seismic response coefficient(s), $C_{S}$.
10. Response modification coefficient(s), $R$.
11. Analysis procedure used.
1603.1.6 Geotechnical information. The design loadbearing values of soils shall be shown on the construction documents.
1603.1.7 Flood design data. For buildings located in whole or in part in flood hazard areas as established in Section 1612.3, the documentation pertaining to design, if required in Section 1612.5, shall be included and the following information, referenced to the datum on the community's Flood Insurance Rate Map (FIRM), shall be shown, regardless of whether flood loads govern the design of the building:
12. In flood hazard areas not subject to high-velocity wave action, the elevation of the proposed lowest floor, including the basement.
13. In flood hazard areas not subject to high-velocity wave action, the elevation to which any nonresidential building will be dry flood proofed.
14. In flood hazard areas subject to high-velocity wave action, the proposed elevation of the bottom of the lowest horizontal structural member of the lowest floor, including the basement.
1603.1.8 Special loads. Special loads that are applicable to the design of the building, structure or portions thereof shall be indicated along with the specified section of this code that addresses the special loading condition.
1603.1.9 Systems and components requiring special inspections for seismic resistance. Construction documents or specifications shall be prepared for those systems and components requiring special inspection for seismic resistance as specified in Section 1705.11 by the registered design professional responsible for their design and shall be submitted for approval in accordance with Section 107.1. Reference to seismic standards in lieu of detailed drawings is acceptable.

## CHAPTER 2 ALLOWABLE STRESS DESIGN OF MASONRY

## CODE

## 2.1 - General

### 2.1.1 Scope

This chapter provides requirements for allowable stress design of masonry. Masonry design in accordance with this chapter shall comply with the requirements of Chapter 1, Sections 2.1.2 through 2.1.7, and either Section 2.2 or 2.3 .

### 2.1.2 Load combinations

When the legally adopted building code does not provide allowable stress load combinations, structures and members shall be designed to resist the combinations of load specified by the building official.

### 2.1.3 Design strength

2.1.3.1 Project drawings shall show the specified compressive strength of masonry, $f_{m}^{\prime}$, for each part of the structure.
2.1.3.2 Each portion of the structure shall be designed based on the specified compressive strength of masonry, $f_{m}^{\prime}$, for that part of the work.
2.1.3.3 Computed stresses shall not exceed the allowable stress requirements of this Chapter.

### 2.1.4 Anchor bolts embedded in grout

2.1.4.1 Design requirements - Anchor bolts shall be designed using either the provisions of Section 2.1.4.2 or, for headed and bent-bar anchor bolts, by the

## COMMENTARY

## 2.1 - General

### 2.1.1 Scope

Historically, a one-third increase in allowable stress has been permitted for load combinations that include wind or seismic loads. The origin and the reason for the one-third stress increase are unclear ${ }^{2.1}$. From a structural reliability standpoint, the one-third stress increase is a poor way to handle load combination effects. Therefore, the one-third stress increase is no longer permitted in this Code. The allowable stresses of this Chapter should not be increased by one-third for wind and load combinations.

### 2.1.2 Load combinations

When there is no legally adopted building code or the legally adopted building code does not have allowable stress load combinations, possible sources of allowable stress load combinations are ASCE $7^{2.2}$ and IBC ${ }^{2,3}$.

### 2.1.3 Design strength

The structural adequacy of masonry construction requires that the compressive strength of masonry equal or exceed the specified strength. The specified compressive strength $f_{m}^{\prime}$ on which design is based for each part of the structure must be shown on the project drawings.

The 1995, 1999, 2002, and 2005 editions of the Code contained provisions to permit use of strength-level load combinations in allowable stress design, to compensate for lack of service-level load combinations in previously referenced load standards. This procedure, which enabled the calculation of 'pseudo-strengths' on the basis of allowable stresses, is no longer included in the Code because recent editions of ASCE 7 include both service-level and strengthlevel load combinations. The 2005 edition of the Code provides guidance for using strength-level load combinations whenever the legally adopted building code does not provide service-level load combinations.

### 2.1.4 Anchor bolts embedded in grout

Allowable Stress Design anchor bolt provisions were obtained by calibrating corresponding Strength Design provisions to produce similar results. See Code

Code Requirements for Steel Construction, AISC $14^{\text {th }} \mathrm{ed}$.
Design by elastic, inelastic or plastic analysis is permitted. Provisions for inelastic and plastic analysis are as stipulated in Appendix 1, Inelastic Analysis and Design.
The provisions for moment redistribution in continuous beams in Appendix 1, The provisions for moment redistribution in continuous beams in Appendix 1,
Section 1.3 are permitted for elastic analysis only.

## Limit States

Design shall be based on the principle that no applicable strength or serviceability limit state shall be exceeded when the structure is subjected to all appropriate load combinations.
3. Design for Strength Using Load and Resistance Factor Design
(LRFD)
Design according to the provisions for Load and Resistance Factor Design (LRFD) satisfies the requirements of this Specification when the design strength of each structural component equals or exceeds the required strength determined on the basis of the LRFD load combinations. All provisions of this Specification, except for those in Section B3.4, shall apply.
Design shall be performed in accordance with Equation B3-1:

$$
R_{u} \leq \phi R_{n}
$$

where

$$
R_{u}=\text { required strength (LRFD) }
$$

$R_{n}=$ nominal strength, specified in Chapters B through K $\phi=$ resistance factor, specified in Chapters B through K
$\phi R_{n}=$ design strength
4. Design for Strength Using Allowable Strength Design (ASD) Design according to the provisions for Allowable Strength Design (ASD) satisfies the requirements of this Specification when the allowable strength of each structural component equals or exceeds the required strength determined on the basis of the ASD load combinations. All provisions of this Specification, except those of Section B3.3, shall apply.
Design shall be performed in accordance with Equation B3-2:

ぶ | $R_{a} \leq R_{n} / \Omega$ |  |
| :--- | :--- |
| where |  |
| $R_{a}$ | $=$ required strength (ASD) |
| $R_{n}$ | $=$ nominal strength, specified in Chapters B through K |
| $\Omega$ | $=$ safety factor, specified in Chapters B through K |
| $R_{n} / \Omega$ | $=$ allowable strength | <br> \title{

CHAPTER B
} <br> \title{
CHAPTER B
}
The general requirements for the analysis and design of steel structures that are applica ble to all chapters of the specification are given in this chapter.
The chapter is organized as follows:
The design of members and connections shall be consistent with the intended behavior of the framing system and the assumptions made in the structural analysis. Unless restricted by the applicable building code, lateral load resistance and stability may be provided by any combination of members and connections.
B2. LOADS AND LOAD COMBINATIONS
The loads and load combinations shall be as stipulated by the applicable building code. In the absence of a building code, the loads and load combinations shall be those stipulated in SEI/ASCE 7. For design purposes, the nominal loads shall be taken as the loads stipulated by the applicable building code.

[^2] 2.4 apply.
B3. DESIGN BASIS
Designs shall be made according to the provisions for Load and Resistance Factor
Design (LRFD) or to the provisions for Allowable Strength Design (ASD).

1. Required Strength
The required strength of structural members and connections shall be determined by structural analysis for the appropriate load combinations as stipulated in Section B2.

## Code Requirements for Structural Concrete, ACI 318-11

## CHAPTER 9 - STRENGTH AND SERVICEABILITY REQUIREMENTS

## CODE

## 9.1-General

9.1.1 - Structures and structural members shall be designed to have design strengths at all sections at least equal to the required strengths calculated for the factored loads and forces in such combinations as are stipulated in this Code.
9.1.2 - Members also shall meet all other requirements of this Code to ensure adequate performance at service load levels.
9.1.3 - Design of structures and structural members using the load factor combinations and strength reduction factors of Appendix $C$ shall be permitted. Use of load factor combinations from this chapter in conjunction with strength reduction factors of Appendix $C$ shall not be permitted.

## 9.2 - Required strength

9.2.1 - Required strength $\boldsymbol{U}$ shall be at least equal to the effects of factored loads in Eq. (9-1) through (9-7). The effect of one or more loads not acting simultaneously shall be investigated.

$$
\begin{gather*}
U=1.4 D  \tag{9-1}\\
U=1.2 D+1.6 L+0.5\left(L_{r} \text { or } S \text { or } R\right)  \tag{9-2}\\
U=1.2 D+1.6\left(L_{r} \text { or } S \text { or } R\right)+(1.0 L \text { or } 0.5 W)  \tag{9-3}\\
U=1.2 D+1.0 W+1.0 L+0.5\left(L_{r} \text { or } S \text { or } R\right)  \tag{9-4}\\
U=1.2 D+1.0 E+1.0 L+0.2 S \tag{9-5}
\end{gather*}
$$

## COMMENTARY

## R9.1 - General

In the 2002 Code, the factored load combinations and strength reduction factors of the 1999 Code were revised and moved to Appendix C. The 1999 combinations were replaced with those of SEI/ASCE 7-02. ${ }^{9.1}$ The strength reduction factors were replaced with those of the 1999 Appendix C, except that the factor for flexure was increased. In the 2011 Code, the factored load combinations were revised for consistency with ASCE/SEI 7-10.9.2

The changes were made to further unify the design profession on one set of load factors and combinations, and to facilitate the proportioning of concrete building structures that include members of materials other than concrete. When used with the strength reduction factors in 9.3 , the designs for gravity loads will be comparable to those obtained using the strength reduction and load factors of the 1999 and earlier Codes. For combinations with lateral loads, some designs will be different, but the results of either set of load factors are considered acceptable.

Chapter 9 defines the basic strength and serviceability conditions for proportioning structural concrete members.

The basic requirement for strength design may be expressed as follows:

Design Strength $\geq$ Required Strength
$\phi($ Nominal Strength $) \geq \boldsymbol{U}$
In the strength design procedure, the margin of safety is provided by multiplying the service load by a load factor and the nominal strength by a strength reduction factor.

## R9.2 - Required strength

The required strength $\boldsymbol{U}$ is expressed in terms of factored loads, or related internal moments and forces. Factored loads are the loads specified in the general building code multiplied by appropriate load factors.

The factor assigned to each load is influenced by the degree of accuracy to which the load effect usually can be calculated and the variation that might be expected in the load during the lifetime of the structure. Dead loads, because they are more accurately determined and less variable, are assigned a lower load factor than live loads. Load factors also account for variability in the structural analysis used to compute moments and shears.

# Code Requirements for Structural Concrete, ACI 318-11 (continued) 

## CODE

$$
\begin{align*}
& U=0.9 D+1.0 W \\
& U=0.9 D+1.0 E \tag{9-7}
\end{align*}
$$

except as follows:
(a) The load factor on the live load $L$ in Eq. (9-3) to (9-5) shall be permitted to be reduced to 0.5 except for garages, areas occupied as places of public assembly, and all areas where $L$ is greater than $100 \mathrm{lb} / \mathrm{ft}^{2}$.
(b) Where $\boldsymbol{W}$ is based on service-level wind loads, 1.6 W shall be used in place of 1.0 W in Eq. (9-4) and (9-6), and $0.8 W$ shall be used in place of $0.5 W$ in Eq. (9-3).
(c) Where $E$ is based on service-level forces, 1.4E shall be used in place of 1.0E in Eq. (9-5) and (9-7).

## COMMENTARY

The Code gives load factors for specific combinations of loads In assigning factors to combinations of loading, some consideration is given to the probability of simultaneous occurrence. While most of the usual combinations of loadings are included, it should not be assumed that all cases are covered.

Due regard is to be given to sign in determining $U$ for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The load combinations with 0.9 D are specifically included for the case where a higher dead load reduces the effects of other loads. The loading case may also be critical for tension-controlled column sections. In such a case, a reduction in axial load and an increase in moment may result in a critical load combination.

Consideration should be given to various combinations of loading to determine the most critical design condition. This is particularly true when strength is dependent on more than one load effect, such as strength for combined flexure and axial load or shear strength in members with axial load.

If unusual circumstances require greater reliance on the strength of particular members than encountered in usual practice, some reduction in the stipulated strength reduction factors $\phi$ or increase in the stipulated load factors may be appropriate for such members.

In 2011, the Code removed the weight of soil and other fill materials as part of the definition of $\boldsymbol{H}$. Consistent with ASCE/SEI 7-10, the weight of these materials is part of dead load, $\boldsymbol{D}$. The load factors for $\boldsymbol{D}$ are appropriate provided the unit weight and thickness of earth or other fill materials are well controlled. If the weight of earth stabilizes the structure, a load factor of zero may be appropriate.

R9.2.1(a) - The load modification factor of 9.2.1(a) is different than the live load reductions based on the loaded area that may be allowed in the legally adopted general building code. The live load reduction, based on loaded area, adjusts the nominal live load ( $L_{0}$ in ASCE/SEI 7) to $\boldsymbol{L}$. The live load reduction as specified in the legally adopted general building code can be used in combination with the 0.5 load factor specified in 9.2.1(a).

R9.2.1(b) - ASCE/SEI 7-10 has converted wind loads to strength level, and reduced the wind load factor to 1.0. ACI 318 requires use of the previous load factor for wind loads. 1.6, when service-level wind loads are used. For serviceability checks, the commentary to Appendix C of ASCE SEI 7-10 provides service-level wind loads, $\boldsymbol{W}_{\boldsymbol{a}}$.

R9.2.1(c) - In 1993, ASCE $7^{9.3}$ converted earthquake forces to strength level, and reduced the earthquake load factor to 1.0. Model building codes ${ }^{9.4-9.6}$ followed. ACI 318 requires use of the previous load factor for earthquake effects, approximately 1.4 , when service-level earthquake effects are used.

## Examples: Load Tracing and Factored Loads

## EXAMPLE (pg. 129 with corrections and additions)

Assume that the average dead plus live load on the structure shown in Figure 3.15 is $60 \mathrm{lbs} / \mathrm{ft}^{2}$. Determine the reactions for Beam D. This is the same structure as shown in Figure 3.1.
${ }^{\wedge} \mathrm{E}, \mathrm{B}$ and A
Assuming all beams are weightless!

## Solution:

Note carefully the directions of the decking span. Beam D carries floor loads from the jecking to the left (see the contributory area and load strip), but not to the right, since the


Figure 3.1


FIGURE 3.15 Load modeling and reaction determination.


By symmetry; $R_{c c 1}=R_{c c 3}=(4893 \mathrm{lb}+4896 \mathrm{lb}) / 2=4896 \mathrm{lb}$

Beam B $\quad \mathrm{R}_{\mathrm{D} 1}=4464 \mathrm{lb} \mathrm{R}_{\mathrm{E} 1}=4464 \mathrm{lb} \quad$ By symmetry; $\mathrm{Rcc} 2=\mathrm{R}_{\mathrm{cc} 4}=(4464 \mathrm{lb}+4464 \mathrm{lb}) / 2+(6 \mathrm{ft})\left(60 \mathrm{lb} / \mathrm{t}^{2}\right)(12 \mathrm{ft}) / 2=6624 \mathrm{lb}$
Additional loads are transferred to the column from the reactions on Beams C and F :

$$
R_{C 1}=R_{C 2}=R_{F 1}=R_{F 2}=w L / 2=(6 \mathrm{ft})\left(60 \mathrm{lb} / \mathrm{ft}^{2}\right)(20 \mathrm{ft}) / 2=3600 \mathrm{lb}
$$

center decking runs parallel to Beam $D$ and is not carried by it. Beam $D$ also picks up the end of Beam G and thus also "carries" the reactive force from Beam G. It is therefore necessary to analyze Beam G first to determine the magnitude of this force. The analysis appears in Figure 3.15. The reactive force from Beam G of 2160 lbs is then treated as a downward force acting on Beam D. The load model for Beam D thus consists of distributed forces from the decking plus the $2160-\mathrm{lb}$ force. It is then analyzed by means of the equations of statics to obtain reactive forces of 4896 lbs and 4464 lbs at its ends.

$$
\begin{aligned}
& \mathrm{C} 1=4896 \mathrm{lb}+3600 \mathrm{lb}=8,496 \mathrm{lb} \\
& \mathrm{C} 2=6624 \mathrm{lb}+3600 \mathrm{lb}=10,224 \mathrm{lb} \\
& \mathrm{C} 3=4896 \mathrm{lb}+3600 \mathrm{lb}=8,496 \mathrm{lb} \\
& \mathrm{C} 4=6624 \mathrm{lb}+3600 \mathrm{lb}=10,224 \mathrm{lb}
\end{aligned}
$$

## Example 2

Determine the controlling load combinations(s) using AISC-LRFD for a building column subject to the following service or nominal (unfactored) axial compressive loads: $D=30 \mathrm{k}, L=50 \mathrm{k}, L_{r}=10 \mathrm{k}, W=25 \mathrm{k}, E=40 \mathrm{k}$

Using a spreadsheet analysis:

| LRFD (ASCE-7) |  | $\begin{aligned} & \hline \text { FACTORED } \\ & \text { LOAD } \end{aligned}$ |
| :---: | :---: | :---: |
| $1.4 D$ |  |  |
| $1.4 D$ | = | 42 kips |
| $\begin{gathered} 1.2 D+1.6 L+0.5\left(L_{r} \text { or } S \text { or } R\right) \\ 1.2 D+1.6 L+0.5 L_{r} \end{gathered}$ | = | 121 |
| $1.2 D+1.6\left(L_{r}\right.$ or $S$ or $\left.R\right)+(L$ or $0.5 W)$ |  |  |
| $1.2 D+1.6 L_{r}+L$ | = | 102 |
| $1.2 D+1.6 L_{r}+0.5 W$ | = | 64.5 |
| $1.2 D+1.6 L_{r}-0.5 W$ | = | 39.5 |
| $1.2 D+1.0 W+L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$ |  |  |
| $1.2 D+1.0 W+L+0.5 L_{r}$ | = | 116 |
| $1.2 \mathrm{D}-1.0 W+L+0.5 L_{r}$ | = | 66 |
| $1.2 D+1.0 E+L+0.2 S$ |  |  |
| $1.2 D+1.0 E+L$ | = | 126 |
| $1.2 D-1.0 E+L$ | = | 46 |
| $0.9 D+1.0 W$ |  |  |
| $0.9 \mathrm{D}+1.0 \mathrm{~W}$ | = | 52 |
| 0.9D-1.0W | $=$ | 2 |
| $0.9 D+1.0 E$ |  |  |
| $0.9 D+1.0 E$ | $=$ | 67 |
| $0.9 D-1.0 E$ | $=$ | -13 |
|  | d Load | $\begin{aligned} & \hline 126 \text { kips (C) } \\ & -13 \text { kips (T) } \end{aligned}$ |

Example 3

## EXAMPLE 2-4

Determine factored loads for the beam shown in Figure 2-16.

## Solution

For the left half of the beam:

$$
\begin{aligned}
& w_{u 1}=1.2 w_{D}+1.6 w_{L} \\
& w_{u 1}=1.2 \times 1.0+1.6 \times 2.0=4.4 \mathrm{kip} / \mathrm{ft}
\end{aligned}
$$

For the right half of the beam:

$$
\begin{aligned}
& w_{u 2}=1.2 w_{D}+1.6 w_{L} \\
& w_{u 2}=1.2 \times 1.0+1.6 \times 0=1.2 \mathrm{kip} / \mathrm{ft}
\end{aligned}
$$



FIGURE 2-16 Example 2-4 (service loads).


FIGURE 2-17 Example 2-4 (factored loads).

The concentrated load is a live load only:

$$
\begin{aligned}
& P_{u}=1.2 P_{D}+1.6 P_{L} \\
& P_{u}=1.2 \times 0+1.6 \times 10=16 \mathrm{kip}
\end{aligned}
$$

The factored loads on the beam are shown in Figure 2-17.

BEAM DIAGRAMS AND FORMULAS For Various Static Loading Conditions, AISC ASD $8^{\text {th }} \mathrm{ed}$.

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER


4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END


7. SIMPLE BEAM - CONCENTRATED LOAD AT CENTER

8. SIMPLE BEAM - CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load

$$
=\frac{8 \mathrm{Pab}}{l^{2}}
$$

$$
\begin{aligned}
& R_{1}=V_{1}(\max . \text { when } a<b) \\
& R_{2}=V_{2}(\max . \text { when } a>b)
\end{aligned}
$$

$$
\begin{aligned}
& R_{2}=V_{\mathbf{2}}(\text { max. when } a>b) \\
& M \text { max. }(\text { at point of load })
\end{aligned}
$$

$$
v_{2} M_{x} \quad(\text { when } x<a \quad)
$$

$$
\Delta \text { max. } \quad\left(\text { at } x=\sqrt{\frac{a(a+2 b)}{3}} \text { when } a>b\right)
$$

$\Delta a \quad$ (at point of load )
$=\frac{\mathrm{Pa}^{2} \mathrm{~b}^{\mathbf{2}}}{3 \mathrm{EI} l}$
$\Delta x \quad($ when $x<a \quad)$
9. SIMPLE BEAM-TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

10. SIMPLE BEAM - TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

11. SIMPLE BEAM-TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHERUNIFORMLY DISTRIBUTED LOAD





26. BEAM OVERHANGING ONE SUPPORT-CONCENTRATED

LOAD AT END OF OVERHANG

$R_{1}$
$R_{2}$
$V_{2}$
$M$
$M_{X}$
$M_{x_{1}}$
$\Delta m$
$\Delta m$
$\Delta x$
$\Delta X_{1}$

| $\mathbf{R}_{\mathbf{2}}=\mathrm{V}_{1}+\mathrm{V}_{\mathbf{2}}$ |  |
| :---: | :---: |
| $\mathbf{V}_{\mathbf{2}} \cdot \cdot \cdot\left(\text { at } \mathbf{R}_{\mathbf{2}}\right)$ |  |
|  |  |
| $\mathrm{M}_{\mathbf{x}} \quad$ (between supports) |  |
| $M_{\mathrm{X}_{1}} \quad$ (for overhang |  |
| $\Delta$ max. (between supports at $\mathrm{x}=\frac{l}{\sqrt{\overline{3}}}$ ) |  |
| $\Delta$ max. (for overhang |  |
|  | (between supports) |
|  | for overhang) |

$=\frac{\mathrm{Pa}}{l}$
$=\frac{\mathrm{P}}{l}(l+\mathrm{a})$
$=\mathrm{P}$
$=\mathrm{Pa}$
$=\frac{\mathrm{Pax}}{l}$
$=\mathrm{P}(\mathrm{a}$
$\Delta_{x_{1}} \quad$ (for overhang)

28. BEAM OVERHANGING ONE SUPPORT-CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS

29. CONTINUOUS BEAM-TWO EQUAL SPANS-UNIFORM LOAD ON ONE SPAN


30. CONTINUOUS BEAM-TWO EQUAL SPANS-CONCENTRATED LOAD AT CENTER OF ONE SPAN

31. CONTINUOUS BEAM-TWO EQUAL SPANS-CONCENTRATED LOAD AT ANY POINT



35. CONTINUOUS BEAM-THREE EQUAL SPANS-END SPANS LOADED

$\Delta \operatorname{Max} .(0.479 l$ from $A$ or $D)=0.0099 w l^{4} / E l$
36. CONTINUOUS BEAM-THREE EQUAL SPANS—ALL SPANS LOADED .

$\Delta \operatorname{Max} .(0.446 l$ from A or $D)=0.0069 \mathrm{wl}^{4} / \mathrm{EI}$
37. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED

$\mathrm{R}_{\mathrm{A}}=0.380 w l \quad \mathrm{R}_{\mathrm{B}}=1.223 w l \quad \mathrm{R}_{\mathrm{C}}=0.357 w l \quad \mathrm{R}_{\mathrm{D}}=0.598 w l \quad \mathrm{R}_{\mathrm{E}}=0.442 w l$

$\Delta$ Max. $(0.475 l$ from $E)=0.0094 w^{4} / E I$
38. CONTINUOUS BEAM-FOUR EQUAL SPANS-LOAD FIRST AND THIRD SPANS

39. CONTINUOUS BEAM-FOUR EQUAL SPANS-ALL SPANS LOADED


## Beam Analysis using Multiframe

1．The software is on the computers in the College of Architecture in Programs under the Windows Start menu（see https：／／wikis．arch．tamu．edu／display／HELPDESK／Computer＋Accounts for lab locations）．Multiframe is under the Multiframe［current version \＃］menu．Or it can be downloaded from the web site：http：／／www．formsys．com／academic／multiframe
2．There are tutorials available on line at http：／／www．formsys．com／mflearning that list the tasks and order in greater detail．The first task is to define the unit system：
－Choose Units．．．from the View menu．Unit sets are available，but specific units can also be selected by double clicking on a unit or format and making a selection from the menu．

| Units |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Unit Ser： | Configuration： |  |  |  |  |  |
| American Australian British Canadian European Japanese |  | Unit Type | Unit | Decimal Places | Format | $\wedge$ |
|  | 1 | Length | ft | 3 | Fixed Decimal |  |
|  | 2 | Angle | deg | 3 | Fixed Decimal | 三 |
|  | 3 | Deflection | in | 3 | Fixed Decimal |  |
|  | 4 | Rotation | deg | 3 | Fixed Decimal |  |
|  | 5 | Force | kip | 3 | Fixed Decimal |  |
|  | 6 | Moment | 1 b －ft | 3 | Fixed Decirnal |  |
|  | 7 | Dist．Force | lbfift | 3 | Fixed Decimal |  |
|  | 8 | Stress | ksi | 3 | Fixed Decimal |  |
|  | 9 | Mass | lb | 3 | Fixed Decimal |  |
|  | 10 | Mass／Length | $\mathrm{lb} / \mathrm{ft}$ | 3 | Fixed Decimal |  |
|  | 11 | Area | $\mathrm{in}^{2}$ | 3 | Fixed Decimal |  |
|  | 12 | Mrnt of Inertia | in ${ }^{\wedge} 4$ | 3 | Fixed Decimal |  |
|  | 13 | Density | $1 \mathrm{l} / \mathrm{ft}^{3}$ | 3 | Fixed Decimal |  |
|  | 14 | Section Modulus | in ${ }^{3}$ | 3 | Fixed Decimal | $\checkmark$ |
|  | $\stackrel{-1}{\square}$ |  | III | － | - |  |
|  |  |  |  |  | OK | Cancel |

3．To see the scale of the geometry，a grid option is available：
－Choose Grid．．．from the View menu


4．To create the geometry，you must be in the Frame window（default）．The symbol is the frame in the window toolbar：


The Member toolbar shows ways to create members：


The Generate toolbar has convenient tools to create typical structural shapes．
－To create a beam with supports at one or both ends，use the add member button：


- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window.
- To create a beam with supports NOT at the ends, use the add connected members button to create segments between supports and ends

- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.
- The geometry can be set precisely by selecting the beam member, bringing up the specific menu (right click), choosing Member Properties to set the length.

- The support types can be set by selecting the joint (drag) and using the Joint Too bar (pin shown), or the Frame / Joint Restraint ... men (right click).
NOTE: If the support appears at both ends of the beam, you had the beam selected rather than the joint. Select the joint to change the suppor for and right click to select the joint restraints menut or select the correct support on the joint toolbar.

The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section proceed to step 6. For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like $\mathrm{t}_{\mathrm{f}}, \mathrm{t}_{\mathrm{w}}$, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:


- Choose the group name and section name:
(STANDARD SHAPES)

(CUSTOM)


7. The beam geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

8. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons)..


- Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):

- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.
- Enter the values of the load and distances (if any). Distances can be entered as a function of the length , i.e. L/2, L/4...


## NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.



Multiframe4D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).


- Choose Linear from the Analyze menu

10. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

11. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):


- To show the moment diagram, Choose the red Moment button

- To show the shear diagram, Choose the green Shear button

- To show the axial force diagram, Choose the purple Axial Force button

- To show the deflection diagram, Choose the blue Deflection button

- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a avi file by checking the box.
- To plot the bending moment on the "top" choose Preferences from the Edit menu and under the Presentation tab Draw moments on the compression face
- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.


12. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.

13. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.


NOTE: Px' refers to the axial load (P) in the local axis $x$ direction ( $x^{\prime}$ ). Vy' refers to the shear perpendicular to the local x axis, and $\mathrm{Mz}^{\prime}$ refers to the bending moment.
14. To save the file Choose Save from the File menu.
15. To load an existing file Choose Open... from the File menu.
16. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot $(\mathrm{Ctrl}+\mathrm{c})$ and paste it in a word processing document $(\mathrm{Ctrl}+\mathrm{v})$.

## Examples: <br> Beams (V, M, Stresses and Design)

## Example 1

## Example Problem 9.5: Section Modulus

 (Figures 9.26 to 9.28)Two $\mathrm{C} 10 \times 15.3$ steel channels are placed back to back to form a 10"-deep beam. Determine the permissible $P$ if $F_{b}=30 \mathrm{ksi}$. Assume A572 grade 50 steel.
Solution:

$$
\begin{aligned}
I_{x} & =67.4 \mathrm{in} . .^{4} \times 2=134.8 \mathrm{in} . .^{4} \\
M_{\max } & =1 / 2(5)(5)+(P / 2)(5) \\
M_{\max } & =12.5+2.5 P \\
& =(12.5 \mathrm{k}-\mathrm{ft} .+2.5 P) \times(12 \mathrm{in} . / \mathrm{ft.}) \\
f & =\frac{M c}{I}=\frac{M}{S} ; \quad \therefore M=F_{b} \times S_{x} \\
S_{x} & =2 \times 13.5 \mathrm{in} .{ }^{3}=27 \mathrm{in} . .^{3}
\end{aligned}
$$



Figure 9.26 Two steel channels.


Figure 9.28 Load, V, and M diagrams.

Example 2 From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College


## CHECK BENDING, SHEAR, BEARING STRESSES AND DEFLECTIONS

Reference: eStructures v1.1, Shodek \& Pollalis, 2000 Simple Beams, Beam Analysis
? Bin Beam Analysis
STEP 2

DRAW SHEAR AND MOMENT DIAGRAMS


SHEAR AND MOMENT DIAGRAMS:

Maximum Shear Force:
$=1000 \mathrm{lbs}$
Maximum Bending Moment:

$$
=10,000 \mathrm{ft}-\mathrm{lbs}=120,000 \mathrm{in} \text {-lbs }
$$

Example 2 (continued)
앙 ? Beam Analysis
DETERMINE BEAM PROPERTIES

Example 2 (continued)


Example 2 (continued)


## DEFLECTIONS



For a simply supported beam with a concentrated load, the maximum deflection is given by $\Delta=\mathrm{PL}^{3} / 48 \mathrm{EI}$ :

$$
\begin{aligned}
\Delta & =\mathrm{PL}^{3} / 48 \mathrm{EI} \\
& =\frac{(2000 \mathrm{lb})(20 \mathrm{ft} \times 12 \mathrm{in} / \mathrm{ft})^{3}}{48\left(1.6 \times 10^{6} \mathrm{Ib} / \mathrm{in}^{2}\right)\left(576 \mathrm{in}^{4}\right)} \\
& =0.625 \text { inches }
\end{aligned}
$$

COMPARE ACTUAL DEFLECTION TO ALLOWABLE DEFLECTION:
$\Delta_{\text {actual }}=0.625$ in
$\Delta_{\text {alallowable }} \mathrm{L} / 360=(20 \mathrm{ft} \times 12 \mathrm{in} / \mathrm{ft}) / 360=0.67 \mathrm{in}$.

$$
\Delta_{\text {actual }}<\Delta_{\text {allowable }}
$$

## Example 3

Using an "approximate" method of analysis (specifically beam diagrams and formulas with superpositioning), find reactions, shears, and moments present in the structure. Verify the solution using a computer-based structural analysis program (Multiframe4D).


## SOLUTION:

The load cases can be divided into the two shown which correspond to beam diagrams 30 and 29 (mirrored).

Because the maximum moments do not occur at the same place, find the reactions to add up and construct the V \& M diagrams. The moment diagram should look like the two diagrams (with one flipped) "added" together:

Diagram 30:
$R_{1}=\frac{13}{32} P=\frac{13}{32}(10 k)=4.06 k \quad R_{2}=\frac{11}{16} P=\frac{11}{16}(10 k)=6.875 k$
$R_{3}=-\frac{3}{32} P=-\frac{3}{32}(10 k)=-0.9375 k$


Diagram 29:
$R_{1}\left(\right.$ was $\left.R_{3}\right)=-\frac{1}{16} w l=-\frac{1}{16}(2 \mathrm{k} / \mathrm{ft}) 10 f t=-1.25 k$

$$
R_{2}=\frac{5}{8} w l=\frac{5}{8}(2 \mathrm{k} / f t) 10 f t=12.5 k
$$

$R_{3}\left(\right.$ was $\left.R_{1}\right)=\frac{7}{16} w l=\frac{7}{16}(2 \mathrm{k} / f t) 10 f t=8.75 k$

Reaction sums:
$R_{1}=4.06+-1.25=2.81 k \quad R_{2}=6.875+12.5=19.375 k \quad R_{3}=-0.9375+8.75=7.8125 k$
Shear calculations:
$\mathrm{V}_{\mathrm{A}}=0$ and $2.81 \mathrm{k} \quad \mathrm{V}_{\mathrm{at} 5 \mathrm{t}}=2.81 \mathrm{k}$ and 2.81-10 $=-7.19 \mathrm{k} \quad \mathrm{V}_{\mathrm{B}}=-7.19 \mathrm{k}$ and $-7.19+19.375=12.185 \mathrm{k}$
$V_{C}=12.185-2 \mathrm{k} / \mathrm{ft}(10 \mathrm{ft})=-7.8125$ and $-7.815+7.815=0 \mathrm{k}$
Moment shapes:
$M_{A}=0 \quad M_{a t} 5 \mathrm{ft}=0+2.81 \mathrm{k}(5 \mathrm{ft})=14.05 \mathrm{k}-\mathrm{ft} \quad \mathrm{M}_{\mathrm{B}}=14.05-7.19 \mathrm{k}(5 \mathrm{ft})=-21.9 \mathrm{k}-\mathrm{ft}$
location of cross over $=12.185 \mathrm{k} /(2 \mathrm{k} / \mathrm{ft})=6.0925 \mathrm{ft}: \quad \mathrm{Mat} 6.1 \mathrm{ftrom} \mathrm{B}=-21.9+12.185 \mathrm{k}(6.0925 \mathrm{ft}) / 2=15.218 \mathrm{k}-\mathrm{ft}$
$\mathrm{Mc}_{\mathrm{C}}=15.218-7.8125 \mathrm{k}(3.9075 \mathrm{ft}) / 2=0$

MULTIFRAME4D:
V:


## Truss Analysis using Multiframe

1．The software is on the computers in the College of Architecture in Programs under the Windows Start menu（see https：／／wikis．arch．tamu．edu／pages／HELPDESK／Computer＋Accounts for lab locations）．Multiframe is under the Multiframe［current version \＃］menu．Or it can be downloaded from the web site：http：／／www．formsys．com／academic／multiframe
2．There are tutorials available on line at http：／／www．formsys．com／mflearning that list the tasks and order in greater detail．The first task is to define the unit system：
－Choose Units．．．from the View menu．Unit sets are available，but specific units can also be selected by double clicking on a unit or format and making a selection from the menu．


3．To see the scale of the geometry，a grid option is available：
－Choose Grid．．．from the View menu


4．To create the geometry，you must be in the Frame window（default）．The symbol is the frame in the window toolbar：


The Member toolbar shows ways to create members：

$$
\|\angle N \% \times \square\|
$$

The Generate toolbar has convenient tools to create typical structural shapes．

```
...? 用回囲四丽
```

－To create a truss，use the add connected members button：


- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing. Continue to use the add connected members button. Any time the cursor is over an existing joint, the joint will be highlighted by a red circle.
- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.
(
- The support types can be/set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint/... menu (right click).
NOTE: If the support appear at both ends of the member, you had the member seleqted rather than the joint. Select the joint to change spipport for and right click to select the joint restraints nhenu or select the correct support on the joint toolbar.

The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section proceed to step 6. For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like $\mathrm{t}_{\mathrm{f}}, \mathrm{t}_{\mathrm{w}}$, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:


- Choose the group name and section name:
(STANDARD SHAPES)

(CUSTOM)


7. In order for Multiframe4D to recognize that the truss members are two-force bodies, all joints must be highlighted and assigned as pins with the Pinned Joints button on the Joint toolbar:

8. The truss geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).


- Choose the joint to be loaded (drag) and select the load type (here shown for point loading):

- Choose the direction by the arrow shown. There is no need to put in negative values for downward loading.
- Enter the values of the load

NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.


Multiframe2D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

10. In order to run the analysis after the geometry, member properties and loading has been defined:

- Choose Linear from the Analyze menu

11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):


- To show the axial force diagram, Choose the purple Axial Force button. Tensile members will have "T" by the value (if turned on), while compression members will have "C" by the value
- To show the deflection diagram, Choose the blue Deflection button


To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a avi file by checking the box.

- To see exact values of axial load and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.


13. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.
14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.


| Static Case: Load Case 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Memb | Label | Joint | $\begin{aligned} & \mathbf{P X}^{*} \\ & \mathbf{l l f f} \end{aligned}$ | $\begin{aligned} & \text { Vy } \\ & \text { Ilbf } \end{aligned}$ | $\begin{gathered} \mathrm{Mz}^{*} \\ \mathrm{lbf}-\mathrm{ft} \end{gathered}$ |
| 1 | 1 |  | 1 | 7.377 | 0.000 | 0.000 |
| 2 | 1 |  | 2 | -7.377 | 0.000 | 0.000 |
| 3 | 2 |  | 2 | -0.681 | 0.000 | 0.000 |
| 4 | 2 |  | 3 | 0.681 | 0.000 | 0.000 |
| 5 | 3 |  | 1 | 1.075 | 0.000 | 0.000 |
| 6 | 3 |  | 3 | -1.075 | 0.000 | 0.000 |
| 7 | 4 |  | 2 | 4.157 | 0.000 | 0.000 |
| 8 | 4 |  | 4 | -4.157 | 0.000 | 0.000 |
|  |  |  |  |  |  |  |
| Ready |  |  |  |  |  |  |

NOTE: Px' refers to the axial load ( P ) in the local axis x direction ( $\mathrm{x}^{\prime}$ ).
15. To save the file Choose Save from the File menu.
16. To load an existing file Choose Open... from the File menu.
17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot $(\mathrm{Ctrl}+\mathrm{c})$ and paste it in a word processing document $(\mathrm{Ctrl}+\mathrm{v})$.

## Examples: <br> Trusses and Columns

## Example 1 <br> Example Problem 4.1 (Method of Joints)

An asymmetrical roof truss, shown in Figure 4.4, supports two vertical roof loads. Determine the support reactions at each end, then, using the method of joints, solve for all member forces. Summarize the results of all member forces on a FBD (this diagram is referred to as a force summation diagram).

1. FBD

2. solve for support forces

$$
\begin{aligned}
& \sum F_{x}=A_{x}=0 \\
& \sum M_{A}=3000^{1 b} \cdot 10^{A t}+1200^{1 b} \cdot 20^{A t}-E \cdot 30^{A t}=0 \quad E=\frac{54000^{1 b-A t}}{30^{A t}}=1800^{1 b} \\
& \sum F_{y}=1800^{1 b}-1200^{1 b}-3000^{1 b}+A_{y}=0 \quad A_{y}=2400^{1 b}
\end{aligned}
$$


3. look for special cases:

4. choose a joint with 2 or less unknowns: E or A will work ( C won't)

need $B D, A B,(A F$ or $D F)$ which leaves joints $B, D \& A(F$ won't work)
5. choose a joint with 2 or less unknowns: B, D or A will work ( F won't)

6. last joint needs only one equation


$$
\left(\sum F_{x}=-2400^{1 b}-\left(-3394^{1 b}\right) \cos 45=0\right) \sqrt{ }
$$



Example 2
Example Problem 4.3 (Method of Sections)
A 64 -foot parallel chord truss (Figure 4.30) supports horizontal and vertical loads as shown. Using the method of sections, determine the member forces $B C, H G$, and $G D$.


1. look for sections
2. FBD

3. solve for support forces

$$
\begin{aligned}
& \sum F_{x}=A_{x}+4^{k}=0 \quad A_{x}=-4^{k} \\
& \sum F_{y}=A_{y}-4^{k}-4^{k}-3^{k}+E=0 \\
& \sum M_{A}=-4^{k} \cdot 12^{A t}-4^{k} \cdot 16^{A t}-4^{k} \cdot 32^{A t}-3^{k} \cdot 48^{A}+E \cdot 64^{A t}=0 \\
& E=\frac{384^{k-A t}}{64^{A t}}=6^{k} \quad \text { and sub: } \quad A_{y}=5^{k}
\end{aligned}
$$

4. draw section

5. look for intersection for summing moments (B or G)
6. write equilibrium equations

$$
\begin{aligned}
& \sum M_{B}=H G \cdot 12^{A}-5^{k} \cdot 16^{A}-4^{k} \cdot 12^{A}=0 \quad H G=\frac{128^{k-A}}{12^{t}}=10.67^{k} \\
& \sum M_{G}=4^{k} \cdot 16^{A t}-5^{k} \cdot 32^{A}-4^{k} \cdot 12^{A}-B C \cdot 12^{A}=0 \\
& B C=\frac{144^{k-t}}{-12^{A}}=-12^{k} \\
& \left(\sum F_{y}=5^{k}-4^{k}-B G\left(\frac{12}{20}\right)=0 \quad B G=-1.67^{k}\right)
\end{aligned}
$$

7. repeat with other section

$$
\Sigma F_{y}=6^{k}-3^{k}-G D\left(\frac{12}{20}\right)=0 \quad G D \neq 5^{k}
$$

Example 3 From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College


## (3) Braced Column

COLUMNS


L,

MEMBER


TIMBER

$$
\begin{aligned}
& \text { Modulus of Elasticity } \\
& E_{\mathrm{T}}=1.6 \times 10^{6} \mathrm{Ib} / \mathrm{in}^{2} \\
& \text { Crushing Stress } \\
& \mathrm{F}_{\mathrm{C}}=2400 \mathrm{lb}_{\mathrm{s}} \mathrm{in}^{2}
\end{aligned}
$$

## OUT OF PLANE BUCKLING: <br> $P_{C R x}$

Length $=$ Overall Physical Length $=L_{x}$
Moment of Inertia: $\mathrm{I}_{\mathrm{x}}$
$\mathrm{I}_{\mathrm{X}}=\frac{\mathrm{bd}^{3}}{12}=\frac{2(3)^{3}}{12}=4.5 \mathrm{kN}^{4}$
$P_{C R_{x}}=\frac{\pi^{2} E_{x}}{L_{x}^{2}}$
$=\frac{\pi^{2}\left(1.6 \times 10^{5}\right)(4.5)}{(144)^{2}}$
$=3,423 \mathrm{LB8}$
Critical Buckling Stress
$f_{C R}=\frac{P_{C R}}{A}=\frac{3423}{(2 \times 3)}=570$ Les $_{\text {in }}{ }^{2}$
$\mathbf{f}_{\mathrm{CA}}<\mathrm{F}_{\mathrm{C}} \therefore$ Member Buckles

Example 3 (continued)

(3) Braced Column ( ( )

## COLUMNS



## Bracing Level

L,
$L_{y}=L_{\alpha} / 2$


Example 3 (continued)


COLUMNS



TIMBER
Modulus of Elasticity $E_{\mathrm{T}}=1.6 \times 10^{5} \mathrm{Jb}_{\mathrm{hn}}{ }^{2}$

Crushing Stress
$\mathrm{F}_{\mathrm{C}}=2400 \mathrm{lb} / \mathrm{in}^{2}$

NOTE THAT IF THE MID-HEIGHT BRACING WERE "REMOVED", THEN THE COLUMN WOULD BUCKLE AT A LOWER LOAD IN THE OTHER DIRECTION


$$
\begin{aligned}
P_{C R_{r}} & =\frac{\pi^{2} E I_{Y}}{L_{\psi}^{2}} \\
& =\frac{\pi^{2}\left(1.6 \times 10^{5}\right)(2.0)}{(144)^{2}} \\
& =1,521 \text { LBs } \\
P_{\mathrm{CR}_{x}} & =3,423 \text { As Before }
\end{aligned}
$$

Since $P_{C R},<P_{C R_{x}}$
The Column buckles as shown at a load of 1,521 Les

## BRICK MASONRY ARCHES INTRODUCTION


#### Abstract

The masonry arch is one of the oldest structural elements. Brick masonry arches have been used for hundreds of years. This Technical Notes is an introduction to brick masonry arches. Many of the different types of brick masonry arches are discussed and a glossary of arch terms is provided. Material selection, proper construction methods, detailing and arch construction recommendations are discussed to ensure proper structural support, durability and weather resistance of the brick masonry arch.


Key Words: arch, brick, reinforced, unreinforced.

## INTRODUCTION

In the latter part of the 19th century, an arch was discovered in the ruins of Babylonia. Archeologists estimate that the arch was constructed about the year 1400 B.C. Built of well-baked, cigar-shaped brick and laid with clay mortar, this arch is probably the oldest known to man. The Chinese, Egyptians and others also made use of the arch before the Christian era. Later, more elaborate arches, vaults and domes with complicated forms and intersections were constructed by Roman builders during the Middle Ages.

The brick arch is the consummate example of form following function. Its aesthetic appeal lies in the variety of forms which can be used to express unity, balance, proportion, scale and character. Its structural advantage results from the fact that under uniform load, the induced stresses are principally compressive. Be-
cause brick masonry has greater resistance to compression than tension, the masonry arch is frequently the most efficient structural element to span openings.

This Technical Notes addresses the detailing and construction of brick masonry arches. The common types of brick masonry arches are presented, along with proper arch terminology. Methods of selecting the type and configuration of brick masonry arches most appropriate for the application are discussed. Proper material selection and construction methods are recommended. Other Technical Notes in this series discuss the structural design of brick masonry arches and lintels.

## ARCH TYPES AND TERMINOLOGY

Many arch forms have been developed during the centuries of use, ranging from the jack arch through the circular, elliptical and parabolic to the Gothic arch. Fig-


## Structural Brick Arches

FIG. 1

ure 1 depicts examples of structural masonry arches used in contemporary construction. An arch is normally classified by the curve of its intrados and by its function, shape or architectural style. Figure 2 illustrates some of the many different brick masonry arch types. Jack, segmental, semicircular and multicentered arches are the most common types used for building arches. For very long spans and for bridges, semicircular arches are often used because of their structural efficiency.

Mainly due to their variety of components and elements, arches have developed their own set of terminology. Following is a glossary of arch terminology. Figure 3 illustrates many of the terms defined in this glossary. Technical Notes in this series will use this terminology.
Abutment: The masonry or combination of masonry
and other structural members which support one end of the arch at the skewback.
Arch: A form of construction in which masonry units span an opening by transferring vertical loads laterally to adjacent voussoirs and, thus, to the abutments. Some common arch types are as follows:

Blind - An arch whose opening is filled with masonry.

Bullseye - An arch whose intrados is a full circle. Also known as a Circular arch.

Elliptical - An arch with two centers and continually changing radii.

Fixed - An arch whose skewback is fixed in position and inclination. Masonry arches are fixed arches by nature of their construction.

Gauged - An arch formed with tapered voussoirs and thin mortar joints.

Gothic - An arch with relatively large rise-to-span ratio, whose sides consist of arcs of circles, the centers of which are at the level of the spring line. Also referred to as a Drop, Equilateral or Lancet arch, depending upon whether the spacings of the centers are respectively less than, equal to or more than the clear span.

Horseshoe - An arch whose intrados is greater than a semicircle and less than a full circle. Also known as an Arabic or Moorish arch.

Jack - A flat arch with zero or little rise.
Multicentered - An arch whose curve consists of several arcs of circles which are normally tangent at their intersections.

Relieving - An arch built over a lintel, jack arch or smaller arch to divert loads, thus relieving the lower arch or lintel from excessive loading. Also known as a Discharging or Safety arch.

Segmental - An arch whose intrados is circular but less than a semicircle.

Semicircular - An arch whose intrados is a semicircle (half circle).

Slanted - A flat arch which is constructed with a keystone whose sides are sloped at the same angle as the skewback and uniform width brick and mortar joints.

Triangular - An arch formed by two straight, inclined
sides.
Tudor - A pointed, four-centered arch of medium rise-to-span ratio whose four centers are all beneath the extrados of the arch.

Venetian - An arch formed by a combination of jack arch at the ends and semicircular arch at the middle. Also known as a Queen Anne arch.
Camber: The relatively small rise of a jack arch.
Centering: Temporary shoring used to support an arch until the arch becomes self-supporting.
Crown: The apex of the arch's extrados. In symmetrical arches, the crown is at the midspan.
Depth: The dimension of the arch at the skewback which is perpendicular to the arch axis, except that the depth of a jack arch is taken to be the vertical dimension of the arch at the springing.
Extrados: The curve which bounds the upper edge of the arch.
Intrados: The curve which bounds the lower edge of the arch. The distinction between soffit and intrados is that the intrados is a line, while the soffit is a surface.
Keystone: The voussoir located at the crown of the arch. Also called the key.
Label Course: A ring of projecting brickwork that forms the extrados of the arch.
Rise: The maximum height of the arch soffit above the level of its spring line.
Skewback: The surface on which the arch joins the supporting abutment.
Skewback Angle: The angle made by the skewback from horizontal.
Soffit: The surface of an arch or vault at the intrados.
Span: The horizontal clear dimension between abutments.
Spandrel: The masonry contained between a horizontal line drawn through the crown and a vertical line drawn through the upper most point of the skewback.
Springing: The point where the skewback intersects the intrados.
Springer: The first voussoir from a skewback.
Spring Line: A horizontal line which intersects the springing.
Voussoir: One masonry unit of an arch.


Arch Terms
FIG. 3

## STRUCTURAL FUNCTION OF ARCHES

The brick masonry arch has been used to span openings of considerable length in many different applications. Structural efficiency is attributed to the curvature of the arch, which transfers vertical loads laterally along the arch to the abutments at each end. The transfer of vertical forces gives rise to both horizontal and vertical reactions at the abutments. The curvature of the arch and the restraint of the arch by the abutments cause a combination of flexural stress and axial compression. The arch depth, rise and configuration can be manipulated to keep stresses primarily compressive. Brick masonry is very strong in compression, so brick masonry arches can support considerable load.

Historically, arches have been constructed with unreinforced masonry. Most brick masonry arches continue to be built with unreinforced masonry. The structural design of unreinforced brick masonry arches is discussed in Technical Notes 31A. Very long span arches and arches with a small rise may require steel reinforcement to resist tensile stresses. Also, reduction in abutment size and arch thickness for economy may require incorporation of reinforcement for adequate load resistance. Refer to the Technical Notes 17 Series for more information on reinforced brick masonry. Elaborate and intricate arches are sometimes prefabricated to avoid the complexity of on-site shoring. Most prefabricated brick masonry arches are reinforced. Prefabricated arches are built off site and transported to the job or built at the site. Cranes are often used to lift the arch into place in the wall. Such fabrication, handling and transportation should be considered in the structural design of the arch. Refer to Technical Notes 40 for a discussion of prefabricated brick masonry.

If an unreinforced or reinforced brick masonry arch is not structurally adequate, the arch will require support. Typically, this support is provided by a steel angle. This is the most common means of supporting brick masonry arches in modern construction. The steel angle is bent to the curvature of the intrados of the arch. Curved sections of steel angle are welded to horizontal steel angles to form a continuous support. The angle either bears on the brickwork abutments or is attached to a structural member behind the wall. One example is shown in Fig. 4. When an arch is supported by a steel angle, the angle is designed to support the entire weight of brick masonry loading the arch, and the structural resistance of the arch is neglected. Consult Technical Notes 31B Revised for a discussion of the structural design of steel angle lintels.

## WEATHER RESISTANCE

Water penetration resistance is a primary concern in most applications of the building arch. In the past, the mass of a multi-wythe brick masonry arch was sufficient to resist water penetration. Today, thinner wall sections are used to minimize material use for economy and efficiency. Still, the arch must provide an effective


Arch Supported by Curved Steel Angle
FIG. 4
weather resistant facade. Some arch applications do not require provisions for water penetration and insulation. For example, arch arcades and arches supported by porch columns typically do not conceal a direct path for water migration to the interior of the building they serve and may not require insulation. If this is the case, provisions for weather resistance need not be included in the arch design and detailing.

Preventing water entry at an arch in an exterior building wall is just as important as at any other wall opening. Water penetration resistance can be provided by using a barrier wall system or a drainage wall system. Refer to Technical Notes 7 Revised for definitions and discussion of barrier and drainage wall systems. A drainage wall system, such as a brick veneer or cavity wall, is the most common brick masonry wall system used today. For either wall system, the arch should be flashed, with weep holes provided above all flashing locations.

## Flashing and Weep Holes

Installation of flashing and weep holes around an arch can be difficult. Installation of flashing is easiest with jack arches because they are flat or nearly flat. Flashing should be installed below the arch and above the window framing or steel angle lintel. Flashing should extend a minimum of 4 in . $(100 \mathrm{~mm})$ past the wall opening at either end and should be turned up to form end dams. This is often termed tray flashing. Weep holes should be provided at both ends of the flashing and should be placed at a maximum spacing of $24 \mathrm{in} .(600 \mathrm{~mm})$ on centers along the arch span, or 16 in. ( 400 mm ) if rope wicks are used. An example of flashing a jack arch in this manner is shown in Fig. 5a. Attachment of the flashing to the backing and formation of end dams should follow standard procedures. If the arch is constructed with reinforced brick masonry, flashing and weep holes can be placed in the first masonry course above the arch.


Flashing Arches
FIG. 5

Installation of flashing with other arch types, such as segmental and semicircular arches, can be more difficult. This is because most rigid flashing materials are
hard to bend around an arch with tight curvature. If the arch span is less than about $3 \mathrm{ft}(0.9 \mathrm{~m})$, one section of tray flashing can be placed in the first horizontal mortar joint above the keystone, as illustrated in Fig. 5b. For arch spans greater than $3 \mathrm{ft}(0.9 \mathrm{~m})$, flashing can be bent along the curve of the arch with overlapping sections, as illustrated in Fig. 4. Alternately, a combination of stepped and tray flashing can be used, as shown in Fig. 5c. To form a step, the end nearest the arch should be turned up to form an end dam, while the opposite end is laid flat. A minimum of No. 15 building paper or equivalent moisture resistant protection should be installed on the exterior face of the backing over the full height of the arch and abutments. The building paper or equivalent should overlap the arch flashing.

The design of a structural masonry arch should include consideration of the effect of flashing on the strength of the arch. Flashing acts as a bond break. If flashing is installed above the arch, the loading on the arch will likely be increased, and the structural resistance of the arch will be reduced. Installation of flashing at the abutments will affect their structural resistance and should also be considered. Consult Technical Notes 31A for a more extensive discussion of arch loads and structural resistance of brick masonry arches.

## DETAILING CONSIDERATIONS

The brick masonry arch should serve its structural purpose and also provide an attractive architectural element to complement its surrounding structure. Careful consideration should be given to the options available for the arch, soffit and skewback. Proper configuration of the abutments and location of expansion joints should be considered for any arch design.

## Arch

Arches can be configured in a variety of arch depths, brick sizes and shapes and bonding patterns. The arch is normally composed of an odd number of units for aesthetic purposes. Some of the more common arch configurations are illustrated in Fig. 6. Arch voussoirs are typically laid in radial orientation and are most often of similar size and color to the surrounding brickwork. However, the arch can be formed with brick which are thinner or wider than the surrounding brickwork and of a different color for variation. Another variation is to project or recess rings of multiple-ring arches to provide shadow lines or a label course.

Brick masonry arches are constructed with two different types of units. The first is tapered or wedgeshaped brick. These brick are tapered in the appropriate manner to obtain mortar joints of uniform thickness along the arch depth. The second is uncut, rectangular brick. When rectangular brick are used, the mortar joints are tapered to obtain the desired arch curvature. In some cases, a combination of these is used. For example, a slanted arch is formed with a tapered keystone and rectangular brick. This arch is similar to a jack


## Typical Arch Configurations <br> FIG. 6

arch, but can be more economical because it requires only one special-shaped brick.

Selection of tapered or rectangular brick can be determined by the arch type, arch dimensions and by the appearance desired. Some arch types require more unique shapes and sizes of brick if uniform mortar joint thickness is desired. For example, the brick in a traditional jack arch or elliptical arch are all different sizes and shapes from the abutment to the keystone. Conversely, the voussoirs of a semicircular arch are all the same size and shape. Arch types with many different brick shapes and sizes should be special ordered from the brickmanufacturer rather than cut in the field.

The arch span should also be considered when selecting the arch brick. For short arch spans, use of tapered brick is recommended to avoid excessively wide mortar joints at the extrados. Larger span arches require less taper of the voussoirs and, consequently, can be formed with rectangular brick and tapered mortar joints. The thickness of mortar joints between arch brick should be a maximum of $3 / 4 \mathrm{in}$. ( 19 mm ) and a minimum of $1 / 8 \mathrm{in}$. (3 mm ). When using mortar joints thinner than $1 / 4 \mathrm{in}$. (6 mm ), consideration should be given to the use of very uniform brick that meet the dimensional tolerance limits of ASTM C 216, Type FBX, or the use of gauged brickwork. Refer to Table 1 for determination of the mini-

TABLE 1
Minimum Radius for Uncut Arch Brick ${ }^{1,2}$

| Nominal Face Dimensions <br> of Arch Brick, in. <br> (height by width) | Minimum Permissible <br> Radius of Arch <br> to Intrados, ft |
| :---: | :---: |
| $4 \times 2^{2 / 3}$ | 3.3 |
| $8 \times 2^{2 / 3}$ | 6.7 |
| $12 \times 2^{2 / 3}$ | 10.0 |
| $16 \times 2^{2 / 3}$ | 13.3 |
| $4 \times 3^{1 / 5}$ | 4.0 |
| $8 \times 3^{1 / 5}$ | 8.0 |
| $12 \times 3^{1 / 5}$ | 12.0 |
| $16 \times 3^{1 / 5}$ | 16.0 |
| $4 \times 4$ | 5.2 |
| $8 \times 4$ | 10.3 |
| $12 \times 4$ | 15.5 |
| $16 \times 4$ | 20.7 |

${ }^{1}$ Based on $1 / 4 \mathrm{in}$. ( 6 mm ) mortar joint width at the intrados and $1 / 2 \mathrm{in}$. ( 13 mm ) mortar joint width at the extrados. If the mortar joint thickness at theextra dos is $3 / 4 \mathrm{in}$. ( 19 mm ), divide minimum radius value by 2 .
${ }^{2} 1 \mathrm{in}$. $=25.4 \mathrm{~mm} ; 1 \mathrm{ft}=0.3 \mathrm{~m}$
mum segmental and semicircular arch radii permitted for rectangular brick and tapered mortar joints. Typically, the use of tapered brick and uniform thickness mortar joints will be more aesthetically appealing.

Depth. The arch depth will depend upon the size and orientation of the brick used to form the arch. Typically, the arch depth is a multiple of the brick's width. For structural arches, a minimum arch depth is determined from the structural requirements. If the arch is supported by a lintel, any arch depth may be used.

The depth of the arch should also be detailed based on the scale of the arch in relation to the scale of the building and surrounding brickwork. To provide proper visual balance and scale, the arch depth should increase with increasing arch span. Because aesthetics of an arch are subjective, there are no hard rules for this. However, the following rules-of-thumb will help provide an arch with proper scale. For segmental and semicircular arches, the arch depth should equal or exceed 1 in . ( 25 mm ) for every foot ( 300 mm ) of arch span or 4 in . ( 100 mm ), whichever is greater. For jack arches, the arch depth should equal or exceed 4 in. (100 mm ) plus 1 in . ( 25 mm ) for every foot ( 300 mm ) of arch span or 8 in . ( 200 mm ), whichever is greater. For example, the minimum arch depth for an $8 \mathrm{ft}(2.4 \mathrm{~m})$ span should be 8 in . $(200 \mathrm{~mm})$ for segmental arches and $12 \mathrm{in} .(300 \mathrm{~mm})$ for jack arches.

The depth of jack arches will also be a function of the coursing of the surrounding brick masonry. The springing and the extrados of the jack arch should coincide with horizontal mortar joints in the surrounding brick masonry. Typically, the depth of a jack arch will equal the height of 3,4 or 5 courses of the surrounding brickwork, depending upon the course height.

Keystone. The keystone may be a single brick, multiple brick, stone, precast concrete or terra cotta. Avoid using a keystone which is much taller than the adjacent voussoirs. A rule-of-thumb is that the keystone should not extend above adjacent arch brick by more than onethird the arch depth. When a keystone is used that is larger than adjacent arch brick or formed with different material, one option is to use springers that match the keystone.
The use of a large keystone has its basis in both purpose and visual effect. With most arch types, the likely location of the first crack when the arch fails is at the mortar joint nearest to the midspan of the arch. Use of a large keystone at this point moves the first mortar joint further from the midspan and increases the resistance to cracking at this point. Aesthetically, a large keystone adds variation of scale and can introduce other masonry materials in the facade for additional color and texture.

If the keystone is formed with more than one masonry unit, avoid placing the smaller unit at the bottom. Such units are more likely to slip when the arch settles under load. Also, it is preferred to have the arch crown (the top of the keystone) coincident with a horizontal mortar joint in the surrounding brickwork to give the arch a neater appearance.

## Soffit

A brick masonry soffit is one attractive feature of a structural brick masonry arch. Many bonding patterns and arrangements can be used to form the arch soffit. Deep soffits are common on building arcades or arched entranceways. In this case, it is common to form a U shaped wall section, as illustrated in Fig. 7. The arches on either wall face should be bonded to the brick masonry forming the soffit. Bonding pattern or metal ties should be used to tie the brick masonry forming the soffit together structurally and to tie the arches on either wall face to the soffit. If metal ties are used to bond the masonry, corrosion resistant box or Z metal wire ties


Structural Arch Soffit Option
FIG. 7
should be placed along the arch span at a maximum spacing of 24 in . ( 600 mm ) on center.

Structural resistance of the arch should be evaluated at sections through the soffit, the exterior wall face and the interior wall face. Deeper soffits may require an increase in arch depth. If the arch is structural, connection of the brick masonry forming the soffit to interior framing members with wall ties or connectors may not be required.

## Skewback

For flat arches and arch types that have horizontal skewbacks, such as jack and semicircular arches, respectively, the most desirable spring line location is coincident with a bed joint in the abutment. For other arch types, it is preferred to have the spring line pass about midway through a brick course in the abutment, as illustrated in Fig. 8, to avoid a thick mortar joint at the springing. The brick in the abutment at the spring-

ing should be cut or be a special cant-shaped brick. This allows vertical alignment with the brick beneath, producing more accurate alignment of the arch.

When two arches are adjacent, such as with a twobay garage or building arcades, intersection of the arches may occur at the skewback. Attention should be given to proper bonding of the arches for both visual appeal and structural bonding. Creation of a vertical line between arches should be avoided. Rather, special shape brick should be used to mesh the two arches properly. One example is illustrated in Fig. 9.

## Abutments

An arch abutment can be a column, wall or combination of wall and shelf angle. Failure of an abutment occurs from excessive lateral movement of the abutment or exceeding the flexural, compressive or shear strength of the abutment. Lateral movement of the abutment is due to the horizontal thrust of the arch. Thrust develops in all arches and the thrust force is greater for flatter arches. The thrust should be resisted so that lateral movement of the abutment does not cause failure in the arch. If the abutment is formed by a combination of brickwork and a non-masonry structural member, rigidity of the non-masonry structural member and rigidity of the ties are very important. Adjustable ties or single or double wire ties are recommended. Corrugated ties should not be used in this application because they do not provide adequate axial stiffness. Consult Technical Notes 31A for further discussion of abutment and tie stiffness requirements.

## Lateral Bracing

In addition to gravity loads, out-of-plane loads should be considered when designing a masonry arch. The arch should have adequate resistance to out-ofplane loads or lateral bracing should be provided. In veneer construction, lateral bracing is provided by the backing through the use of wall ties. Arches which are not laterally braced may require increased masonry thickness or reinforcement to carry loads perpendicular to the arch plane in addition to vertical loads.

## Expansion Joints

Thermal and moisture movements of brick masonry are controlled by the use of expansion joints. Expansion joints avoid cracking of the brickwork and also reduce the size of wall sections. Reduction of wall size has a very important effect upon the performance of structural brick masonry arches. The state of stress in a structural brick arch and the surrounding masonry is very sensitive to the relative movements of the abutments. If an inadequate number of expansion joints are provided, the differential movement of abutments can cause cracking and downward displacement of brick in the masonry arch and surrounding masonry. Proper size and spacing of expansion joints is discussed in Techni cal Notes 18A Revised.

If the arch is structural, care should be taken not to
affect the integrity of the arch by detailing expansion joints too close to the arch and its abutments. Vertical expansion joints should not be placed in the masonry directly above a structural arch. This region of masonry is in compression, so an expansion joint will cause displacement when centering is removed and possible collapse of the arch and surrounding brickwork. In addition, vertical expansion joints should not be placed in close proximity to the springing. The expansion joint will reduce the effective width of the abutment and its ability to resist horizontal thrust from the arch. If the arch is non-structural, placement of expansion joints may be at the arch crown and also at a sufficient distance away from the springing to avoid sliding. While permitted, placement of an expansion joint at the arch crown is not preferred because it disrupts ones tradi-

tional view of the arch as a structural element. Refer to Fig. 10 for suggested expansion joint locations for structural and non-structural arches.

Detailing of expansion joints can be difficult with very long span arches or runs of multiple arches along an arcade. Structural analysis of the arch should consider the location of expansion joints. For the particular case of multiple arches closely spaced, vertical expansion joints should be detailed at a sufficient distance away from the end arches so that horizontal arch thrusts are adequately resisted by the abutments to avoid overturning of the abutments. For long arcades, expansion joints should also be placed along the centerline of abutments between arches when necessary. In this case, horizontal thrusts from adjacent arches will not be counteracting, so the effective abutment length should be halved and overturning of each half of the abutment should be checked. Refer to Technical Notes 31A for further discussion of abutment design for adequate stiffness.

## MATERIAL SELECTION

To provide a weather resistant barrier and maintain its structural resistance, the arch must be constructed with durable materials. The strength of an arch depends upon the compressive strength and the flexural tensile strength of the masonry. Selection of brick and mortar should consider these properties.

## Brick

Solid or hollow clay brick may be used to form the arch and the surrounding brickwork. Solid brick should comply with the requirements of ASTM C 216 Specification for Facing Brick. Hollow brick should comply with the requirements of ASTM C 652 Specification for Hollow Brick. Refer to Technical Notes 9 Series for a discussion of brick selection and classification. The compressive strength of masonry is related to the compressive strength of the brick, the mortar type and the grout strength. For structural arches, brick should be selected with consideration of the required compressive strength of masonry. Typically, compressive strength of the brick masonry will not limit the design of the arch.

Tapered voussoirs can be cut from rectangular units at the job site or special ordered from the brick manufacturer. Before specifying manufactured special arch shapes, the designer should determine the availability of special shapes for the arch type and brick color and texture desired. Many brick manufacturers produce tapered arch brick for the more common arch types as part of their regular stock of special shapes. Be sure to contact the manufacturer as early as possible if special shapes are needed. In many instances, production of the special shapes may require a color matching process and adequate lead time for the manufacturer.

## Mortar

Mortar used to construct brick masonry arches should meet the requirements of ASTM C 270 Standard

Specification for Masonry Mortar. Consult Technical Notes 8 Series for a discussion of mortar types and kinds for brick masonry. For structural arches, the flexural tensile strength of the masonry should be considered when selecting the mortar. The flexural tensile strength of the masonry will affect the load resistance of the arch and the abutments.

## CONSTRUCTION AND WORKMANSHIP

The proper performance of a brick masonry arch depends upon proper methods of construction and attention to workmanship. Layout of the arch prior to construction will help avoid poor spacing of voussoirs, which results in thicker mortar joints and unsymmetrical arches. Some arch applications, such as barrel vaults and domes, can be entirely self-supporting, even during construction. However, most applications of the masonry arch used today require proper shoring and bracing.

## Centering

Both structural and non-structural arches should be properly supported throughout construction. Brick masonry arches are constructed with the aid of temporary shoring, termed centering, or permanent supports, such as a structural steel angle.
Centering is used to carry the weight of a brick masonry arch and the loads being supported by the arch until the arch itself has gained sufficient strength. The term "centering" is used because the shoring is marked for proper positioning of the brick forming the arch.


Centering is typically provided by wood construction. An example of centering for an arch is shown in Fig. 11. Careful construction of the centering will ensure a more pleasing arch appearance and avoid layout problems, such as an uneven number of brick to either side of the keystone.

Immediately after placement of the keystone, very slight downward displacement of the centering, termed easing, can be performed to cause the arch voussoirs to press against one another and compress the mortar joints between them. Easing helps to avoid separation cracks in the arch. In no case should centering be removed until it is certain that the masonry is capable of carrying all imposed loads. Premature removal of the centering may result in collapse of the arch.

Centering should remain in place for at least seven days after construction of the arch. Longer curing periods may be required when the arch is constructed in cold weather conditions and when required for structural reasons. The arch loading and the structural resistance of the arch will depend upon the amount of brickwork surrounding the arch, particularly the brick masonry within spandrel areas. Appropriate time of removal of centering for a structural arch should be determined with consideration of the assumptions made in the structural analysis of the arch. It may be necessary to wait until the brickwork above the arch has also cured before removing the centering.

## Workmanship

All mortar joints should be completely filled, especially in a structural member such as an arch. If hollow brick are used to form the arch, it is very important that all face shells and end webs are completely filled with mortar. Brick masonry arches are sometimes constructed with the units laid in a soldier orientation. It may be difficult to lay units in a soldier position and also obtain
completely filled mortar joints. This is especially true for an arch with tapered mortar joints. In such cases, the use of two or more rings of arch brick laid in rowlock orientation can help ensure full mortar joints.

## SUMMARY

This Technical Notes is an introduction to brick masonry arches. A glossary of arch terms has been provided. Many different types of brick masonry arches are described and illustrated. Proper detailing of brick masonry arches for appearance, structural support and weather resistance is discussed. Material selection and proper construction practices are explained. Other Technical Notes in this Series discuss the structural design of arches.

The information and suggestions contained in this Technical Notes are based on the available data and the experience of the engineering staff of the Brick Institute of America. The information contained herein must be used in conjunction with good technical judgment and a basic understanding of the properties of brick masonry. Final decisions on the use of the information contained in this Technical Notes are not within the purview of the Brick Institute of America and must rest with the project architect, engineer and owner.

## REFERENCES

1. Brickwork Arch Detailing, Ibstock Building Products, Butterworth \& Co. (Publishers) Ltd., London, England, 1989, 114 pp.
2. Lynch, G., Gauged Brickwork, A Technical Handbook, Gower Publishing Company, Aldershot, Hants, England, 1990, 115 pp.
3. Trimble, B.E., and Borchelt, J.G., "Jack Arches in Masonry Construction," The Construction Specifier, Construction Specifications Institute, Alexandria, VA, January 1991, pp. 62-65.

## Examples: <br> Cables and Arches

## Example 1



## SAMPLE PROBLEM 7.9

A light cable is attached to a support at $A$, passes over a small pulley at $B$, and supports a load P. Knowing that the sag of the cable is 0.5 m and that the mass per unit length of the cable is $0.75 \mathrm{~kg} / \mathrm{m}$, determine (a) the magnitude of the $\operatorname{load} \mathbf{P},(b)$ the slope of the cable at $B$, and $(c)$ the total length of the cable from $A$ to $B$. Since the ratio of the sag to the span is small, assume the cable to be parabolic. Also, neglect the weight of the portion of cable from $B$ to $D$.

a. Load P. We denote by $C$ the lowest point of the cable and draw the free-body diagram of the portion $C B$ of cable. Assuming the load to be uniformly distributed along the horizontal, we write

$$
w=(0.75 \mathrm{~kg} / \mathrm{m})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)=7.36 \mathrm{~N} / \mathrm{m}
$$

The total load for the portion $C B$ of the cable is

$$
W=w x_{B}=(7.36 \mathrm{~N} / \mathrm{m})(20 \mathrm{~m})=147.2 \mathrm{~N}
$$

and is applied halfway between $C$ and $B$. Summing moments about $B$, we write

$$
+\uparrow \Sigma M_{B}=0: \quad(147.2 \mathrm{~N})(10 \mathrm{~m})-T_{0}(0.5 \mathrm{~m})=0 \quad T_{0}=2944 \mathrm{~N}
$$

From the force triangle we obtain

$$
\begin{aligned}
T_{B} & =\sqrt{T_{0}^{2}+W^{2}} \\
& =\sqrt{(2944 \mathrm{~N})^{2}+(147.2 \mathrm{~N})^{2}}=2948 \mathrm{~N}
\end{aligned}
$$

Since the tension on each side of the pulley is the same, we find

$$
P=T_{B}=2948 \mathrm{~N}
$$

b. Slope of Cable at B. We also obtain from the force triangle

$$
\tan \theta=\frac{W}{T_{0}}=\frac{147.2 \mathrm{~N}}{2944 \mathrm{~N}}=0.05
$$

$$
\theta=2.9^{\circ}
$$

c. Length of Cable. Applying Eq. (7.10) between $C$ and $B$, we write

$$
\begin{aligned}
s_{B} & =x_{B}\left[1+\frac{2}{3}\left(\frac{y_{B}}{x_{B}}\right)^{2}+\cdots\right] \\
& =(20 \mathrm{~m})\left[1+\frac{2}{3}\left(\frac{0.5 \mathrm{~m}}{20 \mathrm{~m}}\right)^{2}+\cdots\right]=20.00833 \mathrm{~m}
\end{aligned}
$$

The total length of the cable between $A$ and $B$ is twice this value,

$$
\text { Length }=2 s_{B}=40.0167 \mathrm{~m}
$$

Example 2 (Figure 5.19(b))
Using Multiframe4D, verify the axial force, shear and bending moment for the funcular shape with $\mathrm{P}=50,000 \mathrm{lb}, \mathrm{L}=150 \mathrm{ft}$, and $h_{\max }=75 \mathrm{ft}$.

| Joint Coordinates (ft) |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| Joint | Labe2 | $x$ | $y$ | $z$ |
| 1 |  | 0.000 | 0.000 | 0.000 |
| 2 |  | 50.000 | 41.700 | 0.000 |
| 3 | 75.000 | 66.700 | 0.000 |  |
| 4 | 100.000 | 75.000 | 0.000 |  |
| 5 | 125.000 | 66.700 | 0.000 |  |
| 6 | 150.000 | 41.700 | 0.000 |  |
| 7 |  | 0.000 | 0.000 |  |

# Simplified Frame Analysis 

### 2.1 INTRODUCTION

The final design of the structural components in a building frame is based on maximum moment, shear, axial load, torsion and/or other load effects, as generally determined by an elastic frame analysis (ACI 8.3). For building frames of moderate size and height, preliminary and final designs will often be combined. Preliminary sizing of members, prior to analysis, may be based on designer experience, design aids, or simplified sizing expressions suggested in this book.

Analysis of a structural frame or other continuous construction is usually the most time consuming part of the total design. For gravity load analysis of continuous one-way systems (beams and slabs), the approximate moments and shears given by ACI 8.3 .3 are satisfactory within the span and loading limitations stated. For cases when ACI 8.3 .3 is not applicable, a two-cycle moment distribution method is accurate enough. The speed and accuracy of the method can greatly simplify the gravity load analysis of building frames with usual types of construction, spans, and story heights. The method isolates one floor at a time and assumes that the far ends of the upper and lower columns are fixed. This simplifying assumption is permitted by ACI 8.8.3.

For lateral load analysis of a sway frame, the Portal Method may be used. It offers a direct solution for the moments and shears in the beams (or slabs) and columns, without having to know the member sizes or stiffnesses.

The simplified methods presented in this chapter for gravity load analysis and lateral load analysis are considered to provide sufficiently accurate results for buildings of moderate size and height. However, determinations of load effects by computer analysis or other design aids are equally applicable for use with the simplified design procedures presented in subsequent chapters of this book.

### 2.2 LOADING

### 2.2.1 Service Loads

The first step in the frame analysis is the determination of design (service) loads and lateral forces (wind and seismic) as called for in the general building code under which the project is to be designed and constructed. For the purposes of this book, design live loads (and permissible reductions in live loads) and wind loads are based on Minimum Design Loads for Buildings and Other Structures, ASCE7-02.2.1 References to specific ASCE Standard requirements are noted (ASCE 4.2 refers to ASCE 7-02, Section 4.2). For a specific project, however, the governing general building code should be consulted for any variances from ASCE 7-02.

Design dead loads include member self-weight, weight of fixed service equipment (plumbing, electrical, etc.) and, where applicable, weight of built-in partitions. The latter may be accounted for by an equivalent uniform load of not less than 20 psf , although this is not specifically defined in the ASCE Standard (see ASCE Commentary Section 3.2).

Design live loads will depend on the intended use and occupancy of the portion or portions of the building being designed. Live loads include loads due to movable objects and movable partitions temporarily supported by the building during maintenance. In ASCE Table 4-1, uniformly distributed live loads range from 40 psf for residential use to 250 psf for heavy manufacturing and warehouse storage. Portions of buildings, such as library floors and file rooms, require substantially heavier live loads. Live loads on a roof include maintenance equipment, workers, and materials. Also, snow loads, ponding of water, and special features, such as landscaping, must be included where applicable.

Occasionally, concentrated live loads must be included; however, they are more likely to affect individual supporting members and usually will not be included in the frame analysis (see ASCE 4.3).

Design wind loads are usually given in the general building code having jurisdiction. For both example buildings here, the calculation of wind loads is based on the procedure presented in ASCE 6.0. Design for seismic loads is discussed in Chapter 11.

### 2.6 LATERAL LOAD ANALYSIS

For frames without shear walls, the lateral load effects must be resisted by the "sway" frame. For low-to-moderate height buildings, lateral load analysis of a sway frame can be performed by either of two simplified methods: the Portal Method or the Joint Coefficient Method. Both methods can be considered to satisfy the elastic frame analysis requirements of the code (ACI 8.3). The two methods differ in overall approach. The Portal Method considers a vertical slice through the entire building along each row of column lines. The method is well suited to the range of building size and height considered in this book, particularly to buildings with a regular rectangular floor plan. The Joint Coefficient Method considers a horizontal slice through the entire building, one floor at a time. The method can accommodate irregular floor plans, and provision is made to adjust for a lateral loading that is eccentric to the centroid of all joint coefficients (centroid of resistance). The Joint Coefficient Method considers member stiffnesses, whereas the Portal Method does not.

The Portal Method is presented in this book because of its simplicity and its intended application to buildings of regular shape. If a building of irregular floor plan is encountered, the designer is directed to Reference 2.2 for details of the Joint Coefficient Method.

### 2.6.1 Portal Method

The Portal Method considers a two-dimensional frame consisting of a line of columns and their connecting horizontal members (slab-beams), with each frame extending the full height of the building. The frame is considered to be a series of portal units. Each portal unit consists of two story-high columns with connecting slab-beams. Points of contraflexure are assumed at mid-length of beams and mid-height of columns. Figure 2-11 illustrates the portal unit concept applied to the top story of a building frame, with each portal unit shown separated (but acting together).

The lateral load W is divided equally between the three portal units. The shear in the interior columns is twice that in the end columns. In general, the magnitude of shear in the end column is $\mathrm{W} / 2 \mathrm{n}$, and in an interior column it is $\mathrm{W} / \mathrm{n}$, where n is the number of bays. For the case shown with equal spans, axial load occurs only in the end columns since the combined tension and compression due to the portal effect results in zero axial loads in the interior
columns. Under the assumptions of this method, however, a frame configuration with unequal spans will have axial load in those columns between the unequal spans, as well as in the end columns. The general term for axial load in the end columns in a frame of $n$ bays with unequal spans is:

$$
\frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{1}} \text { and } \frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{\mathrm{n}}}, \ell_{\mathrm{n}}=\text { length of bay } \mathrm{n}
$$

The axial load in the first interior column is:

$$
\frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{1}}-\frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{2}}
$$

and, in the second interior column:

$$
\frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{2}}-\frac{\mathrm{Wh}}{2 \mathrm{n} \ell_{3}}
$$

Column moments are determined by multiplying the column shear with one-half the column height. Thus, for joint B in Fig. 2-11, the column moment is $(\mathrm{W} / 3)(\mathrm{h} / 2)=\mathrm{Wh} / 6$. The column moment $\mathrm{Wh} / 6$ must be balanced by equal moments in beams BA and BC, as shown in Fig. 2-12.

Note that the balancing moment is divided equally between the horizontal members without considering their relative stiffnesses. The shear in beam AB or BC is determined by dividing the beam end moment by one-half the beam length, $(\mathrm{Wh} / 12)(\ell / 2)=\mathrm{Wh} / 6 \ell$.

The process is continued throughout the frame taking into account the story shear at each floor level.

### 2.6.2 Examples: Wind Load Analyses for Buildings \#1 and \#2

For Building \#1, determine the moments, shears, and axial forces using the Portal Method for an interior frame resulting from wind loads acting in the $\mathrm{N}-\mathrm{S}$ direction. The wind loads are determined in Section 2.2.1.2.

Moments, shears, and axial forces are shown directly on the frame diagram in Fig. 2-13. The values can be easily determined by using the following procedure:
(1) Determine the shear forces in the columns:

For the end columns:

$$
\begin{aligned}
& \text { 3rd story: } \mathrm{V}=12.0 \mathrm{kips} / 6=2.0 \mathrm{kips} \\
& \text { 2nd story } \mathrm{V}=(12.0 \mathrm{kips}+23.1 \mathrm{kips}) / 6=5.85 \mathrm{kips} \\
& \text { 1st story: } \mathrm{V}=(12.0 \mathrm{kips}+23.1 \mathrm{kips}+21.7 \mathrm{kips}) / 6=9.50 \mathrm{kips}
\end{aligned}
$$

The shear forces in the interior columns are twice those in the end columns.


- Assumed inflection point at mid-length members


Figure 2-11 Portal Method


Figure 2-12 Joint Detail
(2) Determine the axial loads in the columns:

For the end columns, the axial loads can be obtained by summing moments about the column inflection points at each level. For example, for the 2nd story columns:

$$
\begin{aligned}
& \Sigma \mathrm{M}=0: 12(13+6.5)+23.1(6.5)-\mathrm{P}(90)=0 \\
& \mathrm{P}=4.27 \mathrm{kips}
\end{aligned}
$$

For this frame, the axial forces in the interior columns are zero.
(3) Determine the moments in the columns:

The moments can be obtained by multiplying the column shear force by one-half of the column length.
For example, for an exterior column in the 2nd story:

$$
\mathrm{M}=5.85(13 / 2)=38.03 \mathrm{ft}-\mathrm{kips}
$$

(4) Determine the shears and the moments in the beams:

These quantities can be obtained by satisfying equilibrium at each joint. Free-body diagrams for the 2nd story are shown in Fig. 2-14.

As a final check, sum moments about the base of the frame:

$$
\Sigma \mathrm{M}=0: \quad 12.0(39)+23.1(26)+21.7(13)-10.91(90)-2(61.53+123.07)=0 \quad \text { (checks) }
$$

In a similar manner, the wind load analyses for an interior frame of Building \#2 (5-story flat plate), in both the $\mathrm{N}-\mathrm{S}$ and E-W directions are shown in Figs. 2-15 and 2-16, respectively. The wind loads are determined in Section 2.2.1.1.


Figure 2-13 Shear, Moments and Axial Forces Resulting from Wind Loads for an Interior Frame of Building \#1 in the N-S Direction, using the Portal Method

Figure 2-14 Shear Forces, Axial Forces, and Bending Moments at 2nd Story of Building \#1

## Frame Analysis Using Multiframe

1. The software is on the computers in the College of Architecture in Programs under the Windows Start menu (see https://wikis.arch.tamu.edu/display/HELPDESK/Computer+Accounts for lab locations). Multiframe is under the Multiframe [current version \#] menu. Or it can be downloaded from the web site: http://www.formsys.com/academic/multiframe
2. There are tutorials available on line at http://www.formsys.com/mflearning that list the tasks and order in greater detail. The first task is to define the unit system:

- Choose Units... from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.


3. To see the scale of the geometry, a grid option is available:

- Choose Grid... from the View menu


4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:


The Member toolbar shows ways to create members:


The Generate toolbar has convenient tools to create typical structural shapes.

- To create a frame, use the multi-bay frame button:

- Enter the number of bays (horizontally), number of stories (vertically) and the corresponding spacings:

- If the frame does not have regular bays, use the add connected members button to create segments:

- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.
- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.
- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (fixed shown), or the Frame / Joint Restraint ... menu (right click).

NOTE: If the support appears at both ends
 of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.


The support forces will be determined in the analysis.
5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section proceed to step 6. For custom sections the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.


Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don't know or care about. Properties like $\mathrm{t}_{\mathrm{f}}, \mathrm{t}_{\mathrm{w}}$, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:


- Choose the group name and section name:
(STANDARD SHAPES)

(CUSTOM)


7. If there is an area that has a uniformly distributed load, load panels may be defined in the Frame window. Because the loaded area may not be visible in the current view, choose the View button at the lower left of the Frame window. The options for view are shown. (See 3D Frames, last page.)

- Choose the panel type (rectangular, 4-node, or 3-node) from the menu and select the corners. If the area is rectangular, only the
 opposite corners need to be selected.
- Select the panel and from the pop-up menu, or the Frame menu, specify the load panel supports. The default supports are on all sides. If the panel is one way, chose the corresponding picture


8. The frame geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).


- Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):

- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.
- Enter the values of the load and distances (if any). Distances can be entered as a function of the length , i.e. L/2, L/4...
- Area load units may have to be changed in the View Units dialogue.

NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.


Multiframe4D will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).
10. In order to run the analysis after the geometry, member
 properties and loading has been defined:

- Choose Linear from the Analyze menu

11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:
12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):


- To show the moment diagram, Choose the red Moment button

- To show the shear diagram, Choose the green Shear button

- To show the axial force diagram, Choose the purple Axial Force button

- To show the deflection diagram, Choose the blue Deflection button

- To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
- To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.


13. The Data window (D) allows you to view all data "entered" for the geometry, sections and loading. These values can be edited.

14. The Results window ( R ) allows you to view all results of the
 analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

NOTE: Px' refers to the axial load (P) in the local axis $x$ direction ( $\mathrm{x}^{\prime}$ ). Vy' refers to the shear perpendicular to the local x axis, and Mz ' refers to the bending moment.

15. To save the file Choose Save from the File menu.
16. To load an existing file Choose Open... from the File menu. .
17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot $(\mathrm{Ctrl}+\mathrm{c})$ and paste it in a word processing document $(\mathrm{Ctrl}+\mathrm{v})$.

## Example of Combined Stresses:

for member 3: $\mathrm{M}_{\max }=19.6 \mathrm{k}-\mathrm{ft}, \mathrm{P}=1.76 \mathrm{k}$
knowing $\mathrm{A}=21.46 \mathrm{in}^{2}, \mathrm{I}=796.0 \mathrm{in}^{4}, \mathrm{c}=7.08$ in
$f_{\max }=\frac{1.76 k}{21.46 i^{2}}+\frac{19.6^{k-f t} \cdot 7.08 i n}{796 i i^{4}} \cdot \frac{12 i n}{f t}=0.082 k s i+2.092 k s i=2.174 k s i$

Results window:

where Sx' refers to the axial stress, Sy' refers to the bending stress around the local vertical axis and $\mathrm{Sz}^{\prime}$ refers to the bending stress around the local horizontal axis.

## For 3D Frames:

- There is a tutorial in the Help menu (Chapter 1 - 3D Tutorial) that lists the tasks and order in greater detail. It expects that you have been through the 2D tutorial to build on the steps already mastered.
- There are standard 3D frame shapes on the frame toolbar.

- It is very useful to change the view to isometric with the View Button

- If you wish to have additional beams supported by the beams of your frame, choose the beam and use the Subdivide Member menu under Geometry. This will make additional joints, but keep the segments together.

- In order to model a beam end as simply supported, you must release the restraint preventing rotation about the $\mathrm{x}-\mathrm{x}$ axis of the beam. The pinned ends menu is useful for segments or subdivided members.


Or, by selecting a segment and right clicking for a menu, you can use Member Releases (also under the Frame menu) to release the Major Bending ( $\mathrm{M}_{\mathrm{z}}^{\prime}$ ) for one end or both.


- It is necessary to understand the local member axes to assign the correct load direction. Choosing the local loading types will show the member orientation with respect to the load direction.



## Examples:

Rigid Frames
Example $1 \_$From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College


RIGID FRAME STRUCTURES: LATERAL LOADING PINNED BASE CONNECTIONS


Determine axial forces, shear forces, and bending moments in each member of the rigid frame shown.


## RIGID FRAME STRUCTURES

DETERMINE REACTIONS


Assumed directions of reactions:
Horizontal components balance applied force
Vertical components act as shown to prevent overturning
$\Sigma \mathrm{M}_{\mathrm{A}}=0$
$+2000(12)-R_{c_{y}}(30)=0$ $R_{C_{y}}=800 \downarrow$
$\Sigma F_{y}=0$
$+R_{A_{y}}-800=0$ or $R_{A_{y}}=800 \uparrow$
$\Sigma F_{x}=0$

$$
R_{A_{x}}+R_{C_{x}}=2000
$$

This last equation cannot be solved by statics alone. The structure is actually statically indeterminate. As shown on the following slides, an approximate method of analysis can be used to find the unknown reactions.

Example 1 (continued)


Example 1 (continued)


Example 1 (continued)


Example 1 (continued)


Example 1 (continued)
(4) ? Lateral Loading ( 1 sTEP 8

RIGID FRAME STRUCTURES


Example 1 (continued)


## Example 2

The rigid frame shown at the right has the loading and supports as show. Using superpositioning from approximate analysis methods, draw the shear and bending moment diagrams.

## Solution:

Reactions The two loading situations for which approximate reaction values are available are shown below. These values must be calculated and added together (allowed by superpositioning).


$$
\begin{aligned}
& \mathrm{R}_{\mathrm{AH}}=-0.907 \mathrm{wh}+0.0551 \mathrm{Ph} / \mathrm{L}=-0.907\left(10^{\mathrm{kN} / \mathrm{m}}\right)(6 \mathrm{~m})+\frac{0.0551(50 \mathrm{kN})(6 \mathrm{~m})}{5 m}=-51.11 \mathrm{kN} \\
& \mathrm{R}_{\mathrm{AV}}=-0.197 \mathrm{wh}^{2} / \mathrm{L}+0.484 \mathrm{P}=\frac{-0.197\left(10^{k N / m}\right)(6 \mathrm{~m})^{2}}{5 m}+0.484(50 \mathrm{kN})=10.02 \mathrm{kN} \\
& \mathrm{MR}_{\mathrm{A}}=-0.303 \mathrm{wh}^{2}+0.0112 \mathrm{Ph}^{2} / \mathrm{L}=-0.303\left(10^{\mathrm{kN} / \mathrm{m}}\right)(6 \mathrm{~m})^{2}+\frac{0.0112(50 \mathrm{kN})(6 \mathrm{~m})^{2}}{5 m}=-105.05 \mathrm{kN}-\mathrm{m} \\
& \mathrm{R}_{\mathrm{DH}}=-0.093 \mathrm{wh}-0.0551 \mathrm{Ph} / \mathrm{L}=-0.093\left(10^{k N / m}\right)(6 \mathrm{~m})-\frac{0.0551(50 \mathrm{kN})(6 \mathrm{~m})}{5 m}=-8.89 \mathrm{kN} \\
& \mathrm{R}_{\mathrm{DV}}=0.197 \mathrm{wh}^{2} / \mathrm{L}+0.516 \mathrm{P}=\frac{0.197\left(10^{k N / m}\right)(6 \mathrm{~m})^{2}}{5 m}+0.516(50 \mathrm{kN})=39.98 \mathrm{kN}
\end{aligned}
$$

Member End Forces The free-body diagrams of all the members and joints of the frame are shown below. The unknowns on the members are drawn as anticipated, and the opposite directions are drawn on the joint. We can begin the computation of internal forces with either member AB or CD , both of which have only three unknowns.


Member $A B$ With the magnitudes of reaction forces at A know, the unknowns are at end B of $\mathrm{BA}_{\mathrm{x}}, \mathrm{BA}_{\mathrm{y}}$, and $\mathrm{M}_{\mathrm{BA}}$, which can get determined by applying $\sum F_{x}=0, \sum F_{y}=0$, and $\sum M_{B}=0$. Thus,
$\sum F_{x}=-51.11 \mathrm{kN}+10 \mathrm{kN}(6 m)-B A_{x}=0 \quad \mathrm{BA}_{x}=8.89 \mathrm{kN}, \sum F_{y}=10.02 \mathrm{kN}-B A_{y}=0 \quad$ BAy $=10.02 \mathrm{kN}$
$\sum M_{A}=105.05^{k N-m}-10^{k N / m}(6 m)(3 m)+8.89 k N(6 m)+M_{B A}=0 \quad \mathrm{M}_{\mathrm{BA}}=21.16 \mathrm{kN}-\mathrm{m}$
Joint B Because the forces and moments must be equal and opposite, $\mathrm{BC}_{\mathrm{x}}=8.89 \mathrm{kN}, \mathrm{BC}_{\mathrm{y}}=10.02 \mathrm{kN}$ and $\mathrm{M}_{\mathrm{BC}}=21.16 \mathrm{kN} \cdot \mathrm{m}$

Member $C D$ With the magnitudes of reaction forces at D know, the unknowns are at end C of $\mathrm{CD}_{\mathrm{x}}, \mathrm{CD}_{\mathrm{y}}$, and $\mathrm{M}_{\mathrm{CD}}$, which can get determined by applying $\sum F_{x}=0, \sum F_{y}=0$, and $\sum M_{B}=0$. Thus,
$\sum F_{x}=-8.89 \mathrm{kN}+C D_{x}=0 \quad \mathrm{CD}_{\mathrm{x}}=8.89 \mathrm{kN}, \quad \sum F_{y}=39.98 \mathrm{kN}-C D_{y}=0 \quad \mathrm{CD}_{\mathrm{y}}=39.98 \mathrm{kN}$
$\sum M_{D}=-8.89 \mathrm{kN}(6 m)+M_{C D}=0 \quad M_{\mathrm{DC}}=53.34 \mathrm{kN}-\mathrm{m}$
Joint $C$ Because the forces and moments must be equal and opposite, $\mathrm{CB}_{\mathrm{x}}=8.89 \mathrm{kN}, \mathrm{CB}_{\mathrm{y}}=39.98 \mathrm{kN}$ and $\mathrm{M}_{\mathrm{CB}}=53.34 \mathrm{kN}-\mathrm{m}$

Member BC All forces are known, so equilibrium can be checked.
(Remember: To find the point of zero shear with a distributed load, divide the peak $\{$ triangle $\}$ shear by the distributed load; ex. $\left.51.11 \mathrm{kN} /\left(10^{\mathrm{kN} / \mathrm{m}}\right)=5.11 \mathrm{~m}\right)$


## Example 3

Using Multiframe4D, verify the bending moment diagram for the example in Figure 9.9:


Figure 9.9 The moment distribution illustrates the importance of relative stiffness considerations. The values obtained are quite different from those obtained by estimating points of inflection and using hand calculations.

| Joint Coordinates (ft) |  |  |  |  |
| :--- | :---: | ---: | ---: | ---: |
| Joint | Label | $X$ | $Y$ | $z$ |
| 1 |  | 0.000 | 0.000 | 0.000 |
| 2 | 0.000 | 19.500 | 0.000 |  |
| 3 | 120.000 | 19.500 | 0.000 |  |
| 4 | 120.000 | 0.000 | 0.000 |  |

Assuming steel $(\mathrm{E}=29,000 \mathrm{ksi})$

Sections

Section Properties

| Section | A | Ix | Ix |
| ---: | ---: | ---: | ---: |
|  | in $^{2}$ | in $^{\wedge}$ | in^4 |
| mies-slender | 1.000 | 2380.000 | 2380.000 |
| mies-stiff | 1.000 | 58700.001 | 58700.001 |




## Example 3 (continued)

Displacement:


Maximum Actions for all members (column-1, beam-2, column-3):

|  | Memb | Label | Section | Sign | $\begin{aligned} & \mathbf{P x}^{*} \\ & \text { kip } \end{aligned}$ | $\begin{aligned} & \text { Vy }{ }^{*} \\ & \text { kip } \end{aligned}$ | $\begin{aligned} & \text { Vz' } \\ & \text { kip } \end{aligned}$ | $\begin{gathered} \text { Tx' } \\ \text { kip-ft } \end{gathered}$ | $\begin{aligned} & \text { My' } \\ & \text { kip-ft } \end{aligned}$ | $\underset{\text { Mip-ft }}{\text { Mz }}$ | $\begin{aligned} & \text { dy } y^{*} \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \hline \mathbf{d z} \\ & \text { in } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 |  | mies-slender | +ve | 216.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1424.716 | 0.486 | 0.000 |
| 2 | 1 |  | mies-slender | -ve | 0.000 | -109.079 | 0.000 | 0.000 | 0.000 | -702.318 | -0.032 | 0.000 |
| 3 | 1 |  | mies-slender | abs | 216.000 | 109.079 | 0.000 | 0.000 | 0.000 | 1424.716 | 0.486 | 0.000 |
| 4 | 2 |  | mies-stiff | +ve | 109.079 | 216.000 | 0.000 | 0.000 | 0.000 | 1424.716 | 0.000 | 0.000 |
| 5 | 2 |  | mies-stiff | -ve | 0.000 | -216.000 | 0.000 | 0.000 | 0.000 | -5055.282 | -7.326 | 0.000 |
| 6 | 2 |  | mies-stiff | abs | 109.079 | 216.000 | 0.000 | 0.000 | 0.000 | 5055.282 | 7.326 | 0.000 |
| 7 | 3 |  | mies-slender | +ve | 216.000 | 109.079 | 0.000 | 0.000 | 0.000 | 702.318 | 0.032 | 0.000 |
| 8 | 3 |  | mies-slender | -ve | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | -1424.716 | -0.486 | 0.000 |
| 9 | 3 |  | mies-slender | abs | 216.000 | 109.079 | 0.000 | 0.000 | 0.000 | 1424.716 | 0.486 | 0.000 |

(axes orientation reference)



Maximum Stresses for all members (column-1, beam-2, column-3):

|  | Memb | Label | Section | Sign | Sbz' top ksi | Sbz' bot ksi | $\begin{aligned} & S x^{*} \\ & \mathbf{k s i} \end{aligned}$ | $\begin{array}{\|c} \hline \mathbf{S x}^{*}+\mathrm{Sbz} z^{*} \\ \text { top } \\ \text { ksi } \\ \hline \end{array}$ | $\begin{gathered} \mathrm{Sx}^{\prime}+\mathrm{Sb} z^{*} \\ \text { bot } \\ \text { koi } \end{gathered}$ | $\begin{aligned} & \text { dy } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \text { dz' } \\ & \text { in } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 |  | mies-sI | +ve | 42.494 | 86.203 | 7.714 | 50.208 | (93.917 | 0.486 | 0.000 |
| 2 | 1 |  | mies-slen | -ve | -86.203 | -42.494 | 0.000 | -78.489 | 34.780 | -0.032 | 0.000 |
| 3 | 1 |  | mies-slen | abs | 86.203 | 86.203 | 7.714 | 78.489 | 93.917 | 0.486 | 0.000 |
| 4 | 2 |  | mies-sti | +ve | 38.237 | 10.776 | 1.283 | 39.521 | 12.060 | 0.000 | 0.000 |
| 5 | 2 |  | mies-stiff | -ve | -10.776 | -38.237 | 0.000 | -9.493 | -36.954 | -7.326 | 0.000 |
| 6 | 2 |  | mies-stiff | abs | 38.237 | 38.237 | 1.283 | -39.524 | 36.954 | 7.326 | 0.000 |
| 7 | 3 |  | mies-sI | +ve | 86.203 | 42.494 | 7.714 | 93.917 | ) 50.208 | 0.032 | 0.000 |
| 8 | 3 |  | mies-slen | -ve | -42.494 | -86.203 | 0.000 | -34.780 | -78.489 | -0.486 | 0.000 |
| 9 | 3 |  | mies-slen | abs | 86.203 | 86.203 | 7.714 | 93.917 | 78.489 | 0.486 | 0.000 |

Beam-Column stress verification (combined stresses) when d=24in, A=28 in ${ }^{2}$. $\mathrm{I}_{\mathrm{x}}=2380 \mathrm{in}^{4}$ :

$$
f_{\max }=\frac{P}{A}+\frac{M}{S}=\frac{P}{A}+\frac{M c}{I}=\frac{216 k}{28 \mathrm{in}^{2}}+\frac{1425^{k-f t} \cdot(24 \mathrm{in} / 2)}{2380 \mathrm{in}^{4}} \cdot \frac{12 \mathrm{in}}{f t}=7.71 \mathrm{ksi}+86.22 \mathrm{ksi}=93.93 \mathrm{ksi}
$$

Frame Analysis by Coefficients and Live Load Reduction from Simplified Design, $3^{\text {rd }}$ ed., Portland Cement Association, 2004

### 2.2.2 Live Load Reduction for Columns, Beams, and Slabs

Most general building codes permit a reduction in live load for design of columns, beams and slabs to account for the probability that the total floor area "influencing" the load on a member may not be fully loaded simultaneously. Traditionally, the reduced amount of live load for which a member must be designed has been based on a tributary floor area supported by that member. According to ASCE 7-02, the magnitude of live load reduction is based on an influence area rather than a tributary area. The influence area is a function of the tributary area for the structural member. The influence area for different structural members is calculated by multiplying the tributary area for the member $\mathrm{A}_{\mathrm{T}}$, by the coefficients $\mathrm{K}_{\mathrm{LL}}$ given in Table 2-3, see ASCE 4.8.

The reduced live load L per square foot of floor area supported by columns, beams, and two-way slabs having an influence area ( $\mathrm{K}_{\mathrm{LL}} \mathrm{A}_{\mathrm{T}}$ ) of more than 400 sq ft is:

$$
\mathrm{L}=\mathrm{L}_{0}\left(0.25+\frac{15}{\sqrt{\mathrm{~K}_{\mathrm{LL}} \mathrm{~A}_{\mathrm{T}}}}\right) \quad \text { ASCE (Eq. 4-1) }
$$

where $\mathrm{L}_{0}$ is the unreduced design live load per square foot. The reduced live load cannot be taken less than $50 \%$ for members supporting one floor, or less than $40 \%$ of the unit live load $\mathrm{L}_{0}$ otherwise. For other limitations on live load reduction, see ASCE 4.8.

Using the above expression for reduced live load, values of the reduction multiplier as a function of influence area are given in Table 2-4.

Table 2-3 Live Load Element Factor KLL

| Element | $\mathrm{K}_{\mathrm{LL}}$ |
| :--- | :---: |
| Interior columns | 4 |
| Exterior column without cantilever slabs | 4 |
| Edge column with cantilever slabs | 3 |
| Corner columns with cantilever slabs | 2 |
| Edge beams without cantilever slabs | 2 |
| Interior beams | 2 |
| All other members not identified above including: <br> Edge beams with cantilever slabs <br> Cantilever beams <br> Two-way slabs | 1 |

The above limitations on permissible reduction of live loads are based on ASCE 4.8. The governing general building code should be consulted for any difference in amount of reduction and type of members that may be designed for a reduced live load.

### 2.3 FRAME ANALYSIS BY COEFFICIENTS

The ACI Code provides a simplified method of analysis for both one-way construction (ACI 8.3.3) and twoway construction (ACI 13.6). Both simplified methods yield moments and shears based on coefficients. Each method will give satisfactory results within the span and loading limitations stated in Chapter 1. The direct design method for two-way slabs is discussed in Chapter 4.

Table 2-4 Reduction Multiplier (RM) for Live Load $=\left(0.25+\frac{15}{\sqrt{\mathrm{~K}_{\mathrm{LL}} \mathrm{A}_{\mathrm{T}}}}\right)$

| Influence Area $K_{L L} A_{T}$ | RM | Influence Area $\mathrm{K}_{\mathrm{LI}} \mathrm{A}_{\mathrm{T}}$ | RM |
| :---: | :---: | :---: | :---: |
| $40{ }^{2}$ | 1.000 | 5600 | 0.450 |
| 800 | 0.780 | 6000 | 0.444 |
| 1200 | 0.683 | 6400 | 0.438 |
| 1600 | 0.625 | 6800 | 0.432 |
| 2000 | 0.585 | 7200 | 0.427 |
| 2400 | 0.556 | 7600 | 0.422 |
| 2800 | 0.533 | 8000 | 0.418 |
| 3200 | 0.515 | 8400 | 0.414 |
| 3600 | $0.500^{\text {b }}$ | 8800 | 0.410 |
| 4000 | 0.487 | 9200 | 0.406 |
| 4800 | 0.467 | 10000 | $0.400^{\text {c }}$ |
| 5200 | 0.458 |  |  |

### 2.3.1 Continuous Beams and One-Way Slabs

When beams and one-way slabs are part of a frame or continuous construction, ACI 8.3.3 provides approximate moment and shear coefficients for gravity load analysis. The approximate coefficients may be used as long as all of the conditions illustrated in Fig. 2-2 are satisfied: (1) There must be two or more spans, approximately equal in length, with the longer of two adjacent spans not exceeding the shorter by more than 20 percent; (2) loads must be uniformly distributed, with the service live load not more than 3 times the dead load ( $\mathrm{L} / \mathrm{D} \leq 3$ ): and (3) members must have uniform cross section throughout the span. Also, no redistribution of moments is permitted (ACI 8.4). The moment coefficients defined in ACI 8.3.3 are shown in Figs. 2-3 through 2-6. In all cases, the shear in end span members at the interior support is taken equal to $1.15 \mathrm{w}_{\mathrm{u}} \ell_{\mathrm{n}} / 2$. The shear at all other supports is $w_{u} / 2$ (see Fig. 2-7). $w_{u} \ell_{n}$ is the combined factored load for dead and live loads, $w_{u}=1.2 w_{d}+1.6 w_{f}$. For beams, $w_{u}$ is the uniformly distributed load per unit length of beam (plf), and the coefficients yield total moments and shears on the beam. For one-way slabs, $w_{u}$ is the uniformly distributed load per unit area of slab (psf), and the moments and shears are for slab strips one foot in width. The span length $\ell_{n}$ is defined as the clear span of the beam or slab. For negative moment at a support with unequal adjacent spans, $i_{\mathrm{n}}$ is the average of the adjacent clear spans. Support moments and shears are at the faces of supports.


Figure 2-2 Conditions for Analysis by Coefficients (ACl 8.3.3)


Figure 2-3 Positive Moments-All Cases


Figure 2-4 Negative Moments-Beams and Slabs


Figure 2-5 Negative Moments-Slabs with spans $\leq 10$ ft


Figure 2-6 Negative Moments-Beams with Stiff Columns $\left(\Sigma K_{d} / \Sigma K_{b}>8\right)$


Figure 2-7 End Shears-All Cases

### 4.3 TWO-WAY SLAB ANALYSIS BY COEFFICIENTS

For gravity loads, ACI Chapter 13 provides two analysis methods for two-way slab systems: 1) the Direct Design Method (ACI 13.6) and the Equivalent Frame Method (ACI 13.7). The Equivalent Frame Method, using member stiffnesses and complex analytical procedures, is not suitable for hand calculations. Only the Direct Design Method, using moment coefficients, will be presented in this Chapter.

Table 4-1 Minimum Thickness for Two-Way Slab Systems

| Two-Way Slab System | $\alpha_{m}$ | $\beta$ | Minimum h |
| :---: | :---: | :---: | :---: |
| Flat Plate <br> Flat Plate with Spandrel Beams ${ }^{1}$ <br> [Min. $\mathrm{h}=5 \mathrm{in}$.] | - | $\begin{aligned} & \leq 2 \\ & \leq 2 \end{aligned}$ | $\begin{aligned} & \ln / 30 \\ & \ln _{n} / 33 \end{aligned}$ |
| Flat Slab ${ }^{2}$ <br> Flat Slab ${ }^{2}$ with Spandrel Beams ${ }^{1}$ <br> [Min. $\mathrm{h}=4 \mathrm{in}$.] | - | $\begin{aligned} & \leq 2 \\ & \leq 2 \end{aligned}$ | $\begin{aligned} & \ln / 33 \\ & \ln / 36 \end{aligned}$ |
| Two-Way Beam-Supported Slab ${ }^{3}$ | $\begin{gathered} \leq 0.2 \\ 1.0 \\ \geq 2.0 \end{gathered}$ | $\begin{gathered} \leq 2 \\ 1 \\ 2 \\ 1 \\ 2 \\ \hline \end{gathered}$ | $\begin{aligned} & \ln / 30 \\ & \ln ^{n} / 33 \\ & \ln _{n} / 36 \\ & \ln ^{2} / 37 \\ & \ln / 44 \end{aligned}$ |
| Two-Way Beam-Supported Slab ${ }^{\text {1,3 }}$ | $\begin{gathered} \leq 0.2 \\ 1.0 \\ \geq 2.0 \end{gathered}$ | $\begin{gathered} \leq 2 \\ 1 \\ 2 \\ 1 \\ 2 \end{gathered}$ | $\begin{aligned} & \ln / 33 \\ & \ln ^{2} / 36 \\ & \ln ^{2} / 40 \\ & \ln / 41 \\ & \ln / 49 \end{aligned}$ |

${ }^{1}$ Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$ (ACl 9.5.3.3)
${ }^{2}$ Drop panel length $\geq d / 3$, depth $\geq 1.25 \mathrm{~h}$ ( ACl 13.4 .7 )
${ }^{3} \mathrm{Min} . \mathrm{h}=5$ in. for $\alpha_{\mathrm{m}} \leq 2.0$; min. $\mathrm{h}=3.5$ in. for $\alpha_{\mathrm{m}}>2.0$ (ACl 9.5.3.3)
$\alpha$ is the ratio of flexural stiffness of a beam section ti the slab; $\alpha_{m}$ is the average $\alpha$ for all beams on edges of a panel $\beta$ is the ratio of clear spans in long to short direction

The Direct Design Method applies when all of the conditions illustrated in Fig. 4-4 are satisfied (ACI 13.6.1):

- There must be three or more continuous spans in each direction.
- Slab panels must be rectangular with a ratio of longer to shorter span (c/c of supports) not greater than 2.
- Successive span lengths (c/c of supports) in each direction must not differ by more than one-third of the longer span.
- Columns must not be offset more than $10 \%$ of the span (in direction of offset) from either axis between centerlines of successive columns.
- Loads must be due to gravity only and must be uniformly distributed over the entire panel. The live load must not be more than 3 times the dead load ( $\mathrm{L} / \mathrm{D} \leq 3$ ). Note that if the live load exceeds onehalf the dead load ( $\mathrm{L} / \mathrm{D}>0.5$ ), column-to-slab stiffness ratios must exceed the applicable values given in ACI Table 13.6.10, so that the effects of pattern loading can be neglected. The positive factored moments in panels supported by columns not meeting such minimum stiffness requirements must be magnified by a coefficient computed by ACI Eq. (13-5).
- For two-way slabs, relative stiffnesses of beams in two perpendicular directions must satisfy the minimum and maximum requirements given in ACI 13.6.1.6.
- Redistribution of moments by ACI 8.4 shall not be permitted.


Figure 4-4 Conditions for Analysis by Coefficients
In essence, the Direct Design Method is a three-step analysis procedure. The first step is the calculation of the total design moment $\mathrm{M}_{0}$ for a given panel. The second step involves the distribution of the total moment to the negative and positive moment sections. The third step involves the assignment of the negative and positive moments to the column strips and middle strips.

For uniform loading, the total design moment $\mathrm{M}_{0}$ for a panel is calculated by the simple static moment expression, ACI Eq. (13-3):

$$
\mathrm{M}_{\mathrm{o}}=\mathrm{w}_{\mathrm{u}} \mathrm{l}_{2} \mathrm{l}_{\mathrm{n}}{ }^{2} / 8
$$

where $\mathrm{w}_{\mathrm{u}}$ is the factored combination of dead and live loads (psf), $\mathrm{w}_{\mathrm{u}}=1.2 \mathrm{w}_{\mathrm{d}}+1.6 \mathrm{w}_{\ell} \quad$ The clear span $\ell_{\mathrm{n}}$ is defined in a straightforward manner for columns or other supporting elements of rectangular cross section (ACI 12.6.2.5). Note that circular or regular polygon shaped supports shall be treated as square supports with the same area (see ACI Fig. R13.6.2.5). The clear span starts at the face of support. One limitation requires that the clear span never be taken less than $65 \%$ of the span center-to-center of supports (ACI 13.6.2.5). The span $\ell_{2}$ is simply the span transverse to $\ell_{n}$; however, when the span adjacent and parallel to an edge is being considered, the distance from edge of slab to panel centerline is used for $\ell_{2}$ in calculation of $\mathrm{M}_{0}$ (ACI 13.6.2.4).

Division of the total panel moment $\mathrm{M}_{0}$ into negative and positive moments, and then into column and middle strip moments, involves direct application of moment coefficients to the total moment $\mathrm{M}_{0}$. The moment coefficients are a function of span (interior or exterior) and slab support conditions (type of two-way slab system). For design convenience, moment coefficients for typical two-way slab systems are given in Tables 4-2 through 4-6. Tables 4-2 through 4-5 apply to flat plates or flat slabs with various end support conditions. Table 4-6 applies to twoway slabs supported on beams on all four sides. Final moments for the column strip and middle strip are computed directly using the tabulated values. All coefficients were determined using the appropriate distribution factors in ACI 13.6.3 through 13.6.6.

NOTE: The interior column strip is defined by one quarter of the smaller of $\ell_{1}$ and $\ell_{2}$ each side of the column centerline. The exterior column strip is bound by the slab edge and one quarter of the smaller of $\ell_{1}$ and $\ell_{2}$ from the column centerline. The middle strip is the remaining width between column strips.

The column strip and middle strip moments are distributed over an effective slab width as illustrated in Fig. 4-9. The column strip is defined as having a width equal to one-half the transverse or longitudinal span, whichever is smaller (ACI 13.2.1). The middle strip is bounded by two column strips.


Figure 4-9 Definition of Design Strips
Once the slab and beam (if any) moments are determined, design of the slab and beam sections follows the simplified design approach presented in Chapter 3. Slab reinforcement must not be less than that given in Table $3-5$, with a maximum spacing of 2 h or 18 in . (ACI 13.4).

Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns

| ${ }^{+}$ |  |  | 17 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{H}{(1)}$ | End Span (2) | (3) | Interior Span <br> (4) | (5) |  |
|  | End Span |  |  | Interior Span |  |
| Slab Moments |  | 2 <br> Positive | 3 <br> First Interior Negative | Positive | 5 <br> Interior Negative |
| Total Moment | $0.26 \mathrm{M}_{0}$ | $0.52 \mathrm{M}_{0}$ | $0.70 \mathrm{M}_{0}$ | $0.35 \mathrm{M}_{0}$ | $0.65 \mathrm{M}_{0}$ |
| Column Strip | $0.26 \mathrm{M}_{0}$ | $0.31 \mathrm{M}_{0}$ | $0.53 \mathrm{M}_{0}$ | $0.21 \mathrm{M}_{0}$ | $0.49 \mathrm{M}_{0}$ |
| Middle Strip | 0 | $0.21 \mathrm{M}_{0}$ | $0.17 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.16 \mathrm{M}_{0}$ |

Note: All negative moments are at face of support.
Table 4-3 Flat Plate or Flat Slab with Spandrel Beams

|  |  | T] |  |  | $I_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | End Span (2) |  | Interior Span <br> (4) |  |  |
|  | End Span |  |  | Interior Span |  |
| Slab Moments | 1 <br> Exterior Negative | $2$ <br> Positive | $3$ <br> First Interior Negative | Positive |  |
| Total Moment | $0.30 \mathrm{M}_{0}$ | $0.50 \mathrm{M}_{0}$ | $0.70 \mathrm{M}_{0}$ | $0.35 \mathrm{M}_{0}$ | $0.65 \mathrm{M}_{0}$ |
| Column Strip | $0.23 \mathrm{M}_{0}$ | $0.30 \mathrm{M}_{0}$ | $0.53 \mathrm{M}_{0}$ | $0.21 \mathrm{M}_{0}$ | $0.49 \mathrm{M}_{0}$ |
| Middle Strip | $0.07 \mathrm{M}_{0}$ | $0.20 \mathrm{M}_{0}$ | $0.17 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.16 \mathrm{M}_{0}$ |

Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of spandrel beams $\beta_{\mathrm{t}} \geq 2.5$. For values of $\beta_{\mathrm{t}}$ less than 2.5 , exterior negative column strip moment increases to $\left(0.30-0.03 \beta_{\mathrm{t}}\right) \mathrm{M}_{\mathrm{o}}$.

Table 4-4 Flat Plate or Flat Slab with End Span Integral with Wall

(1)
(2)
(3)
(4)
(5)

| Slab Moments | End Span |  |  | Interior Span |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | 1 <br> Exterior <br> Negative | 2 | 3 <br> Positive | First Interior <br> Negative | 5 <br> Positive |
|  | $0.65 \mathrm{M}_{\mathrm{O}}$ | $0.35 \mathrm{M}_{\mathrm{O}}$ | $0.65 \mathrm{M}_{\mathrm{O}}$ | $0.35 \mathrm{M}_{\mathrm{O}}$ | $0.65 \mathrm{M}_{\mathrm{O}}$ |
| Negatior |  |  |  |  |  |$|$

Note: All negative moments are at face of support.

Table 4-5 Flat Plate or Flat Slab with End Span Simply Supported on Wall


| (1) | (2) | (3) |  | (5) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | End Span |  |  | Interior Span |  |
| Slab Moments | 1 <br> Exterior <br> Negative | $2$ <br> Positive | 3 First Interior Negative | Positive | 5 <br> Interior <br> Negative |
| Total Moment | 0 | $0.63 \mathrm{M}_{0}$ | $0.75 \mathrm{M}_{0}$ | $0.35 \mathrm{M}_{0}$ | $0.65 \mathrm{M}_{0}$ |
| Column Strip | 0 | $0.38 \mathrm{M}_{0}$ | $0.56 \mathrm{M}_{0}$ | $0.21 \mathrm{M}_{0}$ | $0.49 \mathrm{M}_{0}$ |
| Middle Strip | 0 | $0.25 \mathrm{M}_{0}$ | $0.19 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.16 \mathrm{M}_{0}$ |

Note: All negative moments are at face of support.

Table 4-6 Two-Way Beam-Supported Slab

|  |  |  | T |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) End Span |  | Interior Span <br> (4) |  |  |  |
|  | Slab Moments | End Span |  |  | Interior Span |  |
| $\begin{aligned} & \text { Span } \\ & \text { ratio } \end{aligned}$ |  | 1 <br> Exterior <br> Negative | Positive | 3 <br> First Interior Negative | 4 Positive | 5 <br> Interior <br> Negative |
| $\mathrm{CO}_{2} 4$ | Total Moment | $0.16 \mathrm{M}_{0}$ | $0.57 \mathrm{M}_{0}$ | $0.70 \mathrm{M}_{0}$ | $0.35 \mathrm{M}_{0}$ | $0.65 \mathrm{M}_{0}$ |
| 0.5 | Column Strip Beam Slab | $\begin{aligned} & 0.12 \mathrm{M}_{\mathrm{O}} \\ & 0.02 \mathrm{M}_{\mathrm{o}} \end{aligned}$ | $\begin{aligned} & 0.43 \mathrm{M}_{\mathrm{O}} \\ & 0.08 \mathrm{Mo} \end{aligned}$ | $\begin{aligned} & 0.54 \mathrm{M}_{\mathrm{O}} \\ & 0.09 \mathrm{M}_{\mathrm{O}} \end{aligned}$ | $\begin{aligned} & 0.27 \mathrm{M}_{\mathrm{O}} \\ & 0.05 \mathrm{M}_{\mathrm{O}} \end{aligned}$ | $\begin{aligned} & 0.50 \mathrm{M}_{\mathrm{O}} \\ & 0.09 \mathrm{M}_{\mathrm{O}} \end{aligned}$ |
|  | Middle Strip | $0.02 \mathrm{M}_{0}$ | $0.06 \mathrm{M}_{0}$ | $0.07 \mathrm{M}_{0}$ | $0.03 \mathrm{M}_{0}$ | $0.06 \mathrm{M}_{0}$ |
|  | Column Strip Beam | $0.10 \mathrm{M}_{0}$ | $0.37 \mathrm{Mo}_{0}$ | $0.45 \mathrm{M}_{0}$ | $0.22 \mathrm{M}_{0}$ | 0.42 Mo |
| 1.0 | Slab | $0.02 \mathrm{Mo}_{0}$ | $0.06 \mathrm{Mo}_{0}$ | $0.08 \mathrm{Mo}_{0}$ | $0.04 \mathrm{M}_{0}$ | $0.07 \mathrm{M}_{0}$ |
|  | Middle Strip | $0.04 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.17 \mathrm{M}_{0}$ | $0.09 \mathrm{M}_{0}$ | $0.16 \mathrm{M}_{0}$ |
|  | Column Strip Beam | $0.06 \mathrm{M}_{0}$ | $0.22 \mathrm{Mo}_{0}$ | $0.27 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.25 \mathrm{M}_{0}$ |
| 2.0 | Slab | $0.01 \mathrm{M}_{0}$ | $0.04 \mathrm{M}_{0}$ | $0.05 \mathrm{M}_{0}$ | $0.02 \mathrm{M}_{0}$ | $0.04 \mathrm{M}_{0}$ |
|  | Middle Strip | $0.09 \mathrm{M}_{0}$ | $0.31 \mathrm{M}_{0}$ | $0.38 \mathrm{M}_{0}$ | $0.19 \mathrm{M}_{0}$ | $0.36 \mathrm{M}_{0}$ |

Notes: (1) Beams and slab satisfy stiffness criteria: $\alpha_{1} \ell_{2} / \ell_{1} \geq 1.0$ and $\beta_{\mathrm{t}} \geq 2.5$.
(2) Interpolate between values shown for different $\ell_{2} / \ell_{1}$ ratios.
(3) All negative moments are at face of support.
(4) Concentrated loads applied directly to beams must be accounted for separately.

# Concrete Floor Systems 

## GUIDE TO ESTIMATING AND ECONOMIZING

By August W. Domel Jr. and S.K. Ghosh


The main objectives of this publication are to:

- Assist in the selection of the most economical cast-in-place concrete floor system for a given plan layout and a given set of loads;
- Provide a preliminary estimate of material quantities for the floor system; and
- Discuss the effect of different variables in the selection process.
Five different floor systems are considered in this publication. These are the flat plate, the flat slab, the one-way joist, the two-way joist or waffle, and the slab supported on beams on all four sides. Material quantity estimates are given for each floor system for various bay sizes.


## Pricing Trends

The total cost to construct a building depends on the use for which the structure is designed, the availability of qualified contractors, and the part of the country in which the structure is built. Figure 1 gives cost comparisons for two different types of uses over the past several years. (The data presented in Figures 1 through 5 and Table 1 were obtained from Means Concrete Cost Data, 1990.) The average price per square foot is considerably greater for office buildings than for apartment buildings. Part of the higher


Figure 1 - Price Comparisons for Different Building Types
cost is because office buildings are designed with more open spaces which in structural terms means costlier, longer clear spans.

Table 1 gives cost indices for many major cities in the United States and Canada. The cost index includes both labor and materials, with the value of 100 representing the average cost for 30 major cities. The table shows the wide variation in costs depending on the locale. In Anchorage, Alaska (127.9) or New York City (126.9) the cost of a building can be as much as $60 \%$ higher than that of a similar building in Charleston, South Carolina (80.2), Jackson, Mississippi (81) or Sioux Falls, South Dakota (82.2). Figure 2 shows the relative change in costs in current dollars of material and labor over the past 40 years.


Figure 2 - Annual Construction Cost Comparisons

The majority of the structural cost of a building typically is the cost of the floor system. This is particularly true of low-rise buildings and buildings in low seismic zones. Therefore, it is imperative to select the most economical floor system.

In this publication, estimated quantities are provided for concrete, reinforcing steel and formwork for the five floor systems discussed in the following sections. Prices for labor and material for these items over the past several years are shown in Figures 3 through 5.

Table 1-Relative Construction Costs for Reinforced Concrete

| ALABAMA (BIRMINGHAM) | 84.0 | NEW HAMPSHIRE (MANCHESTER) | 90.3 |
| :--- | ---: | :--- | ---: |
| ALASKA (ANCHORAGE) | 127.9 | NEW JERSEY (NEWARK) | 104.9 |
| ARIZONA (PHOENIX) | 91.9 | NEW MEXICO (ALBUQUERQUE) | 91.5 |
| ARKANSAS (LITILE ROCK) | 84.5 | NEW YORK (NEW YORK) | 126.9 |
| CALIFORNIA (LOS ANGELES) | 112.0 | NEW YORK (ALBANY) | 94.5 |
| CALIFORNIA (SAN FRANCISCO) | 126.0 | NORTH CAROLINA (CHARLOTIE) | 80.8 |
| COLORADO (DENVER) | 93.5 | OHIO (CLEVELAND) | 107.3 |
| CONNECTICUT (HARTFORD) | 100.1 | OHIO (CINCINNATI) | 95.3 |
| DELAWARE (WILMINGTON) | 100.3 | OKLAHOMA (OKLAHOMA CITY) | 89.4 |
| WASHINGTON, D.C. | 95.4 | OREGON (PORTLAND) | 101.0 |
| FLORIDA (MIAMI) | 89.9 | PENNSYLVANIA (PHILADELPHIA) | 107.2 |
| GEORGIA (ATLANTA) | 89.7 | PENNSYLVANIA (PITTSBURGH) | 100.6 |
| HAWAII (HONOLULU) | 111.1 | RHODE ISLAND (PROVIDENCE) | 100.8 |
| IDAHO (BOISE) | 93.3 | SOUTH CAROLNA (CHARLESTON) | 80.2 |
| ILLINOIS (CHICAGO) | 101.8 | SOUTH DAKOTA (SIOUX FALLS) | 82.2 |
| INDIANA (INDIANAPOLIS) | 97.6 | TENNESSEE (MEMPHIS) | 87.6 |
| IOWA (DES MOINES) | 90.7 | TEXAS (DALAS) | 87.8 |
| KANSAS (WICHITA) | 86.8 | UTAH (SALT LAKE CITY) |  |
| KENTUCKY (LOUISVILE) | 88.3 | VERMONT (BURLINGTON) | 91.7 |
| LOUISIANA (NEW ORLEANS) | 88.6 | VRGINIA (NORFOLK) | 88.1 |
| MAINE (PORTLAND) | 89.8 | WASHINGTON (SEATTLE) | 83.3 |
| MARYLAND (BALTIMORE) | 96.1 | WEST VIRGINIA (CHARLESTON) | 101.6 |
| MASSACHUSETTS (BOSTON) | 115.6 | WSCONSIN (MILWAUKEE) | 97.4 |
| MICHIGAN (DEIROIT) | 106.9 | WYOMING (CHEYENNE) | 97.3 |
| MINNESOTA (MINNEAPOLIS) | 99.4 | CANADA (EDMONTON) | 87.4 |
| MISSISSIPPI (JACKSON) | 81.0 | CANADA (MONTREAL) | 100.2 |
| MISSOURI (ST. LOUIS) | 101.6 | CANADA (QUEBEC) | 100.0 |
| MONTANA (BILLINGS) | 92.1 | CANADA (TORONTO) | 99.0 |
| NEBRASKA (OMAHA) | 88.6 | CANADA (VANCOUVER) | 109.8 |
| NEVADA (LAS VEGAS) | 104.6 | CANADA (WINNIPEG) | 105.5 |
|  |  |  | 101.5 |



Figure 3-Cost of Reinforcing Bars in Place


Figure 4 - Cost of Ready-Mixed Concrete


Figure 5 - Cost of Formwork

## Presentation of Results

The following pages provide discussion and quantity estimates for the five floor systems. These results were obtained using a five bay by five bay structure. Bay sizes are measured from centerline of column to centerline of column. Floors were designed using ACI 318-89 Building Code Requirements for Reinforced Concrete. Concrete, reinforcing steel and formwork quantities are presented for each of the floor systems. An overview of the floor systems is provided, following the discussion of the floor systems.

Included with each floor system is a discussion of the factors that may affect the estimated quantities. The factors discussed are column dimensions, live loads, and aspect ratios. A cost breakdown is also given in each case. Following the discussion for each individual floor system are several tables and graphs. The graphs show the variation in costs for increased bay size and higher concrete strength. The tables give quantities for various bay sizes.

## Fire Resistance of Concrete Floor Systems

Fire resistance rated construction will often be required by the governing building code, or the owner may desire a highly fire resistant structure
in order to qualify for the lowest fire insurance rates.

Concrete floor systems offer inherent fire resistance. Therefore, when the floor system is completed, no additional protective measures are necessary in order to achieve code required fire resistance ratings.

On the other hand, for steel floor systems for instance, additional protection must be provided by special acoustical ceilings, or fireproofing sprayed on the underside of the steel deck and/or beams. In addition, when an acoustical ceiling is an integral part of a rated floor/ceiling assembly, special ceiling suspension systems, and special protective devices at penetrations for light fixtures and HVAC diffusers are required.

These additional costs associated with protecting the structural framing members must be added to the cost of the structural frame to produce an accurate cost estimate. If this is not done, the actual cost of the competing floor system is understated, making a valid comparison with a concrete floor system difficult, if not impossible.

Fire resistance rating requirements vary from zero to four hours, with two hours typically being required for high rise buildings. Before selecting the floor system, the designer should determine the fire resistance rating required by the applicable building code. Except for oneway and two-way joist systems, the minimum slab thickness necessary to satisfy structural requirements (usually 5 in .) will normally provide a floor system that has at least a two hour fire resistance rating.

Table 2 shows minimum slab thicknesses necessary to provide fire resistance ratings from one to four hours, for different types of aggregate. If the thickness necessary to satisfy fire resistance requirements exceeds that required for structural purposes, consideration should be given to using a different type of aggregate that provides higher fire resistance for the same thickness. For example, a one-way joist system may require a 3 in. thick slab to satisfy structural requirements. However, if a two hour fire resistance rating is desired, a 5 in. thick slab will be required if siliceous aggregate normal weight concrete is used. By using lightweight aggre-
gate concrete, the slab thickness can be reduced to 3.6 in . This $28 \%$ reduction in thickness translates into approximately a $45 \%$ reduction in dead load.

## Table 2-Minimum Slab Thickness for Fire Resistance Rating

| Floor <br> Construction <br> Material | Minimum slab <br> thickness (in.) <br> for fire-resistance rating |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 hr | 2 hr | 3 hr | 4 hr |
| Siliceous Aggregate | 3.5 | 5.0 | 6.2 | 7.0 |
| Concrete | 3.2 | 4.6 | 5.7 | 6.6 |
| Carbonate Aggregate <br> Concrete | 2.7 | 3.8 | 4.6 | 5.4 |
| Sand-lightweight <br> Concrete | 2.5 | 3.6 | 4.4 | 5.1 |

Adequate cover must be provided to keep reinforcing steel temperatures within code prescribed limits. The amount of cover depends on the element considered (i.e., slab, joist or beam), and whether the element is restrained against thermal expansion. All elements of cast-in-place concrete framing systems are considered to be restrained.

For positive moment reinforcement in beams spaced at 4 ft or less on center, and in joists and slabs, regardless of the type of concrete aggregate used, the minimum cover required by ACI 318 is adequate for ratings of up to four hours. For beams spaced at more than 4 ft on center, the cover must not be less than the values given in Table 3.

Table 3-Cover Thickness for Fire Resistance Rating for Beams Spaced More than 4 ft on Center

| Beam Width <br> (in.) | Cover thickness (in.) for <br> fire-resistance rating |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | 1 hr | 2 hr | 3 hr | 4 hr |
| 5 | $3 / 4$ | $3 / 4$ | 1 | $11 / 4$ |
| 7 | $3 / 4$ | $3 / 4$ | $3 / 4$ | $3 / 4$ |
| $\geq 10$ | $3 / 4$ | $3 / 4$ | $3 / 4$ | $3 / 4$ |

The cover for an individual bar is the minimum cover between the surface of the bar and the fire-exposed surface of the structural member. When more than one bar is used, the cover is assumed to be the average of the minimum cover to each bar, where the cover for corner bars used in the calculation is one-half the actual value. The actual cover for an individual bar must be not less than one-half the value shown in Table 3, nor less than $3 / 4 \mathrm{in}$. For beam widths between tabulated values, use direct interpolation to determine minimum cover.

The foregoing is intended to give a brief overview of the subject of fire resistance of concrete floor systems. While the information cited is consistent with the three model building codes in use in the United States, the legally adopted building code governing the specific project should be consulted.

## General Discussion

This section provides overall comparisons of the economics of the various floor systems discussed in this publication. It provides a summary of the factors that may influence the costs of cast-in-place concrete floor systems. These factors include column dimensions, live loads, aspect ratios and proper detailing. A few other aspects that have an influence on economy are also discussed.

## Overall Comparisons

Four figures that compare the economics of the different structural floor systems considered are provided at the end of this publication. The figures clearly show that the optimality of the slab system depends on two major factors: the span in the long direction, and the intensity of superimposed dead and live loads. For a given set of loads, the slab system that is optimal for short spans, is not necessarily optimal for longer spans. For a given span, the slab system that is optimal for light superimposed loads, is not necessarily optimal for heavier loads. The four figures should facilitate the section of a structural floor system most appropriate for a certain application.

## Column Dimensions

Analysis shows that the height between floors has very little influence on the material quantities for the floor system. Column cross-sectional properties determine the clear span length and the shear capacity of the slab. The column cross-sectional dimensions used in this publication were representative of 10 - to 20 -story buildings. Increasing or decreasing the column dimensions by 2 in . did not affect the concrete quantities and changed the steel reinforcing quantities by less than $1 \%$.

## Live Loads

The material quantities for the floor system are typically controlled by deflections rather then stresses. Increasing the live load from 50 psf to 100 psf only resulted in a $4 \%$ to $10 \%$ increase in the floor system cost.

## Aspect Ratio

Square bays usually represent the most economical floor layout, since deflection control requirements can be exactly met in both directions. A rectangular bay with an aspect ratio of 1.5 ranges between $4 \%$ to $10 \%$ more in cost than a bay with an aspect ratio of 1.0 and the same floor area. This, however, is not
the case for one-way joist systems. This type of floor system should have the joists span in the short direction, and is almost unaffected by aspect ratios of up to 1.5 .

## Concrete Strengths

Concrete strengths of $4000 \mathrm{psi}, 5000 \mathrm{psi}$, and 6000 psi were used in this publication. Cost analysis shows that for gravity loads, 4000 psi concrete is more economical than higher concrete strengths.

## Cost Breakdown

The formwork for the floor systems will absorb from $50 \%$ to $58 \%$ of the costs. Concrete material, placing and finishing account for $21 \%$ to $30 \%$. The material and placing costs of the reinforcing steel amount to between $17 \%$ and $25 \%$ of the cost.

## Repetition

A cost efficient design utilizes repetition. Changes should be minimized from floor to floor. Changing column locations, joist spacing, or the type of floor system increases the cost of formwork, time of construction and the chance of field mistakes, and therefore should be avoided.

## Column-Beam Intersections

The beams that frame into columns should be at least as wide as the columns. If the beams are narrower than the columns, the beam forms will require costly field labor to pass the formwork around the columns.

## Standard Dimensions

Standard available sizes should be used for structural forming. For instance, joist formwork pans are available in various web depths of 20 in . and from 8 in. to 16 in . in 2 in . increments. Specifying a depth different from these sizes will require the fabrication of costly special formwork. When detailing drop panels or other changes in the floor system depth, actual lumber dimensions should be taken into account.

## Depth of the Ceiling Sandwich

This publication has addressed the economy of the structural slab system only. However, the structural engineer usually has to look beyond. The structural slab system is part of the so-called ceiling sandwich which also includes the mechanical system (HVAC ducts), the lighting fixtures, and the ceiling itself.

The floor-to-floor height of a building is the total depth of the ceiling sandwich plus the clear floor-toceiling height. Any variation in the depth of the ceiling sandwich will have an impact on the total height of: the shearwalls and columns, the mechanical, electrical and plumbing risers, the stairs and interior architectural finishes, and the exterior cladding. It will also have an impact on the total heating, cooling and ventilation volume. To minimize the depth of the ceiling sandwich is very often the goal of the structural engineer. This becomes particularly important in cities like Washington, D.C. that impose a height limit on buildings. Optimization of the ceiling sandwich depth may translate into an extra story or two accommodated within the prescribed height limit.


A number of details have been attempted in the past to accomplish a reduced depth of the ceiling sandwich. The HVAC ducts can pass through the webs of joists or beams. This will reduce the floor-to-floor height, but will increase formwork and field labor costs. Another alternative is to cut notches at the bottom of the joist or beam to allow passage of the upper portions of the HVAC ducts. This alternátive also requires additional forming costs. Further, special detailing would be needed to meet the structural integrity requirements of the ACI 318-89 Code. More importantly, however, such practices take flexibility away from accommodating future changes in the use of the floor space. Such flexibility is becoming more important in view of the shifting emphasis towards consciously designing buildings for a long service life.

OVERVIEW

Live Load = 50 psf
Superimposed Dead Load = 20 psf
$\mathrm{fc}_{\mathrm{c}}=4000 \mathrm{psi}$


ft

Live Load $=100 \mathrm{psf}$
Superimposed Dead Load $=20$ psf
$\mathrm{fc}_{\mathrm{c}}=4000 \mathrm{psi}$


ft

## Openings in Concrete Slab Systems

from Notes on ACI 318-99, Portland Cement Association, 1999

### 11.12.5 Openings in Slabs

The effect of openings (vertical holes through slabs) on the shear strength of slabs must be investigated when the openings are within the column strip areas of slabs or within middle strip areas when the openings are closer than 10 times the slab thickness ( 10 h ) from a column. A reduction in shear strength is made by considering as ineffective that portion of the critical section $b_{o}$ which is enclosed by straight lines projecting from the column centroid to the edges of the opening. Ineffective portions of critical sections $b_{o}$ are illustrated in Fig. 18-10. For slabs with shear reinforcement, the ineffective portion of the perimeter $b_{o}$ is one-half of that without shear reinforcement. The one-half factor is interpreted to apply equally to shearhead reinforcement and bar or wire reinforcement.


Figure 18-10 Effect of Slab Openings on Shear Strength

### 13.4 OPENINGS IN SLAB SYSTEMS

Openings of any size are permitted in slab systems without beams if special analysis indicates that both strength and serviceability of the slab system, considering the effects of the opening, are satisfied. Without special analysis, openings up to a certain size are permitted as illustrated in Fig. 18-11. The size of openings located within intersecting middle strip areas is unlimited. Within the area of the slab common to intersecting column
strips, size of openings is the most restrictive, due to their effect on slab shear strength or load transfer near slabcolumn connections. See discussion on effect of slab openings on shear strength (11.12.5) and Fig. 18-10. Without special analysis, size of openings within intersecting column strips is limited to one-sixteenth of the slab span length in either direction $(1 / 8(\ell / 2)=\ell / 16)$. Within the slab area common to one column and one middle strip, opening size is limited to one-eighth the span length in either direction $(1 / 4(\ell / 2)=\ell / 8)$.

The total amount of reinforcement required for the panel without openings, in both directions, must be maintained; reinforcement interrupted by any opening must be replaced, one-half on each side of the opening.


Figure 18-11 Openings in Slab Systems without Beams

## Examples:

## Plate and Grids

## Example 1

What is the maximum positive and negative bending moments developed in a 52 x 40 ft fully fixed plate that carries a load of $120 \mathrm{lb} / \mathrm{ft}^{2}$ ?

## SOLUTION:

The aspect ratio of the side lengths, $a / b$, must be determined and an appropriate coefficient chart must be found:
$a / b=52 / 40=1.3$ (no units, and $a$ is always the bigger number).


40 ft

BENDING MOMENTS IN RECTANGULAR PLATES

| Aspect ratio $\frac{\mathrm{a}}{\mathrm{b}}$ | Simply supported on all four sides | Fixed on all four sides | Corner slabs fixed on two adjacent sides and free on two sides |
| :---: | :---: | :---: | :---: |
| 1.0 | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0479 \\ & \mathbf{C}_{\mathrm{b}}=+0.0479 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0231, \mathbf{C}_{\mathrm{a}}=-0.0513 \\ & \mathbf{C}_{\mathrm{b}}=+0.0231, \mathbf{C}_{\mathrm{b}}=-0.0513 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=-0.29 \\ & \mathbf{C}_{\mathrm{b}}=-0.29 \end{aligned}$ |
| 1.3 | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0298 \\ & \mathbf{C}_{\mathrm{b}}=+0.0694 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0131, \mathbf{C}_{\mathrm{a}}=-0.0333 \\ & \mathbf{C}_{\mathrm{b}}=+0.0327, \mathbf{C}_{\mathrm{b}}=-0.0687 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=-0.35 \\ & \mathbf{C}_{\mathrm{b}}=-0.35 \end{aligned}$ |
| 1.5 | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0221 \\ & \mathbf{C}_{\mathrm{b}}=+0.0812 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0090, \mathbf{C}_{\mathrm{a}}=-0.0253 \\ & \mathbf{C}_{\mathrm{b}}=+0.0368, \mathbf{C}_{\mathrm{b}}=-0.0757 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=-0.37 \\ & \mathbf{C}_{\mathrm{b}}=-0.37 \end{aligned}$ |
| 2.0 | $\begin{aligned} & \mathrm{C}_{\mathrm{a}}=+0.0116 \\ & \mathrm{C}_{\mathrm{b}}=+0.1017 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=+0.0039, \mathbf{C}_{\mathrm{a}}=-0.0143 \\ & \mathbf{C}_{\mathrm{b}}=+0.0412, \mathrm{C}_{\mathrm{b}}=-0.0829 \end{aligned}$ | $\begin{aligned} & \mathbf{C}_{\mathrm{a}}=-0.43 \\ & \mathbf{C}_{\mathrm{b}}=-0.43 \end{aligned}$ |
| Note | In all cases, $\begin{aligned} & \mathbf{M}_{\mathrm{a}} \\ & \mathbf{M}_{\mathrm{b}} \end{aligned}$ | $\begin{aligned} & =C_{a} w a^{2} \\ & =C_{b} w^{2} \end{aligned}$ |  |

The coefficients for moment for the $a$ side length and $b$ side length for fixed support all sides and $a / b=1.3$ are:
$C_{a}=+0.0131$ and $C_{a}=-0.0333 \quad C_{b}=+0.0327$ and $C_{b}=-0.0687$

The maximum moments are calculated with the formula in the table:

$$
\begin{aligned}
& M_{a}(\text { positive })=C_{a} w a^{2}=0.0131\left(120 \mathrm{lb} / f t^{2}\right)(52 f t)^{2}=4251 \frac{\mathrm{lb-ft}}{f t} \\
& M_{a}(\text { negative })=C_{a} w a^{2}=-0.0333\left(120 \mathrm{lb} / f t^{2}\right)(52 f t)^{2}=-10,805 \frac{\mathrm{lb-ft}}{f t} \\
& M_{b}(\text { positive })=C_{b} w b^{2}=0.0327\left(120 \mathrm{lb} / f t^{2}\right)(40 f t)^{2}=6278 \frac{\mathrm{lb-ft}}{f t} \\
& M_{b}(\text { negative })=C_{b} w b^{2}=-0.0687\left(120 \mathrm{lb} / f^{2}\right)(40 f t)^{2}=-13,190 \frac{\mathrm{lb-ft}}{f t}
\end{aligned}
$$

## Example 2

A two-way interior-bay flat (concrete) slab with the dimensions shown supports a live loading of $80 \mathrm{lb} / \mathrm{ft}^{2}$ and has a dead load of $90 \mathrm{lb} / \mathrm{ft}^{2}$. The columns can be assumed to be 18 inches square. Determine the design moments based on ACI-318, (ASCE-7) and the Direct Design method.

Also compare design moments for an exterior-interior bay

SOLUTION:
Determine the distributed load combinations:
$W_{u}=1.2 D+1.6 L=1.2\left(90 \mathrm{lb} / \mathrm{ft}^{2}\right)+1.6\left(80 \mathrm{lb} / \mathrm{tt}^{2}\right)=236 \mathrm{lb} / \mathrm{tt}^{2}$


Determine the clear span length for the $\mathrm{N}-\mathrm{S}$ direction:
$l_{\mathrm{n}}=l_{1}-1 / 2$ column width $-1 / 2$ column width
$=25 \mathrm{ft}-1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})-1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=23.5 \mathrm{ft}$
Because $l_{2}$ is not the same width on either side of an interior panel, it is taken as the average $=(21 \mathrm{ft}+20 \mathrm{ft}) / 2=20.5 \mathrm{ft}$.

Total moment (to distribute to middle and interior column strip):

$$
M_{o}=\frac{w_{u} \ell_{2} \ell_{n}^{2}}{8}=\frac{\left(236 \mathrm{lb} / \mathrm{ft}^{2}\right)(20.5 f t)(23.5 f t)^{2}}{8}=333,973^{\mathrm{lb}-f t}
$$

## Interior Column Strip $\left(l_{2} \leq l_{1}\right)$ :


(a) Column strip for $\ell_{2} \leq \ell_{1}$

The column strip width is $1 / 4$ the smaller of $l_{2}$ either side of the column:
strip width $=1 / 4(21 \mathrm{ft})+1 / 4(20 \mathrm{ft})=10.25 \mathrm{ft}$
From Table 4.2, the maximum positive moment occurs in an end span:

$$
\begin{aligned}
M(\text { positive }) & =0.31 M_{o}=(0.31)\left(333,973^{\mathrm{lb}-f t}\right)=103,532^{\mathrm{lb}-\mathrm{ft}} \text {, distributed over } 10.25 \mathrm{ft}=103,532 \mathrm{lb}-\mathrm{ft} /(10.25 \mathrm{ft}) \\
& =10,101 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

The positive design moment for an interior span is:
$\begin{aligned} M(\text { positive }) & =0.21 M_{o}=(0.21)\left(333,973^{\text {lb-ft }}\right)=70,134^{\text {lb-ft }} \text {, distributed over } 10.25 \mathrm{ft}=70,134 \mathrm{lb}-\mathrm{ft} /(10.25 \mathrm{ft})= \\ & =6842 \mathrm{lb} \mathrm{ft} / \mathrm{ft}\end{aligned}$ $=6842 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$

Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns

| TT |  |  | 1 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | End Span | $\underset{(3)}{L_{1}}$ | Interior Span <br> (4) | $\frac{L_{5}}{}$ |  |
|  | End Span |  |  | Interior Span |  |
| Slab Moments | Exterior Negative | $\begin{gathered} \hline 2 \\ \text { Positive } \end{gathered}$ | 3 <br> First Interior Negative |  | 5 Interior <br> Negative |
| Total Moment | $0.26 \mathrm{Mo}_{0}$ | 0.52 Mo | $0.70 \mathrm{Mo}_{0}$ | $0.35 \mathrm{M}_{0}$ | $0.65 \mathrm{M}_{0}$ |
| Column Strip | $0.26 \mathrm{Mo}_{0}$ | $0.31 \mathrm{Mo}_{0}$ | $0.53 \mathrm{Mo}_{0}$ | $0.21 \mathrm{Mo}_{0}$ | 0.49 Mo |
| Middle Strip | 0 | $0.21 \mathrm{M}_{0}$ | $0.17 \mathrm{M}_{0}$ | $0.14 \mathrm{M}_{0}$ | $0.16 \mathrm{M}_{0}$ |

Note: All negative moments are at face of support.

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$
\begin{aligned}
M(\text { negative }) & =0.53 M_{o}=(0.53)\left(333,973^{l b-f t}\right)=177,006^{\mathrm{lb}-\mathrm{ft}}, \text { distributed over } 10.25 \mathrm{ft}=177,006 \mathrm{lb}-\mathrm{ft} /(10.25 \mathrm{ft})= \\
& =17,269 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

The negative design moment at the exterior of an end span is:

$$
\begin{aligned}
M(\text { negative }) & =0.26 M_{o}=(0.26)\left(333,973^{\text {lb-ft }}\right)=86,833^{\text {lb-ft }}, \text { distributed over } 10.25 \mathrm{ft}=86,833 \mathrm{lb}-\mathrm{ft} /(10.25 \mathrm{ft})= \\
& =8472 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

The negative design moment for an interior span is:

$$
\begin{aligned}
M(\text { negative }) & =0.49 M_{o}=(0.49)\left(333,973^{l b-f t}\right)=163,647^{l b-f t}, \text { distributed over } 10.25 \mathrm{ft}=163,647 \mathrm{lb}-\mathrm{ft} /(10.25 \mathrm{ft})= \\
& =15,966 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

## Middle Strip:

The width is the remaining width of $l_{2}$ between column strips:
strip width $=21 \mathrm{ft}-1 / 4(20 \mathrm{ft})-1 / 4(21 \mathrm{ft})=10.75 \mathrm{ft}$

From Table 4.2, the maximum positive moment occurs in an end span:
$M($ positive $)=0.21 M_{o}=(0.21)\left(333,973^{\mathrm{lb}-f t}\right)=70,134^{\mathrm{lb}-f t}$, distributed over $10.75 \mathrm{ft}=70,134 \mathrm{lb}-\mathrm{ft} /(10.75 \mathrm{ft})=$

$$
=6524 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
$$

The positive design moment for an interior span is:
$\begin{aligned} M(\text { positive }) & =0.14 M_{o}=(0.14)\left(333,973^{l b-f t}\right)=46,756^{l b-f t} \text {, distributed over } 10.75 \mathrm{ft}=46,756 \mathrm{lb}-\mathrm{ft} /(10.75 \mathrm{ft})= \\ & =4349 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}\end{aligned}$
From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$
\begin{aligned}
M(\text { negative }) & =0.17 M_{o}=(0.17)\left(333,973^{\mathrm{lb-ft}}\right)=56,775^{\mathrm{lb}-\mathrm{ft}}, \text { distributed over } 10.75 \mathrm{ft}=56,775 \mathrm{lb}-\mathrm{ft} /(10.75 \mathrm{ft})= \\
& =5281 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

There is no negative design moment at the exterior of an end span.
The negative design moment for an interior span is:

$$
\begin{aligned}
M(\text { negative }) & =0.16 M_{o}=(0.16)\left(333,973^{\mathrm{lb-ft}}\right)=53,436^{\mathrm{lb}-f t} \text {, distributed over } 10.75 \mathrm{ft}=53,436 \mathrm{lb}-\mathrm{ft} /(10.75 \mathrm{ft})= \\
& =4971 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

## Exterior Column Strip:

The value to use for $l_{2}$ for an edge strip includes the distance to the outside of the columns $=21 \mathrm{ft}+1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=21.75 \mathrm{ft}$
$M_{o}=\frac{w_{u} \ell_{2} \ell_{n}^{2}}{8}=\frac{\left(236 \mathrm{lb} / \mathrm{ft}^{2}\right)(21.75 \mathrm{ft})(23.5 \mathrm{ft})^{2}}{8}=354,337^{\mathrm{lb}-f t}$
The width is $1 / 4 l_{2}$ one side of the column plus the distance to the slab edge:
strip width $=1 / 4(21 \mathrm{ft})+1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=6 \mathrm{ft}$
So a comparison to the interior column strip maximum positive moment occurring in an end span is:
$M($ positive $)=0.31 M_{o}=(0.31)\left(354,337^{l b-f t}\right)=109,844^{l b-f t}$, distributed over $6 \mathrm{ft}=109,844 \mathrm{lb}-\mathrm{ft} /(6 \mathrm{ft})=18,307 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ (as opposed to $10,101 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ )

## For the E-W direction:

Because the adjacent spans are not the same length, the longer span, which is the END span will be larger:

$$
\begin{aligned}
l_{\mathrm{n}} & =l_{1}-1 / 2 \text { column width }-1 / 2 \text { column width } \\
& =21 \mathrm{ft}-1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})-1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=19.5 \mathrm{ft}
\end{aligned}
$$

## Because $l_{2}$ is 25 ft .

Total moment (to distribute to middle and interior column strip):

$$
M_{o}=\frac{w_{u} \ell_{2} \ell_{n}^{2}}{8}=\frac{\left(236 \mathrm{lb} / f t^{2}\right)(25 f t)(19.5 f t)^{2}}{8}=280,434^{l b-f t}
$$

Interior Column Strip END Spans $\left(l_{2}>l_{1}\right)$ :

The column strip width is $1 / 4$ the smaller of $l_{1}$ and $l_{2}$ either side of the column:

(b) Column strip for $\ell_{2}>l_{1}$
strip width $=1 / 4(21 \mathrm{ft})+1 / 4(21 \mathrm{ft})=10.5 \mathrm{ft}$
From Table 4.2, the maximum positive moment occurs in an end span:

$$
\begin{aligned}
M(\text { positive }) & =0.31 M_{o}=(0.31)\left(280,434^{l b-f t}\right)=86,935^{l b-f t} \text {, distributed over } 10.5 \mathrm{ft}=86,935 \mathrm{lb}-\mathrm{ft} /(10.5 \mathrm{ft}) \\
& =8279 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:
$M($ negative $)=0.53 M_{o}=(0.53)\left(280,434^{l b-f t}\right)=148,630^{l b-f t}$, distributed over $10.5 \mathrm{ft}=148,630 \mathrm{lb}-\mathrm{ft} /(10.5 \mathrm{ft})=$

$$
=14,155 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
$$

The negative design moment at the exterior of an end span is:

$$
\begin{aligned}
M(\text { negative }) & =0.26 M_{o}=(0.26)\left(280,434^{\mathrm{lb-ft}}\right)=72,913^{\mathrm{lb}-f t}, \text { distributed over } 10.5 \mathrm{ft}=72,913 \mathrm{lb}-\mathrm{ft} /(10.5 \mathrm{ft})= \\
& =6944 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

## Middle Strip END Spans:

The width is the remaining width of $l_{2}$ between column strips:
strip width $=25 \mathrm{ft}-1 / 4(21 \mathrm{ft})-1 / 4(21 \mathrm{ft})=14.5 \mathrm{ft}$

From Table 4.2, the maximum positive moment occurs in an end span:

$$
\begin{aligned}
M(\text { positive }) & =0.21 M_{o}=(0.21)\left(280,434^{\mathrm{lb-ft}}\right)=58,891^{\mathrm{lb-ft}} \text {, distributed over } 14.5 \mathrm{ft}=58,891 \mathrm{lb}-\mathrm{ft} /(14.5 \mathrm{ft})= \\
& =4061 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$
\begin{aligned}
M(\text { negative }) & =0.17 M_{o}=(0.17)\left(280,434^{l b-f t}\right)=47,674^{l b-f t}, \text { distributed over } 14.5 \mathrm{ft}=47,674 \mathrm{lb}-\mathrm{ft} /(14.5 \mathrm{ft})= \\
& =3288 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}
\end{aligned}
$$

There is no negative design moment at the exterior of an end span.

## Exterior Column Strip END Spans:

The value to use for $l_{2}$ for an edge strip includes the distance to the outside of the columns $=25 \mathrm{ft}+1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=25.75 \mathrm{ft}$

$$
M_{o}=\frac{w_{u} \ell_{2} \ell_{n}^{2}}{8}=\frac{\left(236 \mathrm{lb} / \mathrm{ft}^{2}\right)(25.75 \mathrm{ft})(19.5 \mathrm{ft})^{2}}{8}=288,847^{\mathrm{lb}-f t}
$$

The width is $1 / 4 l_{1}$ (because it is smaller than $l_{2}$ ) one side of the column plus the distance to the slab edge:
strip width $=1 / 4(21 \mathrm{ft})+1 / 2(18 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft})=6 \mathrm{ft}$
So a comparison to the interior column END strip maximum positive moment occurring in an end span is:
$M($ positive $)=0.31 M_{o}=(0.31)\left(288,847^{l b-f t}\right)=89,543^{\text {lb-ft }}$, distributed over $6 \mathrm{ft}=89,543 \mathrm{lb}-\mathrm{ft} /(6 \mathrm{ft})=14,923 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ (as opposed to $8279 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ )

## TABLE OF DESIGN MOMENTS

| slab moments / ft | End Span |  |  | Interior Span |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exterior Negative | Positive | First Interior Negative | Positive | Interior Negative |
| NS column strip interior | $8472 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | 10,101 lb-ft/ft | 17,269 lb-ft/ft | $6842 \mathrm{lb}-\mathrm{ft} / \mathrm{tt}$ | 15,966 lb-ft/ft |
| NS middle strip | 0 | $6524 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | 5281 lb-ft/ft | 4349 lb-ft/ft | 4971 lb-ft/ft |
| NS column strip edge | 15,355 lb-ft/ft | 18,307 lb-ft/ft | 31,300 lb-ft/ft | 12,402 lb-ft/ft | 28,937 lb-ft/ft |
| EW column strip interior | $6944 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | $8279 \mathrm{lb-ft/ft}$ | 14,155 lb-ft/ft | $5048 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | 11,779 lb-ft/ft |
| EW middle strip | 0 | 4061 lb-ft/ft | $3288 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | $2437 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ | $5686 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ |
| EW column strip edge | 12,517 lb-ft/ft | 14,923 lb-ft/ft | 25,515 lb-ft/ft | $6066 \mathrm{lb}-\mathrm{ft} / \mathrm{tt}$ | $6933 \mathrm{lb}-\mathrm{ft} / \mathrm{ft}$ |

## Reinforced Concrete Design

## Notation:

|  | $=$ depth of the effective compression block in a concrete beam |
| :---: | :---: |
| A | = name for area |
| $A_{g}$ | $=$ gross area, equal to the total area ignoring any reinforcement |
| $A_{s}$ | = area of steel reinforcement in concrete beam design |
| $A_{s}^{\prime}$ | $\begin{aligned} & =\text { area of steel compression } \\ & \text { reinforcement in concrete beam } \\ & \text { design } \end{aligned}$ |
| $A_{s t}$ | $=$ area of steel reinforcement in concrete column design |
| $A_{v}$ | $=$ area of concrete shear stirrup reinforcement |
| ACI | = American Concrete Institute |
| $b$ | $=$ width, often cross-sectional |
| $b_{E}$ | $=$ effective width of the flange of a concrete T beam cross section |
| $b_{f}$ | $=$ width of the flange |
| $b_{w}$ | $=$ width of the stem (web) of a concrete T beam cross section |
| cc | $=$ shorthand for clear cover |
| C | = name for centroid |
|  | = name for a compression force |
| $C_{c}$ | $=$ compressive force in the compression steel in a doubly reinforced concrete beam |
| $C_{s}$ | $=$ compressive force in the concrete of a doubly reinforced concrete beam |
| $d$ | $=$ effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel |
| $d^{\prime}$ | $=$ effective depth from the top of a reinforced concrete beam to the centroid of the compression steel |
| $d_{b}$ | $=$ bar diameter of a reinforcing bar |
| D | = shorthand for dead load |
| DL | = shorthand for dead load |
| E | $\begin{aligned} & =\text { modulus of elasticity or Young's } \\ & \text { modulus } \\ & =\text { shorthand for earthquake load } \end{aligned}$ |
| $E_{c}$ | $=$ modulus of elasticity of concrete |
| $E_{s}$ | $=$ modulus of elasticity of steel |
|  | $=$ symbol for stress |

$f_{c} \quad=$ compressive stress
$f_{c}^{\prime}=$ concrete design compressive stress
$f_{s} \quad=$ stress in the steel reinforcement for concrete design
$f_{s}^{\prime}=$ compressive stress in the
compression reinforcement for
concrete beam design
$f_{y} \quad=$ yield stress or strength
$F \quad=$ shorthand for fluid load
$F_{y}=$ yield strength
$G \quad=$ relative stiffness of columns to beams in a rigid connection, as is $\Psi$
$h \quad=$ cross-section depth
$H=$ shorthand for lateral pressure load
$h_{f} \quad=$ depth of a flange in a T section
$I_{\text {transformed }}=$ moment of inertia of a multimaterial section transformed to one material
$k \quad=$ effective length factor for columns
$\ell_{b} \quad=$ length of beam in rigid joint
$\ell_{c} \quad=$ length of column in rigid joint
$l_{d} \quad=$ development length for reinforcing steel
$l_{d h}=$ development length for hooks
$l_{n} \quad=$ clear span from face of support to face of support in concrete design
$L \quad=$ name for length or span length, as is $l$
$=$ shorthand for live load
$L_{r} \quad=$ shorthand for live roof load
$L L=$ shorthand for live load
$M_{n} \quad=$ nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
$M_{u}=$ maximum moment from factored loads for LRFD beam design
$n \quad=$ modulus of elasticity transformation coefficient for steel to concrete
n.a. $=$ shorthand for neutral axis (N.A.)
$p H=$ chemical alkalinity
$P=$ name for load or axial force vector


## Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the Building Code and Commentary (ACI 318-11) published by the American Concrete Institute International, and uses ultimate strength design.

## Materials

$f^{\prime}{ }_{\mathrm{c}}=$ concrete compressive design strength at 28 days (units of psi when used in equations)

Deformed reinforcing bars come in grades $40,60 \& 75$ (for 40 ksi , 60 ksi and 75 ksi yield strengths). Sizes are given as \# of $1 / 8$ " up to \#8 bars. For \#9 and larger, the number is a nominal size (while


GRADE 60 the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be $150 \mathrm{lb} / \mathrm{ft}^{3}$. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

Plane sections of composite materials can still be assumed to be plane (strain is linear), but the stress distribution is not the same in both materials because the modulus of elasticity is different. ( $f=\mathrm{E} \cdot \varepsilon$ )


In order to determine the stress, we can define $n$ as the ratio of the elastic moduli:

$$
n=\frac{E_{2}}{E_{1}}
$$

$n$ is used to transform the width of the second material such that it sees the equivalent element stress.

## Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.

ex: When material 1 above is concrete and material 2 is steel:
to transform steel into concrete $n=\frac{E_{2}}{E_{1}}=\frac{E_{\text {steel }}}{E_{\text {concrete }}}$
to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by $n$
to find the moment of inertia of the equivalent concrete member, $\mathrm{I}_{\text {transformed }}$, use the new geometry resulting from transforming the width of the steel
concrete stress: $f_{\text {concrete }}=-\frac{M y}{I_{\text {transformel }}}$
steel stress: $\quad f_{\text {steel }}=-\frac{M y n}{I_{\text {transformel }}}$

## Reinforced Concrete Beam Members



Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.


Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)


Actual stress distribution near ultimate strength (nonlinear).


Typical stress-strain curve for concrete,


Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

## Ultimate Strength Design for Beams

The ultimate strength design method is similar to LRFD. There is a nominal strength that is reduced by a factor $\phi$ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular "stress" block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, $\mathrm{F}_{\mathrm{y}}$

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment


Figure 8.5: Bending in a concrete bearn without and with steel reinforcing

The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

Because the $\mathrm{n} . \mathrm{a}$. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

$$
b x \cdot \frac{x}{2}-n A_{s}(d-x)=0
$$

x can be solved for when the equation is rearranged into the generic format with $\mathrm{a}, \mathrm{b} \& \mathrm{c}$ in the binomial equation: $\quad a x^{2}+b x+c=0 \quad$ by $\quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## T-sections

If the n.a. is above the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is below the bottom of a flange in a T section, x is found by including the flange and the stem of the web $\left(b_{w}\right)$ in the moment area calculation:


$$
b_{f} h_{f}\left(x-\frac{h_{f}}{h^{2}}\right)+\left(x-h_{f}\right) b_{w} \frac{\left(x-h_{f}\right)}{2}-n A_{s}(d-x)=0
$$

## Load Combinations - (Alternative values allowed)

$1.4 D$
$1.2 D+1.6 L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
$1.2 D+1.6\left(L_{r}\right.$ or $S$ or $\left.R\right)+(1.0 L$ or $0.5 W)$
$1.2 D+1.0 W+1.0 L+0.5\left(L_{r}\right.$ or $S$ or $\left.R\right)$
$1.2 D+1.0 E+1.0 L+0.2 S$
$0.9 D+1.0 W$
$0.9 D+1.0 E$


## Internal Equilibrium

ASTM STANDARD REINFORCING BARS
$\mathrm{C}=$ compression in concrete $=$ stress x area $=0.85 f^{\prime} c b a$
$\mathrm{T}=$ tension in steel $=$ stress x area $=A_{s} f_{y}$

$$
C=T \text { and } M_{n}=T(d-a / 2)
$$

$$
\begin{array}{ll}
\text { where } & \mathrm{f}^{\prime}{ }_{\mathrm{c}}=\text { concrete compression strength } \\
& \mathrm{a}=\text { height of stress block } \\
& \mathrm{b}=\text { width of stress block } \\
& \mathrm{f}_{\mathrm{y}}=\text { steel yield strength } \\
\mathrm{A}_{\mathrm{s}}=\text { area of steel reinforcement } \\
& \mathrm{d}=\text { effective depth of section } \\
& \text { (depth to n.a. of reinforcement) }
\end{array}
$$

| Bar size, no. | Nominal <br> diameter, in. | Nominal area, <br> in. $^{2}$ | Nominal weight, <br> lbfft |
| :---: | :---: | :---: | :---: |
| 3 | 0.375 | 0.11 | 0.376 |
| 4 | 0.500 | 0.20 | 0.668 |
| 5 | 0.625 | 0.31 | 1.043 |
| 6 | 0.750 | 0.44 | 1.502 |
| 7 | 0.875 | 0.60 | 2.044 |
| 8 | 1.000 | 0.79 | 2.670 |
| 9 | 1.128 | 1.00 | 3.400 |
| 10 | 1.270 | 1.27 | 4.303 |
| 11 | 1.410 | 1.56 | 5.313 |
| 14 | 1.693 | 2.25 | 7.650 |
| 18 | 2.257 | 4.00 | 13.600 |

With $\mathrm{C}=\mathrm{T}, A_{S} f y=0.85 f^{\prime} c b a$ so $a$ can be determined with $a=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} b}$

## Criteria for Beam Design

For flexure design:

$$
M_{u} \leq \phi M_{n} \quad \phi=0.9 \text { for flexure }
$$

so, $M_{u}$ can be set $=\phi M_{n}=\phi T(d-a / 2)=\phi A_{S} f y(d-a / 2)$

## Reinforcement Ratio

The amount of steel reinforcement is limited. Too much reinforcement, or over-reinforced will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be under reinforced.
The reinforcement ratio is just a fraction: $\rho=\frac{A_{s}}{b d}$ (or p ) and must be less than a value determined with a concrete strain of 0.003 and tensile strain of 0.004 (minimum). The practical value for the strain in the reinforcement is a value of 0.005 . Previous codes limited the amount to $0.75 \rho_{\text {balanced }}$ where $\rho_{\text {balanced }}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain.

## Flexure Design of Reinforcement

One method is to "wisely" estimate a height of the stress block, $a$, and solve for $A_{s}$, and calculate a new value for $a$ using $M_{u}$.

Maximum Reinforcement Ratio $\rho$ for Singly Reinforced Rectangular Beams (tensile strain $=0.005$ )

1. guess $a$ (less than n.a.)
2. $A_{s}=\frac{0.85 f_{c}^{\prime} b a}{f_{y}}$
3. solve for $a$ from

$$
\begin{gathered}
M_{u}=\phi A_{S} f y(d-a / 2): \\
a=2\left(d-\frac{M_{u}}{\phi A_{s} f_{y}}\right)
\end{gathered}
$$

|  | $f_{c}^{\prime}=3000 \mathrm{psi}$ | $f_{c}^{\prime}=3500 \mathrm{psi}$ | $f_{c}^{\prime}=4000 \mathrm{psi}$ | $f_{c}^{\prime}=5000 \mathrm{psi}$ | $f_{c}^{\prime}=6000 \mathrm{psi}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{y}$ | $\beta_{1}=0.85$ | $\beta_{1}=0.85$ | $\beta_{1}=0.85$ | $\beta_{1}=0.80$ | $\beta_{1}=0.75$ |
| $40,000 \mathrm{psi}$ | 0.0203 | 0.0237 | 0.0271 | 0.0319 | 0.0359 |
| $50,000 \mathrm{psi}$ | 0.0163 | 0.0190 | 0.0217 | 0.0255 | 0.0287 |
| $60,000 \mathrm{psi}$ | 0.0135 | 0.0158 | 0.0181 | 0.0213 | 0.0239 |
|  | $f_{c}^{\prime}=20 \mathrm{MPa}$ | $f_{c}^{\prime}=25 \mathrm{MPa}$ | $f_{c}^{\prime}=30 \mathrm{MPa}$ | $f_{c}^{\prime}=35 \mathrm{MPa}$ | $f_{c}^{\prime}=40 \mathrm{MPa}$ |
| $f_{y}$ | $\beta_{1}=0.85$ | $\beta_{1}=0.85$ | $\beta_{1}=0.85$ | $\beta_{1}=0.81$ | $\beta_{1}=0.77$ |
| 300 MPa | 0.0181 | 0.0226 | 0.0271 | 0.0301 | 0.0327 |
| 350 MPa | 0.0155 | 0.0194 | 0.0232 | 0.0258 | 0.0281 |
| 400 MPa | 0.0135 | 0.0169 | 0.0203 | 0.0226 | 0.0245 |
| 500 MPa | 0.0108 | 0.0135 | 0.0163 | 0.0181 | 0.0196 |

4. repeat from 2. until $a$ found from step 3 matches $a$ used in step 2.

## Design Chart Method:

1. calculate $R_{n}=\frac{M_{n}}{b d^{2}}$
2. find curve for $f^{\prime} c$ and $f_{y}$ to get $\rho$
3. calculate $A_{s}$ and $a$

$$
A_{s}=\rho b d \text { and } a=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} b}
$$

Any method can simplify the size of d using $\mathrm{h}=1.1 \mathrm{~d}$

## Maximum Reinforcement

Based on the limiting strain of
0.005 in the steel, $x($ or $c)=0.375 d$ so
$a=\beta_{1}(0.375 d)$ to find $\mathrm{A}_{s-\max }$
( $\beta_{1}$ is shown in the table above)

## Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.
Minimum required: $A_{s}=\frac{3 \sqrt{f_{c}^{\prime}}}{f_{y}}\left(b_{w} d\right)$ but not less than: $A_{S}=\frac{200}{f_{y}}\left(b_{w} d\right)$


Figure 3.8.1 Strength curves ( $R_{n}$ vs $\rho$ ) for singly reinforced rectangular sections. Upper limit of curves is at $\rho_{\text {max }}$. (tensile strain of 0.004 )
where $f_{c}^{\prime}$ is in psi. $\quad$ This can be translated to $\rho_{\min }=\frac{3 \sqrt{f_{c}^{\prime}}}{f_{y}}$ but not less than $\frac{200}{f_{y}}$

## Compression Reinforcement

If a section is doubly reinforced, it means there is steel in the beam seeing compression. The force in the compression steel at yield is equal to stress x area, $\mathrm{C}_{\mathrm{s}}=A_{c} \cdot F_{y}$. The total compression that balances the tension is now: $T=C_{c}+C_{s}$. And the moment taken about the centroid of the compression
 stress is $M_{n}=T(d-a / 2)+C_{s}\left(a-d^{\prime}\right)$
where $A_{s}{ }^{\text {‘ }}$ is the area of compression reinforcement, and $d^{\prime}$ is the effective depth to the centroid of the compression reinforcement

## $T$-sections (pan joists)

T beams have an effective width, $b_{E}$, that sees compression stress in a wide flange beam or joist in a slab system.

For interior T -sections, $b_{E}$ is the smallest of $L / 4, b_{w}+16 t$, or center to center of beams

For exterior T-sections, $b_{E}$ is the smallest of


Figure 9.3.1 Actual and equivalent stress distribution over flange width.
$b_{w}+L / 12, b_{w}+6 t$, or $b_{w}+1 / 2($ clear distance to next beam)
When the web is in tension the minimum reinforcement required is the same as for rectangular sections with the web width $\left(b_{w}\right)$ in place of $b$.

When the flange is in tension (negative bending), the minimum reinforcement required is the greater value of $\quad A_{s}=\frac{6 \sqrt{f_{c}^{\prime}}}{f_{y}}\left(b_{w} d\right) \quad$ or $\quad A_{s}=\frac{3 \sqrt{f_{c}^{\prime}}}{f_{y}}\left(b_{f} d\right)$
where $f_{c}^{\prime}$ is in psi, $b_{w}$ is the beam width, and $b_{f}$ is the effective flange width

## Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, $3 / 4$ inch is required for slabs, 1.5 inch is typical for beams, and for concrete cast against soil, 3 inches is typical.

## Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement.

## Slabs

One way slabs can be designed as "one unit"wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric. Minimum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding $18 "$. For required flexure reinforcement spacing the limit is three times the slab thickness not exceeding 18 ".

TABLE 9.5(a)-MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

|  |  | Minimum thickness, $\boldsymbol{h}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Simply supported | One end continuous | Both ends continuous | Cantilever |
| Member | Members not supporting or attached to partitions or other construction likely to be damaged by large deflections. |  |  |  |
| Solid oneway slabs | $\ell / 20$ | $\ell / 24$ | $\ell / 28$ | $\ell / 10$ |
| Beams or ribbed oneway slabs | $\ell / 16$ | $\ell / 18.5$ | $\ell / 21$ | $\ell / 8$ |

Notes:
Vahues given shall be used directly for members with normalweight concrete
and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, $w_{c}$, in the range of 90
to $115 \mathrm{lb} / \mathrm{ft}^{3}$, the values shall be multiplied by ( $1.65-0.005 w_{c}$ ) but not less
than 1.09 .
b) For $f_{y}$ other than $60,000 \mathrm{psi}$, the values shall be multiplied by $\left(\mathbf{0 . 4}+\mathrm{f}_{y} \mathrm{H} \mathbf{1 0 0 , 0 0 0 )}\right.$.

Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):
Minimum for slabs with grade 40 or 50 bars:

$$
\begin{aligned}
& \rho=\frac{A_{s}}{b t}=0.002 \text { or } A_{s-m i n}=0.002 b t \\
& \rho=\frac{A_{s}}{b t}=0.0018 \text { or } A_{s-m i n}=0.0018 b t
\end{aligned}
$$

## Shear Behavior



The maximum shear for design, $V_{u}$ is the value at a distance of $d$ from the face of the support.

## Nominal Shear Strength

The shear force that can be resisted is the shear stress $\times$ cross section area: $V_{c}=v_{c} \times b_{w} d$ The shear stress for beams (one way) $v_{c}=2 \sqrt{f_{c}^{\prime}} \quad$ so $\phi V_{c}=\phi 2 \sqrt{f_{c}^{\prime}} b_{w} d$

$$
\text { where } \quad b_{w}=\text { the beam width or the minimum width of the stem. }
$$

One-way joists are allowed an increase of $10 \% \mathrm{~V}_{\mathrm{c}}$ if the joists are closely spaced.
Stirrups are necessary for strength (as well as crack control): $V_{s}=\frac{A_{v} f_{y} d}{s}$
where $\quad A_{v}=$ area of all vertical legs of stirrup
$\mathrm{s}=$ spacing of stirrups
d = effective depth

For shear design:

$$
V_{U} \leq \phi V_{c}+\phi V_{S} \quad \phi=0.75 \text { for shear }
$$

## Spacing Requirements

Stirrups are required when $\mathrm{V}_{\mathrm{u}}$ is greater than $\frac{\phi V_{c}}{2}$
Table 3-8 ACI Provisions for Shear Design*

*Members subjected to shear and flexure only; $\phi \mathrm{V}_{\mathrm{c}}=\phi 2 \sqrt{\mathrm{f}_{c}^{\prime}} \mathrm{b}_{\mathrm{w}} \mathrm{d}, \phi=0.75$ ( ACl 11.3.1.1)
** $A_{v}=2 \times A_{b}$ for $U$ stirrups; $f_{y} \leq 60 \mathrm{ksi}(\mathrm{ACl} 11.5 .2)$
$\dagger$ A practical limit for minimum spacing is $\mathrm{d} / 4$
$\dagger \dagger$ Maximum spacing based on minimum shear reinforcement ( $=\mathrm{A}_{\mathrm{v}} \mathrm{f}_{\mathrm{y}} / 50 \mathrm{~b}_{\mathrm{w}}$ ) must also be considered
( ACl 11.5.5.3).
Economical spacing of stirrups is considered to be greater than $\mathrm{d} / 4$. Common spacings of $d / 4, d / 3$ and $d / 2$ are used to determine the values of $\phi V_{s}$ at which the spacings can be increased.

$$
\phi V_{s}=\frac{\phi A_{v} f_{y} d}{s}
$$

This figure shows the size of $\mathrm{V}_{\mathrm{n}}$ provided by $\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{s}}$ (long dashes) exceeds $\mathrm{V}_{\mathrm{u}} / \phi$ in a step-wise function, while the spacing provided (short dashes) is at or less than the required $s$ (limited by the maximum allowed). (Note that the maximum shear permitted from the stirrups is $8 \sqrt{f_{c}^{\prime}} b_{w} d$


The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

## Torsional Shear Reinforcement

On occasion beam members will see twist along the access caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.


Fig. R11.6.3.6(b)—Definition of $\mathbf{A}_{\mathbf{o h}}$

## Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (both sides) so it won't slip. This is referred to as the development length, $l_{\mathrm{d}}$. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. Detailing reinforcement is a tedious job. Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations are not provided here.

## Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:
\#6 bars and smaller: $\quad l_{d}=\frac{d_{b} F_{y}}{25 \sqrt{f_{c}^{\prime}}}$ or 12 in. minimum
\#7 bars and larger: $\quad l_{d}=\frac{d_{b} F_{y}}{20 \sqrt{f_{c}^{\prime}}} \quad$ or 12 in. minimum

## Development Length in Compression

$$
l_{d}=\frac{0.02 d_{b} F_{y}}{\sqrt{f_{c}^{\prime}}} \leq 0.0003 d_{b} F_{y}
$$

Hook Bends and Extensions
The minimum hook length is $l_{d h}=\frac{1200 d_{b}}{\sqrt{f_{c}^{\prime}}}$


Figure 9-17: Minimum requirements for $90^{\circ}$ bar hooks.
Figure 9-18: Minimum requirements for $180^{\circ}$ bar hooks.

## Modulus of Elasticity \& Deflection

$\mathrm{E}_{\mathrm{c}}$ for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and $\mathrm{E}_{\mathrm{c}}$ is adjusted.

Code values:

$$
E_{c}=57,000 \sqrt{f_{c}^{\prime \prime}} \text { (normal weight) } \quad E_{c}=w_{c}^{1.5} 33 \sqrt{f_{c}^{\prime}}, w_{c}=90 \mathrm{lb} / f t^{3}-160 \mathrm{lb} / f t^{3}
$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 9.5(a) (see Slabs).

## Criteria for Flat Slab \& Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Two way shear at columns is resisted by the thickness of the slab at a perimeter of $d / 2$ away from the face of the support by the shear stress $\times$ cross section area: $V_{c}=v_{c} \times b_{o} d$ The shear stress (two way) $v_{c}=4 \sqrt{f_{c}^{\prime}}$ so $\phi V_{c}=\phi 4 \sqrt{f_{c}^{\prime}} b_{o} d$

$$
\text { where } \quad b_{o}=\text { perimeter length. }
$$

## Criteria for Column Design


(American Concrete Institute) ACI 318-11 Code and Commentary:

$$
\begin{aligned}
& P_{u} \leq \phi_{\mathrm{c}} P_{n} \quad \text { where } \\
& \qquad P_{\mathrm{u}} \text { is a factored load } \\
& \quad \phi \text { is a resistance factor } \\
& \mathrm{P}_{\mathrm{n}} \text { is the nominal load capacity (strength) }
\end{aligned}
$$

Load combinations, ex: $\quad 1.4 \mathrm{D}$ ( D is dead load)

$$
1.2 \mathrm{D}+1.6 \mathrm{~L}(\mathrm{~L} \text { is live load })
$$

$1.2 \mathrm{D}+1.6 \mathrm{~L}+0.5 \mathrm{~W}$
( W is wind load)
$0.90 \mathrm{D}+1.0 \mathrm{~W}$


$$
9.90 \mathrm{D}+1.0 \mathrm{~W}
$$



For compression, $\phi_{c}=0.75$ and $\mathrm{P}_{\mathrm{n}}=0.85 \mathrm{P}_{\mathrm{o}}$ for spirally reinforced, $\phi_{c}=0.65 \mathrm{P}_{\mathrm{n}}=0.8 \mathrm{P}_{\mathrm{o}}$ for tied columns where $P_{o}=0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+f_{y} A_{s t}$ and $\mathrm{P}_{\mathrm{o}}$ is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios, $\rho_{g}=A_{s t} / A_{g}$, in the range of $1 \%$ to $2 \%$ will usually be the most economical, with $1 \%$ as a minimum and $8 \%$ as a maximum by code.
Bars are symmetrically placed, typically.

## Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The interaction diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

## Eccentric Design

The strength interaction diagram is dependant upon the strain developed in the steel reinforcement.

If the strain in the steel is less than the yield stress, the section is said to be compression controlled.

Below the transition zone, where the steel starts to yield, and when the net tensile strain in the reinforcement exceeds 0.005 the section is said to be tension controlled. This is a ductile condition and is preferred.


Figure 5-3 Transition Stages on Interaction Diagram

## Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.


Figure 13.6.1 Typical strength interaction diagram for axial compression and bending moment about one axis. Transition zone is where $\boldsymbol{\epsilon}_{y} \leq \boldsymbol{\epsilon}_{\mathrm{t}} \leq 0.005$.

## Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k , the relative stiffness, G or $\Psi$, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

$$
G=\Psi=\frac{\Sigma E I / l_{c}}{\Sigma E I / l_{b}}
$$

where
$\mathrm{E}=$ modulus of elasticity for a member
I = moment of inertia of for a member
$l_{c}=$ length of the column from center to center

$l_{\mathrm{b}}=$ length of the beam from center to center

- For pinned connections we typically use a value of 10 for $\Psi$.
- For fixed connections we typically use a value of 1 for $\Psi$.


Braced - non-sway frame


Unbraced - sway frame


Factored Moment Resistance of Concrete Beams, $\phi M_{n}(\mathbf{k}-\mathrm{ft})$ with $\boldsymbol{f}^{\prime}{ }_{c}=\mathbf{4 k s i}, f_{y}=\mathbf{6 0} \mathbf{k s i}{ }^{\mathrm{a}}$

| $b x d$ (in) | Approximate Values for $\mathrm{a} / \mathrm{d}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | 0.1 | 0.2 | 0.3 |
|  | Approximate Values for $\rho$ |  |  |
|  | 0.0057 | 0.01133 | 0.017 |
| $10 \times 14$ | 2 \#6 | 2 \#8 | 3 \#8 |
|  | 53 | 90 | 127 |
| $10 \times 18$ | 3 \#5 | 2 \#9 | 3 \#9 |
|  | 72 | 146 | 207 |
| $10 \times 22$ | 2 \#7 | 3 \#8 | (3 \#10) |
|  | 113 | 211 | 321 |
| $12 \times 16$ | 2 \#7 | 3 \#8 | 4 \#8 |
|  | 82 | 154 | 193 |
| $12 \times 20$ | 2 \#8 | 3 \#9 | 4 \#9 |
|  | 135 | 243 | 306 |
| $12 \times 24$ | 2 \#8 | 3 \#9 | (4 \#10) |
|  | 162 | 292 | 466 |
| $15 \times 20$ | 3 \#7 | 4 \#8 | 5 \#9 |
|  | 154 | 256 | 383 |
| $15 \times 25$ | 3 \#8 | 4 \#9 | 4 \#11 |
|  | 253 | 405 | 597 |
| $15 \times 30$ | 3 \#8 | 5 \#9 | (5 \#11) |
|  | 304 | 608 | 895 |
| $18 \times 24$ | 3 \#8 | 5 \#9 | 6 \#10 |
|  | 243 | 486 | 700 |
| $18 \times 30$ | 3 \#9 | 6 \#9 | (6 \#11) |
|  | 385 | 729 | 1074 |
| $18 \times 36$ | 3 \#10 | 6 \#10 | (7 \#11) |
|  | 586 | 1111 | 1504 |
| $20 \times 30$ | 3 \# 10 | 7 \# 9 | 6 \# 11 |
|  | 489 | 851 | 1074 |
| $20 \times 35$ | 4 \#9 | 5 \#11 | (7 \#11) |
|  | 599 | 1106 | 1462 |
| $20 \times 40$ | 6 \#8 | 6 \#11 | (9 \#11) |
|  | 811 | 1516 | 2148 |
| $24 \times 32$ | 6 \#8 | 7 \#10 | (8 \#11) |
|  | 648 | 1152 | 1528 |
| $24 \times 40$ | 6 \#9 | 7 \#11 | (10 \#11) |
|  | 1026 | 1769 | 2387 |
| $24 \times 48$ | 5 \#10 | (8 \#11) | (13 \#11) |
|  | 1303 | 2426 | 3723 |

${ }^{a}$ Table yields values of factored moment resistance in kip-ft with reinforcement indicated. Reinforcement choices shown in parentheses require greater width of beam or use of two stack layers of bars. (Adapted and corrected from Simplified Engineering for Architects and Builders, $11^{\text {th }}$ ed, Ambrose and Tripeny, 2010.

## Beam / One-Way Slab Design Flow Chart



## Beam / One-Way Slab Design Flow Chart - continued



# ONE-WAY CONCRETE JOIST CONSTRUCTION: STEEL LAP PAN FORMING SYSTEM 

## ENGINEERING DATA REPORT NUMBER 43

933 N. Plum Grove Rd., Schaumburg, Illinois 60173-4758

## INTRODUCTION

This type of construction has frequently utilized ready-made steel pan forms of standard sizes. Depths and spacings for one-way concrete joist construction were standardized in 1932 by the U.S. Department of Commerce on the basis of an industry study by a committee of Architects/Engineers, Contractors and Steel Form Suppliers. That early standard has since been replaced by ANSI/CRSI A48.1-1986 "Forms for One-Way Joist Construction," which establishes standard dimensions for one-way joist forms. Standard form widths are 20, 30, 40* 53* and 66* inches, corresponding to structural modules ranging from 2 to 6 feet. Standard depths are $8,10,12,14,16,18$, 20, 22* and 24* inches. Not all depths are manufactured in each form width. Filler forms and tapered endforms are usually available locally to fit varying floor layouts and sizes. This type of construction is well established with a long record of successful use.
J oist construction was developed to reduce dead weight and reinforcement. As desired spans increase, the efficiency of solid slab construction is rapidly offset by the increase in the dead load. J oist construction enables the Architect/Engineer to provide the depth required for adequate stiffness and efficient utilization of the reinforcement without excessively high dead load/live load ratios. Standard size reusable forms make it possible to eliminate unnecessary dead weight with overall economy. Longer spans or relatively heavy loads can be accommodated by using tapered end forms which permit widening of the ribs in areas of high shear.

## DESCRIPTION

One-way concrete joist construction provides a monolithic combination of regularly-spaced joists (ribs) and a thin slab of concrete cast in place to form an integral unit with the supporting beams, columns and walls. In one-way concrete joist construction, the joists are arranged in one direction between parallel supports. Joist rib widths vary from 4 to 6 inches. Standard endforms consist of square endforms. A tapered endform for a 2 -foot module tapers from 20

[^3]to 16 inches wide in a distance of 36 inches. A tapered endform for a 3 -foot module tapers from 30


Figure 1 Tapered Endforms
to 25 inches (in some systems 26 inches) wide in a distance of 36 inches. See Figure 1.
Wide-module joist systems (also referred to as "skip-joist" systems) are defined as joist systems with a clear spacing between the ribs of more than 30 inches. Since this module, for application of the ACI Building Code, exceeds the rib spacing limit for standard joist construction (Section 8.11.3), widemodule joists become repetitive " T " beams and are subject to design requirements for such members.
Joist widths for wide-module joist systems vary from 6 to 8 inches. Standard forms for void spaces between ribs are 40,53 or 66 inches wide and 12, 14, 16, 18, 20, 22 or 24 inches deep. Standard endforms consist of square endforms. Use of square end joist forms simplifies forming. Tapered endforms are generally not available for wide-module systems. See Table 1.

## FORMWORK SELECTION CONSIDERATIONS

Maximum overall economy in concrete joist construction, as in any cast-in-place reinforced concrete design, is achieved by considering the relatively high cost of formwork and construction time versus material costs. Almost invariably overall economy is achieved by the maximum reuse of the same forms throughout the project, not only throughout each floor but also the same layout and

TABLE 1 Standard Dimensions of Forms for One-way Joist Construction¹

| System | Standard Forms |  | Special Filler Forms $^{4}$ |  |
| :---: | :---: | :--- | :--- | :--- |
|  | Width $^{2}$ | Depth |  |  |
|  | Depth $^{3}$ | Width $^{2}$ | Den $^{2}$ | 10,15 |
| 2NOO | 20 | $8,10,12$ | 10,12 |  |
| 3NOO | 30 | $8,10,12,14,16,20$ | $10,15,20$ | $8,10,12,14,16,20$ |
| 4NOO | 40 | $12,14,16,18,20,22,24$ | 20,30 | - |
| 5NOO | 53 | $16,14,16,18,20,22,24$ |  |  |
| 6NOO | 66 | $14,16,20$ | - | - |

## NOTES

1. All dimensions are in inches, except the module designations.
2. Width is the horizontal clear distance, between two consecutive joists, measured at the bottom of the joists.
3. Depth is the vertical distance, measured between two consecutive joists, from the underside of the concrete slab to the bottom of the joists.
4. Special filler forms may be available only in limited quantities. Availability should be investigated before specifying these forms.
5. Tapered endforms are available for the one-way 3NOO module. These forms are 30 inches wide at one end and 25 inches wide at the other end, and they are 36 inches long. Standard depths of these forms are 8, $10,12,14,16$, and 20 inches.
6. Tapered endforms are available for the one-way 4AOO module. These forms are 40 inches wide at one end and 34 inches wide at the other end, and they are 36 inches long. Standard depths of these forms are 12, $14,16,18,20,22$, and 24 inches. These forms are generally available only on the West Coast.
size of forms for all levels of the structure.
The use of the lap-type steel one-way pan system is probably one of the most efficient methods of reinforced concrete construction ever devised in terms of spans and applied loads versus volume of concrete and weight of reinforcing steel. A steel lap pan system has one major drawback: typically it can produce no better than a Class ' $C$ ' finish.
Sectional steel pan forms can adjust to varying site conditions without extensive detailing and fabricating of special shapes. The Architect/E ngineer is allowed great freedom in varying joist widths for accommodating concentrated loads by slightly adjusting the center-to-center spacing of the ribs. Clearing blockouts, drops and other interferences is accomplished by workers simply starting and stopping pan runs as required. Steel pan forms are a proper forming system to consider when evaluating design choices because they provide inherently stiff floor systems for the volume of concrete and reinforcing steel, and the forms are economical to obtain and erect when concrete esthetics are not a concern.

Project specifications are often vague with reference to laps and single one-piece voids. The Architect/E ngineer's expectations are generally different from those of the Contractor. The Contractor should be very sensitive to the Class of finish for which the pan forms are intended. Lap pans are generally inappropriate for exposed work. The Architect/E ngineer's attention should be focused on the end product results during pre-construction meetings as to the finish that these forms are and are not capable of producing. For instance, when pans are lapped, both the joist width and slab thickness vary slightly. ACI 117 tolerances for joists and slabs are $+3 / 8,-1 / 4$ inch in width and thickness (Section 4.4.1). The Contractor needs to ensure that the erection of the formwork is performed with a reasonable degree of accuracy. Finally, the Architect/Engineer may want to recognize the
challenges with this type of forming and specify a joist width one inch larger than required by design. While it is usually better to cast an onsite mockup section, it may be more practical and prudent to have the Architect/E ngineer and Owner participate in a site visit to a structure of similar construction and application to measure both esthetics and performance.

## FABRICATION AND ERECTION

The typical lap pan is a 16 -gauge or a 14 -gauge piece of sheet metal, 3 feet long, bent into one of three traditional shapes (see Figure 2) with varying flange widths dependent on style and Supplier. Both ends are open. A chalkline on the deck or soffit form should be used to align the pans. End caps are placed first and work proceeds toward the center of the member from both ends, overlapping the pans until proper closure is achieved. Flanged pans are nailed into position. After the pans are tightly in place, they should be oiled before other trades proceed with their work. See Figure 3.

The normal procedure for setting pans is to set the end caps first, nailed to the deck form on the line where the coffer begins. A long section of pan is first placed over the end cap. Then, through pre-punched matching holes in the top flange of the end cap and the top surface of the pan section, nails are dropped in to form a bond between the form sections. It is not uncommon to see small machine screws or center pin rivets used. However, form stripping procedures need to be considered with these types of fasteners. This connecting procedure also assists in preventing the end cap from collapsing inward under the pressure of concrete placing. The pan section is then nailed in place and a free standing steel or wood diaphragm (internal brace) is inserted into the form (suggested spacing is 18 inches on center under normal concrete placing conditions and should include the lap point between pans) and nailed in
place. The next pan section is then installed, reasonably lapping ( 1 to 5 inches) the previous section and the previous procedure is repeated until the coffer is completely formed. It should be noted that all pans may require diaphragms to resist lateral pressures. However, 14 -gauge pans with a depth of 16 inches or greater should always be installed with internal bracing. The soffits of all steel pans should be strengthened with some type of permanently attached internal brace, the most common of which is a welded sheet metal angle at least of the same gauge as the body of the pan. Because the steel lap pan system is characterized by offsets, fins and


Figure 2 Lap Pan Shapes


Figure 3 Setting Lap Pans
protrusions as well as chips and dings that result from the removal of the pan sections, the contract documents should include guidance and information on acceptable tolerances for formed surfaces.
It should be pointed out that care must be taken with the installation of any embedded items or mechanical inserts or fixtures. If the attachment of these items is not considered in relationship to the stripping of these forms they may act as anchors preventing the removal of the pan forms. Therefore, it is recommended that only center pin soft rivets be used. The center pin of the rivet will remain exposed
on the underside of the pan form and can be removed prior to stripping. This will allow the body of the rivet to close as the pan is stripped, permitting the easy removal of the forms.

## TOLERANCES

Tolerance guidance can be found in several ACl standards and reports. ACI 117, "Standard Tolerances for Concrete Construction and Materials"; ACI 301, "Specifications for Structural Concrete"; and ACI 347R, "Guide to Formwork for Concrete," provide information on finished surfaces, but do not address pan joist surfaces specifically. ACI 117 and ACI 347R limit offsets and other irregularities based on "Class" of surface finish. See Table 2. The ACI 117 standard might be regarded as the most authoritative. The mandatory specification checklist in ACl 117 requires the Architect/Engineer to designate the intended Class of surface finish and thereby establish the tolerance for form offsets. ACl 301 addresses the finishing of formed surfaces in Chapter 2 and differentiates between rough form finishes (those not exposed to public view) and smooth finishes (exposed to public view). ACI 301 requires: "Patch tie holes and defects. Remove all fins completely." for smooth formed finishes, but permits up to $1 / 4$ inch fins for rough finishes. As a default, Article 5.3.3.5 of ACI 301 calls for the finish to be based on exposure to public view where

| Type of <br> Irregularity | Class of Surface Finish |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D |
| Gradual <br> (ACI 347R) | $1 / 8$ | $1 / 4$ | $1 / 2$ | 1 |
| Abrupt |  |  |  |  |
| (ACI 117) | $1 / 8$ | $1 / 4$ | $1 / 2$ | 1 |
| (ACI 347R) | $1 / 8$ | $1 / 4$ | $1 / 4$ | 1 |


| Class A: | For surfaces prominently exposed to public view where <br> appearance is of special importance. |
| :--- | :--- |
| Class B: | Coarse-textured concrete-formed surfaces intended to <br> receive plaster, stucco, or wainscoting. |
| Class C: | General standard for permanently exposed surfaces where <br> other finishes are not specified. |
| Class D: | Minimum-quality surface where roughness is notobjectionable, <br> usually applied where surfaces will be concealed. |

surface finish is not designated in the contract documents.

TABLE 2 Surface Finish Class
ACl Committee 347 notes that revisions of the 347 R report are in progress to change the limit for abrupt offsets within Class C finish to $1 / 2$ inch, consistent with ACI 117. Although ACI 347R cautions against using pry bars directly against concrete to remove formwork, this is common practice in many areas of the country.

## CONCLUSION

Surface irregularities should be expected in pan joist construction. It is difficult to patch surface spalls successfully. The patch may be more noticeable

## ACKNOWLEDGMENT

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than the spall. The continuing use of the steel lap pan joist form system remains a proven and excellent method of forming site cast reinforced concrete floor systems in non-critically exposed applications.

REFERENCES

1. Formwork For Concrete, SP-4, 5th Edition, American Concrete Institute, Farmington Hills, Michigan, 1995.
2. "Standard Tolerances for Concrete Construction and Materials (ACI 117-90)", American Concrete Institute.
3. "Specifications for Structural Concrete (ACI 30196)", American Concrete Institute.
4. "Guide to Formwork for Concrete (ACI 347R-94)", American Concrete Institute.
5. "Building Code Requirements for Structural Concrete ( $\mathrm{ACl} 318-95$ ) and Commentary ( ACI 318 R 95)", American Concrete Institute.

ARCH 631

# WIDE-MODULE JOIST SYSTEMS - REVISITED 

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## INTRODUCTION

Standard joist construction, as defined in $\mathrm{ACI} 8.11^{*}$, includes a limit of 30 in . on the maximum clear spacing between ribs. According to the accompanying Commentary for ACI 8.11 , the rationale for the limit on rib spacing is that ACI 8.11 includes special provisions for higher design shear strengths of the concrete and less concrete cover over the reinforcement. Dimensions of removable standard size forms for modules of $2^{\prime}-0^{\prime \prime}$ and $3^{\prime}-0^{\prime \prime}$ are given in Table 1.

An increasingly popular type of joist construction is the wide-module joist system. Wide-module joist systems may be defined as joist systems with clear spacings of
ribs exceeding 30 in . Since the rib spacings for widemodules are greater than the ACI 318 Code limit for standard joist construction, wide-module joists must be designed as T-beams. In other words, the design shear strength of concrete and minimum concrete cover requirements for beams are applicable to the T-beams of wide-module joist systems. Economy in construction is achieved through the use of combinations of commonly available, re-usable standard joist forms. Standard form dimensions for modules of $4^{\prime}-0^{\prime \prime}$, $5^{\prime}-0^{\prime \prime}$ and $6^{\prime}-0^{\prime \prime}$ are given in Table 1.

* References in this report to "Building Code Requirements for Structural Concrete ( $\mathrm{ACl} 318-99$ )" are given as " ACl " followed by the appropriate section number.

Table 1 Dimensions of Forms for One-Way Joist Construction ${ }^{(1)}$

| Module | Standard Forms |  | Special Filler Forms ${ }^{(4)}$ |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Width $^{(2)}$ | Depth $^{(3)}$ |  |  |  | Width $^{(2)}$ | Depth ${ }^{(3)}$ |
| Standard Joist Construction |  |  |  |  |  |  |  |
| $2^{\prime}-0^{\prime \prime}$ | 20 | $8,10,12$ | 10,15 | $8,10,12$ |  |  |  |
| $3^{\prime}-0^{\prime \prime(5)}$ | 30 | $8,10,12,14,16,20$ | $10,15,20$ | $8,10,12,14,16,20$ |  |  |  |
| Wide-Module Joist Construction |  |  |  |  |  |  |  |
| $4^{\prime}-0^{\prime \prime(6)}$ | 40 | $12,14,16,18,20,22,24$ <br> $5^{\prime}-0^{\prime \prime}$ <br> $6^{\prime}-0^{\prime \prime}$ | 53 | $16,20,24$ <br> $14,16,20,24$ |  |  |  |

## NOTES

1. All dimensions are in inches, except the module designations.
2. Width is the horizontal clear distance, between two consecutive joists, measured at the bottom of the joists.
3. Depth is the vertical distance, measured between two consecutive joists, from the underside of the concrete slab to the bottom of the joists.
4. Special filler forms may be available only in limited quantities. Availability should be investigated before specifying these forms.
5. Tapered endforms are available for the one-way $3^{\prime}-0^{\prime \prime}$ module. These forms are 30 in . wide at one end and 25 in . wide at the other end, and they are 36 in . long. Standard depths of these forms are 8, 10, 12, 14, 16, and 20 in .
6. Tapered endforms are available for the one-way $4^{\prime}-0^{\prime \prime}$ module. These forms are 40 in . wide at one end and 34 inches wide at the other end, and they are 36 in . long. Standard depths of these forms are 12, 14, 16, 18, 20, 22, and 24 in . These forms are generally available only on the West Coast.

## TYPICAL WIDE-MODULE JOIST DIMENSIONS

Figure 1 shows typical cross-sectional dimensions for $5^{\prime}-0^{\prime \prime}$ and $6^{\prime}-0^{\prime \prime}$ wide modules. As noted in the figure, the modules are formed with single size forms in widths of 53-in. and 66-in.

(Where wider ribs are required, the module (spacing of joist forms is increased. Note that typical single size forms are available in $53-\mathrm{in}$. and 66 -in. widths)

Figure 1 - Typical Wide-Module Joist Dimensions
The wide-module joist system can easily adapt to wider modules where required for architectural purposes, or to provide wider ribs where structural considerations require the use of larger reinforcing bars and higher shear capacity. Where single full-width forms are not readily available, combinations of smaller standard forms may be used with covers over the omitted ribs. See Figure 2 for an example of a $66-\mathrm{in}$. clear spacing of ribs resulting from using $30-\mathrm{in}$. standard joist forms.


Figure 2 - 66-in. Module Using 30-in. Standard Forms

## TWO-WAY JOIST CONSTRUCTION

Two-way joist construction, meeting the requirements of ACl 8.11 , is commonly called waffle slab construction. Waffle slabs are designed as two-way flat slab systems under Chapter 13 of the ACI 318 Building Code. The dimensions of forms for standard two-way joist construction, i.e., waffle slabs, with modules of $2^{\prime}-0^{\prime \prime}, 2^{\prime}-6^{\prime \prime}$ and $3^{\prime}-0^{\prime \prime}$, are given in Table 2.
Table 2 also includes the standard dimensions of forms for two-way joist construction with $4^{\prime}-0^{\prime \prime}$ and $5^{\prime}-0^{\prime \prime}$ modules.

## GENERAL STRUCTURAL CONSIDERATIONS

Top Slab. To meet the fire ratings of the statutory building codes, the required thickness of the top slab is usually about $41 / 2-\mathrm{in}$. In standard joist construction (ACI 8.11), which limits the maximum clear spacing of the ribs to 30 -in., the flexural capacity of a $41 / 2$-in. top slab is underutilized. In contrast, the wide-module joist system takes advantage of the structural value of the slab thickness. A $41 / 2$-in. thick top slab is utilized more fully as a structural element.

## Table 2 Dimensions of Forms for Two-Way Joist Construction ${ }^{(1)}$

| System | Standard Forms |  | Special Filler Forms ${ }^{(4)}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Width ${ }^{(2)}$ | Depth ${ }^{(3)}$ | Width ${ }^{(2)}$ | Depth ${ }^{(3)}$ |
| 2'-0" Module 19" x 19" Square with $21 / 2^{\prime \prime}$ Flanges | $19 \times 19$ | 8, 10, 12, 14, 16 | - | - |
| 2'-6" Module $24^{\prime \prime} \times 24^{\prime \prime}$ Square with $3^{\prime \prime}$ Flanges | $24 \times 24$ | $8,10,12,14,16,20$ | - | - |
| 3'-0" Module $30^{\prime \prime} \times 30^{\prime \prime}$ Square with 3" Flanges | $30 \times 30$ | $8,10,12,14,16,20$ | $\begin{aligned} & 20 \times 20 \\ & 20 \times 30 \end{aligned}$ | $8,10,12,14,16,20$ <br> $8,10,12,14,16,20$ |
| 4'-0" Module $41^{\prime \prime} \times 41^{\prime \prime}$ Square with $31 / 2^{\prime \prime}$ Flanges | $41 \times 41$ | 12, 14, 16, 18, 20, 24 | - | - |
| 5'-0" Module $52^{\prime \prime} \times 52^{\prime \prime}$ Square with 4" Flanges | $52 \times 52$ | 14, 16, 20, 24 | $40 \times 40$ | 14, 16, 20, 24 |

Notes 1 through 4 under Table 1 are also applicable to this Table 2.

Ribs. Since the wide-module "joists" are technically classified as beams, their design must conform to the requirements for T-beams (ACI 8.10). Principal design requirements are:

1. Minimum concrete cover. $1 \frac{1}{2}$ in. to stirrups and main flexural bars (top, bottom, and sides) instead of $3 / 4 \mathrm{in}$. (ACI 7.7.1).
2. Design shear strength of concrete.
$\phi \mathrm{V}_{\mathrm{c}}=\phi 2 \sqrt{6} \mathrm{~b}_{\mathrm{w}} \mathrm{d}$ instead of $\phi \mathrm{V}_{\mathrm{c}}=\phi 2.2 \mathrm{~b}_{\mathrm{w}} \mathrm{d}$
( ACl 11.3.1.1 and 8.11.8).
3. Minimum area of shear reinforcement.
$\mathrm{A}_{\mathrm{v}}=50\left(\mathrm{~b}_{\mathrm{w}} \mathrm{s}\right) / f_{\mathrm{y}}$ where factored shear $\mathrm{V}_{\mathrm{u}}>0.5 \phi \mathrm{~V}_{\mathrm{c}}$ (ACl 11.5.5.3 and 11.5.5.1).
4. Reinforcing steel requirements and recommended details. Alternative arrangements to provide required shear reinforcement include the common open U-stirrup. With minimum rib widths, the maximum size of the main tensile reinforcement becomes limited by concrete cover requirements. And with minimum rib widths, fabricating constraints may require wider U-stirrups. These conditions may require that the U-stirrups be angled to fit. For minimum rib widths, the use of single leg stirrups simplifies placing. A special note should be included on the design drawings and placing drawings to require alternating of the stirrup positions. See Figure 3.


Figure 3 - Details for Single Leg and Common U-Stirrup Applictions for Shear Reinforcement in Wide-Module Joist

Welded wire fabric (plain or deformed) can also be used as shear reinforcement. The vertical wires are developed by two horizontal wires spaced at 2 in. maximum at the top and at the bottom (Figure 4).


Figure 4 - Alternative Stirrup Arrangements
Live-load reduction. A typical wide-module joist system can often be laid out to take maximum live-load reductions as permitted by most national and local building codes. For rectangular bays, where the joists' capacity permits the layout of joists parallel to the longer span, the longer span wide module width provides a larger area to qualify for the reduction.

Formwork economy. If conditions permit, further formwork economy will result from the use of a uniform depth. See Section A-A in Figure 5. The entire procedure of formwork utilizes the same height shores and provides a solid level work platform and the simplest formwork. Placing of reinforcement is facilitated and minimum time is achieved for completion of each floor level. The supporting beam will usually be wide enough to reduce shear reinforcement, often with the use of the higher two-way design shear strength immediately around the column. See layout of typical wide-module joist system in Figure 5.


Figure 5 - Typical Wide-Module Joist Layout

## SUPPORTING REINFORCEMENT

Chapter 3 in the CRSI Manual of Standard Practice contains information on the various types of bar supports used in reinforced concrete construction. Industry practices for the placing of bar supports are presented in the chapter. Recommendations for supporting reinforcing bars in standard one-way and twoway (waffle slabs) joist construction are also included in Chapter 3.

Recently, the CRSI technical committees have prepared recommendations for supporting the shrinkage and temperature reinforcement in the top slabs of wide-module joist construction:

For wide-module joist systems, it is recommended that the shrinkage and temperature reinforcement be supported by placing rows of slab bolsters at right angles to the shrinkage and temperature bars and spaced at $4^{\prime}-0^{\prime \prime}$ on center maximum, unless otherwise shown in the Contract Documents.

Placing practices in certain geographical areas of the country may prefer to substitute individual bar supports (steel wire, all-plastic, or precast concrete) in lieu of continuous bar supports. If individual bar supports are used, they should be placed at a maximum spacing of $4^{\prime}-0^{\prime \prime}$ on center each way.

## DESIGN AND DETAILING AIDS

The following publications provide guidance in designing and detailing reinforced concrete standard joist and wide-module joist systems.

1. CRSI Design Handbook, Concrete Reinforcing Steel Institute, 8th Edition, 1996.
2. Reinforcing Bar Detailing, Concrete Reinforcing Steel Institute, $4^{\text {th }}$ Edition, 2000.
3. ACI Detailing Manual, American Concrete Institute, SP-66, 1994.
4. "HB1JOIST and HB2JOIST, Handbook Computer Programs", Concrete Reinforcing Steel Institute, 1997.
5. "Effective Width of One-Way Monolithic Joist Construction as a Two-Way System", Structural Bulletin No. 8, Concrete Reinforcing Steel Institute, 1983.
6. Workbook for Evaluating Concrete Building Designs, Concrete Reinforcing Steel Institute, 2nd Edition, 1997.

## CLOSING COMMENTS

Potential savings in both materials and construction with the use of wide-module joist systems include:

- Utilization of the top slab required for fire rating,
- Elimination of $50 \%$ of the ribs,
- Uniform height of the deck form with the wide beam,
- Easy adjustments to fit the common range of modular column layouts,
- Less field labor time for construction.

The five preceding items are direct potential savings. Indirect benefits are:

- Elimination of half the ribs reduces dead load and reinforcement, and
- The wider rib spacing creates a larger supported area per rib, thereby increasing the allowable live load reductions and further reducing reinforcement.


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[^4]
## Examples: Reinforced Concrete

## Example 1

A simply supported beam 20 ft long carries a service dead load of $300 \mathrm{lb} / \mathrm{ft}$ and a live load of $500 \mathrm{lb} / \mathrm{ft}$. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi .

SOLUTION:
Find the design moment, $\mathrm{Mu}_{\mathrm{u}}$, from the factored load combination of $1.2 \mathrm{D}+1.6 \mathrm{~L}$. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of $10 \mathrm{in} \times 12 \mathrm{in}$. Self weight for normal weight concrete is the density of $150 \mathrm{lb} / \mathrm{ft}^{3}$ multiplied by the cross section area: self weight $=150 \mathrm{lb} / \mathrm{ft}^{3}(10 \mathrm{in})(12 \mathrm{in}) \cdot\left(\frac{1 \mathrm{ft}}{12 \mathrm{in}}\right)^{2}=125 \mathrm{lb} / \mathrm{ft}$
$W_{u}=1.2(300 \mathrm{lb} / \mathrm{ft}+125 \mathrm{lb} / \mathrm{ft})+1.6(500 \mathrm{lb} / \mathrm{ft})=1310 \mathrm{lb} / \mathrm{ft}^{2}$

Table 3.7.1
" \#14 and \#18 bars are used primarily as column reinforcement and are rarely used in beams.

Total Areas for Various Numbers of Reinforcing Bars

| $\begin{aligned} & \text { Bar } \\ & \text { Size } \end{aligned}$ | Nominal Diameter (in.) | Weight <br> (lb/ft) | Number of Bars |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \# | 0.375 | 0.376 | 0.11 | 0.22 | 0.33 | 0.44 | 0.55 | 0.66 | 0.77 | 0.88 | 0.99 | 1.10 |
| \#4 | 0.500 | 0.668 | 0.20 | 0.40 | 0.60 | 0.80 | 1.00 | 1.20 | 1.40 | 1.60 | 1.80 | 2.00 |
| \#5 | 0.625 | 1.043 | 0.31 | 0.62 | 0.93 | 1.24 | 1.55 | 1.86 | 2.17 | 2.48 | 2.79 | 3.10 |
| \#6 | 0.750 | 1.502 | 0.44 | 0.88 | 1.32 | 1.76 | 2.20 | 2.64 | 3.08 | 3.52 | 3.96 | 4.40 |
| \#7 | 0.875 | 2.044 | 0.60 | 1.20 | 1.80 | 2.40 | 3.00 | 3.60 | 4.20 | 4.80 | 5.40 | 6.00 |
| \#8 | 1.000 | 2.670 | 0.79 | 1.58 | 2.37 | 3.16 | 3.95 | 4.74 | 5.53 | 6.32 | 7.11 | 7.90 |
| \#9 | 1.128 | 3.400 | 1.00 | 2.00 | 3.00 | 4.00 | 5.00 | 6.00 | 7.00 | 8.00 | 9.00 | 10.00 |
| \#10 | 1.270 | 4.303 | 1.27 | 2.54 | 3.81 | 5.08 | 6.35 | 7.62 | 8.89 | 10.16 | 11.43 | 12.70 |
| \#11 | 1.410 | 5.313 | 1.56 | 3.12 | 4.68 | 6.24 | 7.80 | 9.36 | 10.92 | 12.48 | 14.04 | 15.60 |
| \#14 ${ }^{\text {a }}$ | 1.693 | 7.65 | 2.25 | 4.50 | 6.75 | 9.00 | 11.25 | 13.50 | 15.75 | 18.00 | 20.25 | 22.50 |
| $\# 18{ }^{\text {a }}$ | 2.257 | 13.60 | 4.00 | 8.00 | 12.00 | 16.00 | 20.00 | 24.00 | 28.00 | 32.00 | 36.00 | 40.00 |

The maximum moment for a simply supported beam
is $\frac{w l^{2}}{8}$ :
$M_{u}=\frac{w_{u} l^{2}}{8}=\frac{1310 \mathrm{lb} / f \mathrm{ft}(20 f t)^{2}}{8} 65,500 \mathrm{lb}-\mathrm{ft}$
$\mathrm{M}_{\mathrm{n}}$ required $=\mathrm{M}_{\mathrm{u}} / \phi=\frac{65,500^{\mathrm{lb}-\mathrm{ft}}}{0.9}=72,778 \mathrm{lb}-\mathrm{ft}$

To use the design chart aid, find $\mathrm{R}_{\mathrm{n}}=\frac{M_{n}}{b d^{2}}$,
estimating that d is about 1.75 inches less than h :
$d=12$ in -1.75 in $=10.25$ in
$R_{n}=\frac{72,778^{\text {lb-ft }}}{(10 \mathrm{in})(10.25 \mathrm{in})^{2}} \cdot(12 \mathrm{in} / \mathrm{ft})=831 \mathrm{psi}$
$\rho$ corresponds to approximately 0.023 , so the estimated area required, $A_{s,}$ can be found:
$\mathrm{A}_{s}=\rho b d=(0.023)(10 \mathrm{in})(10.25 \mathrm{in})=2.36 \mathrm{in}^{2}$
The number of bars for this area can be found from handy charts.
(Whether the number of bars actually fit for the width with cover and space between bars must also be considered.)
Try $\mathrm{A}_{\mathrm{s}}=2.37 \mathrm{in}^{2}$ from 3\#8 bars
$d=12$ in -1.5 in (cover) $-1 / 2(8 / 8$ in diameter bar) $=10$ in
Find the moment capacity of the beam as designed, $\phi \mathrm{M}_{\mathrm{n}}$

$$
\begin{aligned}
& \mathrm{a}=\mathrm{A}_{\mathrm{sf}} \mathrm{f} / 0.85 \mathrm{f}^{\prime} \mathrm{cb}=2.37 \mathrm{in}^{2}(40 \mathrm{ksi}) /[0.85(5 \mathrm{ksi}) 10 \mathrm{in}]=2.23 \mathrm{in} \\
& \phi \mathrm{M}_{\mathrm{n}}=\phi \mathrm{A}_{\mathrm{sf}}(\mathrm{~d}-\mathrm{a} / 2)=0.9\left(2.37 \mathrm{in}^{2}\right)(40 \mathrm{ksi})\left(10 \mathrm{in}-\frac{2.23 \mathrm{in}}{2}\right) \cdot\left(\frac{1}{12 \mathrm{in} / \mathrm{ft}}\right)=63.2 \mathrm{k} \text {-ft } \ngtr 64 \mathrm{k} \text {-ft needed (not OK) }
\end{aligned}
$$

So, we can increase d to 13 in , and $\phi \mathrm{M}_{\mathrm{n}}=70.3 \mathrm{k}$-ft (OK). Or increase $\mathrm{As}_{\mathrm{s}}$ to 2 \# 10 's ( 2.54 in 2), for $\mathrm{a}=2.39$ in and $\phi \mathrm{M}_{\mathrm{n}}$ of 67.1 k-ft (OK).

Example 2
Determine the capacity of a 16 " x 16 " column with 8 - \#10 bars, tied. Grade 40 steel and 4000 psi concrete.

## SOLUTION:

Find $\phi \mathrm{P}_{\mathrm{n}}$, with $\phi=0.65$ and $\mathrm{P}_{\mathrm{n}}=0.80 \mathrm{P}_{0}$ for tied columns and

$$
P_{o}=0.85 f_{c}^{\prime}\left(A_{g}-A_{s t}\right)+f_{y} A_{s t}
$$



Steel area (found from reinforcing bar table for the bar size):

$$
\mathrm{A}_{\text {st }}=8 \text { bars } \times\left(1.27 \mathrm{in}^{2}\right)=10.16 \mathrm{in}^{2}
$$

Concrete area (gross):

$$
\mathrm{A}_{\mathrm{g}}=16 \text { in } \times 16 \text { in = } 256 \mathrm{in}^{2}
$$

Grade 40 reinforcement has $\mathrm{f}_{\mathrm{y}}=40,000$ psi and $f_{c}^{\prime}=4000 \mathrm{psi}$

| ASTM STANDARD REINFORCING BARS |  |  |  |
| :---: | :---: | :---: | :---: |
| Bar size, no. | Nominal <br> diameter, in. | Nominal area, <br> in. $^{2}$ | Nominal weight, <br> lb/ft |
| 3 | 0.375 | 0.11 | 0.376 |
| 4 | 0.500 | 0.20 | 0.668 |
| 5 | 0.625 | 0.31 | 1.043 |
| 6 | 0.750 | 0.44 | 1.502 |
| 7 | 0.875 | 0.60 | 2.044 |
| 8 | 1.000 | 0.79 | 2.670 |
| 9 | 1.128 | 1.00 | 3.400 |
| 10 | 1.270 | 1.27 | 4.303 |
| 11 | 1.410 | 1.56 | 5.313 |
| 14 | 1.693 | 2.25 | 7.650 |
| 18 | 2.257 | 4.00 | 13.600 |

$\phi \mathrm{P}_{\mathrm{n}}=(0.65)(0.80)\left[0.85(4000 \mathrm{psi})\left(256 \mathrm{in}^{2}-10.16 \mathrm{in}^{2}\right)+(40,000 \mathrm{psi})\left(10.16 \mathrm{in}^{2}\right)\right]=646,026 \mathrm{lb}=646 \mathrm{kips}$

## Case Study in Reinforced Concrete

adapted from Simplified Design of Concrete Structures, James Ambrose, $7^{\text {th }}$ ed.

## Building description

The building is a three-story office building intended for speculative rental. Figure 17.37 presents a full-building section and a plan of the upper floor. The exterior walls are permanent. The design is a rigid perimeter frame to resist lateral loads.

Loads (UBC 1994)

## Live Loads:

Roof: $20 \mathrm{lb} / \mathrm{ft}^{2}$
Floors: Office areas: $50 \mathrm{lb} / \mathrm{ft}^{2}(2.39 \mathrm{kPa})$
Corridor and lobby: $100 \mathrm{lb} / \mathrm{ft}^{2}(4.79 \mathrm{kPa})$ Partitions: $20 \mathrm{lb} / \mathrm{ft}^{2}(0.96 \mathrm{kPa})$

Wind: map speed of $80 \mathrm{mph}(190 \mathrm{~km} / \mathrm{h})$; exposure B

## Assumed Construction Loads:

Floor finish: $5 \mathrm{lb} / \mathrm{ft}^{2}(0.24 \mathrm{kPa})$
Ceilings, lights, ducts: $15 \mathrm{lb} / \mathrm{ft}^{2}(0.72 \mathrm{kPa})$
Walls (average surface weight):
Interior, permanent: $10 \mathrm{lb} / \mathrm{ft}^{2}(0.48 \mathrm{kPa})$
Exterior curtain wall: $15 \mathrm{lb} / \mathrm{ft}^{2}(0.72 \mathrm{kPa})$

## Materials



Use $f^{\prime}{ }_{c}=3000 \mathrm{psi}(20.7 \mathrm{MPa})$ and grade 60 reinforcement ( $f_{y}=60 \mathrm{ksi}$ or 414 MPa ).

## Structural Elements/Plan

Case 1 is shown in Figure 17.44 and consists of a flat plate supported on interior beams, which in turn, are supported on girders supported by columns. We will examine the slab, and a four-span interior beam.

Case 2 will consider the bays with flat slabs, no interior beams with drop panels at the columns and an exterior rigid frame with spandrel (edge) beams. An example of an edge bay is shown to the right. We will examine the slab and the drop panels.

For both cases, we will examine the exterior frames in the 3-bay direction.


## Case 1:

Slab:
The slabs are effectively $10 \mathrm{ft} \times 30 \mathrm{ft}$, with an aspect ratio of 3 , making them one-way slabs. Minimum depths (by ACI) are a function of the span. Using Table 3-1 for one way slabs the minimum is $\frac{l_{n}}{24}$ with

|  |  |  |  | Minimum thickness, $\boldsymbol{h}$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Simply sup- <br> ported | One end <br> continuous | Both ends <br> continuous | Cantilever |  |  |
|  | Members not supporting or attached to partitions or <br> other construction likely to be damaged by large <br> deflections. |  |  |  |  |  |
| Member |  |  |  |  |  |  |
| Solid one- <br> way slabs | $\ell / \mathbf{2 0}$ | $\ell / \mathbf{2 4}$ | $\ell / \mathbf{2 8}$ | $\ell / \mathbf{1 0}$ |  |  |
| Beams or <br> ribbed one- <br> way slabs | $\ell / \mathbf{1 6}$ | $\ell / \mathbf{1 8 . 5}$ | $\ell / \mathbf{2 1}$ | $\ell / \mathbf{8}$ |  |  | 5 inches minimum for fire rating. We'll presume the interior beams are 12 " wide, so

$$
\begin{aligned}
& l_{n}=10 \mathrm{ft}-1 \mathrm{ft}=9 \mathrm{ft} \\
& \text { minimum } \mathrm{t}(\text { or h })=\frac{9^{\text {ft }} \cdot 12^{\text {in/ ft }}}{24}=4.5 \mathrm{in}
\end{aligned}
$$

Use 5 in.
dead load from slab $=\frac{150^{\mathrm{lb} / \mathrm{ft}^{3}} \cdot 5^{\text {in }}}{12^{\text {in/ft }}}=62.5 \mathrm{lb} / \mathrm{ft}^{2}$
total dead load $=(5+15+62.5) \mathrm{lb} / \mathrm{ft}^{2}+2$ " of lightweight concrete topping with weight of $18 \mathrm{lb} / \mathrm{ft}^{2}(0.68 \mathrm{KPa})$ (presuming interior wall weight is over beams \& girders)

$$
\text { dead load }=100.5 \mathrm{lb} / \mathrm{ft}^{2}
$$



FIGURE 17.44 Building Five: Framing plan for the concrete structure for the upper floor.
live load $($ worst case in corridor $)=100 \mathrm{lb} / \mathrm{ft}^{2}$
total factored distributed load (ASCE-7) of 1.2D+1.6L:

$$
\mathrm{w}_{u^{\prime}}=1.2\left(100.5 \mathrm{lb} / \mathrm{ft}^{2}\right)+1.6\left(100 \mathrm{lb} / \mathrm{ft}^{2}\right)=280.6 \mathrm{lb} / \mathrm{ft}^{2}
$$

Maximum Positive Moments from Figure 2-3, end span (integral with support) for a $\mathbf{1} \mathbf{f t}$ wide strip:

$$
\mathrm{M}_{\mathrm{u}}(\text { positive })=\frac{w_{u} \ell^{2} n}{14}=\frac{w_{u}^{\prime} \cdot 1 f t \cdot \ell^{2}{ }_{n}}{14}=\frac{\left(280.6^{\mathrm{lb} / f^{2}}\right)(1 f t)\left(9^{f t}\right)^{2}}{14} \cdot \frac{1 \mathrm{k}}{1000 \mathrm{lb}}=1.62 \mathrm{k}-\mathrm{ft}
$$



Figure 2-3 Positive Moments-All Cases

Maximum Negative Moments from Figure 2-5, end span (integral with support) for a $\mathbf{1} \mathbf{f t}$ wide strip:

$$
\mathrm{M}_{\mathrm{u}-}(\text { negative })=\frac{w_{u} \ell^{2}{ }_{n}}{12}=\frac{w_{u} \cdot 1 f t \cdot \ell^{2}{ }_{n}}{12}=\frac{\left(280.6^{\left.\mathrm{lb} /{f t^{2}}^{2}\right)(1 f t)\left(9^{f t}\right)^{2}}\right.}{12} \cdot \frac{1 k}{1000 \mathrm{lb}}=1.89 \mathrm{k}-\mathrm{ft}
$$



Figure 2-5 Negative Moments-Slabs with spans $\leq 10$ fi
The design aid (Figure 3.8.1) can be used to find the reinforcement ratio, $\rho$, knowing $R_{n}=M_{n} / b d^{2}$ with $\mathrm{M}_{\mathrm{n}}=\mathrm{M}_{\mathrm{u}} / \phi_{f}$, where $\phi_{f}=0.9$. We can presume the effective depth to the centroid of the reinforcement, d , is 1.25 " less than the slab thickness (with $3 / 4$ " cover and $1 / 2$ of a bar diameter for a \#8 (1") bar) $=3.75$ ".

$$
\mathrm{R}_{\mathrm{n}}=\frac{1.89^{k-f t}}{(0.9)\left(12^{i n}\right)\left(3.75^{i n}\right)^{2}} \cdot 12^{i n / f t} \cdot 1000^{l b / k}=149.3 \mathrm{psi}
$$

so $\rho$ for $f^{\prime}{ }_{c}=3000 \mathrm{psi}$ and $f_{y}=60,000 \mathrm{psi}$ is the minimum. For slabs, $\mathrm{A}_{\mathrm{s}}$ minimum is 0.0018 bt for grade 60 steel.

$$
\mathrm{A}_{\mathrm{s}}=0.0018(12 \mathrm{in})(5 \mathrm{in})=0.108 \mathrm{in}^{2} / \mathrm{ft}
$$



Pick bars and spacing off Table 3-7. Use \#3 bars @ 12 in $\left(\mathrm{A}_{\mathrm{s}}=0.11 \mathrm{in}^{2}\right)$.
Table 3-7 Areas of Bars per Foot Width of Slab-A $\mathrm{A}_{\mathrm{s}}\left(\mathrm{in}^{2} / \mathrm{ft}\right)$

| $\begin{aligned} & \hline \text { Bar } \\ & \text { size } \end{aligned}$ | Bar spacing (in.) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| \#3 | 0.22 | 0.19 | 0.17 | 0.15 | 0.13 | 0.12 | 0.11 | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 |
| \#4 | 0.40 | 0.34 | 0.30 | 0.27 | 0.24 | 0.22 | 0.20 | 0.18 | 0.17 | 0.16 | 0.15 | 0.14 | 0.13 |
| \#5 | 0.62 | 0.53 | 0.46 | 0.41 | 0.37 | 0.34 | 0.31 | 0.29 | 0.27 | 0.25 | 0.23 | 0.22 | 0.21 |
| \#6 | 0.88 | 0.75 | 0.66 | 0.59 | 0.53 | 0.48 | 0.44 | 0.41 | 0.38 | 0.35 | 0.33 | 0.31 | 0.29 |
| \#7 | 1.20 | 1.03 | 0.90 | 0.80 | 0.72 | 0.65 | 0.60 | 0.55 | 0.51 | 0.48 | 0.45 | 0.42 | 0.40 |
| \#8 | 1.58 | 1.35 | 1.18 | 1.05 | 0.95 | 0.86 | 0.79 | 0.73 | 0.68 | 0.63 | 0.59 | 0.56 | 0.53 |
| \#9 | 2.00 | 1.71 | 1.50 | 1.33 | 1.20 | 1.09 | 1.00 | 0.92 | 0.86 | 0.80 | 0.75 | 0.71 | 0.67 |
| \#10 | 2.54 | 2.18 | 1.91 | 1.69 | 1.52 | 1.39 | 1.27 | 1.17 | 1.09 | 1.02 | 0.95 | 0.90 | 0.85 |
| \#11 | 3.12 | 2.67 | 2.34 | 2.08 | 1.87 | 1.70 | 1.56 | 1.44 | 1.34 | 1.25 | 1.17 | 1.10 | 1.04 |

Check the moment capacity. d is actually 5 in -0.75 in (cover) $-1 / 2(3 / 8 \mathrm{in}$ bar diameter $)=4.06$ in

$$
\begin{aligned}
& \mathrm{a}=\mathrm{A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}} / 0.85 \mathrm{f}^{\prime}{ }_{\mathrm{c}} \mathrm{~b}=0.11 \mathrm{in}^{2}(60 \mathrm{ksi}) /[0.85(3 \mathrm{ksi}) 12 \mathrm{in}]=0.22 \mathrm{in} \\
& \phi \mathrm{M}_{\mathrm{n}}=\phi \mathrm{A}_{\mathrm{s}} \mathrm{f}_{\mathrm{y}}(\mathrm{~d}-\mathrm{a} / 2)=0.9\left(0.11 \mathrm{in}^{2}\right)(60 \mathrm{ksi})\left(4.06 \mathrm{in}-\frac{0.22 \mathrm{in}}{2}\right) \cdot\left(\frac{1}{12 \mathrm{in} / \mathrm{ft}}\right)=1.96 \mathrm{k}-\mathrm{ft}>1.89 \mathrm{k}-\mathrm{ft} \text { needed }
\end{aligned}
$$

(OK)
Maximum Shear: Figure 2-7 shows end shear that is $w_{u} l_{n} / 2$ except at the end span on the interior column which sees a little more and you must design for $15 \%$ increase:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{u}-\mathrm{max}}=1.15 \mathrm{w}_{\mathrm{u}} l_{n} / 2=\frac{1.15\left(280.6^{1 \mathrm{lb/ft}}\right)\left(\cdot \mathrm{l}^{\mathrm{ft}}\right)\left(9^{f t}\right)}{2}=1452 \mathrm{lb}(\text { for a } 1 \mathrm{ft} \text { strip }) \\
& \mathrm{V}_{\mathrm{u}} \text { at d away from the support }=\mathrm{V}_{\mathrm{u}-\max }-\mathrm{w}(\mathrm{~d})=1452 \mathrm{lb}-\frac{\left(280.6^{1 \mathrm{~b} / \mathrm{ft}^{2}}\right)\left(\cdot \cdot^{f t}\right)(4.06 \mathrm{in})}{12^{i n / f t}}=1357 \mathrm{lb}
\end{aligned}
$$



Figure 2-7 End Shears-All Cases
Check the one way shear capacity: $\phi_{v} \mathrm{~V}_{\mathrm{c}}=\phi_{v} 2 \sqrt{f_{c}^{\prime}}$ bd $\left(\phi_{v}=0.75\right)$ :

$$
\phi_{v} \mathrm{~V}_{\mathrm{c}}=0.75(2) \sqrt{3000} p s i\left(12^{i n}\right)\left(4.06^{i n}\right)=4003 \mathrm{lb}
$$

Is $\mathrm{V}_{\mathrm{u}}$ (needed) $<\phi_{v} \mathrm{~V}_{\mathrm{c}}$ (capacity)? YES: $1357 \mathrm{lb} \leq 4003 \mathrm{lb}$, so we don't need to make the slab thicker.

Interior Beam (effectively a T-beam):
Tributary width $=10 \mathrm{ft}$ for an interior beam.

$$
\text { dead load }=\left(100.5 \mathrm{lb} / \mathrm{ft}^{2}\right)(10 \mathrm{ft})=1005 \mathrm{lb} / \mathrm{ft}(14.7 \mathrm{kN} / \mathrm{m})
$$



Reduction of live load is allowed, with an influence area, $\mathrm{A}_{\mathrm{I}}$, of 2 panels beside an interior beam, assuming the girder is 12 " wide. The live load is $100 \mathrm{lb} / \mathrm{ft}^{2}$ :

$$
\mathrm{L}=L_{o}\left(0.25+\frac{15}{\sqrt{A_{I}}}\right)=100^{\mathrm{l} / f^{2}}\left(0.25+\frac{15}{\sqrt{(30 f t-1 f t)(2 \times 10 f t)}}\right)=87.3 \mathrm{lb} / \mathrm{ft}^{2}
$$

$($ Reduction Multiplier $=0.873)$
live load $=87.3 \mathrm{lb} / \mathrm{ft}^{2}(10 \mathrm{ft})=873 \mathrm{lb} / \mathrm{ft}(12.7 \mathrm{kN} / \mathrm{m})$

Estimating a 12 " wide $\times 24$ " deep beam means the additional dead load from self weight ( $w=\gamma \cdot A$ in units of load/length) can be included. The top 5 inches of slab has already been included in the dead load:

$$
\begin{aligned}
& \text { dead load from self weight }=150^{\mathrm{b} / \mathrm{f}^{3}}(12 \text { in wide })(24-5 \text { in deep }) \cdot\left(\frac{1 f t}{12 \text { in }}\right)^{2}=237.5 \mathrm{lb} / \mathrm{ft}(3.46 \mathrm{kN} / \mathrm{m}) \\
& \mathrm{w}_{\mathrm{u}}=1.2(1005 \mathrm{lb} / \mathrm{ft}+237.5 \mathrm{lb} / \mathrm{ft})+1.6(873 \mathrm{lb} / \mathrm{ft})=2888 \mathrm{lb} / \mathrm{ft}(4.30 \mathrm{kN} / \mathrm{m})
\end{aligned}
$$

The effective width, $\mathrm{b}_{\mathrm{E}}$, of the T part is the smaller of $\frac{\ell_{n}}{4}, b_{w}+16 t$, or center-center spacing

$$
\mathrm{b}_{\mathrm{E}}=\operatorname{minimum}\{29 \mathrm{ft} / 4=7.25 \mathrm{ft}=87 \mathrm{in}, 12 \mathrm{in}+16 \times 5 \mathrm{in}=92 \mathrm{in}, 10 \mathrm{ft}=120 \mathrm{in}\}=87 \mathrm{in}
$$

The clear span for the beam is

$$
l_{\mathrm{n}}=30 \mathrm{ft}-1 \mathrm{ft}=29 \mathrm{ft}
$$

Maximum Positive Moments from Figure 2-3, end span (integral with support):

$$
\mathrm{M}_{\mathrm{u}}(\text { positive })=\frac{w_{u} \ell^{2}{ }_{n}}{14}=\frac{2888^{l b / f t}\left(29^{f t}\right)^{2}}{14} \cdot \frac{1 k}{1000 l b}=173.5 \mathrm{k}-\mathrm{ft}
$$

Maximum Negative Moments from Figure 2-4, end span (integral with support):


Figure 2-4 Negative Moments-Beams and Slabs

Figure 3.8.1 can be used to find an approximate $\rho$ for top reinforcement if $R_{n}=M_{n} / b d^{2}$ and we set $\mathrm{M}_{\mathrm{n}}=\mathrm{M}_{\mathrm{u}} / \phi_{\mathrm{f}}$. We can presume the effective depth is 2.5 " less than the 24 " depth (for 1.5 " cover and $1 / 2$ bar diameter for a \#10 (10/8)" bar + \#3 stirrups ( $3 / 8$ " more) ), so $d=21.5 "$.

$$
\mathrm{R}_{\mathrm{n}}=\frac{1000^{\text {lb/k }} \cdot 242.9^{k-f t}}{0.9 \cdot\left(12^{i n}\right)\left(21.5^{-i n}\right)^{2}} \cdot 12^{\text {in/ft }}=584 \mathrm{psi}
$$

so $\rho$ for $f^{\prime}{ }_{c}=3000 \mathrm{psi}$ and $f_{y}=60,000 \mathrm{psi}$ is about 0.011
Then we pick bars and spacing off Table 3-7 to fit in the effective flange width in the slab.
For bottom reinforcement (positive moment) the effective flange is so wide at 87 in , that it resists a lot of compression, and needs very little steel to stay under-reinforced ( $a$ is between 0.6 " and 0.5 "). We'd put in bottom bars at the minimum reinforcement allowed and for tying the stirrups to.

Maximum Shear: $\mathrm{V}_{\max }=\mathrm{w}_{\mathrm{u}} / / 2$ normally, but the end span sees a little more and you must design for $15 \%$ increase. But for beams, we can use the lower value of V that is a distance of d from the face of the support

$$
\mathrm{V}_{\mathrm{u}-\mathrm{design}}=1.15 \mathrm{w}_{\mathrm{u}} l_{n} / 2-\mathrm{w}_{\mathrm{u}} \mathrm{~d}=\frac{1.15\left(2888^{l b / f t}\right)\left(29^{f t}\right)}{2}-\frac{2888^{\mathrm{lb/ft}}\left(21.5^{i n}\right)}{12^{\text {in } / f t}}=42,983 \mathrm{lb}=43.0 \mathrm{k}
$$

Check the one way shear capacity $=\phi_{v} \mathrm{~V}_{\mathrm{c}}=\phi_{v} 2 \sqrt{f_{c}^{\prime}}$ bd, where $\phi_{v}=0.75$

$$
\phi_{v} \mathrm{~V}_{\mathrm{c}}=0.75(2) \sqrt{3000} p s i\left(12^{i n}\right)\left(21.5^{i n}\right)=21,197 \mathrm{lb}=21.2 \mathrm{k}
$$

Is $\mathrm{V}_{\mathrm{u}}$ (needed) $<\phi_{v} \mathrm{~V}_{\mathrm{c}}$ (capacity) ?
NO: 43.0 k is greater than 21.2 k , so stirrups are needed

$$
\phi_{v} \mathrm{~V}_{\mathrm{s}}=\mathrm{V}_{\mathrm{u}}-\phi_{v} \mathrm{~V}_{\mathrm{c}}=43.0 \mathrm{k}-21.2 \mathrm{k}=21.8 \mathrm{k}(\text { max needed })
$$

Using \#3 bars (typical) with two legs means $\mathrm{A}_{\mathrm{v}}=2\left(0.11 \mathrm{in}^{2}\right)=0.22 \mathrm{in}^{2}$.
To determine required spacing, use Table 3-8. For $\mathrm{d}=21.5$ " and $\phi_{v} \mathrm{~V}_{\mathrm{s}} \leq \phi_{v} 4 \sqrt{f_{c}^{\prime}}$ bd (where $\left.\phi_{v} 4 \sqrt{f_{c}^{\prime}} \mathrm{bd}=2 \phi V_{c}=2(21.2 \mathrm{k})=42.4 \mathrm{k}\right)$, the maximum spacing is $\mathrm{d} / 2=10.75 \mathrm{in}$. or $24^{\prime \prime}$.

$$
\mathrm{S}_{\text {required }}=\frac{\phi A_{v} f_{y} d}{V_{u}-\phi V_{c}}=\frac{\phi A_{v} f_{y} d}{\phi V_{s}}=\frac{0.75 \cdot 0.22^{i n} \cdot 60^{k s i} \cdot 21.5^{\text {in }}}{21.8^{k}}=9.75 \mathrm{in}, \text { so use } 9 \mathrm{in} .
$$

We would try to increase the spacing as the shear decreases, but it is a tedious job. We need stirrups anywhere that $\mathrm{V}_{\mathrm{u}}>\phi_{v} \mathrm{~V}_{\mathrm{c}} / 2$. One recommended intermediate spacing is $\mathrm{d} / 3$.

Table 3-8 ACI Provisions for Shear Design*

|  |  | $\mathrm{V}_{\mathrm{u}} \leq \frac{\phi \mathrm{V}_{\mathrm{c}}}{2}$ | $\phi V_{c} \geq V_{u}>\frac{\phi V_{c}}{2}$ | $\mathrm{V}_{\mathrm{u}}>\phi \mathrm{V}_{\mathrm{c}}$ |
| :---: | :---: | :---: | :---: | :---: |
| Required area of stirrups, $A_{V}{ }^{* *}$ |  | none | $\frac{50 b_{w} s}{\mathrm{fy}_{\mathrm{y}}}$ | $\frac{\left(V_{u}-\phi V_{c}\right) \mathrm{S}}{\phi \mathrm{f}_{\mathrm{y}} \mathrm{~d}}$ |
| Stirrup spacing, s | Required | - | $\frac{A_{v} f_{y}}{50 b_{w}}$ | $\frac{\phi A_{v} f_{y} d}{V_{u}-\phi V_{c}}$ |
|  | Recommended $\text { Minimum }{ }^{\dagger}$ | - | - | 4 in. |
|  | Maximum ${ }^{\dagger} \dagger$ (ACl 11.5.4) | - | $\frac{\mathrm{d}}{2} \text { or } 24 \mathrm{in} .$ | $\frac{\mathrm{d}}{2}$ or 24 in. for $\left(\mathrm{V}_{u}-\phi \mathrm{V}_{\mathrm{c}}\right) \leq \phi 4 \sqrt{\mathrm{f}_{c}^{\prime}} \mathrm{b}_{w} \mathrm{~d}$ |
|  |  |  |  | $\frac{\mathrm{d}}{4}$ or 12 in. for $\left(\mathrm{V}_{u}-\phi V_{c}\right)>\phi 4 \sqrt{\mathrm{f}_{c}^{\prime}} \mathrm{b}_{w} \mathrm{~d}$ |

*Members subjected to shear and flexure only; $\phi \mathrm{V}_{\mathrm{c}}=\phi 2 \sqrt{\mathrm{f}_{c}^{\prime}} \mathrm{b}_{\mathrm{w}} \mathrm{d}, \phi=0.75$ ( ACl 11.3.1.1)
** $A_{v}=2 \times A_{b}$ for $U$ stirrups; $f_{y} \leq 60 \mathrm{ksi}(\mathrm{ACl} 11.5 .2)$
$\dagger$ A practical limit for minimum spacing is $\mathrm{d} / 4$
$\dagger \dagger$ Maximum spacing based on minimum shear reinforcement ( $=\mathrm{A}_{\mathrm{v}} \mathrm{f}_{\mathrm{y}} / 50 \mathrm{~b}_{\mathrm{w}}$ ) must also be considered
( ACl 11.5 .5 .3 ).

The required spacing where stirrups are needed for crack control $\left(\phi_{v} \mathrm{~V}_{\mathrm{c} \geq} \mathrm{V}_{\mathrm{u}}>1 / 2 \phi_{v} \mathrm{~V}_{\mathrm{c}}\right)$ is $\mathrm{S}_{\text {required }}=\frac{A_{v} f_{y}}{50 b_{w}}=\frac{0.22 \mathrm{in}^{2}(60,000 \mathrm{psi})}{50(12 \mathrm{in})}=22 \mathrm{in}$ and the maximum spacing is $\mathrm{d} / 2=10.75 \mathrm{in}$. or $24 "$. Use 10 in.
A recommended minimum spacing for the first stirrup is 2 in . from the face of the support. A distance of one half the spacing near the support is often used.


END SPAN STIRRUP LAYOUT

## Spandrel Girders:

Because there is a concentrated load on the girder, the approximate analysis can't technically be used. If we converted the maximum moment (at midspan) to an equivalent distributed load by setting it equal to $\mathrm{w}_{\mathrm{u}} \mathrm{l}^{2} / 8$ we would then use:


Maximum Positive Moments from Figure 2-3, end span (integral with support):

$$
\mathrm{M}_{\mathrm{u}+}=\frac{w_{u} \ell^{2}{ }_{n}}{14}
$$

Maximum Negative Moments from Figure 2-4, end span (column support):

$$
\mathrm{M}_{\mathrm{u}-}=\frac{w_{u} \ell^{2}{ }_{n}}{10}\left(\text { with } \frac{w_{u} \ell^{2}{ }_{n}}{16} \text { at end }\right)
$$

## Column:

An exterior or corner column will see axial load and bending moment. We'd use interaction charts for $P_{u}$ and $M_{u}$ for standard sizes to determine the required area of steel. An interior column sees very little bending. The axial loads come from gravity. The factored load combination is $1.2 \mathrm{D}+1.6 \mathrm{~L}+0.5 \mathrm{~L}_{\mathrm{r}}$.

The girder weight, presuming 1 ' x 4 ' girder at $150 \mathrm{lb} / \mathrm{ft}^{3}=600 \mathrm{lb} / \mathrm{ft}$
Top story: presuming $20 \mathrm{lb} / \mathrm{ft}^{2}$ roof live load, the total load for an interior column (tributary area of $30^{\prime} \times 30^{\prime}$ ) is:

| $\mathrm{DL}_{\text {roof* }}$ assuming | $1.2 \times 100.5 \mathrm{lb} / \mathrm{ft}^{2} \times 30 \mathrm{ft} \times 30 \mathrm{ft}$ <br> he same live load and materials as the floors | $=108.5 \mathrm{k}$ |
| :---: | :---: | :---: |
| $\mathrm{DL}_{\text {beam }}$ | $1.2 \times 237.5 \mathrm{lb} / \mathrm{ft} \times 30 \mathrm{ft} \times 3$ beams | $=25.6 \mathrm{k}$ |
| DL $\mathrm{g}_{\text {girder }}$ | $1.2 \times 600 \mathrm{lb} / \mathrm{ft} \times 30 \mathrm{ft}$ | $=21.6 \mathrm{k}$ |
| $\mathrm{LL}_{\mathrm{r}}$ : | $0.5 \times 20 \mathrm{lb} / \mathrm{ft}^{2} \times 30 \mathrm{ft} \times 30 \mathrm{ft}$ | $=9.0 \mathrm{k}$ |
| Total |  | $=164.7 \mathrm{k}$ |

Lower stories:

| DL $_{\text {floor }}:$ | $1.2 \times 100.5 \mathrm{lb} / \mathrm{ft}^{2} \times 30 \mathrm{ft} \times 30 \mathrm{ft}$ | $=108.5 \mathrm{k}$ |
| :--- | :--- | :--- |
| $\mathrm{DL}_{\text {beam }}$ | $1.2 \times 237.5 \mathrm{lb} / \mathrm{ft} \times 30 \mathrm{ft} \times 3$ beams | $=25.6 \mathrm{k}$ |
| DL $_{\text {girder }}$ | $1.2 \times 600 \mathrm{lb} / \mathrm{ft} \times 30 \mathrm{ft}$ | $=21.6 \mathrm{k}$ |
| LL $_{\text {floor }}:$ | $1.6 \times(0.873) \times 100 \mathrm{lb} / \mathrm{ft}^{2} \times 30 \mathrm{ft} \times 30 \mathrm{ft}$ | $=125.7 \mathrm{k}$ |
| Total |  | $=281.4 \mathrm{k}$ |

$2^{\text {nd }}$ floor column sees $\mathrm{P}_{\mathrm{u}}=164.7+281.4=446.1 \mathrm{k}$
$1^{\text {st }}$ floor column sees $\mathrm{P}_{\mathrm{u}}=446.1+281.4=727.5 \mathrm{k}$
Look at the example interaction diagram for an 18 " x 18 " column (Figure 5-20 - ACI 318-02) using $\dot{\dot{L}_{\underline{c}}^{\prime}}=4000$ psi and $f_{y}=60,000$ psi for the first floor having $\mathrm{P}_{\mathrm{u}}=727.5 \mathrm{k}$, and $\mathrm{M}_{\mathrm{u}}$ to the column being approximately $10 \%$ of the beam negative moment $=0.1 * 242.9 \mathrm{k}-\mathrm{ft}=24.3 \mathrm{k}-\mathrm{ft}$ : (See maximum negative moment calculation for an interior beam.) The chart indicates the capacity for the reinforcement amounts shown by the solid lines.

For $\mathrm{P}_{\mathrm{u}}=727.5 \mathrm{k}$ and $\mathrm{Mu}=24.3 \mathrm{k}$-ft, the point plots below the line marked $4-\# 10(1.57 \%$ area of steel to an 18 in x 18 in area).


Lateral Force Design:
The wind loads from the wind speed, elevation, and exposure we'll accept as shown in Figure 17.42 given on the left in psf. The wind is acting on the long side of the building. The perimeter frame resists the lateral loads, so there are two with a tributary width of $1 / 2[(30 \mathrm{ft}) \mathrm{x}(4$ bays $)+2 \mathrm{ft}$ for beam widths and cladding] = $122 \mathrm{ft} / 2=61 \mathrm{ft}$

The factored combinations with dead and wind load are:
$1.2 \mathrm{D}+1.6 \mathrm{~L}_{\mathrm{r}}+0.5 \mathrm{~W}$
$1.2 \mathrm{D}+1.0 \mathrm{~W}+\mathrm{L}+0.5 \mathrm{~L}_{\mathrm{r}}$
The tributary height for each floor is half the distance to the next floor (top and bottom):


Exterior frame (bent) loads:
$\mathrm{H}_{1}=195^{\mathrm{lb} / f t}\left(61^{f t}\right)=11,895 \mathrm{lb}=11.9 \mathrm{k} /$ bent
$\mathrm{H}_{2}=\frac{234^{\mathrm{lb} / \mathrm{ft}}\left(61^{\mathrm{ft}}\right)}{1000^{\mathrm{lb} / k}}=14.3 \mathrm{k} / \mathrm{bent}$
$\mathrm{H}_{3}=\frac{227^{\mathrm{lb} / \mathrm{ft}}\left(61^{f t}\right)}{1000^{\mathrm{lb} / k}}=13.8 \mathrm{k} / \mathrm{bent}$
Using Multiframe4D, the axial force, shear and bending moment diagrams can be determined using the load combinations, and the largest moments, shear and axial forces for each member determined.

| $\mathrm{M}=218.8 \mathrm{k}-\mathrm{ft}$ |  |  | $\mathrm{M}=237.8 \mathrm{k}-\mathrm{ft}$ |  | $\mathrm{M}=236.3 \mathrm{k}-\mathrm{ft}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & M=203.7 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=26.6 \mathrm{k} \\ & \mathrm{P}=54.7 \mathrm{k} \end{aligned}$ | $\mathrm{V}=45.4 \mathrm{k}$ | $\begin{aligned} & \mathrm{M}=39.3 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=4.7 \mathrm{k} \\ & \mathrm{P}=91.0 \mathrm{k} \end{aligned}$ | $\mathrm{V}=45.5 \mathrm{k}$ | $\begin{aligned} & M=32.4 k-f t \\ & V=3.7 k \\ & P=91.0 k \end{aligned}$ | $\mathrm{V}=46.6 \mathrm{k}$ | $\begin{aligned} & M=183.9 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=24.8 \mathrm{k} \\ & \mathrm{P}=53.5 \mathrm{k} \end{aligned}$ |
|  | $\mathrm{P}=26.6 \mathrm{k}$ |  | $\mathrm{P}=28.9 \mathrm{k}$ |  | $\mathrm{P}=30.8 \mathrm{k}$ |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | $\mathrm{M}=332.4 \mathrm{ft}$ |  | $\mathrm{M}=330.9 \mathrm{k}-\mathrm{ft}$ |  | $\mathrm{M}=317.9 \mathrm{k}-\mathrm{ft}$ |  |
| $\begin{aligned} & M=190.8 k-f t \\ & V=28.0 k \\ & P=121.6 k \end{aligned}$ | $\mathrm{V}=60.5 \mathrm{k}$ | $\begin{aligned} & M=70.2 \mathrm{k}-\mathrm{ft} \\ & V=9.2 \mathrm{k} \\ & \mathrm{P}=215.5 \mathrm{k} \end{aligned}$ | $\mathrm{V}=60.0 \mathrm{k}$ | $\begin{aligned} & M=51.1 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=7.7 \mathrm{k} \\ & \mathrm{P}=215.7 \mathrm{k} \end{aligned}$ | $\mathrm{V}=59.7 \mathrm{k}$ | $\begin{aligned} & M=142.7 \mathrm{k}-\mathrm{ft} \\ & V=21.2 \mathrm{k} \\ & \mathrm{P}=120.3 \mathrm{k} \end{aligned}$ |
|  | $\mathrm{P}=1.9 \mathrm{k}$ |  | $\mathrm{P}=6.4 \mathrm{k}$ |  | $\mathrm{P}=10.4 \mathrm{k}$ |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | $\mathrm{M}=338.5 \mathrm{k}-\mathrm{ft}$ |  | $\mathrm{M}=349.5 \mathrm{k}-\mathrm{ft}$ |  | $\mathrm{M}=347.9 \mathrm{k}-\mathrm{ft}$ |  |
| $\begin{aligned} & M=165.1 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=21.6 \mathrm{k} \\ & \mathrm{P}=192.8 \mathrm{k} \end{aligned}$ | $\begin{aligned} & \mathrm{V}=60.9 \mathrm{k} \\ & \mathrm{P}=6.5 \mathrm{k} \end{aligned}$ | $\begin{aligned} & M=104.4 \mathrm{k}-\mathrm{ft} \\ & \mathrm{~V}=11.6 \mathrm{k} \\ & \mathrm{P}=340.6 \mathrm{k} \end{aligned}$ | $\begin{aligned} & \mathrm{V}=61.0 \mathrm{k} \\ & \mathrm{P}=4.1 \mathrm{k} \end{aligned}$ | $\begin{array}{ll} & \begin{array}{l}V=62.0 \mathrm{k} \\ \\ \mathrm{P}=1.6 \mathrm{k}\end{array} \\ \mathrm{M}=95.5 \mathrm{k}-\mathrm{ft} & \\ \mathrm{V}=10.3 \mathrm{k} \\ \mathrm{P}=341.7 \mathrm{k} & \\ \end{array}$ |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\mathrm{M}=102.6 \mathrm{k}$-ft |
|  |  |  |  |  |  | $\mathrm{V}=8.5 \mathrm{k}$ |
|  |  |  |  |  |  | $\mathrm{P}=185.7 \mathrm{k}$ |
|  |  |  |  |  |  |  |

(This is the summary diagram of force, shear and moment magnitudes refer to the maximum values in the column or beams, with the maximum moment in the beams being negative over the supports, and the maximum moment in the columns being at an end.)

Axial force diagram:


Bending moment diagram:

${ }^{\mathrm{Px}}$ Shear diagram:

${ }^{* 1}$ Displacement:


## Beam-Column loads for design:

The bottom exterior columns see the largest bending moment on the lee-ward side (left):

$$
\mathrm{P}_{\mathrm{u}}=192.8 \mathrm{k} \text { and } \mathrm{M}_{\mathrm{u}}=165.1 \mathrm{k}-\mathrm{ft} \quad \text { (with large axial load) }
$$

The interior columns see the largest axial forces:

$$
\mathrm{P}_{\mathrm{u}}=341.7 \mathrm{k} \text { and } \mathrm{M}_{\mathrm{u}}=95.5 \mathrm{k}-\mathrm{ft} \text { and } \mathrm{P}_{\mathrm{u}}=340.6 \mathrm{k} \text { and } \mathrm{M}_{\mathrm{u}}=104.4 \mathrm{k}-\mathrm{ft}
$$

Refer to an interaction diagram for column reinforcement and sizing.

## Case 2

## Slab:

The slabs are effectively $30 \mathrm{ft} \times 30 \mathrm{ft}$, making them two-way slabs. Minimum thicknesses (by ACI) are a function of the span. Using Table 4-1 for two way slabs, the minimum is the larger of $l_{\mathrm{n}} / 36$ or 4 inches. Presuming the columns are $18^{\prime \prime}$ wide, $l_{n}=30 \mathrm{ft}-(18 \mathrm{in}) /(12 \mathrm{ft} / \mathrm{in})=28.5 \mathrm{ft}$,

$$
\mathrm{h}=l_{n} / 36=(28.5 \times 12) / 36=9.5 \text { in }
$$

Table 4-1 Minimum Thickness for Two-Way Slab Systems

${ }^{1}$ Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$ (ACl 9.5.3.3)
${ }^{2}$ Drop panel length $\geq \ell 3$, depth $\geq 1.25 h(A C l ~ 13.4 .7)$
${ }^{3} \mathrm{Min} . \mathrm{h}=5 \mathrm{in}$. for $\alpha_{m} \leq 2.0 ; \mathrm{min} . \mathrm{h}=3.5 \mathrm{in}$. for $\alpha_{m}>2.0(\mathrm{ACl} 9.5 .3 .3)$

(a) Column strip for $l_{2} \leq l_{1}$

The table also says the drop panel needs to be $1 / 3$ long $=28.5 \mathrm{ft} / 3=9.5 \mathrm{ft}$, and that the minimum depth must be $1.25 \mathrm{~h}=1.25(9.5 \mathrm{in})=12 \mathrm{in}$.

For the strips, $l_{2}=30 \mathrm{ft}$, so the interior column strip will be $30 \mathrm{ft} / 4+30 \mathrm{ft} / 4=15 \mathrm{ft}$, and the middle strip will be the remaining 15 ft .
dead load from slab $=\frac{150^{\mathrm{lb} / \mathrm{ft}^{3}} \cdot 9.5^{\text {in }}}{12^{\text {in/ft }}}=118.75 \mathrm{lb} / \mathrm{ft}^{2}$
total dead load $=5+15+118.75 \mathrm{lb} / \mathrm{ft}^{2}+2$ " of lightweight concrete topping @ $18 \mathrm{lb} / \mathrm{ft}^{2}(0.68 \mathrm{KPa})$ (presuming interior wall weight is over beams \& girders)
total dead load $=156.75 \mathrm{lb} / \mathrm{ft}^{2}$
live load with reduction, where the influence area, $\mathrm{A}_{l}$, for two way slabs is one panel:

$$
\mathrm{L}=L_{o}\left(0.25+\frac{15}{\sqrt{A_{I}}}\right)=100 \mathrm{lb} / f_{t^{2}}\left(0.25+\frac{15}{\sqrt{(30 f t)(30 f t)}}\right)=75 \mathrm{lb} / \mathrm{ft}^{2}
$$

total factored distributed load:

$$
\mathrm{w}_{\mathrm{u}}=1.2\left(156.75 \mathrm{lb} / \mathrm{ft}^{2}\right)+1.6\left(75 \mathrm{lb} / \mathrm{ft}^{2}\right)=308.1 \mathrm{lb} / \mathrm{ft}^{2}
$$

total panel moment to distribute:

$$
\mathrm{M}_{\mathrm{o}}=\frac{w_{u} l_{2} l_{n}^{2}}{8}=\frac{308.1^{\mathrm{lb} / f^{2}}\left(30^{f t}\right)\left(28.5^{f t}\right)^{2}}{8} \cdot \frac{1 k}{1000 l b}=938.4 \mathrm{k}-\mathrm{ft}
$$

## Column strip, end span:

Maximum Positive Moments from Table 4-3, (flat slab with spandrel beams):
$\mathrm{M}_{\mathrm{u}+}=0.30 \mathrm{M}_{\mathrm{o}}=0.30 \cdot(938.4 \mathrm{k}-\mathrm{ft})=281.5 \mathrm{k}-\mathrm{ft}$
Maximum Negative Moments from Table 4-3, (flat slab with spandrel beams):
$\mathrm{M}_{\mathrm{u}-}=0.53 \mathrm{M}_{\mathrm{o}}=0.53 \cdot(938.4 \mathrm{k}-\mathrm{ft})=497.4 \mathrm{k}-\mathrm{ft}$
Table 4-3 Flat Plate or Flat Slab with Spandrel Beams


Notes: (1) All negative moments are at face of support.
(2) Torsional stiffness of spandrel beams $\beta_{t} \geq 2.5$. For values of $\beta_{t}$ less than 2.5 , exterior negative column strip moment increases to $\left(0.30-0.03 \beta_{t}\right) \mathrm{M}_{\mathrm{o}}$.

## Middle strip, end span:

Maximum Positive Moments from Table 4-3, (flat slab with spandrel beams):
$\mathrm{M}_{\mathrm{u}+}=0.20 \mathrm{M}_{\mathrm{o}}=0.20 \cdot(938.4 \mathrm{k}-\mathrm{ft})=187.9 \mathrm{k}-\mathrm{ft}$
Maximum Negative Moments from Table 4-3, (flat slab with spandrel beams):
$\mathrm{M}_{\mathrm{u}-}=0.17 \mathrm{M}_{\mathrm{o}}=0.17 \cdot(938.4 \mathrm{k}-\mathrm{ft})=159.5 \mathrm{k}-\mathrm{ft}$

Design as for the slab in Case 1, but provide steel in both directions distributing the reinforcing needed by strips.

Shear around columns: The shear is critical at a distance $\mathrm{d} / 2$ away from the column face. If the drop panel depth is 12 inches, the minimum d with two layers of 1 " diameter bars would be $12 "-3 / 4 "$ (cover) $-(1 ")-1 / 2\left(1^{\prime \prime}\right)=$ about 9.75 in (to the top steel).

tributary area for column

The shear resistance is $\phi_{v} \mathrm{~V}_{\mathrm{c}}=\phi_{v} 4 \sqrt{f_{c}^{\prime}} \mathrm{b}_{0} \mathrm{~d}, \phi_{v}=0.75$ where $\mathrm{b}_{0}$, is the perimeter length.
The design shear value is the distributed load over the tributary area outside the shear perimeter, $\mathrm{V}_{\mathrm{u}}=\mathrm{w}_{\mathrm{u}}$ (tributary area $-\mathrm{b}_{1} \times \mathrm{b}_{2}$ ) where b 's are the column width plus $\mathrm{d} / 2$ each side.

$$
\begin{aligned}
& \mathrm{b}_{1}=\mathrm{b}_{2}=18^{\prime \prime}+9.75^{\prime \prime} / 2+9.75^{\prime \prime} / 2=27.75 \mathrm{in} \\
& \mathrm{~b}_{1} \times \mathrm{b}_{2}=(27.75 i n)^{2} \cdot\left(\frac{1 \mathrm{ft}}{12 i n}\right)^{2}=5.35 \mathrm{ft}^{2} \\
& \mathrm{~V}_{\mathrm{u}}=\left(284.7^{l b / f t^{2}}\right)\left(30^{f t} \cdot 30^{f t}-4.97^{f t^{2}}\right) \cdot \frac{1 \mathrm{k}}{1000 \mathrm{lb}}=254.7 \mathrm{k}
\end{aligned}
$$

Shear capacity:

$$
\begin{aligned}
& \mathrm{b}_{\mathrm{o}}=2\left(\mathrm{~b}_{1}\right)+2\left(\mathrm{~b}_{2}\right)=4(27.75 \mathrm{in})=111 \mathrm{in} \\
& \phi_{V} \mathrm{~V}_{\mathrm{c}}=0.75 \cdot 4 \cdot \sqrt{3000} p s i \cdot 111^{\text {in }} \cdot 9.75^{\text {in }}=177,832 \mathrm{lb}=177.8 \mathrm{ksi}<\mathrm{V}_{\mathrm{u}}!
\end{aligned}
$$

The shear capacity is not large enough. The options are to provide shear heads or a deeper drop panel, or change concrete strength, or even a different system selection...

There also is some transfer by the moment across the column into shear.

## Deflections:

Elastic calculations for deflections require that the steel be turned into an equivalent concrete material using $\mathrm{n}=\frac{E_{s}}{E_{c}}$. $\mathrm{E}_{\mathrm{c}}$ can be measured or calculated with respect to concrete strength. For normal weight concrete $\left(150 \mathrm{lb} / \mathrm{ft}^{3}\right): \quad E_{c}=57,000 \sqrt{f_{c}^{\prime}}$

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{c}}=57,000 \sqrt{3000} p s i=3,122,019 \mathrm{psi}=3122 \mathrm{ksi} \\
& \mathrm{n}=29,000 \mathrm{psi} / 3122 \mathrm{ksi}=9.3
\end{aligned}
$$

Deflection limits are given in Table 9.5(b)
TABLE 9.5(b) - MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

| Type of member | Deflection to be considered | Deflection limitation |
| :---: | :---: | :---: |
| Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections | Immediate deflection due to live load $L$ | 2180* |
| Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections | Immediate deflection due to live load $L$ | /360 |
| Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections | That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load) ${ }^{\dagger}$ | //480 |
| Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections |  | /240 ${ }^{8}$ |
| *Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage. <br> ${ }^{\dagger}$ Long-term deflection shall be determined in accordance with 9.5 .2 .5 or 9.5 .4 .2 , but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered. <br> $\ddagger$ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements. <br> ${ }^{\S}$ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit. |  |  |

# Materials for Membrane Structures 

R. Houtman, M. Orpana

The most prominent material for stressed membrane structures is obviously the fabric. It is prominently present, attracts much attention and looks very simple. To obtain this pleasant charisma there has been done a lot of research. The material is analysed and specific properties are defined and adapted. Properties like transparency, durability, fire retardance but also elasticity, strength.
In this paper the fabric is discussed to get a better understanding of those properties. The composition of the fabric is explained, followed by a discussion of the most common fabrics available today. Finally the structural behaviour of the fabric is discussed.

## Threads

A thread is built up out of fibres. There are natural fibres and chemical fibres. Natural fibres have a restricted length and are bound up in strands. These are the socalled spin fibres. Chemical fibres theoretically have an endless length and are called filaments. The crosssection of natural fibres is smaller than 0.1 mm , where chemical fibres can have larger cross-sections. The shape of the cross-section is round for natural fibres but can have any shape in chemical fibres. For membrane structures it is best to have a yarn with a circular crosssection.
The mechanical properties of materials in the building industry are normally specified in $\mathrm{N} / \mathrm{mm}^{2}$. In technical textiles this is not common because it is not easy to determine the cross-section of a very small fibre. Therefore it is usual to determine the weight of a fibre with a certain length. When the specific mass is known from the fibre, it is possible to determine an average cross-section of the material. This mass-per-length unit is indicated with Titer with the symbol Tex: 1 Tex weight in grams per 1000 m length. In synthetic fibres it is common to use decitex: 1 dtex $=$ weight in grams per 10000 m length [7].
A Polyester fibre for example with a Titer of 8.35 dtex has a weight of 8.35 grams at a length of 10000 m . When the product is that small, it is very difficult to use it in industrial processes. Therefore it is spun into threads. One thread possibly consists out of hundreds of fibres. When a thread only has one fibre, it is called monofil.

Spin fibres need to be stabilised by twisting around the centre of the thread. Filaments do not need it, but it facilitates the handling. The twisting influences the stressstrain behaviour of the threads. The more the thread is twisted the more the elasticity decreases compared to the elasticity of the fibre. With the adjustment of the twisting the mechanical properties of the thread can be determined precisely.
The characterisation of a filament thread is according to the System Tex, where the number of fibres and twists are added. A thread for example which is called 2200 dtex f 200 z 60 has a total Titer of 2200 dtex , made out of 200 fibres, the thread is twisted 60 times per meter in z direction [7].
There are several fibres that can be applied in membrane structures. For each project it is necessary to consider which type of fabric can be used. Several fibres do have the potential to be applied, however the high costs of it prevent a wide utilisation.

## Cotton fibre

This type of fibre is the only organic fibre, which is being used in membrane structures. Frei Otto used it for his early garden show structures and nowadays it still is applied in some rental tents. As of its organic properties the material is subject to fungi and moisture. When used permanently it has an expected lifetime of about 4 years.

## Polyamide 6.6 (Nylon)

The nylon fibre has a bad resistance against UV light, swells in length direction when it gets wet and is herewith of little importance for textile architecture. It is frequently applied in the sailing industry because of the little weight and high strength.

## Polyester

Polyester fibre together with fibreglass is the most common fibre in textile architecture and regarded as a standard product. The fibre has a good tensile strength and elasticity. Because of its considerable elongation before yield, the material is "forgiving". It enables to make small corrections during installation. The mechanical properties of the material decrease by sunlight and there is ageing.

| Material | Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | Tensile strength ( $\mathrm{N} / \mathrm{mm}^{2}$ ) | Tensile strain (\%) | Elasticity (N/mM2) | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cotton | 1.5-1.54 | 350-700 | 6-15 | 4500-9000 | Only for temporary use of interest |
| Polyamid 6.6 <br> (Nylon) | 1.14 | Until 1000 | 15-20 | 5000-6000 | - When exposed to light only average resistance to ageing <br> - Swelling when exposed to moisture <br> - Only of little importance in textile architecture |
| Polyester fibre (Trevira, Teryiene, Dacron, Diolen) | $\begin{aligned} & 1.38- \\ & 1.41 \end{aligned}$ | 1000-1300 | 10-18 | $\begin{gathered} 10000- \\ 15000 \end{gathered}$ | - Widely spread, together with fibreglass a standard product in textile architecture |
| Fibreglass | 2.55 | Until 3500 | 2.0-3.5 | $\begin{gathered} 70000- \\ 90000 \end{gathered}$ | - When exposed to moisture, reduction of breaking strength <br> - Brittle fibres, therefore is spun into filaments of $3 \mu \mathrm{~m}$ diameter <br> - Together with Polyester a standard product in textile architecture |
| Aramid fibre (Kevlar, Arenka Twaron) | 1.45 | Until 2700 | 2-4 | $\begin{gathered} 130000- \\ 150000 \end{gathered}$ | - Special fibre for high-tech products |
| Polytetrafluorethylen (Teflon, Hostaflon Polyflon, Toyoflon etc.) | 2.1-2.3 | 160-380 | 13-32 | 700-4000 | - High moisture resistance <br> - Remarkable anti adhesive <br> - In air non-combustible <br> - Chemical inert |
| Carbon fibres (Celion, Carbolon, Sigrafil, Thornel) | 1.7-2.0 | 2000-3000 | < 1 | $\begin{gathered} 200000- \\ 500000 \end{gathered}$ | - Special fibres for high-tech products <br> - Very low expansion coefficient <br> - Non-combustible |

Table 1: Material properties of the base material of fabrics [7]

## Fibreglass

The material where fibreglass is made of is of course glass, where threads are spun from, which have a certain bending capacity. The fibreglass has a high tensile strength, but remains brittle and has low elastic strain. Because of the brittleness the material needs to be handled carefully and needs very accurate manufacturing. Ageing exerts little influence on the material what has a tremendous impact on the expected lifetime of the structure. But the tensile strength of the material decreases when it is subjected to moisture.

## Aramid fibre

This is a relative new type of fibre, discovered simultaneously by Akzo (Twaron fibre) and DuPont (Keviar fibre). The material has a high tensile strength and is chemically resistant. A drawback is the low elastic strain and the bad resistance against high temperature and UV-light

## Composition of the base material

Fabric that is used normally for membrane structures is built up out of a woven structural base material, which has a covering on both sides to protect it from water and pollutants, the so-called coating. There are several ways to establish a coherent woven cloth. The basic method of weaving is called basket bond, where the weft threads pass the warp threads alternating above and underneath. There are a lot of varieties possible, like passing three warp threads underneath and one above.


Fig. 1 Basket bond (left) and Panama bond
Doing this, all kinds of patterns occur like is done in the carpet industry. But for structural use this it is not very sufficient and therefore only the basket bond and
panama bond is used for membrane structures. Panama bond indicates that the weave operation is done with more than one thread at a time. 12*12 panama means that one cm of fabric contains 12 warp and 12 weft threads. At the other hand it is also usual to say $2-2$ panama or 3-3 panama which means that the weaving operation is done with two, respectively 3 threads at a time. Panama bond has a better mechanical behaviour than basket weave because of the multiple yarns that are used.

## Coatings

In the table above the fibres are described from which the fabric is woven. To create durable and water tight cloths most of the fibres need a coating on both sides. There are several coatings available. The most common ones are PVC coatings, Teflon coatings and silicone coatings. Sometimes not a coating is applied, but a foil is laminated upon the fabric.
The coating often is used to weld the different parts of the membrane together. The adhesion of the coating to the fabric is an indication for the strength of seams. The adhesion of a lamination to the fabric is much lower and therefore requires other connection methods for the seams.

## PVC coating on Polyester cloth

This type of coating is used mostly on Polyester fabric. It is either coated or laminated upon the cloth. Dozens of different manufacturers provide such a material, which range from laminated fabrics for party rental tents to heavy coated fabrics for permanent (15-20 year replacement cycle) architectural installations. The fabric comes in numerous colours, has three different top coatings (PVDF, PVF, Acrylic) and is considered a fireresistive material (see figure 2a).


Fig. 2a PVC coated Polyester structure in the Netherlands After local fire a hole occurs in the membrane, but the fabric itself is not destroyed.

## PVC coating on Aramid weave

Another interesting lightweight building material is Aramid fibre used for air tubes. These high-pressure air tubes can take on the support function of a beam, an arch or a grid becoming a type of frame structure. The Aramid fibres are braided into curved forms and bonded to an inner urethane membrane to create seamless inflatable arches of approximate 30 psi. The Aramid fabric is enclosed with a PVC cover to protect the fibres from UVdegradation [1].

## PTFE coating on fibreglass weave

Teflon coated fibreglass fabric is the most permanent of the coated architectural fabrics. First employed for a roof in 1973 for the La Verne College Student Centre in California (figure 2 b ) it has a lifetime of over 30 years. It can be used only for permanent applications and is not relocatable. The fabric is considered non-combustible and as such meets the most stringent building codes worldwide. Off the role it has an oatmeal appearance, which bleaches out to white in the sun after a couple of months. With translucency's up to $25 \%$ it has been used in such projects as the Georgia dome, Denver Airport and currently used on the Millennium Dome.

## Silicone coatings on fibreglass weave

Silicone coated fibreglass, which dates from 1981, has been used for Callaway Gardens in Georgia and the tensegrity domes for the Seoul Olympics. Silicone rubber is more flexible than Teflon, and fibreglass coated with it is less likely to be damaged during shipment and erection than fibreglass coated with Teflon. The greatest advantage, however, is that the fabric can be made very translucent, which is claimed to be as much as $25 \%$ translucency for the architectural membrane and $90 \%$


Fig. 2b Oldest commercial PTFE/Fibreglass roof - The LA Verne College Student Centre

| ARCH631 Note Set 13.1 |  |  |  |  | F2008abn |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
|  | Polyester fabric | Fibreglass fabric |  |  |  |  |
| Coating | PVC | PVF-lamination | PVDF-merging |  | Si |  |
| Top coating | Acrylic | $12-15$ years | $12-15$ years | $>30$ years | $>30$ |  |
| Expected <br> lifetime | $8-10$ years | Good | Good | Very good | Very good |  |
| Ageing <br> Resistance | Average | Good | Good | Very good | Average |  |
| Self-cleaning | Average | Good | Good | Good | Very good |  |
| Transparency | Good | Average | Good | Very good | Very good |  |
| Fire-retardant | Good | Average | Good | Bad | Average |  |
| Foldable | Very good |  |  |  |  |  |

Table 2 Properties of fabrics [4]
translucency for the thin liner material. With multiple layers of translucent membrane and glass fibre there can be both daylight illumination and very high heat retention. Silicone ( Si ) is one of the most abundant of the earth's elements, and forms the basis both of the fibreglass threads of the fabric and the silicone rubber of the coating. This similarity in chemical structure allows the design of highly translucent fabrics, while the water protection provided by the silicone coating assures long life span for the fibreglass. With regard to cost and handling, silicone coated fibreglass can be positioned somewhere between Teflon coated fibreglass and PVC coated Polyester.
Recent advantages have partly or wholly resolved early concerns about building with silicone coated fibreglass. Normally the seams are glued, which needs to be done under controlled circumstances. It is said that seams can now be chemically bonded (heat accelerated) to be stronger than the material itself, as with Teflon-coated fibreglass. Some engineers still question whether this process can be adequately applied with patch kits, used on site. The self-cleaning properties have been improved and are said to be equal to Teflon's, yet a once-a-year cleaning is recommended.

## Silicone coating on Polyester weave

An ideal fabric would combine the low cost, easy handling and excellent structural behaviour of PVCcoated Polyester with the translucency and long life of Silicone- coated fibreglass, and the high reflectivity and resistance to dirt of Teflon. Is that maybe a membrane with a fabric of Polyester, a coating of Silicone and a top coating of ETFE?

## Mechanical properties of fabrics [8]

The fabric behaves in a special way due to the weaving process. Conventional building materials are characterised by their linear elastic and isotropic behaviour. Only when the elastic limit is reached and yield area starts, different rules need to be applied. Materials used in textile architecture have a completely different behaviour and act as following:

- Non-linear, that means that the stress-strain behaviour of the material can not be modelled with a linearization of the curve
- Anisotropy, that means that the material itself has two dominant head directions, which makes all the important mechanical properties direction-dependent.
- Non-elastic, that means that the behaviour of the material is dependent on the added loading.


## Non-linearity

At first the non-linearity will be explained. A fabric sample is tested in an uni-axial testing machine. In figure 3 a typical result is displayed from such a test. The stress and strain are displayed.


Fig. 3 Typical stress-strain curve uni-axial loaded [8]

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It is clear that there is no linear relation between the stress and strain.
Only with a lot of creativity it is possible to draw a straight line along the curve.
Next is the anisotropy to be explained. Therefore several strips are cut out of the fabric, but a different orientation of the fibres is regarded (see figure 4).



Fig. 4 Anisotropy shown in different fibre orientations [8]

It is obvious that in the different fibre directions there is a distinctive behaviour. This behaviour is caused by the presence of the woven base material in the fabric. During the weaving process, the warp threads are tensioned in the weaving machine and therefore initially straight. The weft threads sneak around them in alternate patterns, as a result of a weaving process in which alternate warp threads are pulled upwards or downwards and a weft thread is shuttled in between them. In the resulting long rolls of fabric, the weft threads running side to side, are kinked around the straight warp threads, which run the full length. In most coating processes this configuration is maintained. One fabricator of Polyester fabric, Ferrari, stretches the weft threads before coating.


Fig. 5 Left: warp and weft configuration before stressing; right: warp and weft configuration after stressing

Note Set $13.1 \quad$ F2008abn
The effect of this configuration on the mechanical properties is that the strain is not the same in warp and weft direction. When the warp direction is tensioned, there will be little deformation because the fibres are straight already. When the weft fibres are tensioned, they are kinky, but become straight and therefore have a large deformation compared to the warp direction. In figure 5 the configuration is shown before tensioning and after tensioning.
The last aspect, the non-elasticity is explained by means of the same test examples but then carried out more than once on the same sample (see figure 6).


Fig. 6 Non-elastic behaviour of woven material [8]

It can be seen that the loading curve is different from the unloading curve. When the second loading cycle starts, it differs from the first one, as well as the second unloading curve differs from the first one. When the loading cycles are repeated, each loading and unloading cycle is different, although the differences are getting smaller. The difference remains between loading and unloading, which results in a permanent elongation of the fabric. The size of the elongation depends on the previous applied loads. All these aspects act simultaneously. Therefore it is very difficult to describe the mechanical behaviour of fabric with one model. To get a better understanding of those aspects, a short overview is given of the design process. This makes it easier to explain when the different material aspects need to be regarded.

## Design process

The design of a membrane structure starts with the formfinding. Since there is a double opposite curvature, there need to be found equilibrium between the
pretension in the membrane and the boundary conditions. This is normally done by means of computer software. Modelling the membrane as a two-way net is a very representative basis for computer analysis. One direction of the, mesh can be seen as the warp threads, the other direction of the mesh can be seen as the weft threads. When the boundary conditions are set, a first shape is obtained. This can serve as an image to explain the customer what the shape looks like and if it fulfils the needs. When is decided to go on with the structure, it is necessary to think about the patterning layout. The membrane is built up out of small strips because the fabric comes with rolls of a certain width. The strips are welded together and form the membrane. Because of the anisotropy of the material, it is necessary to orient the warp and weft threads in the head directions of the curvature. The load bearing behaviour is influenced considerably when the head direction of the fabric does not correspond with the head direction of the curvature (see figure 7). There is much more deflection possible as the mesh does not have shear stiffness. So the stiffness of the shape is depending on the adhesion from the coating to the fabric.


Fig. 7 Two ways of mesh orientation: These result in the same shape but with different load bearing behaviour ( $1 \mathrm{kN} / \mathrm{m}^{2}$ upward load)

When the main direction of the anisotropy is known, the points of departure for the stiffness of the structure can be determined. With these values a statical analysis is made, which results in forces in the primary structure and the membrane. The results of the statical analysis on stresses and deformation are used to check the loading limits and failure modes. For membranes the following failure modes are critical:

- Failure of the bi-axial loaded membrane within the assumed lifetime of the structure
- Failure of a seam or connection of membrane to primary structure
- Tear failure during installation or because of vandalism.

The first failure mode is depending on the safety factors used upon the ultimate strength of the material. The difficulty of the non-elastic material property is dealt with in a very simple way. Just a small amount of the strip tensile capacity is used. Depending on the used fabric, there is the risk of brittle failure (fibreglass) or large plastic deformation (Polyester). So for permanent loading sometime a ratio $f 1 / 8$ is used, for windloads $1 / 4$ is used and for snowload $1 / 5$ is used because it can last for several weeks and therefore is a semi-permanent loading. According to the DIN, the design load cannot be larger than $0.85 / 3.1$ *strip tensile strength. Another approach is to stay under the tear strength of the material to prevent tear failure. This results in a ratio of $1 / 5-1 / 6$.

| Fabric/ Coating | Weight $\left[\mathrm{g} / \mathrm{m}^{2}\right]$ | Fire retardant | Tensile strength Warp/weft [ $\mathrm{N} / 50 \mathrm{~mm}$ ] | Tensile strain Warp/weft [\%] | Tear strength <br> [ N ] | Bending capacity | Seam strength <br> [ $\mathrm{N} / 50 \mathrm{~mm}$ ] |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polyester/PVC |  | B1 |  |  |  | Very good |  |
| Type 1 | 800 |  | 3000/3000 | 15/20 | 350 |  | 2400 (30mm, 70'C) |
| Type 2 | 900 |  | 4400/3950 | 15/20 | 580 |  | 2850 (60mm, 70 'C) |
| Type 3 | 1050 |  | 5750/5100 | 15/25 | 950 |  | 3350 (60mm, 70 'C) |
| Type 4 | 1300 |  | 7450/6400 | 15/30 | 1400 |  | 4600 (60mm, 70 'C) |
| Type 5 | 1450 |  | 9800/8300 | 20/30 | 1800 |  | 4600 (60mm, 70 'C) |
| Fibreglass/PTFE | 800 | A2 | 3500/3000 | 7/10 | 300 | Sufficient | 6000 (60mm, 70 'C) |
|  | 1270 | A2 | 6600/6000 | 7/10 | 570 |  |  |
| Fibreglass/Si | 800 | A2 | 3500/3000 | 7/10 | 300 | Good |  |
|  | 1270 | A2 | 6600/6000 | 7/10 | 570 |  |  |
| Aramid/PVC | 900 | B1 | 7000/9000 | 5/6 | 700 | Good | 4800 (30mm, 70 'C) |
|  | 2020 | B1 | 24500/24500 | 5/6 | 4450 |  |  |
| PTFE/- | 520 | Non combustible | 2000/2000 | 40/30 | 500 | Very good |  |
| Cotton- Polyester/ - | 350 | B2 | 1700/1000 | 35/18 | 60 | Very good |  |
|  | 520 | B2 | 2500/2000 | 38/20 | 80 |  |  |

Table 3 Mechanical properties of common fabrics [7]
determined.
The second failure mode, failure of a seam, should be avoided by testing which seam width is needed at which temperature. When the temperature rises, the seams get weaker. Above 70* the strength of the seam gets considerably lower.
Tear failure (the third mode) often occurs during installation. It starts at an open edge or at a hole in the fabric. It is critical, therefore, that the fabric panels are contained all around the edges, with a continuity that is meticulously maintained. Most commonly, edge ropes in continuous sleeves, which are connected cables or other structural members, achieve this. Another cause for tear failure is the acting of tangential forces in the membrane. When no proper take-up of these forces is provided, the fabric can tear under heavy loading. When the quality of the membrane is determined, the cutting patterns can be made from the final shape. The shape has a certain pretension, and the patterns need to be compensated for that. The needed compensation is depending on the strain of the fabric under the prestress in the membrane. This strain needs to be investigated by means of biaxial tests on the fabric under similar prestress conditions as present in the membrane.


Fig. 8 Possible design scheme
[1] Nicholas Goldsmith: "Materials for the new Millennium", Proceedings of Conference on Large span structures, Bath, 2000.
[2] Michael Haist, Christoph Niklasch, Yahya Bayraktarli: "Vorgespannte Membrantragwerke", Seminar Leichte Flächentragwerke, TU Berlin 1998/99.
[3] Horst Berger.- "Light structures, structures of light" Birk- hduser Verlag Berlin, 1996.
[4] Rogier Houtman: "From computer model to realised structure", TU Delft, 1996.
[5] Matti Orpana: "Detailing" proceedings of Textile roofs 1995, Berlin.
[6] Tony Robbin: "Engineering a new architecture", Yale University Press New Haven and London, 1996.
[7] Wemer Sobek, Martin Speth: "Von der Faser zum Gewebe" page 74-81 DB nr 9 Sept. 1993.
[8] Rainer Blum: Out of "Leicht und Weit" page 200-224, Deutsche Forschungsgemeinschaft Weinheim, 1990.
(Revised version of the proceedings of the workshop "Textile Roofs 2000" $22^{\text {nd }}$ to $24^{\text {th }}$ June at TU Berlin)

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## Examples: <br> Membranes, Nets \& Shells

## Example 1

An inflatable structure used by a traveling circus has the shape of a half-circular cylinder with closed ends. The fabric and plastic structure is inflated by a small blower and has a radius of 40 ft when fully inflated. A longitudinal seam runs the entire length of the "ridge" of the structure.

If the seam tears open when it is subjected to a tensile load of 540 pounds per inch of seam, what is the factor of safety against tearing when the internal pressure is 0.5 psi and the structure is fully inflated?


What is the force on the seam at the intersection with the quarter spheres? If the thickness of the membrane is 0.025 in , what is the stress?

SOLUTION:
Find the tensile load for a one inch section of the membrane structure. A free body diagram is helpful to show the pressure:

T for a circular membrane for a unit width carrying an internal pressure $p_{r}$ is: $\mathrm{T}=p_{\mathrm{r}} \mathrm{R}$

It doesn't matter where we cut a section, the force will still be T .
$\mathrm{T}=0.5 \mathrm{lb} / \mathrm{in}^{2}(40 \mathrm{ft}) \cdot\left(\frac{12 \mathrm{in}}{1 f t}\right)=240 \mathrm{lb} / \mathrm{in}$

The factor of safety is the ratio of limit load to actual load:
F.S. $=\frac{540 \mathrm{lb} / \mathrm{in}}{240 \mathrm{lb} / \mathrm{in}}=2.25$

The ends are spherical, so the equation for force is $T=p_{r} R / 2$.
The force will be $1 / 2(240 \mathrm{lb} / \mathrm{in})=120 \mathrm{lb} / \mathrm{in}$
The stress is equal to the force per length divided by the thickness, $f=\mathrm{T} / \mathrm{t}$

$\mathrm{f}=\mathrm{T} / \mathrm{t}=(120 \mathrm{lb} / \mathrm{in}) /(0.025 \mathrm{in})=4,800 \mathrm{lb} / \mathrm{in}^{2}$

## Example 2

Investigate with computer modeling the stresses and behavior of a hyperbolic paraboloid under uniform roof loading with column supports away from the edges, as actually built for a residence with glazing between columns. (Ref.Architectural Structures, Wayne Place, 2007, Wiley, NJ.)


IIGure 8.261 Residence with hyperbolic paraboloid roof, showing the ample overhang of the roof, the boundary members, and ullions in the glass walls.

## SOLUTION:

The axial force diagram (b) shows that the axial forces appear to be uniform, as the discussion in the text indicates, but that the edge members have higher axial forces.

The deflection diagram (b) indicates negative bending over the columns, which indicates there are probably significant bending moments (which should be minimal in a shell), verified by the bending moment diagram (d).

This house had significant problems.


Figure 8.262 Hyperbolic paraboloid under uniform load, showing stabilizing columns (a), axial forces (b), deflection (c), and bending stress (d).

## Areas Where Teaching Structures Should Be Strengthened

## William L. Thoen

Retired principal, LeMessurier Consultants, Cambridge, MA

For nearly fifty years I have been pleased to provide structural consulting to architects on building projects throughout the United States and the Mideast, and in size from smaller than a house to as large as a city ${ }^{\text {b }}$

Most of the preliminary designs an architect brings to me for structural services are pretty well thought out in terms of appropriate column spacing and allowance for beam depths, and have suitable locations to accommodate the structural frame. In subsequent discussions an appropriate framing scheme usually develops without a great deal of conflict. Sometimes, knowing what the architect is trying to achieve, a unique structural arrangement becomes obvious, and if the architect can incorporate that in his plans, a strikingly new form evolves ${ }^{c}$.

Having said that, there are some common planning weaknesses that occur frequently. They are: 1) Building stability and lateral bracing, 2) Structural frame vertical organization, 3) Tolerances between the structural frame and the architectural finish, 4) Site considerations, and 5) Floor vibration and comfort performance.

## Lateral Bracing

If he has thought about lateral forces at all, the architect will often say, "Well, I will allow you bracing in the core," as if that were the end of the matter.
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Core bracing alone makes the width or depth of the core become the structural depth of the building, regardless as to how wide or how long the building is. Accordingly, the core becomes a flagpole, or mast, that braces the entire building, and which may be too slender for acceptable sway performance in taller structures. In addition, lateral forces eccentric from the core may twist the building back and forth uncomfortably because the core alone cannot provide sufficient torsional stiffness. Even though the building may have sufficient strength, the inability of the core alone to provide sufficient stiffness can result in undesirable building motion, slapping of elevator cables against sidewalls, sloshing of water in toilet bowls, swinging doors, binding windows, squeaks, groans and mal-de-mer.

Another popular, but ineffective, location for lateral bracing is the exterior wall corner bays of the building, which are the worst exterior wall locations because the corner columns are the most lightly loaded and therefore have the least gravity weight to offset overturning uplift.

## Vertical Alignment

Another common planning weakness is structural frame discontinuity in the vertical direction. Think of a building with

Continued on page 4

## Continued from page 3

upper level apartments above lower level office spaces, all over ground floor commercial spaces with basement parking underneath. Each occupancy has its own optimum structural module, which, if rigorously applied, results in massive transfer girders or story-deep trusses and each change of occupancy.

Teaching should include planning of an efficient structural module that can be threaded through the differing occupancy levels.


## Tolerance

The need for tolerance between the structure and the architectural finish is often not considered. The actual depth of a steel column may be as much as two inches larger than the nominal depth. Splice plates, connections and bolts can make the structural cross-section even deeper, and fireproofing, where required, adds to that. Base plates will be larger than the column they support for welding and area requirements, and commonly sit atop a bed of grout.

Remember, too, that concrete has a way of hardening up and a slightly misplaced wall or anchor bolts have to be accommodated.

Teaching should include the necessity for providing "float" between the structure and the architectural finish when preparing preliminary sketches, especially where concrete work joins the superstructure.

## Site Considerations

Site constraints may influence the choice of structural module or type. Most architects practicing within a region are aware of its special requirements. Hurricanes (Southeast), earthquakes (West coast), tornadoes (Midwest), expansive clays (Texas), permafrost (Alaska), and extreme temperature or humidity variation (Mideast) represent localities with special requirements. Sometimes availability of materials or lack of skilled labor will govern the design vernacular.

As important, site constraints or subgrade conditions may strongly affect the structural system and even the architectural form.

On good soils, the structure can be founded on simple footings. Where the building location is underlain with organic material, soft clays and the like, special foundation systems are required, differential settlements and control of groundwater considered, and these may influence the choice of the superstructure system, including column spacing, to achieve an optimal system.

## Floor Vibration and serviceability

Today's buildings are lighter and more gossamer than their ancestors. With today's high-strength materials, composite construction, and lightweight concretes, floor spans can be made longer, and stiffness, rather than strength, often governs the structural depth. As a consequence, floor vibration, cambering, and careful deflection control become important factors for occupancy comfort, especially in large, column-free spaces without damping partitions. Often floor vibrations are not sensible to walkers, but become intolerable to a person sitting, as in an office.

All too often thin, long span floors are envisioned, but which must be deepened (stiffened), or otherwise damped against excessive vibration.

## How these can be taught

A good way to teach these areas of structural planning (and structures in general), I think, is to choose a building of interest that has been built and study its structure.

How has the architect and engineer collaborated to make it successful? What constitutes the lateral bracing system? What were the site constraints, if any, and how did they influence the design? What is the column grid module and what is the floor system depth for its spans?

If a student studies two or three built projects a semester, each illustrating a type of building the student is likely to encounter in practice, and if the student is made to keep a notebook of sketches and notes relative to each type of building, he will have then studied structural solutions in context with the architectural problem, and will also have a useful future reference upon graduation.

## Endnotes:

a. King Kahlid Military City; Saudi Arabia
b. Citicorp, NYC; Gymnasium, Philips Exeter Academy; Exeter, NH; Fiduciary Trust Bldg, Boston, MA

## Buildings at Risk: Wind Design Basics for Practicing Architects, AIA, 1998

## Wind Impacts on Buildings

### 3.1 WIND FORCES

Buildings are continually subjected to wind forces. Generally, these wind forces are at levels that the structure is capable of resisting, whether that capability is based on an engineered design using building code-specified wind loads, or, as is the case with most residential construction, it is based on standard construction practices that have developed over time. Periodically, structures are subjected to wind forces that cause damage. In some instances, the damage is due to wind loads exceeding design criteria. In most cases, the damage results from a weakness in the building itself. ${ }^{1}$

## Damaging Winds

Damaging wind forces usually are associated with extreme weather phenomena, such as tornadoes, hurricanes, or thunderstorms. Maps indicating wind speeds for 50 -year mean return periods have been used in building codes to establish wind loads for building design. The maps and other factors in design standards take into account the varying wind loads experienced in different environments, i.e. near the coast, inland, open terrain and urban environment. Building codes and standards generally use gust and other factors that are applied to the basic wind speed to account for the dynamic effects of wind.

In practice, the actual wind loads on a building rarely exceed the design wind load. Even in cases where design-level winds are somewhat exceeded, a well-designed and constructed building should sustain relatively little damage to the structural frame.' The building envelope (roof, walls, and openings) is another story: Breaches to the envelope have been observed to be the major cause of damage in high wind events, and envelope systems have sustained considerable damage even at wind speeds below design levels.

Many buildings would suffer severe damage if struck directly by a moderate to strong tornado. This damage results not only from the extreme


Figure 3.1: Flow of air around a high-rise building.


Figure 3.2: Wind tunnel analysis of the World Trade Center buildings and code approach.
wind speeds, but from the dynamically changing wind directions and the impact of wind-borne debris. Similarly, structures along the coast in the path of a hurricane may be simultaneously subjected to the severe forces of both wind and water, the greatest magnitude of each occurring at approximately the same time. The wind velocoties in a hurricane may exceed design levels and may subject the building to high winds first from one direction and then the other. ${ }^{3}$

## Wind Loads

Wind loads on buildings can be calculated using the formula contained in the American Society of Civil Engineers (ASCE 7-95) Standard for Minimum Design Loads for Buildings and Other Structures. The wind load is an expression of the formula:

$$
\begin{aligned}
& \mathrm{p}=\mathrm{qGC} \\
& \mathrm{q}=0.00256 \mathrm{~K}_{\mathrm{z}} \mathrm{~K}_{\mathrm{zt}} \mathrm{~V}^{2} \mathrm{I}
\end{aligned}
$$

where:
$\mathrm{p}=$ design pressure in psf
$\mathrm{q}=$ velocity pressure in psf
$0.00256=$ constant for mass density of air and appropriate conversion con-
stants so that V may be given in mph
$\mathrm{K}_{\mathrm{z}}=$ velocity pressure exposure coefficient
$\mathrm{K}_{\mathrm{zt}}=$ topographic factor
$\mathrm{V}=$ basic 3 -second peak gust wind speed in mph
$\mathrm{I}=$ importance factor, defines the level of risk depending on occupancy
$\mathrm{G}=$ gust effect factor, which considers spatial size of gust relative to the size
of buildings, gust frequency relative to natural frequency and damping of
structure, basic reference design speed, and terrain exposure

Use of this formula by an architect is relatively rare, as most wind load analysis is conducted by engineers and specialists. However, it is important for architects to be familiar with the formula so that they understand the impact of wind on the building's design and can discuss it with the engineer. Regardless of who performs the wind load calculations, it is imperative that loads be determined for the building envelope as well as the structure.

## Vibration

Wind-induced structural vibration can be a concern in specialty structures such as tensile roofs, bridges, and other unusual configurations.

Buffeting vibration is produced by the unsteady loading of a building due to turbulence (velocity fluctuations in magnitude and direction) in the approaching free flow wind field. If the turbulence is generated by an upwind neighboring structure or obstacle, the unsteady loading is called wake buffeting or interference. The World Trade Center Twin Towers in New York City (Figure 3.2) and the John Hancock building in Boston are examples of buildings that experience the latter type.

Most building codes (e.g. ASCE 7) treat the along-wind vibration but do not address across-wind or torsional buffeting vibration.

The flow behind a long cylinder held perpendicular to wind is characterized by the periodic shedding of vortices (whirling air flows). Vortex shedding creates periodic lateral forces that can cause vibration of slender structures such as towers and tall buildings. Although vortex shedding is most noticeable for cylindrical buildings, it also happens to a lesser degree to tall buildings of other shapes. ${ }^{4}$

Vortex-shedding vibration takes place when the wind speed is such that the shedding frequency becomes approximately equal to the natural frequency of the cylinder-a condition that causes resonance. When resonance takes place, further increase in wind speed by a few percent will not alter the shedding frequency. This phenomenon is called "lock-in." Because the structure vibrates excessively only in the lock-in range, having a wind speed either below or above the lock-in range will not cause serious vibration. If the shedding frequency is the same as the natural period of the building, it can have a load impact on the structure, pulling the building back and forth in an across-wind direction. ${ }^{5}$ (Figures 3.3 and 3.4)

Classical flutter (or simply flutter) is a two-degrees-of-freedom vibration involving simultaneous lateral (across-wind translational) and torsional (rotational) vibrations. It occurs in structures that have approximately the same magnitude of natural frequencies for both the translational and the rotational modes. Similar to galloping and torsional divergence, flutter is produced by aerodynamic instability completely unrelated to vortex shedding. ${ }^{6}$

## Damage Mechanisms

The four primary damage mechanisms associated with severe windstorms involve:
(1) aerodynamic pressures created by flow of air around a structure;
(2) induced internal pressure fluctuations due to a breach in the building envelope;
(3) impact forces created by wind-borne debris; and
(4) pressures created by rapid atmospheric pressure fluctuations (associated primarily with tornadoes).

Examinations of building damage caused by various types of windstorms suggest that most winds produce damage due to a combination of aerodynamic pressures and internal pressure fluctuations and, for hurricanes and tornadoes, debris impacts. Amospheric pressure fluctuations have little or no effect on the performance of ordinary structures because most ordinary structures have sufficient building envelope permeability (or venting) th allow equalization of pressures induced by atmospheric pressure changes. In airtight structures such as nuclear containment vessels, atmospheric pressure changes can impore ignificant loading to the building envelope.


Figure 3.3: Vortex shedding.


Figure 3.4: The Karman Vortex Phenomenon.


Figure 3.5: Building in wind flow.

Wind pressures acting on buildings are distributed loads that are assumed to act normal to the building surface. Positive wind pressures act toward the surface of the building element and negative pressures (suction) act away from the building surface. The fundamental characteristics of wind pressures are described below based on the building component affected and the orientation of the building in the wind environment. ${ }^{8}$

As winds increase, pressure against objects is added at a non-linear rate. Pressure force against a wall mounts with the square of the wind speed so that a three-fold increase in wind speed, for example, results in a nine-fold increase in pressure. A 25 mph wind causes about 1.6 pounds of pressure per square foot. Therefore a $4 \times 8$ sheet of plywood will be pushed by a force of about 50 pounds. In 75 mph winds, that force becomes 450 pounds, and at 125 mph , it becomes 1,250 pounds. ${ }^{9}$

### 3.2 AERODYNAMIC PRESSURE IMPACTS



Figure 3.6: Relative wind pressure on walls.

## Impacts on Walls

Figure 3.5 presents a plan view of a simple rectangular building that is submerged in a wind flow as shown. Each wall of the structure is identified as a windward, side, or leeward wall depending upon its location with respect to the direction of wind flow. The windward wall is the wall facing the wind; the leeward wall is on the side opposite to the windward wall; and the side walls are parallel to the wind flow. ${ }^{10}$

Because the windward wall is perpendicular to the wind flow, the wind impinges directly on the windward wall producing positive pressures (Figure 3.6). As the wind flows around the windward corners, the local wind speed increases and the flow lines have a tendency to separate from the corner of the building. This causes the side walls to be subjected to negative pressures as shown. In addition, the turbulence and flow separations that occur at the windward corners of the building induce high negative pressures for short distances along the side walls. The leeward wall is also subjected to negative wind pressures that tend to be relatively uniformly distributed. ${ }^{11}$

## Impacts on Roofs

Wind creates a greater load on the roof covering than on any other element of a building. When a FEMA team investigated wind damage to buildings in Florida in the wake of Hurricane Andrew, their field observations concluded that the loss of roof covering was the most pervasive type of damage to buildings in southern Dade County. To varying degrees, all of the different roof types observed suffered damage due to the failure of the method of attachment and/or material, inadequate design, inadequate workmanship, or debris impact. Similar damage has been observed in the aftermath of other windstorms (Figure 3.7).

## Design Wind Pressures - Envelope Procedure SEI/ASCE 7-10:

Velocity pressure, $p$, irrespective of terrain and height above ground or recurrence probability is related to the wind speed, $V$, by $p=0.00256 V^{2}$. Wind codes also consider the effect of the geometry of the building and location on the surface, wind gusts or turbulence, the local terrain, and annual probability of exceeding the design wind speed.


| Main Wind Force Resisting System - Method 2 |  |  |  |  |  |  |  | $\mathrm{h} \leq 60 \mathrm{ft}$. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Figure 28.6-1 (cont'd) <br> Enclosed Buildings |  | Design Wind Pressures |  |  |  |  |  | Walls \& Roofs |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Simplified Design Wind Pressure , $\mathrm{p}_{\mathbf{S} 30}$ (psf) (Exposure B at $h=30 \mathrm{ft}$. with $I=1.0$ ) |  |  |  |  |  |  |  |  |  |  |  |  |
| Basic Wind Speed (mph) | RoofAngle(degrees) | $$ | Zones |  |  |  |  |  |  |  |  |  |
|  |  |  | Horizontal Pressures |  |  |  | Vertical Pressures |  |  |  | Overhangs |  |
|  |  |  | A | B | C | D | E | F | G | H | EOH | GOH |
| 110 | 0 to $5^{\circ}$ | 1 | 19.2 | -10.0 | 12.7 | -5.9 | -23.1 | -13.1 | -16.0 | -10.1 | -32.3 | -25.3 |
|  | $10^{\circ}$ | 1 | 21.6 | -9.0 | 14.4 | -5.2 | -23.1 | -14.1 | -16.0 | -10.8 | -32.3 | -25.3 |
|  | $15^{\circ}$ | 1 | 24.1 | -8.0 | 16.0 | -4.6 | -23.1 | -15.1 | -16.0 | -11.5 | -32.3 | -25.3 |
|  | $20^{\circ}$ | 1 | 26.6 | -7.0 | 17.7 | -3.9 | -23.1 | -16.0 | -16.0 | -12.2 | -32.3 | -25.3 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | 24.1 -----1. | 3.9 | 17.4 ----17. | ${ }^{4.0}$ | $\begin{gathered} \hline-10.7 \\ -4.1 \\ \hline \end{gathered}$ | $\begin{gathered} \hline-14.6 \\ -7.9 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-7.7 \\ & -1.1 \\ & \hline \end{aligned}$ | $\begin{array}{r} \hline-11.7 \\ -5.1 \\ \hline \end{array}$ | -19.9 | -17.0 ----7 |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & \hline 21.6 \\ & 21.6 \end{aligned}$ | $\begin{aligned} & 14.8 \\ & 14.8 \end{aligned}$ | $\begin{aligned} & \hline 17.2 \\ & 17.2 \end{aligned}$ | $\begin{aligned} & 11.8 \\ & 11.8 \end{aligned}$ | $\begin{aligned} & 1.7 \\ & 8.3 \end{aligned}$ | $\begin{gathered} \hline-13.1 \\ -6.5 \end{gathered}$ | $0.6$ | $\begin{gathered} \hline-11.3 \\ -4.6 \end{gathered}$ | $\begin{aligned} & \hline-7.6 \\ & -7.6 \end{aligned}$ | $\begin{aligned} & \hline-8.7 \\ & \hline-8.7 \end{aligned}$ |
| 115 | 0 to $5^{\circ}$ | 1 | 21.0 | -10.9 | 13.9 | -6.5 | -25.2 | -14.3 | -17.5 | -11.1 | -35.3 | -27.6 |
|  | $10^{\circ}$ | 1 | 23.7 | -9.8 | 15.7 | -5.7 | -25.2 | -15.4 | -17.5 | -11.8 | -35.3 | -27.6 |
|  | $15^{\circ}$ | 1 | 26.3 | -8.7 | 17.5 | -5.0 | -25.2 | -16.5 | -17.5 | -12.6 | -35.3 | -27.6 |
|  | $20^{\circ}$ | 1 | 29.0 | -7.7 | 19.4 | -4.2 | -25.2 | -17.5 | -17.5 | -13.3 | -35.3 | -27.6 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | $26.3$ | $4.2$ | 19.1 | ${ }^{4.3}$ | $\begin{gathered} -11.7 \\ -4.4 \end{gathered}$ | $\begin{gathered} \hline-15.9 \\ -8.7 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-8.5 \\ & -1.2 \end{aligned}$ | $\begin{gathered} \hline-12.8 \\ -5.5 \\ \hline \end{gathered}$ | -21.8 | -18.5 |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & \hline 23.6 \\ & 23.6 \end{aligned}$ | $\begin{aligned} & \hline 16.1 \\ & 16.1 \end{aligned}$ | $\begin{aligned} & 18.8 \\ & 18.8 \end{aligned}$ | $\begin{aligned} & 12.9 \\ & 12.9 \end{aligned}$ | $\begin{aligned} & 1.8 \\ & 9.1 \end{aligned}$ | $\begin{gathered} \hline-14.3 \\ -7.1 \end{gathered}$ | $\begin{aligned} & 0.6 \\ & 7.9 \end{aligned}$ | $\begin{gathered} -12.3 \\ -5.0 \end{gathered}$ | $\begin{aligned} & \hline-8.3 \\ & -8.3 \end{aligned}$ | $\begin{aligned} & \hline-9.5 \\ & -9.5 \end{aligned}$ |
| 120 | 0 to $5^{\circ}$ | 1 | 22.8 | -11.9 | 15.1 | -7.0 | -27.4 | -15.6 | -19.1 | -12.1 | -38.4 | -30.1 |
|  | $10^{\circ}$ | 1 | 25.8 | -10.7 | 17.1 | -6.2 | -27.4 | -16.8 | -19.1 | -12.9 | -38.4 | -30.1 |
|  | $15^{\circ}$ | 1 | 28.7 | -9.5 | 19.1 | -5.4 | -27.4 | -17.9 | -19.1 | -13.7 | -38.4 | -30.1 |
|  | $20^{\circ}$ | 1 | 31.6 | -8.3 | 21.1 | -4.6 | -27.4 | -19.1 | -19.1 | -14.5 | -38.4 | -30.1 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | 28.6 | 4.6 --17.6 | 20.7 - | ${ }^{4.7}$ | $\begin{aligned} & -12.7 \\ & -4.8 \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-17.3 \\ -9.4 \\ \hline \end{gathered}$ | $\begin{aligned} & -9.2 \\ & -1.3 \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-13.9 \\ -6.0 \\ \hline \end{gathered}$ | -23.7 ----7 | -20.2 ----3 |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 25.7 \\ & 25.7 \end{aligned}$ | $\begin{aligned} & \hline 17.6 \\ & 17.6 \end{aligned}$ | $\begin{aligned} & \hline 20.4 \\ & 20.4 \end{aligned}$ | $\begin{aligned} & \hline 14.0 \\ & 14.0 \end{aligned}$ | $\begin{aligned} & \hline 2.0 \\ & 9.9 \end{aligned}$ | $\begin{gathered} -15.6 \\ -7.7 \end{gathered}$ | $\begin{aligned} & 0.7 \\ & 8.6 \end{aligned}$ | $\begin{gathered} \hline-13.4 \\ -5.5 \end{gathered}$ | $\begin{aligned} & \hline-9.0 \\ & -9.0 \end{aligned}$ | $\begin{aligned} & \hline-10.3 \\ & -10.3 \end{aligned}$ |
| 130 | 0 to $5^{\circ}$ | 1 | 26.8 | -13.9 | 17.8 | -8.2 | -32.2 | -18.3 | -22.4 | -14.2 | -45.1 | -35.3 |
|  | $10^{\circ}$ | 1 | 30.2 | -12.5 | 20.1 | -7.3 | -32.2 | -19.7 | -22.4 | -15.1 | -45.1 | -35.3 |
|  | $15^{\circ}$ | 1 | 33.7 | -11.2 | 22.4 | -6.4 | -32.2 | -21.0 | -22.4 | -16.1 | -45.1 | -35.3 |
|  | $20^{\circ}$ | 1 | 37.1 | -9.8 | 24.7 | -5.4 | -32.2 | -22.4 | -22.4 | -17.0 | -45.1 | -35.3 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | 33.6 <br> ..-- | 5.4 | 24.3 ---3. | 5. ${ }^{-}$. | $\begin{gathered} -14.9 \\ -5.7 \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-20.4 \\ & -11.1 \\ & \hline \end{aligned}$ | $\begin{gathered} -10.8 \\ -1.5 \\ \hline \end{gathered}$ | $\begin{gathered} \hline-16.4 \\ -7.1 \\ \hline \end{gathered}$ | -27.8 .----2. | -23.7 .---2. |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 30.1 \\ & 30.1 \end{aligned}$ | $\begin{aligned} & 20.6 \\ & 20.6 \end{aligned}$ | $\begin{aligned} & \hline 24.0 \\ & 24.0 \end{aligned}$ | $\begin{aligned} & 16.5 \\ & 16.5 \end{aligned}$ | $\begin{gathered} \hline 2.3 \\ 11.6 \end{gathered}$ | $\begin{gathered} -18.3 \\ -9.0 \end{gathered}$ | $\begin{gathered} \hline 0.8 \\ 10.0 \end{gathered}$ | $\begin{gathered} -15.7 \\ -6.4 \end{gathered}$ | $\begin{aligned} & -10.6 \\ & -10.6 \end{aligned}$ | $\begin{aligned} & \hline-12.1 \\ & -12.1 \end{aligned}$ |
| 140 | 0 to $5^{\circ}$ | 1 | 31.1 | -16.1 | 20.6 | -9.6 | -37.3 | -21.2 | -26.0 | -16.4 | -52.3 | -40.9 |
|  | $10^{\circ}$ | 1 | 35.1 | -14.5 | 23.3 | -8.5 | -37.3 | -22.8 | -26.0 | -17.5 | -52.3 | -40.9 |
|  | $15^{\circ}$ | 1 | 39.0 | -12.9 | 26.0 | -7.4 | -37.3 | -24.4 | -26.0 | -18.6 | -52.3 | -40.9 |
|  | $20^{\circ}$ | 1 | 43.0 | -11.4 | 28.7 | -6.3 | -37.3 | -26.0 | -26.0 | -19.7 | -52.3 | -40.9 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $39.0$ | $6.3$ | 28.2 - | ${ }^{6.4}$ | $\begin{gathered} -17.3 \\ -6.6 \\ \hline \end{gathered}$ | $\begin{aligned} & -23.6 \\ & -12.8 \end{aligned}$ | $\begin{gathered} -12.5 \\ -1.8 \end{gathered}$ | $\begin{gathered} -19.0 \\ -8.2 \\ \hline \end{gathered}$ | -32.3 | -27.5 ---3 |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 35.0 \\ & 35.0 \end{aligned}$ | $\begin{aligned} & \hline 23.9 \\ & 23.9 \end{aligned}$ | $\begin{aligned} & 27.8 \\ & 27.8 \end{aligned}$ | $\begin{aligned} & 19.1 \\ & 19.1 \end{aligned}$ | $\begin{gathered} 2.7 \\ 13.4 \end{gathered}$ | $\begin{aligned} & -21.2 \\ & -10.5 \end{aligned}$ | $\begin{gathered} 0.9 \\ 11.7 \end{gathered}$ | $\begin{gathered} -18.2 \\ -7.5 \end{gathered}$ | $\begin{aligned} & -12.3 \\ & -12.3 \end{aligned}$ | $\begin{aligned} & -14.0 \\ & -14.0 \end{aligned}$ |
| 150 | 0 to $5^{\circ}$ | 1 | 35.7 | -18.5 | 23.7 | -11.0 | -42.9 | -24.4 | -29.8 | -18.9 | -60.0 | -47.0 |
|  | $10^{\circ}$ | 1 | 40.2 | -16.7 | 26.8 | -9.7 | -42.9 | -26.2 | -29.8 | -20.1 | -60.0 | -47.0 |
|  | $15^{\circ}$ | 1 | 44.8 | -14.9 | 29.8 | -8.5 | -42.9 | -28.0 | -29.8 | -21.4 | -60.0 | -47.0 |
|  | $20^{\circ}$ | 1 | 49.4 | -13.0 | 32.9 | -7.2 | -42.9 | -29.8 | -29.8 | -22.6 | -60.0 | -47.0 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | 44.8 ----- | 7.2 ---8 | 32.4 ----- | 7.4 ----2 | $\begin{gathered} -19.9 \\ -7.5 \end{gathered}$ | $\begin{aligned} & -27.1 \\ & -14.7 \end{aligned}$ | $\begin{gathered} \hline-14.4 \\ -2.1 \\ \hline \end{gathered}$ | $\begin{gathered} -21.8 \\ -9.4 \\ \hline \end{gathered}$ | -37.0 ----1. | -31.6 ----3 |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 40.1 \\ & 40.1 \end{aligned}$ | $\begin{array}{r} 27.4 \\ 27.4 \\ \hline \end{array}$ | $\begin{aligned} & 31.9 \\ & 31.9 \end{aligned}$ | $\begin{aligned} & \hline 22.0 \\ & 22.0 \end{aligned}$ | $3.1$ | $\begin{array}{r} \hline-24.4 \\ -12.0 \\ \hline \end{array}$ | $\begin{aligned} & 1.0 \\ & 134 \end{aligned}$ | $\begin{gathered} -20.9 \\ -8.6 \end{gathered}$ | $\begin{aligned} & -14.1 \\ & -14.1 \end{aligned}$ | $\begin{aligned} & \hline-16.1 \\ & -16.1 \end{aligned}$ |
| Unit Conversions - $1.0 \mathrm{ft}=0.3048 \mathrm{~m} ; 1.0 \mathrm{psf}=0.0479 \mathrm{kN} / \mathrm{m}^{2}$ |  |  |  |  |  |  |  |  |  |  |  |  |


| Main Wind Force Resisting System - Method 2 |  |  |  |  |  |  |  |  | $\mathrm{h} \leq 60 \mathrm{ft}$. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Figure 28.6-1 (cont'd) |  |  | Design Wind Pressures |  |  |  |  |  | Walls \& Roofs |  |  |  |  |
| Enclosed Buildings |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Simplified Design Wind Pressure, $\mathbf{p s} 30^{\text {(psf) }}$ (Exposure B at $\mathrm{h}=30 \mathrm{ft}$.) |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Basic Wind Speed (mph) | Roof Angle (degrees) | $$ | Zones |  |  |  |  |  |  |  |  |  |  |
|  |  |  | Horizontal Pressures |  |  |  |  | Vertical Pressures |  |  |  | Overhangs |  |
|  |  |  | A | B | C |  | D | E | F | G | H | EOH | GoH |
| 160 | 0 to $5^{\circ}$ | 1 | 40.6 | -21.1 | 26.9 |  | -12.5 | -48.8 | -27.7 | -34.0 | -21.5 | -68.3 | -53.5 |
|  | $10^{\circ}$ | 1 | 45.8 | -19.0 | 30.4 |  | -11.1 | -48.8 | -29.8 | -34.0 | -22.9 | -68.3 | -53.5 |
|  | $15^{\circ}$ | 1 | 51.0 | -16.9 | 34.0 |  | -9.6 | -48.8 | -31.9 | -34.0 | -24.3 | -68.3 | -53.5 |
|  | $20^{\circ}$ | 1 | 56.2 | -14.8 | 37.5 |  | -8.2 | -48.8 | -34.0 | -34.0 | -25.8 | -68.3 | -53.5 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | 50.9 ----7 | 8.2 | 36.9 -----1. |  | 8.4 | $\begin{gathered} -22.6 \\ -8.6 \\ \hline \end{gathered}$ | $\begin{array}{r} -30.8 \\ -16.8 \\ \hline \end{array}$ | $\begin{gathered} \hline-16.4 \\ -2.3 \\ \hline \end{gathered}$ | $\begin{aligned} & -24.8 \\ & -10.7 \\ & \hline \end{aligned}$ | -42.1 $---{ }^{-1}$ | $-35.9$ |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 45.7 \\ & 45.7 \end{aligned}$ | $\begin{aligned} & 31.2 \\ & 31.2 \end{aligned}$ | $\begin{aligned} & 36.3 \\ & 36.3 \end{aligned}$ |  |  | $\begin{aligned} & 3.5 \\ & 17.6 \end{aligned}$ | $\begin{aligned} & -27.7 \\ & -13.7 \end{aligned}$ | $\begin{aligned} & 1.2 \\ & 15.2 \end{aligned}$ | $\begin{gathered} -23.8 \\ -9.8 \end{gathered}$ | $\begin{aligned} & \hline-16.0 \\ & -16.0 \end{aligned}$ | $\begin{aligned} & \hline-18.3 \\ & -18.3 \end{aligned}$ |
| 180 | 0 to $5^{\circ}$ | 1 | 51.4 | -26.7 | 34.1 |  | -15.8 | -61.7 | -35.1 | -43.0 | -27.2 | -86.4 | -67.7 |
|  | $10^{\circ}$ | 1 | 58.0 | -24.0 | 38.5 |  | -14.0 | -61.7 | -37.7 | -43.0 | -29.0 | -86.4 | -67.7 |
|  | $15^{\circ}$ | 1 | 64.5 | -21.4 | 43.0 |  | -12.2 | -61.7 | -40.3 | -43.0 | -30.8 | -86.4 | -67.7 |
|  | $20^{\circ}$ | 1 | 71.1 | -18.8 | 47.4 |  | -10.4 | -61.7 | -43.0 | -43.0 | -32.6 | -86.4 | -67.7 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $64.5$ | 10.4 ---2 | 46.7 ----1 |  | 10.6 | $\begin{aligned} & -28.6 \\ & -10.9 \end{aligned}$ | $\begin{aligned} & -39.0 \\ & -21.2 \end{aligned}$ | $\begin{gathered} -20.7 \\ -3.0 \end{gathered}$ | $\begin{array}{r} -31.4 \\ -13.6 \\ \hline \end{array}$ | $\xrightarrow{-53.3}$ | -45.4 <br> - |
|  | 30 to 45 | $\begin{aligned} & 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & 57.8 \\ & 57.8 \end{aligned}$ | $\begin{aligned} & 39.5 \\ & 39.5 \end{aligned}$ | $\begin{aligned} & 45.9 \\ & 45.9 \end{aligned}$ |  | $\begin{aligned} & 31.6 \\ & 31.6 \end{aligned}$ | $\begin{gathered} \hline 4.4 \\ 22.2 \end{gathered}$ | $\begin{aligned} & \hline-35.1 \\ & -17.3 \end{aligned}$ | $\begin{gathered} \hline 1.5 \\ 19.3 \end{gathered}$ | $\begin{aligned} & -30.1 \\ & -12.3 \end{aligned}$ | $\begin{aligned} & \hline-20.3 \\ & -20.3 \end{aligned}$ | $\begin{aligned} & -23.2 \\ & -23.2 \end{aligned}$ |
| 200 | 0 to $5^{\circ}$ | 1 | 63.4 | -32.9 | 42.1 |  | -19.5 | -76.2 | -43.3 | -53.1 | -33.5 | -106.7 | -83.5 |
|  | $10^{\circ}$ | 1 | 71.5 | -29.7 | 47.6 |  | -17.3 | -76.2 | -46.5 | -53.1 | -35.8 | -106.7 | -83.5 |
|  | $15^{\circ}$ | 1 | 79.7 | -26.4 | 53.1 |  | -15.0 | -76.2 | -49.8 | -53.1 | -38.0 | -106.7 | -83.5 |
|  | $20^{\circ}$ | 1 | 87.8 | -23.2 | 58.5 |  | -12.8 | -76.2 | -53.1 | -53.1 | -40.2 | -106.7 | -83.5 |
|  | $25^{\circ}$ | $\begin{aligned} & 1 \\ & 2 \\ & \hline \end{aligned}$ | $79.6$ | 12.8 ---8. | $57.6$ |  | 13.1 ----1 | $\begin{aligned} & -35.4 \\ & -13.4 \\ & \hline \end{aligned}$ | $\begin{array}{r} -48.2 \\ -26.2 \\ \hline \end{array}$ | $\begin{aligned} & -25.6 \\ & -3.7 \\ & \hline \end{aligned}$ | $\begin{array}{r} -38.7 \\ -16.8 \\ \hline \end{array}$ | -65.9 | $-56.1$ |
|  | 30 to 45 | $\begin{aligned} & 2 \\ & \hline 1 \\ & 2 \end{aligned}$ | $\begin{aligned} & \hline 71.3 \\ & 71.3 \end{aligned}$ | $\begin{aligned} & \hline 48.8 \\ & 48.8 \end{aligned}$ | $\begin{aligned} & \hline 56.7 \\ & 56.7 \end{aligned}$ |  | $\begin{aligned} & 39.0 \\ & 39.0 \end{aligned}$ | $\begin{gathered} 5.5 \\ 27.4 \end{gathered}$ | $\begin{aligned} & -43.3 \\ & -21.3 \end{aligned}$ | $\begin{gathered} 1.8 \\ 23.8 \end{gathered}$ | $\begin{aligned} & -37.2 \\ & -15.2 \end{aligned}$ | $\begin{aligned} & \hline-25.0 \\ & -25.0 \end{aligned}$ | $\begin{aligned} & \hline-28.7 \\ & -28.7 \end{aligned}$ |
|  |  |  |  | Adjustment Factor |  |  |  |  |  |  |  |  |  |
|  |  |  | Mean roof height (ft) |  | Exposure |  |  |  |  |  |  |  |  |
|  |  |  |  |  | B |  |  | C | D |  |  |  |  |
|  |  |  |  | 15 | 1.00 |  |  | 1.21 | 1.47 |  |  |  |  |
|  |  |  |  | 20 | 1.00 |  |  | 1.29 | 1.55 |  |  |  |  |
|  |  |  |  | 25 | 1.00 |  |  | 1.35 | 1.61 |  |  |  |  |
|  |  |  |  | 30 | 1.00 |  |  | 1.40 | 1.66 |  |  |  |  |
|  |  |  |  | 35 | 1.05 |  |  | 1.45 | 1.70 |  |  |  |  |
|  |  |  |  | 40 | 1.09 |  |  | 1.49 | 1.74 |  |  |  |  |
|  |  |  |  | 45 | 1.12 |  |  | 1.53 | 1.78 |  |  |  |  |
|  |  |  |  | 50 | 1.16 |  |  | 1.56 | 1.81 |  |  |  |  |
|  |  |  |  | 55 | 1.19 |  |  | 1.59 | 1.84 |  |  |  |  |
|  |  |  |  | 60 | 1.22 |  |  | 1.62 | 1.87 |  |  |  |  |

Unit Conversions - $\mathbf{1 . 0} \mathrm{ft}=\mathbf{0 . 3 0 4 8} \mathbf{m} ; 1.0 \mathrm{psf}=0.0479 \mathrm{kN} / \mathrm{m}^{2}$

Table 1.5-1 Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

| Use or Occupancy of Buildings and Structures |  |  |  | Risk Category |
| :--- | :---: | :---: | :---: | :---: |
| Buildings and other structures that represent a low risk to human life in the event of failure | I |  |  |  |
| All buildings and other structures except those listed in Risk Categories I, III, and IV | II |  |  |  |
| Buildings and other structures, the failure of which could pose a substantial risk to human life. | III |  |  |  |
| Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial |  |  |  |  |
| economic impact and/or mass disruption of day-to-day civilian life in the event of failure. |  |  |  |  |
| Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that |  |  |  |  |
| manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous |  |  |  |  |
| chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity |  |  |  |  |
| exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat |  |  |  |  |
| to the public if released. |  |  |  |  |
| Buildings and other structures designated as essential facilities. |  |  |  |  |

Buildings and other structures, the failure of which could pose a substantial hazard to the community.
Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to the public if released. ${ }^{a}$
Buildings and other structures required to maintain the functionality of other Risk Category IV structures.
${ }^{a}$ Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.2 that a release of the substances is commensurate with the risk associated with that Risk Category.


# Residential Building Loads: Review and Roadmap for Future Progress, Crandell, Kenney \& Rosowsky, ed., 2006 (ASCE) 

APPENDIX B - Simplified ASCE 7-02 Wind Loads For Typical Low-Rise Buildings January 31, 2004

A. 1 General. This appendix provides simplified wind loads that result in designs reasonably consistent with the requirements ASCE 7. It is intended for use by qualified design professionals and is subject to the limitations of Section A.2. In this method, a single wind pressure for each roof and wall vertical projected area and the roof horizontal projected area is used to determine main wind force resisting system loads. For components and cladding loads, surface pressures are determined for specific building elements such that multiple pressure zones are not required to be separately evaluated.
A. 2 Limitations. These provisions are applicable to buildings meeting the following conditions:

- Light-frame, concrete, or masonry construction using shear walls and horizontal diaphragms to resist lateral loads.
- Mean roof height of 40 feet or less.
- One- and two-family dwellings, apartments, commercial buildings, and other building uses or occupancies with a wind load importance factor of 1.0 .


## A. 3 Wind Design Criteria

## A.3.1 Basic Wind Speed

The basic (design) wind speed shall be determined in accordance with Figure A1 or as required by the local governing building code.

## A3.2 Wind Exposure and

 Topography. The provisions of this Appendix are based on wind exposure category B (suburban, urban, or wooded terrain) as defined in ASCE 7. For buildings located in wind exposure category C (open or coastal terrain), tabulated exposure B wind loads shall be increased by a factor 1.4 (see table footnotes as applicable in Section A4). Buildings sited within 10 building heights from the top edge of a prominent topographic feature shall be designed in accordance with ASCE 7. A prominent topographic feature has a ground slope of greater than 15 percent and a vertical rise of greater than 50 feet, and is separated from features of similar or greater height by a distance of more than approximately 100 times the height of the topographic feature.
## A3.3 Wind-borne Debris Region.

The wind-borne debris region shall be defined in accordance with the Figure A1 for Atlantic Ocean and Gulf of Mexico coastal areas as follows:
Basic Wind Speed $\geq \mathbf{1 2 0} \mathbf{~ m p h}$ - all areas.
$110 \mathrm{mph} \leq$ Basic Wind Speed $<120$
$m p h$ - all areas within 1 mile of coastline.
A3.4 Building Enclosure Condition Building enclosure condition shall be classified in accordance with Table A1 for the purpose of determining wind loads in accordance with Section A4.2 and A4.3.
A3.5 Counteracting Dead Load
When dead load is used to counteract
$\mathfrak{F}$



[^5]\mu=0.30
A354 Grade BC

``` & & & & & & & & & \\
\hline \multicolumn{3}{|l|}{A449} & & up A & & & & & \\
\hline & \multirow[t]{6}{*}{Loading} & \multicolumn{8}{|l|}{Nominal Bolt Diameter，d，in．} \\
\hline \multirow[t]{5}{*}{Hole Type} & & \multicolumn{2}{|l|}{5／8} & & & \multicolumn{2}{|l|}{7／8} & \multicolumn{2}{|l|}{1} \\
\hline & & \multicolumn{8}{|l|}{Minimum Group A Bolt Pretension，kips} \\
\hline & & \multicolumn{2}{|l|}{19} & \multicolumn{2}{|l|}{28} & \multicolumn{2}{|l|}{39} & \multicolumn{2}{|l|}{51} \\
\hline & & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) \\
\hline & & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD \\
\hline \multirow[t]{2}{*}{STD／SSLT} & S & 4.29 & 6.44 & 6.33 & 9.49 & 8.81 & 13.2 & 11.5 & 17.3 \\
\hline & D & 8.59 & 12.9 & 12.7 & 19.0 & 17.6 & 26.4 & 23.1 & 34.6 \\
\hline \multirow[t]{2}{*}{OVS／SSLP} & S & 3.66 & 5.47 & 5.39 & 8.07 & 7.51 & 11.2 & 9.82 & 14.7 \\
\hline & D & 7.32 & 10.9 & 10.8 & 16.1 & 15.0 & 22.5 & 19.6 & 29.4 \\
\hline \multirow[t]{2}{*}{LSL} & S & 3.01 & 4.51 & 4.44 & 6.64 & 6.18 & 9.25 & 8.08 & 12.1 \\
\hline & D & 6.02 & 9.02 & 8.87 & 13.3 & 12.4 & 18.5 & 16.2 & 24.2 \\
\hline \multirow[t]{6}{*}{Hole Type} & \multirow[t]{6}{*}{Loading} & \multicolumn{8}{|l|}{Nominal Bolt Diameter，\(d\) ，in．} \\
\hline & & \multicolumn{2}{|l|}{11／8} & \multicolumn{2}{|l|}{11／4} & \multicolumn{2}{|l|}{13／8} & \multicolumn{2}{|l|}{11／2} \\
\hline & & \multicolumn{8}{|l|}{Minimum Group A Bolt Pretension，kips} \\
\hline & & \multicolumn{2}{|l|}{56} & \multicolumn{2}{|l|}{71} & \multicolumn{2}{|l|}{85} & \multicolumn{2}{|l|}{103} \\
\hline & & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\mathrm{P}_{\boldsymbol{n}}\) \\
\hline & & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD \\
\hline \multirow[t]{2}{*}{STD／SSLT} & S & 12.7 & 19.0 & 16.0 & 24.1 & 19.2 & 28.8 & 23.3 & 34.9 \\
\hline & D & 25.3 & 38.0 & 32.1 & 48.1 & 38.4 & 57.6 & 46.6 & 69.8 \\
\hline \multirow[t]{2}{*}{OVS／SSLP} & S & 10.8 & 16.1 & 13.7 & 20.5 & 16.4 & 24.5 & 19.8 & 29.7 \\
\hline & D & 21.6 & 32.3 & 27.4 & 40.9 & 32.7 & 49.0 & 39.7 & 59.4 \\
\hline \multirow[t]{2}{*}{LSL} & S & 8.87 & 13.3 & 11.2 & 16.8 & 13.5 & 20.2 & 16.3 & 24.4 \\
\hline & D & 17.7 & 26.6 & 22.5 & 33.7 & 26.9 & 40.3 & 32.6 & 48.9 \\
\hline \multicolumn{10}{|l|}{\begin{tabular}{ll} 
STD \(=\) standard hole & \(\mathrm{S}=\) single shear \\
OVS \(=\) oversized hole & \(\mathrm{D}=\) double shear \\
SSLT \(=\) short－slotted hole transverse to the line of force & \\
SSLP＝short－slotted hole parallel to the line of force & \\
LSL \(=\) long－slotted hole transverse or parallel to the line of force &
\end{tabular}} \\
\hline Hole Type & ASD & LRFD & \multicolumn{7}{|l|}{\multirow[t]{4}{*}{\begin{tabular}{l}
Note：Slip－critical bolt values assume no more than one filler has been provided or bolts have been added to distribute loads in the fillers． \\
See AISC Specification Sections J3．8 and J5 for provisions when fillers are present． \\
For Class B faying surfaces，multiply the tabulated available strength by 1.67 ．
\end{tabular}}} \\
\hline STD and SSLT & \(\Omega=1.50\) & \(\phi=1.00\) & & & & & & & \\
\hline OVS and SSLP & \(\Omega=1.76\) & \(\phi=0.85\) & & & & & & & \\
\hline LSL & \(\Omega=2.14\) & \(\phi=0.70\) & & & & & & & \\
\hline
\end{tabular}

For bearing of plate material at bolt holes:
\(R_{a} \leq R_{n} / \Omega\) or \(R_{u} \leq \phi R_{n}\)
where \(R_{u}=\Sigma \gamma_{i} R_{i}\)
- deformation at bolt hole is a concern
\[
R_{n}=1.2 L_{c} t F_{u} \leq 2.4 d t F_{u}
\]
- deformation at bolt hole is not a concern
\[
R_{n}=1.5 L_{c} t F_{u} \leq 3.0 d t F_{u}
\]


Figure 10.11 End tear-out.
- long slotted holes with the slot perpendicular to the load
\[
\begin{aligned}
& R_{n}=1.0 L_{c} t F_{u} \leq 2.0 d t F_{u} \\
& \text { where } \quad \mathrm{R}_{\mathrm{n}}=\text { the nominal bearing strength } \\
& \mathrm{F}_{\mathrm{u}}=\text { specified minimum tensile strength } \\
& \mathrm{L}_{\mathrm{c}}=\text { clear distance between the edges of the hole and the next hole or edge in } \\
& \text { the direction of the load } \\
& \mathrm{d}=\text { nominal bolt diameter } \\
& t=\text { thickness of connected material }
\end{aligned}
\]
\[
\phi=0.75(\mathrm{LRFD}) \quad \Omega=2.00(\mathrm{ASD})
\]

The minimum edge desistance from the center of the outer most bolt to the edge of a member is generally \(13 / 4\) times the bolt diameter for the sheared edge and \(11 / 4\) times the bolt diameter for the rolled or gas cut edges.
The maximum edge distance should not exceed 12 times the thickness of thinner member or 6 in.
Standard bolt hole spacing is 3 in . with the minimum spacing of \(22 / 3\) times the diameter of the bolt, \(d_{b}\). Common edge distance from the center of last hole to the edge is \(1 \frac{1}{4} \mathrm{in}\)..



\section*{Tension Member Design}

In steel tension members, there may be bolt holes that reduce the size of the cross section.
\(g\) refers to the row spacing or gage

\(p\) refers to the bolt spacing or pitch
\(s\) refers to the longitudinal spacing of two consecutive holes

\section*{Effective Net Area:}

The smallest effective are must be determined by subtracting the bolt hole areas. With staggered holes, the shortest length must be evaluated.

A series of bolts can also transfer a portion of the tensile force, and some of the effective net areas see reduced stress.

The effective net area, \(A_{e}\), is determined from the net area, \(A_{n}\), multiplied by a shear lag factor, \(U\), which depends on the element type and connection configuration. If a portion of a connected member is not fully connected (like the leg of an angle), the unconnected part is not subject to the full stress and the shear lag factor can range from 0.6 to 1.0: \(\quad A_{e}=A_{n} U\)

The staggered hole path area is determined by:
\[
A_{n}=A_{g}-A_{\text {of all holes }}+t \Sigma \frac{s}{4 g}
\]

where \(t\) is the plate thickness, \(s\) is each stagger spacing, and \(g\) is the gage spacing.

For tension elements:
\(R_{a} \leq R_{n} / \Omega\) or \(R_{u} \leq \phi R_{n}\)
where \(R_{u}=\Sigma \gamma_{i} R_{i}\)
1. yielding
\[
\phi=0.90(\mathrm{LRFD}) \quad \Omega=1.67(\mathrm{ASD})
\]
2. rupture
\[
R_{n}=F_{u} A_{e}
\]
\[
\phi=0.75(\mathrm{LRFD}) \quad \Omega=2.00(\mathrm{ASD})
\]

where \(A_{g}=\) the gross area of the member (excluding holes)
\(\mathrm{A}_{\mathrm{e}}=\) the effective net area (with holes, etc.)
\(\mathrm{F}_{\mathrm{y}}=\) the yield strength of the steel
\(\mathrm{F}_{\mathrm{u}}=\) the tensile strength of the steel (ultimate)

\section*{For shear elements:}
\[
\begin{aligned}
& R_{a} \leq R_{n} / \Omega \text { or } R_{u} \leq \phi R_{n} \\
& \text { where } R_{u}=\Sigma \gamma_{i} R_{i}
\end{aligned}
\]
1. yielding
\[
R_{n}=0.6 F_{y} A_{g}
\]
\[
\phi=1.00(\mathrm{LRFD}) \quad \Omega=1.50(\mathrm{ASD})
\]
2. rupture
\[
\begin{array}{ll} 
& R_{n}=0.6 F_{u} A_{n v} \\
\phi=0.75(\mathrm{LRFD}) & \Omega=2.00(\mathrm{ASD})
\end{array}
\]

where \(\quad \mathrm{A}_{\mathrm{g}}=\) the gross area of the member (excluding holes)
\(\mathrm{A}_{\mathrm{nv}}=\) the net area subject to shear (with holes, etc.)
\(\mathrm{F}_{\mathrm{y}}=\) the yield strength of the steel
\(\mathrm{F}_{\mathrm{u}}=\) the tensile strength of the steel (ultimate)

\section*{Welded Connections}

Weld designations include the strength in the name, i.e. E70XX has \(\mathrm{F}_{\mathrm{y}}=70 \mathrm{ksi}\). Welds are weakest in shear and are assumed to always fail in the shear mode.

The throat size, T , of a fillet weld is determined trigonometry by: \(\mathrm{T}=0.707 \times\) weld size*
* When the submerged arc weld process is used, welds over \(3 / 8\) " will have a throat thickness of 0.11 in . larger than the formula.


Weld sizes are limited by the size of the parts being put together and are given in AISC manual table J2.4 along with the allowable strength per length of fillet weld, referred to as \(S\).


The maximum size of a fillet weld permitted along edges of connected parts shall be:
- Material less than \(1 / 4 \mathrm{in}\). thick, not greater than the thickness of the material.
- Material \(1 / 4 \mathrm{in}\). or more in thickness, not greater than the thickness of the material minus \(1 / 16\) in., unless the weld is especially designated on the drawings to be built out to obtain fullthroat thickness.

The minimum length of a fillet weld is 4 times the nominal size. If it is not, then the weld size used for design is \(1 / 4\) the length.

TABLE J2.4
Minimum Size of Fillet Welds
Intermittent fillet welds cannot be less than four times the weld size, not to be less than \(1 \frac{1}{2}\) ".
\begin{tabular}{|c|c|}
\hline \begin{tabular}{c} 
Material Thickness of Thicker \\
Part Joined (in.)
\end{tabular} & \begin{tabular}{c} 
Minimum Size of Fillet \\
Weld \\
(in.)
\end{tabular} \\
\hline To \(1 / 4\) inclusive & \(1 / 8\) \\
Over \(1 / 4\) to \(1 / 2\) & \(3 / 16\) \\
Over \(1 / 2\) to \(3 / 4\) & \(1 / 4\) \\
Over \(3 / 4\) & \(5 / 16\) \\
\hline \multicolumn{2}{|c|}{ Leg dimension of fillet welds. Single-pass welds must be used. } \\
\hline
\end{tabular}

American Institute of Steel Construction

For fillet welds:
\[
\begin{gathered}
R_{a} \leq R_{n} / \Omega \text { or } R_{u} \leq \phi R_{n} \\
\text { where } R_{u}=\Sigma \gamma_{i} R_{i}
\end{gathered}
\]
for the weld metal: \(\quad R_{n}=0.6 F_{E X X} T l=S l\)
\[
\phi=0.75(\mathrm{LRFD}) \quad \Omega=2.00(\mathrm{ASD})
\]
where:
\(T\) is throat thickness
\(l\) is length of the weld
For a connected part, the other limit states for the base metal, such as tension yield, tension rupture, shear yield, or shear rupture must be considered.
\begin{tabular}{ccc}
\hline \multicolumn{3}{c}{\begin{tabular}{c} 
Available Strength of Fillet Welds \\
per inch of weld \((\phi S)\)
\end{tabular}} \\
\hline \begin{tabular}{c} 
Weld Size \\
(in.)
\end{tabular} & \begin{tabular}{c} 
E60XX \\
(k/in.)
\end{tabular} & \begin{tabular}{c} 
E70XX \\
(k/in.)
\end{tabular} \\
\(3 / 16\) & 3.58 & 4.18 \\
\(1 / 4\) & 4.77 & 5.57 \\
\(5 / 16\) & 5.97 & 6.96 \\
\(3 / 8\) & 7.16 & 8.35 \\
\(7 / 16\) & 8.35 & 9.74 \\
\(1 / 2\) & 9.55 & 11.14 \\
\(5 / 8\) & 11.93 & 13.92 \\
\(3 / 4\) & 14.32 & 16.70 \\
\hline
\end{tabular}
(not considering increase in throat with submerged arc weld process)

\section*{Framed Beam Connections}

Coping is the term for cutting away part of the flange to connect a beam to another beam using welded or bolted angles.


AISC provides tables that give bolt and angle available strength knowing number of bolts, bolt type, bolt diameter, angle leg thickness, hole type and coping, and the wide flange beam being connected. For the connections the limit-state of bolt shear, bolts bearing on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles, and bolt bearing on the beam web are considered.

Group A bolts include A325, while Group B includes A490.
here are also tables for bolted/welded doubleangle connections and all-welded doubleangle connections.

(a)

(b)

\section*{Sample AISC Table for Bolt and Angle Available Strength in All-Bolted Double-Angle Connections}


\section*{Limiting Strength or Stability States}

In addition to resisting shear and tension in bolts and shear in welds, the connected materials may be subjected to shear, bearing, tension, flexure and even prying action. Coping can significantly reduce design strengths and may require web reinforcement. All the following must be considered:
- shear yielding
- shear rupture
- block shear rupture -
failure of a block at a beam as a result of shear and tension
- tension yielding
- tension rupture

- local web buckling
- lateral torsional buckling

Block Shear Strength (or Rupture):
\[
R_{a} \leq R_{n} / \Omega \text { or } R_{u} \leq \phi R_{n}
\]
where \(R_{u}=\Sigma \gamma_{i} R_{i}\)
\(R_{n}=0.6 F_{u} A_{n v}+U_{b s} F_{u} A_{n t} \leq 0.6 F_{y} A_{g v}+U_{b s} F_{u} A_{n t}\) \(\phi=0.75(\mathrm{LRFD}) \quad \Omega=2.00(\mathrm{ASD})\)
where:
\(A_{n v}\) is the net area subjected to shear
\(A_{n t}\) is the net area subjected to tension
\(A_{g v}\) is the gross area subjected to shear
\(U_{b s}=1.0\) when the tensile stress is uniform (most cases)
\(=0.5\) when the tensile stress is non-uniform

\section*{Local Buckling in Steel I Beams- Web Crippling or Flange Buckling}

Concentrated forces on a steel beam can cause the web to buckle (called web crippling). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.


The maximum support load and interior load can be determined from:
\[
\begin{aligned}
& P_{n(\text { max-end })}=(2.5 k+N) F_{y w} t_{w} \\
& P_{n \text { (interior) })}=(5 k+N) F_{y w} t_{w}
\end{aligned}
\]
where \(\quad t_{w}=\) thickness of the web \(N=\) bearing length
 \(k=\) dimension to fillet found in beam section tables
\[
\phi=1.00(\mathrm{LRFD}) \quad \Omega=1.50(\mathrm{ASD})
\]

\section*{Examples: \\ Connections and Tension Members}

\section*{Example 1}

A nominal \(4 \times 6\) in. redwood beam is to be supported by two \(2 \times 6\) in. members acting as a spaced column. The minimum spacing and edge distances for the \(1 / 2\) inch bolts are shown. How many \(1 / 2 \mathrm{in}\). bolts will be required to safely carry a load of 1500 lb ? Use the chart provided.

SOLUTION:
The table requires that the length of the bolt in the main wood member be known, along with the diameter of bolt in inches, and if the bolt is seeing single shear or double shear and what direction it is bearing on the grain.

The main member is the beam. The 4 in . nominal size is actually \(31 / 2 \mathrm{in}\). finished:


The bolt is \(1 / 2\) inches in diameter, and sees two planes of shear at the interfaces with the \(2 \times 6\) 's. This means double shear.

The vertical force is pushing the beam down onto the bolt, so the bolt is in contact with the grain running horizontally. That means the bolt is bearing perpendicular to the grain, and we should look up \(q\).

The allowable load per bolt multiplied by the number of bolts will determine the capacity, which we need to be at least 1500 lb :
\(q \times n \geq P\)
knowing \(q\) \& \(P\), the equation for \(n\) becomes:
\(\mathrm{n} \geq \frac{P}{q}=\frac{1500 \mathrm{lb}}{980 \mathrm{lb} / \text { bolt }}=1.5\) bolt

\section*{rounded up \(=2\) bolts required}

Table: Holding Power of Bolts
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{10}{|l|}{\begin{tabular}{l}
\(p=\) Safe loads parallel to grain in pounds \\
\(q=\) Safe loads perpendicular to grain in pounds
\end{tabular}} \\
\hline \multirow[t]{2}{*}{Length of Boit in Main Wood Member \({ }^{3}\) (In Inches)} & \multicolumn{9}{|c|}{DIAMETER OF BOLT (IN INCHES)} \\
\hline & 36 & 1/2 & 5\% & \(7 / 4\) & 7/8 & ( & 11/8 & 11/4 & 11/2 \\
\hline \begin{tabular}{l}
Single \(p\) \\
Shear \(q\)
\end{tabular} & \[
\begin{aligned}
& 325 \\
& 185
\end{aligned}
\] & \[
\begin{aligned}
& 470 \\
& 215
\end{aligned}
\] & \[
\begin{aligned}
& 590 \\
& 245
\end{aligned}
\] & \[
\begin{aligned}
& 710 \\
& 270
\end{aligned}
\] & \[
\begin{aligned}
& 830 \\
& 300
\end{aligned}
\] & \[
\begin{aligned}
& 945 \\
& 325
\end{aligned}
\] & & & \\
\hline \begin{tabular}{l}
Double \(p\) \\
Shear \(q\)
\end{tabular} & \[
\begin{aligned}
& 650 \\
& 370
\end{aligned}
\] & \[
\begin{aligned}
& 940 \\
& 430
\end{aligned}
\] & \[
\begin{array}{r}
1180 \\
490
\end{array}
\] & \[
\begin{array}{r}
1420 \\
540
\end{array}
\] & \[
\begin{array}{r}
1660 \\
600
\end{array}
\] & \[
\begin{array}{r}
1890 \\
650
\end{array}
\] & & & \\
\hline \[
\begin{aligned}
& \quad \begin{array}{l}
\text { Single } p \\
\text { Shear } q
\end{array} \\
& \hline
\end{aligned}
\] & & \[
\begin{aligned}
& 630 \\
& 360
\end{aligned}
\] & \[
\begin{aligned}
& 910 \\
& 405
\end{aligned}
\] & \[
\begin{array}{r}
1155 \\
450
\end{array}
\] & \[
\begin{array}{r}
1370 \\
495
\end{array}
\] & \[
\begin{array}{r}
1575 \\
540
\end{array}
\] & & & \\
\hline \begin{tabular}{rl}
\(21 / 2\) & \begin{tabular}{l} 
Double \(p\) \\
Shear \(q\)
\end{tabular} \\
\hline
\end{tabular} & \[
\begin{aligned}
& 710 \\
& 620 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
1260 \\
720 \\
\hline
\end{array}
\] & \[
\begin{array}{r}
1820 \\
810
\end{array}
\] & \[
\begin{array}{r}
2310 \\
900
\end{array}
\] & \[
\begin{array}{r}
2740 \\
990
\end{array}
\] & \[
\begin{aligned}
& 3150 \\
& 1080
\end{aligned}
\] & & & \\
\hline Single \(p\) Shear \(q\) & & & \[
\begin{aligned}
& 990 \\
& 565
\end{aligned}
\] & \[
\begin{array}{r}
1400 \\
630
\end{array}
\] & \[
\begin{array}{r}
1790 \\
695
\end{array}
\] & \[
\begin{array}{r}
2135 \\
760
\end{array}
\] & \[
\begin{array}{r}
2455 \\
825
\end{array}
\] & \[
\begin{array}{r}
2740 \\
895
\end{array}
\] & \[
\begin{aligned}
& 3305 \\
& 1020
\end{aligned}
\] \\
\hline \[
\begin{aligned}
& \text { Double } p \\
& \text { Shear } q
\end{aligned}
\] & \[
\begin{aligned}
& 710 \\
& 640
\end{aligned}
\] & \[
\begin{array}{r}
1270 \\
980
\end{array}
\] & \[
\begin{aligned}
& 1980 \\
& 1130
\end{aligned}
\] & \[
\begin{aligned}
& 2800 \\
& 1260
\end{aligned}
\] & \[
\begin{aligned}
& 3580 \\
& 1390
\end{aligned}
\] & \[
\begin{aligned}
& 4270 \\
& 1520
\end{aligned}
\] & \[
\begin{aligned}
& 4910 \\
& 1650
\end{aligned}
\] & \[
\begin{aligned}
& 5480 \\
& 1780
\end{aligned}
\] & \[
\begin{aligned}
& 6610 \\
& 2040
\end{aligned}
\] \\
\hline
\end{tabular}
\({ }^{1}\) Tabulated values are on a normal load-duration basis and apply to joints made of seasoned lumber used in dry locations. See U.B.C. Standard No. 25-17 for other service conditions.
\({ }^{2}\) Double shear values are for joints consisting of three wood members in which the side members are one half the thickness of the main member. Single shear values are for joints consisting of two wood members having a minimum thickness not less than that specified.
\({ }^{3}\) The length specified is the length of the bolt in the main member of double shear joints or the length of the bolt in the thinner member of single shear joints.
\({ }^{4}\) See U.B.C. Standard No. 25-17 for wood-to-metal bolted joints.

\section*{Example 2}
8.11 A built-up plywood box beam with \(2 \times 4 \mathrm{~S} 4 \mathrm{~S}\) top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 \#/ft. along the 26 -foot span. Assume the nails have a shear capacity of \(80 \#\) each.


\section*{Solution:}

Construct the shear ( \(V\) ) diagram to obtain the critical shear condition and its location
Note that the condition of shear is critical at the supports, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing \(P\) varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.
\[
f_{v}=\frac{V Q}{I b}
\]
\[
I_{x}=\frac{\left(4.5^{\prime \prime}\right)\left(18^{\prime \prime}\right)^{3}}{12}-\frac{\left(3.5^{\prime \prime}\right)\left(15^{\prime \prime}\right)^{3}}{12}=1,202.6 \text { in. }{ }^{4}
\]

\[
Q=\Sigma A \bar{y}=\left(9^{\prime \prime}\right)\left(1 / 22^{\prime \prime}\right)\left(4.5^{\prime \prime}\right)+\left(9^{\prime \prime}\right)\left(1 / 22^{\prime \prime}\right)\left(4.5^{\prime \prime}\right)+\left(1.5^{\prime \prime}\right)\left(3.5^{\prime \prime}\right)\left(8.255^{\prime \prime}\right)=83.8 \mathrm{in}^{3}
\]

\[
f_{v-\max }=\frac{(2,600 \#)\left(83.3 \mathrm{in.} .^{3}\right)}{\left(1,202.6 \text { in. }^{4}\right)\left(1 / 2^{\prime 2}+1 / 2^{\prime \prime}\right)}=180.2 \mathrm{psi}
\]

Assume:
\(F=\) Capacity of two nails (one each side) at the flange; representing two shear surfaces

\[
Q=A \bar{y}=\left(5.25 \mathrm{in} .^{2}\right)\left(8.25^{\prime \prime}\right)=43.3 \mathrm{in} .^{3}
\]

Shear force \(=f_{v} \times A_{v}\)
where:
\[
A_{v}=\text { shear area }
\]


At the maximum shear location (support) where \(V=2,600\) \#
\[
p=\frac{(2 \text { nails } \times 80 \# / \text { nail })\left(1,202.6 \text { in. } .^{4}\right)}{(2,600 \#)\left(43.3 \text { in. }^{3}\right)}=1.71^{\prime \prime}
\]

\section*{Example 3}
10.2 The butt splice shown in Figure 10.22 uses two \(8 \times\) \(3 / 8^{\prime \prime}\) plates to "sandwich" in the \(8 \times 1 / 2\) " plates being joined. Four \(7 / 8^{\prime \prime} \phi\) A325-SC bolts are used on both sides of the splice. Assuming A36 steel and standard round holes, determine the allowable capacity of the connection.


SOLUTION:
Shear, bearing and net tension will be checked to determine the critical conditions that governs the capacity of the connection. (The edge distance to the holes is presumed to be adequate.)

Shear: Using the AISC available shear in Table 7-3 (Group A):
\[
\phi R_{n}=26.4 \text { k/bolt x } 4 \text { bolts }=105.6 \mathrm{k}
\]

Bearing: Using the AISC available bearing in Table 7-4:
There are 4 bolts bearing on the center ( \(1 / 2^{\prime \prime}\) ) plate, while there are 4 bolts bearing on a total width of two sandwich plates (3/4" total). The thinner bearing width will govern. Assume 3 in. spacing (center to center) of bolts. For A36 steel, \(\mathrm{F}_{\mathrm{u}}=58 \mathrm{ksi}\).
\[
\phi R_{n}=91.4 \text { k/bolt/in. x } 0.5 \text { in. x } 4 \text { bolts }=182.8 \mathrm{k}
\]

Tension: The center plate is critical, again, because its thickness is less than the combined thicknesses of the two outer plates. We must consider tension yielding and tension rupture:

\[
\phi R_{n}=\phi F_{y} A_{g} \text { and } \phi R_{n}=\phi F_{u} A_{e} \text { where } A_{e}=A_{n e t} U
\]
\(A_{g}=8\) in. \(x^{1 / 2}\) in. \(=4\) in \(^{2}\)
The holes are considered \(1 / 8 \mathrm{in}\). larger than the bolt hole diameter \(=(7 / 8+1 / 8)=1.0 \mathrm{in}\).
\(A_{n}=\left(8 \mathrm{in} .-2\right.\) holes \(\times 1.0 \mathrm{in}\).) \(\times 1 / 2 \mathrm{in}\). \(=3.0 \mathrm{in}^{2}\)
The whole cross section sees tension, so the shear lag factor \(U=1\)
\[
\begin{aligned}
& \phi F_{y} A_{g}=0.9 \times 36 \mathrm{ksi} \times 4 \mathrm{in}^{2}=129.6 \mathrm{k} \\
& \phi F_{u} A_{e}=0.75 \times 58 \mathrm{ksi} \times(1) \times 3.0 \mathrm{in}^{2}=130.5 \mathrm{k}
\end{aligned}
\]

Block Shear Rupture: It is possible for the center plate to rip away from the sandwich plates
 leaving the block (shown hatched) behind:
\[
\phi R_{n}=\phi\left(0.6 F_{u} A_{n v}+U_{b s} F_{u} A_{n t}\right) \leq \phi\left(0.6 F_{y} A_{g v}+U_{b s} F_{u} A_{n t}\right)
\]
where \(A_{n v}\) is the area resisting shear, \(A_{n t}\) is the area resisting tension, \(A_{g v}\) is the gross area resisting shear, and \(U_{b s}=1\) when the tensile stress is uniform.
\[
\begin{aligned}
& A_{g v}=(4+2 \mathrm{in} .) \times 1 / 2 \mathrm{in} .=3 \mathrm{in}^{2} \\
& A_{n v}=A_{g v}-1 \frac{1}{2} \text { holes area }=3 \mathrm{in}^{2}-1.5 \times 1 \mathrm{in} . \times 1 / 2 \mathrm{in} .=2.25 \mathrm{in}^{2} \\
& A_{n t}=3.5 \mathrm{in} . \mathrm{xt}-1 \text { holes }=3.5 \mathrm{in} . \times 1 / 2 \mathrm{in}-1 \times 1 \mathrm{in} . \times 1 / 2 \mathrm{in} .=1.25 \mathrm{in}^{2} \\
& \phi\left(0.6 F_{u} A_{n v}+U_{b s} F_{u} A_{n t}\right)=0.75 \times\left(0.6 \times 58 \mathrm{ksi} \times 2.25 \mathrm{in}^{2}+1 \times 58 \mathrm{ksi} \times 1.25 \mathrm{in}^{2}\right)=113.1 \mathrm{k} \\
& \phi\left(0.6 F_{y} A_{g v}+U_{b s} F_{u} A_{n t}\right)=0.75 \times\left(0.6 \times 36 \mathrm{ksi} \times 3 \mathrm{in}^{2}+1 \times 58 \mathrm{ksi} \times 1.25 \mathrm{in}^{2}\right)=103.0 \mathrm{k}
\end{aligned}
\]

The maximum connection capacity (smallest value) is governed by block shear rupture.
```

\phiRn}=103.0\textrm{k

```

\section*{Example 4}
10.7 Determine the capacity of the connection in Figure 10.44 assuming A36 steel with E70XX electrodes.

\section*{Solution:}

Capacity of weld:
For a \(5 / 16^{\prime \prime}\) fillet weld, \(\phi S=6.96 \mathrm{k} / \mathrm{in}\)
Weld length \(=8\) in +6 in +8 in \(=22 \mathrm{in}\).
Weld capacity \(=22^{\prime \prime} \times 6.96 \mathrm{k} /\) in \(=153.1 \mathrm{k}\)

Capacity of plate: \(\quad 0.9 \times 36 \mathrm{k} / \mathrm{in}^{2} \times 3 / 8^{\prime \prime} \times 6^{\prime \prime}=72.9 \mathrm{k}\)
\(\phi P_{n}=\phi F_{y} A_{g} \quad \phi=0.9\)
Plate capacity \(=0.9 \times 36 \mathrm{k} / \mathrm{in}^{2} \times 3 / 8^{\prime \prime} \times 6^{\prime \prime}=72.9 \mathrm{k}\)
\(\therefore\) Plate capacity governs, \(P_{u}=72.9 \mathrm{k}\)
\begin{tabular}{ccc}
\hline \multicolumn{3}{c}{\begin{tabular}{c} 
Available Strength of Fillet Welds \\
per inch of weld \((\phi S)\)
\end{tabular}} \\
\hline \begin{tabular}{c} 
Weld Size \\
\((\mathrm{in})\).
\end{tabular} & \begin{tabular}{c} 
E60XX \\
\((\mathrm{k} / \mathrm{in})\).
\end{tabular} & \begin{tabular}{c} 
E70XX \\
\((\mathrm{k} / \mathrm{in})\).
\end{tabular} \\
\(3 / 16\) & 3.58 & 4.18 \\
\(1 / 4\) & 4.77 & 5.57 \\
\(5 / 16\) & 5.97 & 6.96 \\
\(3 / 8\) & 7.16 & 8.35 \\
\(7 / 16\) & 8.35 & 9.74 \\
\(1 / 2\) & 9.55 & 11.14 \\
\(5 / 8\) & 11.93 & 13.92 \\
\(3 / 4\) & 14.32 & 16.70 \\
\hline
\end{tabular}
(not considering increase in throat with submerged arc weld process)


The weld size used is obviously too strong. What size, then, can the weld be reduced to so that the weld strength is more compatible to the plate capacity? To make the weld capacity \(\approx\) plate capacity:
\[
\begin{aligned}
& 22^{\prime \prime} \times(\text { weld capacity per in. })=72.9 \mathrm{k} \\
& \text { Weld capacity per inch }=\frac{72.9 \mathrm{k}}{22 \mathrm{in} .}-3.31 \mathrm{k} / \mathrm{in} \text {. }
\end{aligned}
\]

From Available Strength table, use \(3 / 16^{\prime \prime}\) weld
\[
(\phi S=4.18 \mathrm{k} / \mathrm{in} .)
\]

Minimum size fillet \(=3 / 16^{\prime \prime}\) based on a \(3 / 8^{\prime \prime}\) thick plate.


\section*{Example 5}

The steel used in the connection and beams is A992 with \(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}\), and \(\mathrm{F}_{\mathrm{u}}=65 \mathrm{ksi}\). Using A490-N bolt material, determine the maximum capacity of the connection based on shear in the bolts, bearing in all materials and pick the number of bolts and angle length (not staggered). Use A36 steel for the angles.

W21x93: \(\mathrm{d}=21.62 \mathrm{in}, \mathrm{t}_{\mathrm{w}}=0.58 \mathrm{in}, \mathrm{t}_{\mathrm{f}}=0.93 \mathrm{in}\) \(\mathrm{W} 10 \times 54: \mathrm{t}_{\mathrm{f}}=0.615\) in

\section*{SOLUTION:}


The maximum length the angles can be depends on how it fits between the top and bottom flange with some clearance allowed for the fillet to the flange, and getting an air wrench in to tighten the bolts. This example uses 1 " of clearance:
\[
\begin{aligned}
\text { Available length } & =\text { beam depth }- \text { both flange thicknesses }-1^{\prime \prime} \text { clearance at top } \& 1 " \text { at bottom } \\
& =21.62 \mathrm{in}-2(0.93 \mathrm{in})-2(1 \mathrm{in})=17.76 \mathrm{in} .
\end{aligned}
\]

With the spaced at 3 in. and \(1 \frac{1}{4}\) in. end lengths (each end), the maximum number of bolts can be determined:
\[
\begin{aligned}
\text { Available length } & \geq 1.25 \text { in. }+1.25 \text { in. }+3 \text { in. } x \text { (number of bolts }-1) \\
& \text { number of bolts } \leq(17.76 \text { in }-2.5 \text { in. }-(-3 \text { in. })) / 3 \text { in. }=6.1 \text {, so } 6 \text { bolts. }
\end{aligned}
\]

It is helpful to have the All-bolted Double-Angle Connection Tables \(10-1\). They are available for \(3 / 4^{\prime \prime}, 7 / 8^{\prime \prime}\), and 1 " bolt diameters and list angle thicknesses of \(1 / 4\) ", \(5 / 16 ", 3 / 8 "\) ", and \(1 / 2^{\prime \prime}\). Increasing the angle thickness is likely to increase the angle strength, although the limit states include shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

For these diameters the available shear (double) from Table \(7-1\) for 6 bolts is ( 6 ) \(41.5 \mathrm{k} /\) bolt \(=270.6\) kips, (6) \(61.3 \mathrm{k} / \mathrm{bolt}=367.8 \mathrm{kips}\), and (6) \(80.1 \mathrm{k} / \mathrm{bolt}=\) 480.6 kips.

Tables 10-1 (not all provided here) list a bolt and angle available strength of 271 kips for the \(3 / 4\) " bolts, 296 kips for the 7/8" bolts, and 281 kips for the 1 " bolts. It appears that increasing the bolt diameter to 1 " will not gain additional load. Use 7/8" bolts.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \[
\begin{array}{|c|c}
\underset{W}{E} & F_{y}=50 \mathrm{ksi} \\
\underset{\sim}{\infty} & F_{u}=65 \mathrm{ksi}
\end{array}
\] & \multicolumn{8}{|r|}{\multirow[t]{2}{*}{Table 10-1 (continued) All-Bolted Double-Angle Connections}} & \multicolumn{3}{|r|}{\multirow[t]{2}{*}{7/8-in.
Bolts}} \\
\hline \multirow[t]{2}{*}{\[
\]} & & & & & & & & & & & \\
\hline & \multicolumn{11}{|c|}{Bolt and Angle Available Strength, kips} \\
\hline & \multirow{3}{*}{\[
\begin{aligned}
& \text { Bolt } \\
& \text { Group }
\end{aligned}
\]} & \multirow{3}{*}{Thread Cond.} & \multirow{3}{*}{\[
\begin{aligned}
& \text { Hole } \\
& \text { Type }
\end{aligned}
\]} & \multicolumn{8}{|c|}{Angle Thickness, in.} \\
\hline \multirow[t]{2}{*}{\[
\begin{gathered}
\hline \text { W40, } 36,33,30,27, \\
24,21
\end{gathered}
\]} & & & & \multicolumn{2}{|l|}{\(1 / 4\)} & \multicolumn{2}{|r|}{5/16} & \multicolumn{2}{|l|}{3/8} & \multicolumn{2}{|r|}{1/2} \\
\hline & & & & ASD & LRFD & ASD & LR & ASD & LRFD & ASD & LRFD \\
\hline \multirow{14}{*}{} & \multirow{7}{*}{\[
\left.\begin{gathered}
\text { Group } \\
A
\end{gathered} \right\rvert\,
\]} & N & STD & 98.6 & 148 & 123 & 185 & 148 & 222 & 195 & 292 \\
\hline & & x & STD & 98.6 & 148 & 123 & 185 & 148 & 222 & 197 & 296 \\
\hline & & & STD & 8.6 & 148 & \({ }^{106}\) & 159 & \({ }^{106}\) & 159 & \({ }^{106}\) & 159 \\
\hline & & Class A & \[
\begin{aligned}
& \text { ovs } \\
& \text { SSLT }
\end{aligned}
\] & \[
\begin{array}{|l|l|}
90.1 \\
97.3
\end{array}
\] & \[
\begin{aligned}
& 135 \\
& 146
\end{aligned}
\] & \[
\begin{array}{|c|}
\hline 90.1
\end{array}
\] & 135
159 & \[
\begin{gathered}
90.1 \\
106
\end{gathered}
\] & 135
159 & \[
\begin{gathered}
90.1 \\
106
\end{gathered}
\] & 135
159 \\
\hline & & & STD & 98.6 & 148 & 123 & 185 & 148 & 222 & 176 & 264 \\
\hline & & Class B & OVS & 93.5 & 140 & 117 & 175 & 140 & 210 & 150 & 225 \\
\hline & & & SSLT & 97.3 & 146 & 122 & 182 & 146 & 219 & 176 & 264 \\
\hline & \multirow{7}{*}{\[
\left|\begin{array}{c}
\text { Group } \\
B
\end{array}\right|
\]} & \begin{tabular}{l}
N \\
X \\
\hline
\end{tabular} & STD
STD & 98.6
98.6 & \[
\begin{aligned}
& 148 \\
& 148
\end{aligned}
\] & \[
\begin{aligned}
& 123 \\
& 123
\end{aligned}
\] & 185
185 & \[
\begin{aligned}
& 148 \\
& 148
\end{aligned}
\] & 222
222 & \[
\begin{aligned}
& 197 \\
& 197
\end{aligned}
\] & 296
296 \\
\hline & & & & 98.6 & 148 & 123 & 185 & 133 & 199 & 133 & 199 \\
\hline & & sc & ovs & 93.5 & 140 & 113 & 169 & 113 & 169 & 113 & 169 \\
\hline & & & SSLT & 97.3 & 146 & 122 & 182 & 133 & 199 & 133 & 199 \\
\hline & & & \({ }^{\text {STD }}\) & 98.6 & 148 & 123 & 185 & 148 & 222 & 197 & 296 \\
\hline & & \[
\underset{\text { Class BC }}{\text { SC }}
\] & OVS & 93.5 & 140 & 117 & 175 & 140 & 210 & 187 & 281 \\
\hline & & & SSLT & 97.3 & 146 & 122 & 182 & 146 & 219 & 195 & 292 \\
\hline
\end{tabular}
\(\phi R_{n}=368.7\) kips for double shear of \(7 / 8^{\prime \prime}\) bolts
\(\phi R_{n}=296\) kips for limit state in angles

We also need to evaluate bearing of bolts on the beam web, and column flange where there are bolt holes. Table 7-5 provides available bearing strength for the material type, bolt diameter, hole type, and spacing per inch of material thicknesses.
a) Bearing for beam web: There are 6 bolt holes through the beam web. This is typically the critical bearing limit value because there are two angle legs that resist bolt bearing and twice as many bolt holes to the column. The material is A 992 ( \(\mathrm{F}_{\mathrm{u}}=65 \mathrm{ksi}\) ), \(0.58^{\prime \prime}\) thick, with \(7 / 8^{\prime \prime}\) bolt diameters at 3 in . spacing.
\[
\phi R_{n}=6 \text { bolts } \cdot(102 \mathrm{k} / \mathrm{bolt} / \mathrm{inch}) \cdot(0.58 \mathrm{in})=355.0 \mathrm{kips}
\]
b) Bearing for column flange: There are 12 bolt holes through the column. The material is A992 ( \(\mathrm{F}_{\mathrm{u}}=65 \mathrm{ksi}\) ), 0.615 " thick, with 1 " bolt diameters.
\[
\phi R_{n}=12 \text { bolts } \cdot(102 \mathrm{k} / \mathrm{bolt} / \mathrm{inch}) \cdot(0.615 \mathrm{in})=752.8 \mathrm{kips}
\]

Although, the bearing in the beam web is the smallest at 355 kips, with the shear on the bolts even smaller at 324.6 kips, the maximum capacity for the simple-shear connector is 296 kips limited by the critical capacity of the angles.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{11}{|l|}{\begin{tabular}{l}
Based on Bolt Spacing \\
kips/in. thickness
\end{tabular}} \\
\hline \multirow[t]{4}{*}{Hole Type} & \multirow[t]{4}{*}{Bolt
Spacing, \(s\), in.} & \multirow[t]{4}{*}{\(F_{l}, \mathrm{ksi}\)} & \multicolumn{8}{|l|}{Nominal Bolt Diameter, \(d\), in.} \\
\hline & & & \multicolumn{2}{|l|}{5/8} & \multicolumn{2}{|l|}{\(3 / 4\)} & \multicolumn{2}{|l|}{7/8} & \multicolumn{2}{|l|}{1} \\
\hline & & & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi \boldsymbol{r}_{\boldsymbol{n}}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) & \(r_{n} / \Omega\) & \(\phi r_{n}\) \\
\hline & & & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD \\
\hline \multirow[t]{2}{*}{\[
\begin{aligned}
& \text { STD } \\
& \text { SSLT }
\end{aligned}
\]} & \(2^{2} / 3 d_{b}\) & \[
\begin{aligned}
& 58 \\
& 65 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
34.1 \\
38.2 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& \hline 51.1 \\
& 57.3 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 41.3 \\
& 46.3 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& \hline 62.0 \\
& 69.5 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& \hline 48.6 \\
& 54.4
\end{aligned}
\] & \[
\begin{array}{r}
72.9 \\
81.7
\end{array}
\] & \[
\begin{aligned}
& 55.8 \\
& 62.6
\end{aligned}
\] & \[
\begin{aligned}
& 83.7 \\
& 93.8
\end{aligned}
\] \\
\hline & 3 in . & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{array}{r}
43.5 \\
48.8 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
52.2 \\
58.5 \\
\hline
\end{array}
\] & \[
\begin{array}{r}
\hline 78.3 \\
87.8 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 60.9 \\
& 68.3 \\
& \hline
\end{aligned}
\] & \[
\begin{gathered}
91.4 \\
102 \\
\hline
\end{gathered}
\] & \[
67.4
\] & \[
\begin{aligned}
& 101 \\
& 113
\end{aligned}
\] \\
\hline \multirow[t]{2}{*}{SSLP} & \(2^{2} / 3 d_{b}\) & \[
\begin{aligned}
& 58 \\
& 65 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
27.6 \\
30.9
\end{array}
\] & \[
\begin{aligned}
& 41.3 \\
& 46.3
\end{aligned}
\] & \[
\begin{aligned}
& 34.8 \\
& 39.0
\end{aligned}
\] & \[
\begin{aligned}
& \hline 52.2 \\
& 58.5
\end{aligned}
\] & \[
\begin{aligned}
& 42.1 \\
& 47.1
\end{aligned}
\] & \[
\begin{aligned}
& 63.1 \\
& 70.7
\end{aligned}
\] & \[
\begin{aligned}
& 47.1 \\
& 52.8
\end{aligned}
\] & \[
\begin{aligned}
& 70.7 \\
& 79.2
\end{aligned}
\] \\
\hline & 3 in . & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{array}{r}
43.5 \\
48.8
\end{array}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{aligned}
& 52.2 \\
& 58.5
\end{aligned}
\] & \[
\begin{aligned}
& \hline 78.3 \\
& 87.8 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 60.9 \\
& 68.3
\end{aligned}
\] & \[
\begin{gathered}
91.4 \\
102 \\
\hline
\end{gathered}
\] & \[
\begin{aligned}
& 58.7 \\
& 65.8
\end{aligned}
\] & \[
\begin{aligned}
& 88.1 \\
& 98.7
\end{aligned}
\] \\
\hline \multirow[t]{2}{*}{OVS} & \(2{ }^{2} / 3 d_{b}\) & \[
\begin{aligned}
& 58 \\
& 65 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 29.7 \\
& 33.3
\end{aligned}
\] & \[
\begin{aligned}
& 44.6 \\
& 50.0
\end{aligned}
\] & \[
\begin{aligned}
& 37.0 \\
& 41.4
\end{aligned}
\] & \[
\begin{aligned}
& \hline 55.5 \\
& 62.2
\end{aligned}
\] & \[
\begin{aligned}
& 44.2 \\
& 49.6
\end{aligned}
\] & \[
\begin{aligned}
& 66.3 \\
& 74.3
\end{aligned}
\] & \[
\begin{aligned}
& 49.3 \\
& 55.3
\end{aligned}
\] & \[
\begin{aligned}
& 74.0 \\
& 82.9
\end{aligned}
\] \\
\hline & 3 in . & \[
\begin{aligned}
& 58 \\
& 65 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 43.5 \\
& 48.8
\end{aligned}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{array}{r}
52.2 \\
58.5 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& \hline 78.3 \\
& 87.8 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& \hline 60.9 \\
& 68.3 \\
& \hline
\end{aligned}
\] & \[
\begin{gathered}
91.4 \\
102
\end{gathered}
\] & \[
\begin{aligned}
& 60.9 \\
& 68.3 \\
& \hline
\end{aligned}
\] & \[
\begin{gathered}
91.4 \\
102
\end{gathered}
\] \\
\hline \multirow[t]{2}{*}{LSLP} & \(2{ }^{2} / 3 d_{b}\) & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{aligned}
& 3.62 \\
& 4.06
\end{aligned}
\] & \[
\begin{aligned}
& 5.44 \\
& 6.09
\end{aligned}
\] & \[
\begin{aligned}
& 4.35 \\
& 4.88
\end{aligned}
\] & \[
\begin{aligned}
& 6.53 \\
& 7.31
\end{aligned}
\] & \[
\begin{aligned}
& 5.08 \\
& 5.69
\end{aligned}
\] & \[
\begin{aligned}
& 7.61 \\
& 8.53
\end{aligned}
\] & \[
\begin{aligned}
& 5.80 \\
& 6.50
\end{aligned}
\] & \[
\begin{aligned}
& 8.70 \\
& 9.75
\end{aligned}
\] \\
\hline & 3 in . & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{aligned}
& 43.5 \\
& 48.8
\end{aligned}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{aligned}
& 39.2 \\
& 43.9
\end{aligned}
\] & \[
\begin{aligned}
& \hline 58.7 \\
& 65.8
\end{aligned}
\] & \[
\begin{aligned}
& 28.3 \\
& 31.7
\end{aligned}
\] & \[
\begin{aligned}
& \hline 42.4 \\
& 47.5
\end{aligned}
\] & \[
\begin{aligned}
& 17.4 \\
& 19.5
\end{aligned}
\] & \[
\begin{aligned}
& 26.1 \\
& 29.3
\end{aligned}
\] \\
\hline \multirow[t]{2}{*}{LSLT} & \(2{ }^{2} / 3 d_{b}\) & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{aligned}
& 28.4 \\
& 31.8
\end{aligned}
\] & \[
\begin{aligned}
& 42.6 \\
& 47.7
\end{aligned}
\] & \[
\begin{aligned}
& 34.4 \\
& 38.6
\end{aligned}
\] & \[
\begin{aligned}
& \hline 51.7 \\
& 57.9
\end{aligned}
\] & \[
\begin{aligned}
& 40.5 \\
& 45.4
\end{aligned}
\] & \[
\begin{aligned}
& 60.7 \\
& 68.0
\end{aligned}
\] & \[
\begin{aligned}
& 46.5 \\
& 52.1
\end{aligned}
\] & \[
\begin{aligned}
& 69.8 \\
& 78.2
\end{aligned}
\] \\
\hline & 3 in . & \[
\begin{aligned}
& 58 \\
& 65 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 36.3 \\
& 40.6
\end{aligned}
\] & \[
\begin{aligned}
& 54.4 \\
& 60.9
\end{aligned}
\] & \[
\begin{aligned}
& 43.5 \\
& 48.8
\end{aligned}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{aligned}
& 50.8 \\
& 56.9
\end{aligned}
\] & \[
\begin{aligned}
& 76.1 \\
& 85.3
\end{aligned}
\] & \[
\begin{aligned}
& 56.2 \\
& 63.0
\end{aligned}
\] & \[
\begin{aligned}
& 84.3 \\
& 94.5 \\
& \hline
\end{aligned}
\] \\
\hline \[
\begin{gathered}
\hline \text { STD, SSLT, } \\
\text { SSLP, OVS, } \\
\text { LSLP } \\
\hline
\end{gathered}
\] & \(s \geq s_{\text {full }}\) & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{aligned}
& 43.5 \\
& 48.8
\end{aligned}
\] & \[
\begin{aligned}
& 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{aligned}
& 52.2 \\
& 58.5
\end{aligned}
\] & \[
\begin{aligned}
& 78.3 \\
& 87.8
\end{aligned}
\] & \[
\begin{aligned}
& 60.9 \\
& 68.3
\end{aligned}
\] & \[
\begin{gathered}
91.4 \\
102
\end{gathered}
\] & \[
\begin{aligned}
& 69.6 \\
& 78.0
\end{aligned}
\] & \[
\begin{aligned}
& 104 \\
& 117
\end{aligned}
\] \\
\hline LSLT & \(s \geq s_{\text {full }}\) & \[
\begin{aligned}
& 58 \\
& 65
\end{aligned}
\] & \[
\begin{aligned}
& 36.3 \\
& 40.6
\end{aligned}
\] & \[
\begin{aligned}
& 54.4 \\
& 60.9
\end{aligned}
\] & \[
\begin{aligned}
& 43.5 \\
& 48.8
\end{aligned}
\] & \[
\begin{aligned}
& \hline 65.3 \\
& 73.1
\end{aligned}
\] & \[
\begin{aligned}
& 50.8 \\
& 56.9
\end{aligned}
\] & \[
\begin{array}{r}
76.1 \\
85.3 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 58.0 \\
& 65.0 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 87.0 \\
& 97.5 \\
& \hline
\end{aligned}
\] \\
\hline \multicolumn{2}{|l|}{\multirow[t]{4}{*}{Spacing for full bearing strength \(s_{\text {full }}{ }^{\text {a }}\), in.}} & \[
\begin{aligned}
& \text { STD, } \\
& \text { SSLT, } \\
& \text { LSLT }
\end{aligned}
\] & \multicolumn{2}{|l|}{15/16} & \multicolumn{2}{|l|}{25/16} & \multicolumn{2}{|l|}{211/16} & \multicolumn{2}{|l|}{\(3^{1 / 16}\)} \\
\hline & & OVS & \multicolumn{2}{|l|}{21/16} & \multicolumn{2}{|l|}{\(2^{7 / 16}\)} & \multicolumn{2}{|l|}{\(2^{13 / 16}\)} & \multicolumn{2}{|l|}{\(31 / 4\)} \\
\hline & & SSLP & \multicolumn{2}{|l|}{\(21 / 8\)} & \multicolumn{2}{|l|}{\(21 / 2\)} & \multicolumn{2}{|l|}{27/8} & \multicolumn{2}{|l|}{\(35 / 16\)} \\
\hline & & LSLP & \multicolumn{2}{|l|}{\(2^{13 / 16}\)} & \multicolumn{2}{|l|}{33/8} & \multicolumn{2}{|l|}{\(3^{15 / 16}\)} & \multicolumn{2}{|l|}{\(41 / 2\)} \\
\hline \multicolumn{3}{|l|}{Minimum Spacing \({ }^{2}=2^{2 / 3} d\), in.} & \multicolumn{2}{|l|}{111/16} & \multicolumn{2}{|l|}{2} & \multicolumn{2}{|l|}{\(25 / 16\)} & \multicolumn{2}{|l|}{\(2^{11 / 16}\)} \\
\hline \multicolumn{11}{|l|}{\[
\begin{aligned}
& \text { STD = standard hole } \\
& \text { SSLT = short-slotted hole oriented transverse to the line of force } \\
& \text { SSLP = short-slotted hole oriented parallel to the line of force } \\
& \text { OVS = oversized hole } \\
& \text { LSLP = long-slotted hole oriented parallel to the line of force } \\
& \text { LSLT = long-slotted hole oriented transverse to the line of force }
\end{aligned}
\]} \\
\hline ASD & LRFD & \multicolumn{9}{|l|}{\multirow[t]{2}{*}{\begin{tabular}{l}
Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC Specification Section J3.10. \\
\({ }^{\text {a }}\) Decimal value has been rounded to the nearest sixteenth of an inch.
\end{tabular}}} \\
\hline \(\Omega=2.00\) & \(\phi=0.75\) & & & & & & & & & \\
\hline
\end{tabular}

\section*{Wood Design}

\section*{Notation:}
\(a \quad=\) name for width dimension
\(A=\) name for area
\(A_{\text {req' }{ }^{\prime} \text {-adj }}=\) area required at allowable stress
when shear is adjusted to include
self weight
\(b \quad=\) width of a rectangle
\(=\) name for height dimension
\(c_{1}=\) coefficient for shear stress for a rectangular bar in torsion
\(C_{C}=\) curvature factor for laminated arches
\(C_{D}=\) load duration factor
\(C_{f u}=\) flat use factor for other than decks
\(C_{F}=\) size factor
\(C_{H}=\) shear stress factor
\(C_{i}=\) incising factor
\(C_{L}=\) beam stability factor
\(C_{M}=\) wet service factor
\(C_{p}=\) column stability factor for wood design
\(C_{r} \quad=\) repetitive member factor for wood design
\(C_{V} \quad=\) volume factor for glue laminated timber design
\(C_{t} \quad=\) temperature factor for wood design
\(d \quad=\) name for depth
\(d_{\text {min }}=\) dimension of timber critical for buckling
\(D L=\) shorthand for dead load
\(E=\) modulus of elasticity
\(f \quad=\) stress (strength is a stress limit)
\(f_{b} \quad=\) bending stress
\(f_{\text {from table }}=\) tabular strength (from table)
\(f_{p}=\) bearing stress
\(f_{r} \quad=\) radial stress for a glulam timber
\(f_{v} \quad=\) shear stress
\(f_{v-\max }=\) maximum shear stress
\(F_{b}=\) tabular bending strength
= allowable bending stress
\(F_{b}^{\prime}=\) allowable bending stress (adjusted)
\(F_{c}=\) tabular compression strength parallel to the grain
\(F_{c}^{\prime}=\) allowable compressive stress (adjusted)
\(F^{*}{ }_{c}=\) intermediate compressive stress for column design dependent on load duration
\(F_{C E}=\) theoretical allowed buckling stress
\(F_{c \perp}=\) tabular compression strength perpendicular to the grain
\(F_{p} \quad=\) tabular bearing strength parallel to the grain
= allowable bearing stress
\(F_{R} \quad=\) allowable radial stress
\(F_{t}=\) tabular tensile strength
\(F_{u}=\) ultimate strength
\(F_{v}=\) tabular bending strength
= allowable shear stress
\(h \quad=\) height of a rectangle
\(I \quad=\) moment of inertia with respect to neutral axis bending
\(I_{t r i a l}=\) moment of inertia of trial section
\(I_{\text {req'd }}=\) moment of inertia required at limiting deflection
\(I_{y} \quad=\) moment of inertia with respect to an \(y\)-axis
\(J \quad=\) polar moment of inertia
\(K_{c E}=\) material factor for wood column design
\(L_{e} \quad=\) effective length that can buckle for column design, as is \(\ell_{e}\)
\(L \quad=\) name for length or span length
\(L L \quad=\) shorthand for live load
\(L R F D=\) load and resistance factor design
\(M=\) internal bending moment
\(M_{\text {max }}=\) maximum internal bending moment
\(M_{\text {max-adj }}=\) maximum bending moment adjusted to include self weight
\(P \quad=\) name for axial force vector
\(R \quad=\) radius of curvature of a deformed beam
\(=\) radius of curvature of a laminated arch
\(=\) name for a reaction force
\(S \quad=\) section modulus
\(S_{\text {req'd }}=\) section modulus required at allowable stress
\begin{tabular}{|c|c|}
\hline \(S_{\text {req'd-adj }}=\) section modulus required at allowable stress when moment is adjusted to include self weight & \begin{tabular}{l}
\(w_{\text {self } w t}=\) name for distributed load from self weight of member \\
\(\Delta_{\text {allowable }}=\) allowable beam deflection
\end{tabular} \\
\hline \(T \quad=\) torque (axial moment) & \(\Delta_{\text {limit }}=\) allowable beam deflection limit \\
\hline \(V \quad=\) internal shear force & \(\Delta_{\max }=\) maximum beam deflection \\
\hline \[
\begin{aligned}
& V_{\max }=\text { maximum internal shear force } \\
& V_{\max -a d j}=\text { maximum internal shear force }
\end{aligned}
\] & \(\kappa \quad=\) slenderness ratio limit for long columns \\
\hline adjusted to include self weight & \(\gamma \quad=\) density or unit weight \\
\hline \(w \quad=\) name for distributed load & \(\rho \quad=\) radial distance \\
\hline
\end{tabular}

\section*{Wood or Timber Design}

Structural design standards for wood are established by the National Design Specification (NDS) published by the National Forest Products Association. There is a combined specification (from 2005) for Allowable Stress Design and limit state design (LRFD).

Tabulated wood strength values are used as the base allowable strength (ASD) and modified by appropriate adjustment factors:
\[
f=C_{D} C_{M} C_{F} \ldots \times f_{\text {fromtable }}
\]

Adjustment Factors
\(\mathrm{C}_{\mathrm{D}}\) load duration factor
\(\mathrm{C}_{\mathrm{M}}\) wet service factor
( 1.0 dry < \(16 \%\) moisture content)
\(\mathrm{C}_{\mathrm{t}}\) temperature factor (at high temperatures strength decreases)
\(\mathrm{C}_{\mathrm{L}}\) beam stability factor (for beams without full lateral support)
\(\mathrm{C}_{\mathrm{F}} \quad\) size factor for visually graded sawn lumber and round timber \(>12\) " depth
\[
C_{F}=(12 / d)^{1 / 9} \leq 1.0
\]
\(\mathrm{C}_{\mathrm{V}} \quad\) volume factor for glued laminated timber (similar to \(\mathrm{C}_{\mathrm{F}}\) )


DURATION OF LOAD (TIME)
\(\mathrm{C}_{\mathrm{fu}} \quad\) flat use factor (excluding decking)
\(\mathrm{C}_{\mathrm{r}} \quad\) repetitive member factor ( 1.15 for three or more parallel members of Dimension lumber spaced not more than 24 in . on center, connected together by a loaddistributing element such as roof, floor, or wall sheathing)
\(\mathrm{C}_{\mathrm{c}} \quad\) curvature factor for glued laminated timber (1.0 straight \& cambered)
\(\mathrm{t} / \mathrm{R} \leq 1 / 100\) for hardwoods \& southern pine or \(1 / 125\) other softwoods
\[
C_{c}=1-2000(t / R)^{2}
\]
\(\mathrm{C}_{\mathrm{i}} \quad\) incising factor ( 0.85 incised sawn lumber, 1 for sawn lumber not incised and glulam)
\(\mathrm{C}_{\mathrm{H}} \quad\) shear stress factor (amount of splitting)
\(\mathrm{C}_{\mathrm{P}} \quad\) column stability factor (1.0 for fully supported columns)

\section*{Design Values}
\(\mathrm{F}_{\mathrm{b}}\) : bending stress
\(\mathrm{F}_{\mathrm{t}}\) : tensile stress
\(\mathrm{F}_{\mathrm{v}}\) : horizontal shear stress
\(\mathrm{F}_{\mathrm{c} \perp}\) : compression stress (perpendicular to grain)
\(\mathrm{F}_{\mathrm{c}}\) : compression stress (parallel to grain)
E: modulus of elasticity
\(\mathrm{F}_{\mathrm{p}}\) : bearing stress (parallel to grain)
Wood is significantly weakest in shear and strongest along the direction of the grain (tension and compression).

\section*{Load Combinations and Deflection}

The critical load combination is determined by the largest of either:
\[
\frac{\text { dead load }}{0.9} \text { or } \frac{(\text { dead load }+ \text { any combination of live load })}{C_{D}}
\]

The deflection limits may be increased for less stiffness with total load: LL + 0.5(DL)

\section*{Criteria for Beam Design}

Allowable normal stress or normal stress from LRFD should not be exceeded:
Knowing M and \(\mathrm{F}_{\mathrm{b}}\), the minimum section modulus fitting the limit is: \(\quad S_{r e q^{\prime} d} \geq \frac{M}{F_{b}}\)

Besides strength, we also need to be concerned about serviceability. This involves things like limiting deflections \& cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

\section*{Determining Maximum Bending Moment}

Drawing V and M diagrams will show us the maximum values for design. Computer applications are very helpful.

\section*{Determining Maximum Bending Stress}

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a non-prismatic member, the stress varies with the cross section AND the moment.

\section*{Deflections}

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well.

Elastic curve equations can be superpositioned ONLY if the stresses are in the elastic range. The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.

\section*{Allowable Deflection Limits}

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.
\begin{tabular}{|c|l|l|}
\hline \multicolumn{1}{|c|}{ Use } & \multicolumn{1}{|c|}{ LL only } & \multicolumn{1}{|c|}{ DL+LL } \\
\hline Roof beams: & & \\
\hline Industrial & \(\mathrm{L} / 180\) & \(\mathrm{~L} / 120\) \\
\hline Commercial & & \\
\hline plaster ceiling & \(\mathrm{L} / 240\) & \(\mathrm{~L} / 180\) \\
\hline no plaster & \(\mathrm{L} / 360\) & \(\mathrm{~L} / 240\) \\
\hline Floor beams: & & \\
\hline Ordinary Usage & \(\mathrm{L} / 360\) & \(\mathrm{~L} / 240\) \\
\hline Roof or floor (damageable elements) & \(\mathrm{L} / 480\) \\
\hline
\end{tabular}

\section*{Lateral Buckling}

With compression stresses in the top of a beam, a sudden "popping" or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger \(\mathrm{I}_{\mathrm{y}}\).

\section*{Beam Loads \& Load Tracing}

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the tributary area that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element ad infinitum, to the ground.

\section*{Design Procedure}

The intent is to find the most light weight member satisfying the section modulus size.
1. Know \(\mathrm{F}_{\text {all }}\) for the material or \(\mathrm{F}_{\mathrm{U}}\) for LRFD.
2. Draw \(V\) \& \(M\), finding \(M_{\max }\).
3. Calculate \(\mathrm{S}_{\text {req'd. }}\). This step is equivalent to determining \(f_{b}=\frac{M_{\max }}{S} \leq F_{b}^{\prime}\)
4. For rectangular beams \(S=\frac{b h^{2}}{6}\)
- For timber: use the section charts to find S that will work and remember that the beam self weight will increase \(S_{\text {req'd }}\).
****Determine the "updated" \(V_{\max }\) and \(M_{\max }\) including the beam self weight, and verify that the updated \(S_{\text {req'd }}\) has been met. \({ }^{* * * * * * ~}\)
5. Consider lateral stability.
6. Evaluate horizontal shear stresses using \(\mathrm{V}_{\max }\) to determine if \(f_{v} \leq F_{v}^{\prime}\)

For rectangular beams \(\quad f_{v-\max }=\frac{3 \mathrm{~V}}{2 \mathrm{~A}}=1.5 \frac{\mathrm{~V}}{\mathrm{~A}}\)
7. Provide adequate bearing area at supports: \(\quad f_{p}=\frac{P}{A} \leq F_{p}^{\prime}\)
8. Evaluate shear due to torsion \(\quad f_{v}=\frac{T \rho}{J}\) or \(\frac{T}{c_{1} a b^{2}} \leq F_{v}^{\prime}\)
(circular section or rectangular)
9. Evaluate the deflection to determine if \(\Delta_{\text {maxLL }} \leq \Delta_{L L-a l l o w e d ~}\) and/or \(\Delta_{\text {max Total }} \leq \Delta_{\text {Totat-allowed }}\)
\(\begin{aligned} & * * * * \text { note: when } \Delta_{\text {calculuted }}>\Delta_{\text {limit }} I_{\text {required }} \text { can be found with: } \\ & \text { and } S_{\text {req'd }} \text { will be satisfied for similar self weight } * * * * *\end{aligned} I_{\text {req'd }} \geq \frac{\Delta_{\text {too big }}}{\Delta_{\text {limit }}} I_{\text {trial }}\)

\section*{FOR ANY EVALUATION:}

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

\section*{Column Design}

National Design Specification for Wood Construction (1992):
Any slenderness ratio, \(l_{2} \underline{d} \leq 50\) :
\[
f_{c}=\frac{P}{A} \leq F_{c}^{\prime} \quad F_{c}^{\prime}=F_{c}\left(C_{D}\right)\left(C_{M}\right)\left(C_{t}\right)\left(C_{F}\right)\left(C_{p}\right)
\]

For preliminary column design:
\[
F_{c}^{\prime}=F_{c}^{*} C_{p}=\left(F_{c} C_{D}\right) C_{p}
\]

\section*{Procedure}
1. Obtain \(\mathrm{F}_{\mathrm{c}}\)
find \(l_{\mathrm{e}} / \mathrm{d}\) or assume \(\left(l_{\mathrm{e}} / \mathrm{d} \leq 50\right)\)
compute \(F_{c E}=\frac{K_{c E} E}{\left(c_{c} / d\right)^{2}}\) with \(\mathrm{K}_{\mathrm{cE}}=0.3\) for sawn, \(=0.418\) for glu-lam
compute \(F_{c}^{*} \cong F_{c} C_{D}\) with \(\mathrm{C}_{\mathrm{D}}=1\), normal, \(\mathrm{C}_{\mathrm{D}}=1.25\) for 7 day roof...
find \(F_{c E} / F_{c}^{*}\) and get \(\mathrm{C}_{\mathrm{p}}\)
2. Select a section

If the load and area of the column are known, set the stress equal to the allowable stress, and solve for \(l_{\mathrm{e}}, l\), or \(\mathrm{d}_{\text {min }}\)

If the load and length of the column are known, set the stress equal to the allowable stress, and solve for A or \(\mathrm{d}_{\text {min }}\) and select a section that satisfies the values found.
3. Continue from 2 until \(F_{c}^{\prime}\) is satisfied: \(F_{c}^{\prime}=F_{c}^{*} C_{p}\)

\section*{Alternate Column Allowable Stress}

For intermediate length columns with \(11<\mathrm{L} / \mathrm{d}<\kappa\), where \(\kappa=0.67 \sqrt{E / F_{c}}\) :
\[
F_{c}^{\prime}=F_{c}\left\{1-(1 / 3)\left[\left(L_{e} / d\right) \kappa\right]^{4}\right\}
\]

For long columns with \(\mathrm{L} / \mathrm{d}>\kappa\), and an assumed safety factor of 2.73: the allowable stress is:
\[
F_{c}=\frac{0.3 E}{\left(L_{e} / d\right)^{2}}
\]

Table 9.3 Column stability factor \(\mathrm{C}_{\mathrm{p}}\).
Statics and Strength of Materials for Architecture and Building Construction, 2nd ed., Onouye \& Kane
Column Stability Factor \(C_{p}\)


Table developed and permission for use granted by Professor Ed Lebert, Dept. of Architecture, University of Washington.

SECTION PROPERTIES/STANDARD SIZES To the extent that other considerations will permit, the finished sizes of structural glued laminated timber as given in Table B constitute normal industry practice. Industry standards do, however, permit the use of any depth or width of glued laminated timber. Dimension lumber of \(11 / 2 \mathrm{in}\). net thickness is normally used for laminating straight members. The modified section modulus includes size factor \(\left(C_{f}\right)\), and no further reduction of bending stress for size is needed.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline  & \[
\stackrel{a}{\stackrel{a}{<}}
\] & \multirow[t]{3}{*}{} &  &  & \[
\begin{aligned}
& \stackrel{a}{\underset{~}{<}} \\
& \stackrel{\rightharpoonup}{\underset{\alpha}{\alpha}}
\end{aligned}
\] &  &  & \[
\begin{aligned}
& \text { த் } \\
& 0 \\
& \text { E } \\
& \frac{1}{3}
\end{aligned}
\] & \multicolumn{2}{|r|}{} &  \\
\hline \multicolumn{2}{|l|}{\multirow[b]{2}{*}{31/8" WIDTH}} & & & 24.0 & 162.0 & 600.0 & 7,776 & 54.0 & 472.5 & 3,598.0 & 114,818 \\
\hline & & & & 25.5 & 172.1 & 672.8 & 9,327 & 55.5 & 485.6 & 3,789.1 & 124,654 \\
\hline 6.0 & 18.8 & 18.8 & 56 & 27.0 & 182.3 & 749.5 & 11,072 & 57.0 & 498.8 & 3,984.9 & 135,037 \\
\hline 7.5 & 23.4 & 29.3 & 110 & 28.5 & 192.4 & 830.0 & 13,021 & 58.5 & 511.9 & 4,185.3 & 145,980 \\
\hline 9.0 & 28.1 & 42.2 & 190 & 30.0 & 202.5 & 914.5 & 15,188 & 60.0 & 525.0 & 4,390.3 & 157,500 \\
\hline 10.5 & 32.8 & 57.4 & 302 & 31.5 & 212.6 & 1,002.8 & 17,581 & \multicolumn{3}{|l|}{\multirow[t]{2}{*}{10 \(1 / 4 "\) WIDTH}} & \\
\hline 12.0 & 37.5 & 75.0 & 450 & 33.0 & 222.8 & 1,094.9 & 20,215 & & & & \\
\hline 13.5 & 42.2 & 93.7 & 641 & 34.5 & 232.9 & 1,190.8 & 23,098 & 15.0 & 161.3 & 393.3 & - 3,023 \\
\hline 15.0 & 46.9 & 114.3 & 879 & 36.0 & 243.0 & 1,290.5 & 26,244 & 16.5 & 177.4 & 470.8 & 4,024 \\
\hline 16.5 & 51.6 & 136.9 & 1,170 & 37.5 & 253.1 & 1,393.9 & 29,663 & 18.0 & 193.5 & 554.9 & 5,224 \\
\hline 18.0 & 56.3 & 161.3 & 1,519 & 39.0 & 263.3 & 1,501.1 & 33,367 & 19.5 & 209.6 & 645.5 & 6,642 \\
\hline 19.5 & 60.9 & 187.6 & 1,931 & 40.5 & 273.4 & 1,612.0 & 37,367. & 21.0 & 225.8 & 742.5 & 8,296 \\
\hline 21.0 & 65.6 & 215.8 & 2,412 & 42.0 & 283.5 & 1,726.6 & 41,674 & 29.5 & 241.9 & 845.8 & 10,204 \\
\hline 22.5 & 70.3 & 245.9 & 2,966 & 43.5 & 293.6 & 1,845.0 & 46,301 & 24.0 & 258.0 & 955.5 & 12,384 \\
\hline 24.0 & 75.0 & 277.8 & 3,600 & 45.0 & 303.8 & 1,967.0 & 51,258 & 25.5 & 274.1 & 1.071 .4 & 14,854 \\
\hline \multicolumn{2}{|l|}{\multirow[b]{2}{*}{5\%" WIDTH}} & & & 46.5 & 313.9 & 2,092.6 & 56,556 & 27.0 & 290.3 & 1,193.6 & 17,633 \\
\hline & & & & 48.0 & 324.0 & 2,222.0 & 62,208 & 28.5 & 306.4 & 1,321.9 & 20,738 \\
\hline 7.5 & 38.4 & 48.0 & 180 & \multicolumn{2}{|l|}{\multirow[b]{2}{*}{83/4" WIDTH}} & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{}} & 30.0 & 322.5 & 1,456.4 & 24,188 \\
\hline 9.0 & 45.1 & 69.2 & 311 & & & & & 31.5 & 338.6 & 1,597.0 & 28,000 \\
\hline 10.5 & 53.8 & 94.2 & 494 & 12.0 & 105.0 & 210.0 & 1,260 & 33.0 & 354.8 & 1,743.7. & 32,194 \\
\hline 12.0 & 61.5 & 123.0 & 738 & 13.5 & 118.1 & 262.3 & 1,794 & 34.5 & 370.9 & 1,886.4 & 36,786 \\
\hline 13.5 & 69.2 & 153.6 & 1,051 & 15.0 & 131.3 & 320.1 & 2,461 & 36.0 & 387.0 & 2,055.2 & 41,796 \\
\hline 15.0 & 76.9 & 187.5 & 1,441 & 16.5 & 144.4 & 383.2 & 3,276 & 37.5 & 403.1 & 2,219.9 & 47,241 \\
\hline 16.5 & 84.6 & 224.5 & 1,919 & 18.0 & 157.5 & 451.7 & 4,252 & 39.0 & 419.3 & 2,390.6 & 53,140 \\
\hline 18.0 & 92.3 & 264.6 & 2,491 & 19.5 & 170.6 & 525.4 & 5,407 & 40.5 & 435.4 & 2,567.3 & 59,510 \\
\hline 19.5 & 99.9 & 307.7 & 3,167 & 21.0 & 183.8 & 604.4 & 6,753 & 42.0 & 451.5 & 2,749.8 & 56,370 \\
\hline 21.0 & 107.6 & 3540 & 3,955 & 22.5 & 196.9 & 688.5 & 8,306 & 43.5 & 467.6 & 2,938.3 & 73,739 \\
\hline 22.5 & 115.3 & 403.2 & 4,865 & 24.0 & 210.0 & 777.7 & 10,080 & 45.0 & 483.8 & 3,132.6 & 81,033 \\
\hline 24.0 & 123.0 & 455.5 & 5,904 & 25.5 & 223.1 & 872.1 & 12,091 & 46.5 & 499.9 & 3,332.7 & 90,071 \\
\hline 25.5 & 130.7 & 510.8 & 7,082 & 27.0 & 236.3 & 971.5 & 14,352 & 48.0 & 516.0 & 3,538.7 & 99,072 \\
\hline 27.0 & 138.4 & 569.0 & 8,406 & 28.5 & 249.4 & 1,076.0 & 16,880 & 49.5 & 532.1 & 3,750.5 & 108,653 \\
\hline 28.5 & 146.1 & 630.2 & 9,887 & 30.0 & 262.5 & 1,185.5 & 19,688 & 51.0 & 548.3 & 3,968.0 & 118,833 \\
\hline 30.0 & 153.8 & 694.3 & 11,531 & 31.5 & 275.6 & 1,299.9 & 22,791 & 52.5 & 564.4 & 4,191.4 & 129,630 \\
\hline 31.5 & 161.4 & 761.4 & 13,349 & 33.0 & 288.8 & 1,419.3 & 26,204 & 54.0 & 580.5 & 4,420.4 & 141,062 \\
\hline 33.0 & 139.1 & 331.3 & 15,348 & 34.5 & 301.9 & 1,543.6 & 29,942 & 55.5 & 596.6 & 4,655.2 & 153,146 \\
\hline 34.5 & 175.8 & 904.1 & 17,538 & 36.0 & 315.0 & 1,672.8 & 34,020 & 57.0 & 612.8 & 4,895.7 & 165,902 \\
\hline 36.0 & 184.5 & 979.8 & 19,926 & 37.5 & 328.1 & 1,806.9 & 38,452 & 58.5 & 628.9 & 5,141.9 & [179,347 \\
\hline \multicolumn{2}{|l|}{\multirow[b]{2}{*}{S \(1 /{ }^{\prime \prime}\) WIDTH}} & & & 39.0 & 341.3 & 1,945.9 & 43,253 & 60.0 & 645.0 & 5,398.8 & 193,500 \\
\hline & & & & 40.5 & 354.4 & 2,089.6 & 48,439 & 61.5 & 661.1 & 5,651.4 & 208,379 \\
\hline 120 & 31.0 & 162.0 & 972 & 42.0 & 367.5 & 2,238.2 & 54,022 & 63.0 & 677.3 & 5,914.5 & 224,000 \\
\hline 13.5 & 91.1 & 202.4 & 1,384 & 43.5 & 380.6 & 2,391.6 & 60,020 & 64.5 & 693.4 & 6,183.3 & 240,384 \\
\hline 150 & 101.3 & 246.9 & 1,898 & 45.0 & 393.8 & 2,549.8 & 66.445 & 66.0 & 709.5 & 6,457.8 & 257,548 \\
\hline 165 & 111.4 & 295.6 & 2,527 & 46.5 & 406.9 & 2,712.7 & 73,314 & 67.5 & 725.6 & 6,737.8 & 275,511 \\
\hline 180 & 121.5 & 348.4 & 3,280 & 48.0 & 420.0 & 2,880.3 & 80,640 & 69.0 & 741.8 & 7.023 .4 & 294,289 \\
\hline 99.5 & :31.6 & 4053 & 4,171 & 49.5 & 433.1 & 3,052.7 & 88,439 & 70.5 & 757.9 & 7,314.6 & 313,902 \\
\hline 21.0 & 141.8 & 466.2 & 5,209 & 51.0 & 446.3 & 3,229.8 & 96,725 & 72.0 & 774.0 & 7,611.3 & 334,368 \\
\hline 22.5 & 151.9 & 531.1 & .6,407 & 52.5 & 459.4 & 3,411.6 & 105,513 & 73.5 & 790.1 & 7.913.6 & 355.704 \\
\hline
\end{tabular}

\section*{Glue Laminated Timber}

These members come in nominal widths of \(3,4,6,8,10,12,14\) and 16 inches. The depth can exceed 12 inches, so the size factor, \(\mathrm{C}_{\mathrm{F}}\) must be used. The formula is based on a uniformly loaded beam, simply supported with an \(1 / d\) ratio of 21 . With a single midspan load, multiply \(\mathrm{C}_{\mathrm{F}}\) by 1.078 . With two loads at third points, multiply \(\mathrm{C}_{\mathrm{F}}\) by 0.968 . (Note: the table on page 4 provides section modulus that include \(\mathrm{C}_{\mathrm{F}}\) ).
\[
C_{F}=(12 / d)^{1 / 9} \leq 1.0
\]

If a glulam is subject to lateral buckling, the slenderness factor is used, and the size factor is not.
Bending of a curved glulam causes radial stresses (like membrane pressures) in tension and compression which can be evaluated for an arc with a radius of R at the neutral axis from:
\[
\begin{aligned}
& f_{r}=3 M / 2 R b d \text { for constant rectangular cross section } \\
& f_{r} \leq F_{R} \text { where } F_{R}=\left\{\begin{array}{c}
F_{C \perp} \\
1 / 3 F_{V}
\end{array}\right.
\end{aligned}
\]

\section*{ASD Beam Design Flow Chart}


\section*{Laminated Timber}

\section*{Design Guide}


\section*{American Institute of Timber Construction (AITC)}

Representing the glued laminated timber industry since 1952, AITC provides technical support to manufacturers and the design community, and third party quality control manufacturing plants. AITC members design, manufacture, fabricate,or erect wood structural systems.

\section*{Glued Laminated Timber}

Glued laminated timber, often referred to as glulam, permits new uses, enhances the natural beauty and extends the enduring qualities of wood. The laminating process makes possible the production of structural timber in a wide variety of sizes and shapes and allows design creativity. The advantages of using glued laminated timber are as varied as your imagination and your specific applications.

\section*{Product Standards}

AITC recommends and establishes standards and specifications that guide building officials and industry professionals in the design or use of laminated timber.
AITC is the sponsor of the
American National Standard, ANSI/AITC AI90. This includes plant qualifications, a quality control system, inspection, testing, certification and identification.
AITC's certification and quality assurance programs have proven effective for over 40 years.

Boathouse, Boston University, Cambridge, MA.; Architect-Architectural Resources.; Structural Engineer-John Born Associates; Contractor-Walsh Brothers Construction.


\section*{Product Identification}

Laminated structural members manufactured to the Industry Standard are identified with the AITC Quality Inspection Mark. To assure compliance to the Standard, AITC maintains a staff of highly experienced inspectors.

\section*{Species, Sizes and Grades}

Species: Laminated timber is manufactured in many species, including softwoods and hardwoods. The most popular softwood species are Douglas Fir/ Larch, Southern Pine and Alaskan Yellow Cedar. Hem-Fir, Spruce-Pine-Fir (SPF) and Ponderosa Pine are also frequently used. AITC Standard 117 Design Specifications for Structural Glued Laminated Timber of Softwood Species, provides detailed design information.
Sizes: Standard widths for Douglas-Fir are \(3^{1} / 8^{\prime \prime}, 5^{1} / 8^{\prime \prime}, 6^{3} / 4^{\prime \prime}\), \(8^{3} / 4^{\prime \prime}, 10^{3} / 4^{\prime \prime}, 12^{1 / 4 "}\) and \(14^{1} / 4^{\prime \prime}\). Standard widths for Southern Pine are \(3^{\prime \prime}, 3^{1 /} / 8^{\prime \prime}, 5^{\prime \prime}, 5^{1 / 8^{\prime \prime}}, 6^{3} / 4^{\prime \prime}\), \(8^{\prime} / 2^{\prime \prime}, 10^{1 / 2 ",} 12^{\prime \prime}\) and \(14^{\prime \prime}\). Other widths are available upon request.

Depths and lengths of glulam members are limited only by the capability of the individual manufacturer.
Grades: There are four appearance grades -- Industrial, Framing (formerly Industrial S), Architectural and Premium. Industrial grade is suitable where appearance is not a primary concern, or the members will not be exposed to view. Framing grade matches the width of conventional framing for use as window and door headers where appearance is not a concern. Architectural grade is suitable for construction where appearance is an important requirement. Premium is the highest grade and is specified where appearance is of utmost importance. Appearance grades do not modify design values, grades of lumber used or other provisions governing the manufacture or use of glued laminated timber.
Textured surfaces, such as rough sawn, are also available from most manufacturers. See AITC Standard IIO for detailed specifications.

\section*{Strong, Durable and Beautiful}

Because glued laminated timber is fabricated from dry lumber, the resulting higher dimensional stablility reduces checking, twisting, warping and shrinkage. The result is a stable and beautiful installation.

\section*{Easy To Install}

Laminated timbers can be prefabricated at a plant so they arrive at the job site ready for immediate installation. Most timbers are installed with mobile construction equipment and connections are made by using conventional power and hand tools.

\section*{Cost Effective}

Laminated timber construction is economical. It does not require the added expense of false ceilings to cover structural framework. Accurate manufacturing reduces the need for onsite fabrication, minimizing waste and installation costs. Equally important, Engineered Timber is more adaptable to construction design changes than are other framing systems.

\section*{Availablity}

AITC manufacturing plants are located throughout the country. Many straight beam sizes are available through local building material suppliers. Custom glued laminated members can be obtained from a laminator or a local representative.


Custom fabrication for assembly with power tools.


Glued laminated timber truss.

\section*{Energy Efficient}

Wood's natural insulating properties help reduce building heating and cooling expense. Wood has less thermal expansion than steel or concrete, and its insulating value is many times higher. It also has excellent electrical insulating qualities.

\section*{High Resilience}

Wood absorbs shocks and provides high resistance to hurricane force winds and earthquake forces.


A naturally renewable resource.

\section*{A Renewable Resource}

Only one primary building material comes from a renewable resource; cleans the air and water, providing habitat, scenic beauty and recreation as it grows; utilizes nearly \(100 \%\) of its resource for products; is the lowest of all in energy requirements for its manufacturing; creates fewer air and water emissions than any of its alternatives; and is totally reusable, recyclable and 100\% biodegradable: wood. And it has been increasing in U.S. net reserves since 1952, with growth exceeding harvest in the U.S. by more than \(30 \%\).

\section*{Availability}

Straight beams in most tabulated sizes are mass produced and readily available at many building products and lumber distribution centers across the country.

Typical structural uses:
- Complete structural systems
- Ridge beams
- Garage door headers
- Door and window headers
- Long span girders
- Stair treads and stringers

Laminated timbers permit large rooms with minimal columns while providing the warmth of wood for living or working environments.
Renovating with laminated timber is easy as beams can be modified at the jobsite to fit existing conditions. Laminated timber can be textured, stained, or painted to match or meet traditional or historic appearance requirements.


Historic Preservation Award, REI, Denver, CO; Architect-Mithun Partners; Structural Engineer-Skilling Ward Magnusson Barkshire, Inc.


Garage door header.

Residence, Eagle, ID; Architect-Olsen and Associates; Contractor-Gordon Jensen Construction


\section*{}


Residence, Aurora, OR; Architect-Jack Smith F.A.I.A.; Engineer-Bouiss and Associates;
Contractor-Busic Construction Company.


Sports Complex, Coronado, CA; ArchitectSHWC Architects; Engineer--Ramierez and Associates Engineers; Contractor--Taylor Ball Contractors, Inc.


Inventory readily available from local distributors for prompt delivery to job site.


Field cutting a stock beam.


Installing a residential ridge beam.


Saw textured beams add warmth and beauty, and are available from most manufacturers.


Sun Valley, ID Residence; Architect-Jack Smith, F.A.I.A.; Structural Engineer-Purdy and Associates; Contractor-Premier Construction Company.


Albertson Shopping Center, Jackson, WY; Architect--Jeffrey A. Shneider; Structural EngineerRex Harrison Engineering; Contractor-Bateman Hall


Ceiling beams compliment rustic design of this McCall, ID home.


Airport Terminal, Jackson, WY


Office Building Remodel, Jackson, WY.

\section*{Custom Laminated Timber}

Laminated timber permits long, clear spans, majestic soaring arches -- tudor, radial, gothic, or parabolic, and many special shapes.
Cut to size and framed for connections at the plant to exact specifications and shapes, laminated timber requires less on-site fabrication which minimizes waste and installation costs.


Great Buddha Hall, Carmel, NY; ArchitectEdward A. Valeri; Structural Engineer-Enterprise Engineering Consultants, Ltd.


Ross High School, East Hampton, Long Island, NY; Architect-Richard Cook \& Associates; Contractor-Telemar Construction.


Grant Creek Shopping Mall, Missoula, MT; Architect--Fehlman-Labarre Architects; Contractor--Quality Construction


Newport Beach, CA, 30,000 sq. ft. residence; Architect-Brian Jeannete and Associates; Structural Engineer--Omnispan Corporation; Contractor--Buwalda Construction


Animal Science Center, I72 ft. span arches, Univ. of Arkansas, Little Rock,AR; Architect-AMR Architect; Structural Engineer-Engineering Consultants, Inc.; Contractor-Harrison Davis Construction


St. Anastasia Catholic Church, St.Augustine, FL; Architect--Richard L. San Giovanni; Engineer--C. Louis Structural Engineers; Contractor--Hall Construction Company, Inc.


Cabella Outdoor Recreation Store, Owatonna, MN; Architect-Nielsen and Mayne Architects; Engineer-Kirkham Michael Engineers; Contractor-Kraus-Anderson.

\section*{Custom Shapes}

Laminated timber arches or pitched and curved beams can be made in almost any shape. A Tudor type three-hinge arch is favored for many ecclesiastical designs. Radial arches are well suited to large unobstructed clear spans, as are pitched and tapered curved beams.


North Syracuse Baptist Church, Syracuse, NY; Architect-RSA Architects; ContractorButterfield Construction

\section*{Long Span Structures}

Laminated timber beams, arches and other shapes are widely used to provide efficient enclosure of large areas such as gymnasiums, auditoriums and indoor pools. While indoor pools generate high humidity, pressure treatment is not required when the building is adequately ventilated to control humidity, or where a highly durable species, such as Alaska Yellow cedar is used.


Ross High School, East Hampton, Long Island, NY; Architect--Richard Cook \& Associates; Contractor-Telemar Construction.


Wood ceilings and beams were selected for acoustical control. Performing Arts Center, North Texas University, Denton, TX; Architect-KVG Gideon Toll Architects; Engineer-Freese and Nichols, Inc.; Contractor-Huber, Hunt and Nichols, Inc.


Exploration Place, Wichita, KS; Achitect-Moshe Safdie \& Associates with Schaeffer, Johnson, Cox, Frey; Structural Engineer-Dudley Williams \& Associates; Contractor-Dondlinger \& Sons Construction


YMCA Pool, Brewton, AL; Architect--Dampier and Associates; Structural Engineer--Joseph and Spain; Contractor--Stuart Construction


Michigan arch bridge; Engineer--Northwest Design Group, Inc.


Bridge in Nature Park, Portland, OR, pressure treated with environmentally friendly copper naphthenate. Structural Engineer-Ceccacci Associates.


Golf course bridge.

\section*{Water Oriented Installations}

Experience shows that wood is one of the materials most suitable for construction in and around the water.

Wood is resilient enough to resist battering by the ocean and docking ships, and it is naturally resistant to the destructiveness of salt water. It won't rust or spall, and is not affected by corrosion.
Where wood is fully exposed to weather, or where weather protection cannot ensure a moisture content of less than \(20 \%\), pressure treatment is required. Buildings housing wet processes, or where wood is in direct contact with the ground or water also require pressure treatment.
See AITC Standard 109 for specific recommendations.


New covered bridge with laminated trusses; Engineer-USDA Forest Service.

\section*{Load Tables}

Span and load tables are available on AITC's web site or may be obtained by calling AITC. See back page.

\section*{Design Properties}

Bending members are typically specified on the basis of the maximum allowable bending stress of the member.
A 24 F designation indicates a member with an allowable bending stress of 2400 psi.
See AITC Standard 117 for allowable design stresses.

\section*{Cantilevered Beams}

Cantilever beam systems are highly efficient for large flat roofs as the continuity across supports permits smaller beams than required for simple spans.
For most residential applications where cantilever lengths are relatively short, a stock unbalanced glulam can be used. Cantilever roof overhangs up to 20 percent of the main span can be accommodated using an unbalanced beam without special layups. For longer length cantilevers, balanced beams should be specified.

\section*{Balanced and Unbalanced Sections}

Glued laminated timbers are manufactured with both balanced and unbalanced layups. Balanced layups are made with identical lumber grades in the outer

Equivalent Glulam Sections for Dimension Lumber/Timber Beams
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Sawn \({ }^{4}\) & & Roof & eams \({ }^{1,2}\) & & & Floor B & eams \({ }^{1,3}\) & \\
\hline Sections & Select S & uctural & No. & & Select St & uctural & No. & \\
\hline Nominal & Douglas & Southern & Douglas & South & Douglas & Souther & Douglas & Southern \\
\hline Size & Fir/Larch & Pine & Fir/Larch & Pine & Fir/Larch & Pine & Fir/Larch & Pine \\
\hline \(3 \times 8\) & \(3^{1 / 8 \times 6}\) & \(3 \times 6^{7 / 8}\) & 31/8×6 & \(3 \times 51 / 2\) & \(3^{1 / 8 \times 71 / 2}\) & \(3 \times 6^{7 / 8}\) & \(31 / 8 \times 7^{1 / 2}\) & \(3 \times 6{ }^{7 / 8}\) \\
\hline \(3 \times 10\) & 31/8×71/2 & \(3 \times 81 / 4\) & 31/8×6 & \(3 \times 6{ }^{7 / 8}\) & \(31 / 8 \times 9\) & \(3 \times 95 / 8\) & \(31 / 8 \times 9\) & \(3 \times 95 / 8\) \\
\hline \(3 \times 12\) & \(31 / 8 \times 9\) & \(3 \times 95 / 8\) & \(3^{1 / 8 \times 71 / 2}\) & \(3 \times 81 / 4\) & \(31 / 8 \times 12\) & \(3 \times 11\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 11\) \\
\hline \(3 \times 14\) & 31/8x9 & \(3 \times 11\) & \(31 / 8 \times 71 / 2\) & \(3 \times 95 / 8\) & \(31 / 8 \times 13^{1 / 2}\) & \(3 \times 13^{3 / 4}\) & \(31 / 8 \times 13^{1 / 2}\) & \(3 \times 12^{3 / 8}\) \\
\hline \(4 \times 6\) & \(3^{1 / 8 \times 6}\) & \(3 \times 6^{7 / 8}\) & \(3^{1 / 8 \times 6}\) & \(3 \times 51 / 2\) & \(31 / 8 \times 6\) & \(3 \times 6^{7 / 8}\) & \(3^{1 / 8 \times 6}\) & \(3 \times 6^{7 / 8}\) \\
\hline \(4 \times 8\) & \(31 / 8 \times 71 / 2\) & \(3 \times 8{ }^{1 / 4}\) & \(3^{1 / 8 \times 6}\) & \(3 \times 6^{7 / 8}\) & \(3^{1 / 8 \times 9}\) & \(3 \times 8{ }^{1 / 4}\) & \(3^{1 / 8 \times 7} 1 / 2\) & \(3 \times 81 / 4\) \\
\hline \(4 \times 10\) & \(31 / 8 \times 9\) & \(3 \times 11\) & \(3^{1 / 8 \times 7}{ }^{1 / 2}\) & \(3 \times 81 / 4\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 11\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 9\) 9/8 \\
\hline \(4 \times 12\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 12^{3 / 8}\) & \(31 / 8 \times 9\) & \(3 \times 95 / 8\) & \(31 / 8 \times 12\) & \(3 \times 12^{3 / 8}\) & \(31 / 8 \times 12\) & \(3 \times 12^{3 / 8}\) \\
\hline \(4 \times 14\) & \(31 / 8 \times 12\) & \(3 \times 13^{3 / 4}\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 11\) & \(31 / 8 \times 15\) & \(3 \times 151 / 8\) & \(31 / 8 \times 15\) & \(3 \times 13^{3 / 4}\) \\
\hline \(4 \times 16\) & \(31 / 8 \times 13^{1 / 2}\) & \(3 \times 15^{1 / 8}\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 12^{3 / 8}\) & \(3^{1 / 8 \times 161 / 2}\) & \(3 \times 16^{1 / 2}\) & \(31 / 8 \times 16^{1 / 2}\) & \(3 \times 16^{1 / 2}\) \\
\hline \(6 \times 8\) & \(51 / 8 \times 71 / 2\) & \(5 \times 6{ }^{7 / 8}\) & \(51 / 8 \times 71 / 2\) & \(5 \times 6^{7 / 8}\) & \(51 / 8 \times 71 / 2\) & \(5 \times 81 / 4\) & \(51 / 8 \times 71 / 2\) & \(5 \times 81 / 4\) \\
\hline \(6 \times 10\) & \(51 / 8 \times 9\) & \(5 \times 88^{1 / 4}\) & \(51 / 8 \times 71 / 2\) & \(5 \times 88^{1 / 4}\) & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 9\) \%/8 & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 9^{5 / 8}\) \\
\hline \(6 \times 12\) & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 95 / 8\) & \(51 / 8 \times 9\) & \(5 \times 95 / 8\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) \\
\hline \(6 \times 14\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 11\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) \\
\hline \(6 \times 16\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) & \(51 / 8 \times 16^{1 / 2}\) & \(5 \times 151 / 8\) & \(51 / 8 \times 16^{1 / 2}\) & \(5 \times 151 / 8\) \\
\hline \(6 \times 18\) & \(51 / 8 \times 15\) & \(5 \times 151 / 8\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) & \(51 / 8 \times 18\) & \(5 \times 17^{7 / 8}\) & \(51 / 8 \times 18\) & \(5 \times 17^{7 / 8}\) \\
\hline \(6 \times 20\) & \(51 / 8 \times 18\) & \(5 \times 161 / 2\) & \(51 / 8 \times 16^{1 / 2}\) & \(5 \times 15^{1 / 8}\) & \(51 / 8 \times 19^{1 / 2}\) & \(5 \times 191 / 4\) & \(51 / 8 \times 19^{1 / 2}\) & \(5 \times 191 / 4\) \\
\hline \(8 \times 10\) & \(6^{3 / 4 \times 9}\) & \(6^{3 / 4 \times 81 / 4}\) & \(6^{3 / 4 \times 9}\) & \(6^{3} / 4 \times 8^{1 / 4}\) & \(6^{3 / 4 \times 10^{1 / 2}}\) & \(6^{3} / 4 \times 95 / 8\) & \(6^{3 / 4 \times 10^{1 / 2}}\) & \(6^{3} / 4 \times 95 / 8\) \\
\hline \(8 \times 12\) & \(6^{3 / 4 \times 10^{1 / 2}}\) & \(6^{3 / 4} \times 9^{5 / 8}\) & \(6^{3 / 4 \times 10^{1 / 2}}\) & \(6^{3} / 4 \times 95 / 8\) & \(6^{3} / 4 \times 12\) & \(6^{3 / 4 \times 12^{3 / 8}}\) & \(6^{3} / 4 \times 12\) & \(6^{3 / 4 \times 12^{3 / 8}}\) \\
\hline \(8 \times 14\) & \(6^{3} / 4 \times 12\) & \(6^{3 / 4 \times 12^{3 / 8}}\) & \(6^{3} / 4 \times 12\) & \(6^{3} / 4 \times 11\) & \(6^{3 / 4 \times 13^{1 / 2}}\) & \(6^{3 / 4 \times 13^{3 / 4}}\) & \(6^{3 / 4 \times 13^{1 / 2}}\) & \(6^{3 / 4 \times 13^{3 / 4}}\) \\
\hline \(8 \times 16\) & \(6^{3 / 4 \times 13^{1 / 2}}\) & \(6^{3 / 4 \times 13^{3 / 4}}\) & \(6^{3 / 4 \times 13^{1 / 2}}\) & \(6^{3 / 4 \times 12^{3 / 8}}\) & \(6^{3 / 4 \times 16^{1 / 2}}\) & \(6^{3 / 4 \times 15} \times 1 / 8\) & \(6^{3 / 4 \times 16^{1 / 2}}\) & \(6^{3 / 4 \times 15} \times 1 / 8\) \\
\hline \(8 \times 18\) & \(6^{3} / 4 \times 16^{1 / 2}\) & \(6^{3 / 4 \times 15^{1 / 8}}\) & \(6^{3} / 4 \times 15\) & \(6^{3 / 4 \times 13^{3 / 4}}\) & \(6^{3} / 4 \times 18\) & \(6^{3 / 4 \times 17} \times 1 / 8\) & \(6^{3} / 4 \times 18\) & \(6^{3 / 4 \times 17^{7 / 8}}\) \\
\hline \(8 \times 20\) & \(6^{3 / 4 \times 18}\) & \(6^{3 / 4 \times 16^{1 / 2}}\) & \(6^{3 / 4 \times 16^{1 / 2}}\) & \(6^{3 / 4 \times 161 / 2}\) & \(6^{3 / 4 \times 191 / 2}\) & \(6^{3 / 4 \times 191 / 4}\) & \(6^{3 / 4 \times 191 / 2}\) & \(63 / 4 \times 191 / 4\) \\
\hline \(8 \times 22\) & \(6^{3} / 4 \times 191 / 2\) & \(6^{3 / 4 \times 17^{7 / 8}}\) & \(6^{3} / 4 \times 18\) & \(6^{3 / 4 \times 17^{7 / 8}}\) & \(6^{3} / 4 \times 22^{1 / 2}\) & \(6^{3} / 4 \times 22\) & \(6^{3} / 4 \times 22^{1 / 2}\) & \(6^{3} / 4 \times 22\) \\
\hline
\end{tabular}
laminations, placed symmetrically about the neutral axis. Consequently, balanced layups have equal bending strength for both positive and negative bending. Balanced layups are recommended for beams that are continuous across supports and for cantilevered beams.

Unbalanced layups utilize higher grade lumber in the bottom (tension) side of the beam and are stamped with the word "TOP" on the upper surface. This unsymmetrical configuration results in higher strength for positive bending (tension on bottom) than for negative bending. Unbalanced layups are primarily intended for simple span beams, but can also be used for short cantilevers.


AITC mark of quality. The word "TOP" identifies beams with unbalanced sections.


AITC is approved for inspection under this Japanese Agricultural Standard. Laminated timber exported to Japan is identified with this label.

Equivalent Glulam Sections for Steel Beams
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Steel \({ }^{5}\) & \multicolumn{4}{|c|}{Roof Beams \({ }^{1,2}\)} & \multicolumn{4}{|c|}{Floor Beams \({ }^{1,3}\)} \\
\hline Section & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} \\
\hline W 6x9 & \(31 / 8 \times 10^{1 / 2}\) & r \(51 / 8 \times 9\) & \(3 \times 11\) & \(5 \times 81 / 4\) & \(31 / 8 \times 10^{1 / 2}\) & 51/8x9 & \(3 \times 11\) & \(5 \times 95 / 8\) \\
\hline W \(8 \times 10\) & \(31 / 8 \times 12\) & \(51 / 8 \times 9\) & \(3 \times 12^{3 / 8}\) & \(5 \times 9^{5 / 8}\) & \(3^{1 / 8 \times 13^{1 / 2}}\) & \(51 / 8 \times 12\) & \(3 \times 13^{3 / 4}\) & \(5 \times 11\) \\
\hline W \(12 \times 14\) & \(3^{1 / 8 \times 161 / 2}\) & \(5^{1 / 8 \times 131 / 2}\) & \(3 \times 16^{1 / 2}\) & \(5 \times 13^{3 / 4}\) & \(31 / 8 \times 18\) & \(51 / 8 \times 15\) & \(3 \times 17^{7 / 8}\) & \(5 \times 15^{1 / 8}\) \\
\hline W \(12 \times 16\) & \(31 / 8 \times 18\) & \(51 / 8 \times 131 / 2\) & \(3 \times 17^{7 / 8}\) & \(5 \times 13^{3 / 4}\) & \(31 / 8 \times 19^{1 / 2}\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 191 / 4\) & \(5 \times 161 / 2\) \\
\hline W 12x19 & \(31 / 8 \times 191 / 2\) & \(51 / 8 \times 161 / 2\) & \(3 \times 20^{5 / 8}\) & \(5 \times 151 / 8\) & \(31 / 8 \times 21\) & \(51 / 8 \times 18\) & \(3 \times 205 / 8\) & \(5 \times 17^{7 / 8}\) \\
\hline W 10x22 & \(31 / 8 \times 21\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 20^{5 / 8}\) & \(5 \times 161 / 2\) & \(31 / 8 \times 191 / 2\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 205 / 8\) & \(5 \times 17^{7 / 8}\) \\
\hline W 12x22 & 51/8x 18 & \(63 / 4 \times 15\) & \(3 \times 22\) & \(5 \times 17^{7 / 8}\) & \(51 / 8 \times 19^{1 / 2}\) & \(6^{3 / 4 \times 161 / 2}\) & \(5 \times 191 / 4\) & \(6^{3} / 4 \times 16^{1 / 2}\) \\
\hline W 14x22 & \(51 / 8 \times 18\) & \(6^{3 / 4 \times 161 / 2}\) & \(3 \times 23^{3 / 8}\) & \(5 \times 17^{7 / 8}\) & \(51 / 8 \times 21\) & \(63 / 4 \times 18\) & \(5 \times 205 / 8\) & \(6^{3 / 4 \times 17^{7 / 8}}\) \\
\hline W 12x26 & \(51 / 8 \times 191 / 2\) & \(63 / 4 \times 18\) & \(5 \times 191 / 4\) & \(6^{3 / 4 \times 161 / 2}\) & \(51 / 8 \times 21\) & \(6^{3 / 4 \times 191 / 2}\) & \(5 \times 205 / 8\) & \(6^{3 / 4 \times 191 / 4}\) \\
\hline W \(14 \times 26\) & \(51 / 8 \times 21\) & \(63 / 4 \times 18\) & \(5 \times 20^{5 / 8}\) & \(63 / 4 \times 17^{7 / 8}\) & \(51 / 8 \times 21\) & \(6^{3 / 4 \times 191 / 2}\) & \(5 \times 22\) & \(6^{3 / 4 \times 191 / 4}\) \\
\hline W \(16 \times 26\) & \(51 / 8 \times 21\) & \(63 / 4 \times 19^{1 / 2}\) & \(5 \times 20^{5 / 8}\) & \(63 / 4 \times 17^{1 / 8}\) & 51/822 \(1 / 2\) & \(63 / 4 \times 21\) & \(5 \times 23^{3 / 8}\) & \(6^{3 / 4 \times 20^{5 / 8}}\) \\
\hline W 12x30 & \(51 / 8 \times 21\) & \(63 / 4 \times 19^{1 / 2}\) & \(5 \times 20^{5 / 8}\) & \(63 / 4 \times 17^{1 / 8}\) & \(51 / 8 \times 21\) & \(6^{3 / 4 \times 191 / 2}\) & \(5 \times 22\) & \(6^{3 / 4 \times 191 / 4}\) \\
\hline W \(14 \times 30\) & \(51 / 8 \times 22^{1 / 2}\) & \(63 / 4 \times 19^{1 / 2}\) & \(5 \times 22\) & \(6^{3 / 4 \times 191 / 4}\) & \(5^{1 / 8 \times 22^{1 / 2}}\) & \(6^{3} / 4 \times 21\) & \(5 \times 23^{3 / 8}\) & \(6^{3 / 4 \times 20^{5} / 8}\) \\
\hline W 16x31 & 51/8×24 & \(63 / 4 \times 21\) & \(5 \times 23^{3 / 8}\) & \(6^{3 / 4 \times 205 / 8}\) & \(51 / 8 \times 25^{1 / 2}\) & \(6^{3 / 4 \times 22^{1 / 2}}\) & \(5 \times 243 / 4\) & \(6^{3 / 4 \times 23^{3 / 8}}\) \\
\hline W 14x \({ }^{\text {d }}\) & \(51 / 8 \times 24\) & \(63 / 4 \times 21\) & \(5 \times 23^{3 / 8}\) & \(6^{3 / 4 \times 20^{5} / 8}\) & \(51 / 8 \times 24\) & \(6^{3 / 4 \times 22^{1 / 2}}\) & \(5 \times 243 / 4\) & \(6^{3} / 4 \times 22\) \\
\hline W \(18 \times 35\) & \(51 / 8 \times 27\) & \(6^{3 / 4 \times 24}\) & \(5 \times 261 / 8\) & \(6^{3 / 4 \times 22}\) & \(51 / 8 \times 27\) & \(6^{3 / 4 \times 251 / 2}\) & \(5 \times 271 / 2\) & \(6^{3 / 4 \times 243 / 4}\) \\
\hline W 16x40 & \(51 / 8 \times 28^{1 / 2}\) & \(6^{3} / 4 \times 25^{1 / 2}\) & \(5 \times 27^{1 / 2}\) & \(6^{3} / 4 \times 23^{3 / 8}\) & 51/8×27 & \(6^{3 / 4 \times 25} 1 / 2\) & \(5 \times 27^{1 / 2}\) & \(6^{3} / 4 \times 24^{3 / 4}\) \\
\hline W 21x44 & \(51 / 8 \times 33\) & \(63 / 4 \times 28^{1 / 2}\) & \(5 \times 315\) & \(63 / 4 \times 27^{1 / 2}\) & \(51 / 8 \times 33\) & \(63 / 4 \times 30\) & \(5 \times 33\) & \(6^{3 / 4 \times 301 / 4}\) \\
\hline W 18x50 & \(51 / 8 \times 341 / 2\) & \(63 / 4 \times 30\) & \(5 \times 33\) & \(6^{3 / 4 \times 28^{7 / 8}}\) & \(51 / 8 \times 311 / 2\) & \(6^{3 / 4 \times 281 / 2}\) & \(5 \times 315 / 8\) & \(6^{3 / 4 \times 28^{7 / 8}}\) \\
\hline W 21x50 & \(51 / 8 \times 341 / 2\) & \(6^{3 / 4 \times 311 / 2}\) & \(5 \times 343 / 8\) & \(6^{3 / 4 \times 28^{7 / 8}}\) & \(51 / 8 \times 34^{1 / 2}\) & \(6^{3 / 4 \times 311 / 2}\) & \(5 \times 343 / 8\) & \(6^{3 / 4 \times 315 / 8}\) \\
\hline W 18×55 & 51/8×36 & \(63 / 4 \times 311 / 2\) & \(5 \times 34{ }^{3 / 8}\) & \(63 / 4 \times 301 / 4\) & \(51 / 8 \times 33\) & \(63 / 4 \times 30\) & \(5 \times 33\) & \(6^{3 / 4 \times 301 / 4}\) \\
\hline W \(21 \times 62\) & & \(63 / 4 \times 36\) & & \(6^{3} / 4 \times 34^{3 / 8}\) & \(51 / 8 \times 371 / 2\) & \(6^{3 / 4 \times 341 / 2}\) & & \(6^{3 / 4 \times 343 / 8}\) \\
\hline
\end{tabular}

\section*{Equivalent Glulam Sections for Laminated Veneer Lumber (LVL)}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline LV & \multicolumn{4}{|c|}{Roof Beams \({ }^{1,2}\)} & \multicolumn{4}{|c|}{Floor Beams \({ }^{1,3}\)} \\
\hline Sections & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} \\
\hline \(2 \mathrm{pcs} \mathrm{I}^{3} / 4 \times 9^{1 / 2}\) & \(31 / 8 \times 10^{1 / 2}\) & 51/8×9 & \(3 \times\) & \(5 \times 81 / 4\) & \(3^{1 / 8 \times 10^{1 / 2}}\) & \(51 / 8 \times 9\) & \(3 \times 11\) or & \(5 \times 9\) /8 \\
\hline \(2 \mathrm{pcs} 1^{3 / 4 \times\left. 1\right|^{7 / 8}}\) & \(31 / 8 \times 13^{1 / 2}\) & \(51 / 8 \times 10^{1 / 2}\) & \(3 \times 13^{3} /\) & \(5 \times\) & \(31 / 8 \times\) & 51/8x & \(3 \times 13^{3}\) & \(5 \times 11\) \\
\hline \(2 \mathrm{pcs} 13 / 4 \times 14\) & \(31 / 8 \times 16^{1 / 2}\) & \(51 / 8 \times 12\) & \(3 \times 16^{1 / 2}\) & \(5 \times 12^{3}\) & \(31 / 8 \times 16^{1 / 2}\) & \(51 / 8 \times 13^{1 / 2}\) & \(3 \times 161 / 2\) & \(5 \times 13^{3 / 4}\) \\
\hline \(2 \mathrm{pcs} 13 / 4 \times 16\) & 31/8×18 & 5 & \(3 \times\) & \(5 \times\) & 31 & \(51 / 8 \times 15\) & \(3 \times 17^{7 / 8}\) & \(5 \times 15^{1 / 8}\) \\
\hline \(2 \mathrm{pcs} 13 / 4 \times 18\) & \(31 / 8 \times 21\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 20^{5 / 8}\) & \(5 \times 16^{1 /}\) & \(31 / 8 \times 19^{1 / 2}\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 20^{5 / 8}\) & \(5 \times 17^{7 / 8}\) \\
\hline \(3 \mathrm{pcs} 1^{3 / 4 \times 9}\) 9 \(1 / 2\) & \(31 / 8 \times 13^{1 / 2}\) & \(51 / 8 \times 10^{1 / 2}\) & \(3 \times 13^{3}\) & \(5 \times 1\) & \(31 / 8 \times 12\) & \(51 / 8 \times 1\) & \(3 \times 12^{3 / 8}\) & x 11 \\
\hline \(3 \mathrm{pcs} 1^{3 / 4 \times\left. 1\right|^{7 / 8}}\) & \(31 / 8 \times 16^{1 / 2}\) & \(51 / 8 \times 13^{1 / 2}\) & \(3 \times 161 / 2\) & \(5 \times 13^{3 / 4}\) & \(31 / 8 \times 15\) & \(51 / 8 \times 13^{1 / 2}\) & \(3 \times 15^{1}\) & \(5 \times 13^{3 / 4}\) \\
\hline \(3 \mathrm{pcs} 13 / 4 \times 14\) & \(31 / 8 \times 19^{1 / 2}\) & \(51 / 8 \times 15\) & \(3 \times 19^{1 / 4}\) & \(5 \times 15^{1}\) & \(31 / 8 \times 18\) & \(51 / 8 \times 15\) & \(3 \times 17^{7 / 8}\) & \(5 \times 151 / 8\) \\
\hline \(3 \mathrm{pcs} 13 / 4 \times 16\) & \(31 / 8 \times 22^{1 / 2}\) & \(51 / 8 \times 18\) & \(3 \times 22\) & \(5 \times 177 / 8\) & \(31 / 8 \times 21\) & \(51 / 8 \times 18\) & \(3 \times 20^{5 / 8}\) & \(5 \times 177 / 8\) \\
\hline \(3 \mathrm{pcs} 13 / 4 \times 18\) & \(31 / 8 \times 25^{1 / 2}\) & \(51 / 8 \times 191 / 2\) & \(3 \times 243 / 4\) & \(5 \times 191 / 4\) & \(31 / 8 \times 221 / 2\) & \(51 / 8 \times 191 / 2\) & \(3 \times 23^{3 / 8}\) & \(5 \times 191 / 4\) \\
\hline
\end{tabular}

Equivalent Glulam Sections for Parallel Strand Lumber (PSL)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline PSL \({ }^{7}\) & \multicolumn{4}{|c|}{Roof Beams \({ }^{1,2}\)} & \multicolumn{4}{|c|}{Floor Beams \({ }^{1,3}\)} \\
\hline Sections & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} & \multicolumn{2}{|l|}{Douglas Fir/Larch} & \multicolumn{2}{|l|}{Southern Pine \({ }^{8}\)} \\
\hline \(31 / 2 \times 91 / 2\) & \(31 / 8 \times 10^{1 / 2}\) or & 5 \({ }^{1 / 8 \times 9}\) & \(3 \times 11\) or & \(5 \times 95 / 8\) & \(31 / 8 \times 10^{1 / 2}\) or & or \(51 / 8 \times 9\) & \(3 \times 11\) or & \(5 \times 95 / 8\) \\
\hline \(31 / 2 \times 11^{7 / 8}\) & \(31 / 8 \times 13^{1 / 2}\) & \(51 / 8 \times 10^{1 / 2}\) & \(3 \times 13^{3 / 4}\) & \(5 \times 11\) & \(31 / 8 \times 13^{1 / 2}\) & \(51 / 8 \times 12\) & \(3 \times 13^{3 / 4}\) & \(5 \times 11\) \\
\hline \(31 / 2 \times 14\) & \(31 / 8 \times 16^{1 / 2}\) & \(51 / 8 \times 12\) & \(3 \times 161 / 2\) & \(5 \times 12^{3 / 8}\) & \(31 / 8 \times 161 / 2\) & \(51 / 8 \times 13^{1 / 2}\) & \(3 \times 161 / 2\) & \(5 \times 13^{3 / 4}\) \\
\hline \(31 / 2 \times 16\) & \(31 / 8 \times 18\) & \(51 / 8 \times 13^{1 / 2}\) & \(3 \times 17^{7 / 8}\) & \(5 \times 13^{3 / 4}\) & \(31 / 8 \times 18\) & \(5^{1 / 8 \times 15}\) & \(3 \times 17^{7 / 8}\) & \(5 \times 151 / 8\) \\
\hline \(31 / 2 \times 18\) & \(31 / 8 \times 21\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 20^{5 / 8}\) & \(5 \times 161 / 2\) & \(31 / 8 \times 19^{1 / 2}\) & \(51 / 8 \times 16^{1 / 2}\) & \(3 \times 20^{5 / 8}\) & \(5 \times 17^{7} / 8\) \\
\hline \(51 / 4 \times 91 / 2\) & \(31 / 8 \times 13^{1 / 2}\) & \(51 / 8 \times 10^{1 / 2}\) & \(3 \times 13^{3 / 4}\) & \(5 \times 11\) & \(31 / 8 \times 12\) & \(51 / 8 \times 10^{1 / 2}\) & \(3 \times 12^{3 / 8}\) & \(5 \times 11\) \\
\hline \(51 / 4 \times 11^{7 / 8}\) & \(31 / 8 \times 16^{1 / 2}\) & \(51 / 8 \times 131 / 2\) & \(3 \times 161 / 2\) & \(5 \times 13^{3 / 4}\) & \(31 / 8 \times 15\) & \(51 / 8 \times 131 / 2\) & \(3 \times 151 / 8\) & \(5 \times 13^{3 / 4}\) \\
\hline \(51 / 4 \times 14\) & \(31 / 8 \times 19^{1 / 2}\) & \(51 / 8 \times 15\) & \(3 \times 191 / 4\) & \(5 \times 151 / 8\) & \(31 / 8 \times 18\) & \(51 / 8 \times 15\) & \(3 \times 17^{7 / 8}\) & \(5 \times 151 / 8\) \\
\hline \(51 / 4 \times 16\) & \(31 / 8 \times 22^{1 / 2}\) & \(51 / 8 \times 18\) & \(3 \times 22\) & \(5 \times 17^{7 / 8}\) & \(31 / 8 \times 21\) & 51/8x18 & \(3 \times 20^{5 / 8}\) & \(5 \times 17{ }^{7} / 8\) \\
\hline \(51 / 4 \times 18\) & \(31 / 8 \times 25^{1 / 2}\) & \(51 / 8 \times 191 / 2\) & \(3 \times 243 / 4\) & \(5 \times 191 / 4\) & \(31 / 8 \times 22^{1 / 2}\) & \(51 / 8 \times 191 / 2\) & \(3 \times 23^{3 / 8}\) & \(5 \times 191 / 4\) \\
\hline \(7 \times 9^{1 / 2}\) & \(51 / 8 \times 12\) & \(63 / 4 \times 10^{1 / 2}\) & \(5 \times 12^{3 / 8}\) & \(6^{3 / 4 \times 11}\) & \(51 / 8 \times 12\) & \(63 / 4 \times 10^{1 / 2}\) & \(5 \times 12^{3 / 8}\) & \(6^{3 / 4 \times 11}\) \\
\hline \(7 \times 11^{7 / 8}\) & \(51 / 8 \times 15\) & \(63 / 4 \times 13^{1 / 2}\) & \(5 \times 15^{1 / 8}\) & \(63 / 4 \times 13^{3 / 4}\) & \(51 / 8 \times 15\) & \(63 / 4 \times 13^{1 / 2}\) & \(5 \times 151 / 8\) & \(63 / 4 \times 13^{3 / 4}\) \\
\hline \(7 \times 14\) & \(51 / 8 \times 18\) & \(6^{3 / 4} \times 16^{1 / 2}\) & \(5 \times 17^{7 / 8}\) & \(63 / 4 \times 15^{1 / 8}\) & \(51 / 8 \times 16^{1 / 2}\) & \(6^{3 / 4 \times 15}\) & \(5 \times 161 / 2\) & \(63 / 4 \times 15^{1 / 8}\) \\
\hline \(7 \times 16\) & \(51 / 8 \times 21\) & \(6^{3} / 4 \times 18\) & \(5 \times 20^{5} / 8\) & \(63 / 4 \times 17^{7} / 8\) & \(51 / 8 \times 19^{1 / 2}\) & \(6^{3} / 4 \times 18\) & \(5 \times 19^{1 / 4}\) & \(6^{3} / 4 \times 17^{7 / 8}\) \\
\hline \(7 \times 18\) & \(51 / 8 \times 22^{1 / 2}\) & \(6^{3 / 4 \times 21}\) & \(5 \times 22\) & \(63 / 4 \times 191 / 4\) & \(51 / 8 \times 21\) & \(63 / 4 \times 191 / 2\) & \(5 \times 22\) & \(63 / 4 \times 191 / 4\) \\
\hline
\end{tabular}

\section*{Footnotes For All Tables:}
I. Glued laminated timber beam sizes are based on a span to depth ratio, L/d of 21 . When the span to depth ratio is different, sizes should be determined by engineering calculations.
2. Roof beam sections are compared on the basis of equivalent bending resistance only.
These sizes use a dry service condition and a I.I5 increase for duration of load (as for snow loading) as applicable to wood members. Sizes shown should also be checked for shear, deflection, and other applicable strength properties and design considerations. For determining glulam roof beam sections, the bending design value, \(F_{b}\) was adjusted by the volume factor. 3. Floor beam sections are compared on the basis of equivalent stiffness (EI) only, using a dry service condition for the wood members. Sizes shown should also be checked for shear, bending, and other applicable strength properties and design considerations.
4. Solid sawn sections are shown for select structural or No. I grade. Design values used are from the 1997 NDS.
5. Steel sections were selected as the most economical from the "Manual of Steel Construction," AISC, 9th Edition. Design values used were:
\[
\begin{aligned}
& \mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}, \\
& \mathrm{~F}_{\mathrm{b}}=0.66 \times \mathrm{F}_{\mathrm{y}}=29,000 \mathrm{ksi} .
\end{aligned}
\]
6. LVL sections are based on the following design values:
\(\mathrm{F}_{\mathrm{b}}=2350\) psi (adjusted for \(\mathrm{C}_{\mathrm{f}}=(12 / \mathrm{d})^{1 / 7.5}\)
for depths greater than 12 in .)
\(E=2,000,000\) psi.
7. PSL sections are based on the following design values:
\(F_{b}=2900\) psi (adjusted for \(C_{f}=(12 / d)^{1 / 9.0}\)
for depths greater than 12 in .)
\(E=2,000,000 \mathrm{psi}\).
8. \(3^{1 / 8 "}\) width Southern Pine beams are also available.

Glulam beam sections are based on the following design values:
\[
\mathrm{F}_{\mathrm{bx}}=2400 \text { psi (dry service conditions) }
\]
\[
E_{x}^{b x}=1,800,000 \mathrm{psi}
\]
\(30 \mathrm{~F}, 3000\) psi beams are also available.
While these design conversions have been prepared in accordance with recognized engineering principles, and are based on accurate tech-nical data available, conversions should not be used without competent professional examination and verification of the accuracy, suitability, and applicability by a licensed design professional.

Any user of this information assumes all risks and liability arising from such use.

\section*{Posts and Columns}

Laminated posts and columns are available in long length members, eliminating the need to splice short timber sections.
Due to its dimensional stability and close manufacturing tolerances, a glued laminated timber column will remain straight and square. Other framing members, such as beams, can easily be attached with simple connection detailing.

\section*{Other Applications}

The use of laminated stair stringers is a good choice when long stringers are required, or when the stair framing will be exposed. Custom curved members are an option when special architectural considerations need to be met.
Stair stringers should not be notched for installation of risers, because it could compromise the stringers structural performance. Steel angles or ledgers may be used to support risers.

\section*{Connection Details}

Some typical connection details are shown on this page. For more information, request AITC Standard IO4, Typical Construction Details.

\section*{Corrosion Resistance}

Wood has excellent chemical and corrosion resistance and is used in installations such as fertilizer storage buildings.


Beam hangers.


Cantilever connection.


Column connection.


Beam Intersection connection.


Corner support.


Truss connection.

\section*{Heavy Timber Construction}

Heavy timber construction has long been recognized by the model building codes as fire resistant. To receive building code acceptance as "heavy timber," limitations are placed upon size and thickness or composition of all load carrying wood members. Heavy timber also avoids concealed spaces under floors and roofs and requires the use of approved fastenings, construction details and adhesives.
The performance of heavy timber construction under fire conditions is markedly superior to most unprotected noncombustible structures. Fire fighting is simpler and safer due to elimination of concealed spaces and the inherent structural integrity of large glued laminated timbers.
Unprotected metals lose their strength quickly and collapse suddenly under extreme heat. Steel weakens dramatically as its temperature climbs above \(450^{\circ}\) Fahrenheit, retaining only \(10 \%\) of its strength at about \(1380^{\circ} \mathrm{F}\). The average building fire temperatures range from \(1290^{\circ} \mathrm{F}\) to \(1650^{\circ} \mathrm{F}\).
Wood retains a significantly higher percentage of its original strength for a longer period of time, losing strength only as material is lost through surface charring.


Heary timber contruction.

\section*{Fire Resistance}

The fire resistance rating is the time a member can support full design load without collapsing or spreading fire, either directly or indirectly through heat transfer. For example, a one-hour rating means the assembly should be capable of supporting its full load without collapsing for at least one hour after the fire starts.

\section*{Fire Design Method}

Fire tests jointly sponsored by the American Forest \& Paper
Association and AITC led to a fire design methodology which allows the designer to calculate a specific fire rating for a glulam member.
See AITC Technical Note No. 7.


A typical glulam beam following a fire test. The outer surface of the beam has charred, while the inner areas remain unburned. The charred outer material acts as an insulator during fire, reducing the rate at which the inner material burns.

\section*{Quality Control and Inspection}

As a service to the construction industry, AITC provides a quality control and inspection system based on three elements:

\section*{I. Licensing of manufactur-}
ers. AITC licenses qualified laminators whose personnel procedures and facilities have complied with the requirements of ANSI/AITC Al90.I.

\section*{2. Quality control mainte-} nance. Each licensee agrees to accept responsibility for maintaining a quality control system which is in compliance with ANSI/AITC AI90.I, AITC standards, and AITC 200-Inspection Manual.
3. Periodic plant inspection.

AITC's Inspection Bureau, a nationwide team of qualified inspectors, conducts frequent, unannounced inspection and verification checks of laminators' in-plant quality control system, procedures and production.


\section*{AITC Publications}
- Timber Construction Manual - This 904 page handbook for timber design includes design methods and examples for laminated beams, columns, arches, trusses, single and double tapered beams, curved beams, and pitched and tapered curved beams.
- Bridge Systems Manual
- Structural Glued Laminated Timber in Religious Structures
- Glued Laminated Timbers for Residential Construction
- Glulam-Superior Fire Resistance
- Pitched and Curved Glulam Beams
- Pitched and Tapered Curved Beams
- WoodWorks \({ }^{\circledR}\) Sizer for AITC Software

See our web site for publication prices.

\section*{AITC Standards}

AITC 104-84 Typical Construction Details. AITC 109-98 Standard for Preservative Treatment of Structural Glued Laminated Timber.
AITC I 10-97 Standard Appearance Grades for Structural Glued Laminated Timber. AITC 111-79 Recommended Practice for Protection of Structural Glued Laminated Timber During Transit, Storage and Erection. AITC 113-93 Standard for Dimensions of Structural Glued Laminated Timber. AITC 117-93 (Design) Standard Specifications for Structural Glued Laminated Timber of Softwood Species, Design Requirements. AITC 119-96 Standard Specifications for Structural Glued Laminated Timber of Hardwood Species.

A number of Technical Notes cover subjects such as checking, drilling, notching and fire performance. These are available through our web site.

Cover Photo--Poynter Institute, St. Petersburg, FL.; Architect-Jung/Brannen \& Assoc.; Structural Engineer--Weidlinger Assoc. Consulting Engineers; Contractor--Federal Construction Co.


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\section*{BE CONSTRUCTIVE" \\ WOOD}

\section*{Examples:}

\section*{Timber}

\section*{Example 1}

Design a Flat Roof joist, 16 in . on center (o.c.), 18 ft span with Douglas firlarch No. 2. Snow load is 30 psf . Dead load (including ballast, roofing, sheathing, joists \& ceiling \()=18.9 \mathrm{psf} . \mathrm{C}_{\mathrm{r}}=1.15\) for bending only.
\[
\mathrm{F}_{\mathrm{b}}=875 \mathrm{psi} ; \mathrm{F}_{\mathrm{v}}=95 \mathrm{psi} ; \mathrm{E}=1.6 \times 10^{6} \mathrm{psi}
\]

Also design the glulam girder supporting the joists if it spans 35 ft (simply supported) and \(\mathrm{F}_{\mathrm{b}}=2400\) psi.

Assume the density of the glulam timber is \(32 \mathrm{lb} / \mathrm{ft}^{3}\).


SOLUTION:
The load case that is most likely to govern the design is Dead + Live. Because the live load is from snow, \(C_{D}=1.15\) :
\(\frac{18.9 p s f}{0.9}=21 p s f<\frac{(18.9 p s f+30 p s f)}{1.15}=42.5\)
Joist
The distributed load for each joist needs to be found by multiplying the area load by the tributary width:
\(w=\left(30 \mathrm{lb} / \mathrm{ft}^{2}+18.9 \mathrm{lb} / \mathrm{ft}^{2}\right)(16 \mathrm{in})(1 \mathrm{ft} / 12 \mathrm{in})=65.2 \mathrm{lb} / \mathrm{ft}\)
\(M_{\text {max }}=\frac{w l^{2}}{8}=\frac{\left(65.2 \mathrm{lb} /{ }_{f t}\right)(18 f t)^{2}}{8}=2641^{l b-f t}\)

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be \(\mathrm{C}_{\mathrm{D}}\) and \(\mathrm{C}_{\mathrm{r}}\) :
\(F_{b}^{\prime}=F_{b} C_{D} C_{r}=875 \mathrm{lb} / \mathrm{in}^{2}(1.15)(1.15)=1157 \mathrm{lb} / \mathrm{in}^{2}\)
\(\mathrm{S}_{\text {req'd }} \geq \frac{M}{F_{b}^{\prime}}=\frac{2641^{l b-f t}}{1157 \mathrm{lb} / \mathrm{in}^{2}} \cdot(12 \mathrm{in} / f t)=27.4 \mathrm{in}^{3}\)

\section*{SECTION PROPERTIES JOISTS AND BEAMS}


Shear can quite often govern the design of timber beams:
\(V_{\text {max }}=\frac{w l}{2}=\frac{(65.2 \mathrm{lb} / f t)(18 f t)}{2}=587^{\mathrm{lb}}\)

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be \(C_{D}\) only:
\(F_{v}^{\prime}=F_{v} C_{D}=95 \mathrm{lb} / \mathrm{in}^{2}(1.15)=109 \mathrm{lb} / \mathrm{in}^{2}\)
Shear stress in a rectangular beam is found from 3V/2A:
\(\mathrm{A}_{\text {req'd }} \geq \frac{3 V}{2 F_{v}^{\prime}}=\frac{3\left(587^{l b}\right)}{2\left(109 \mathrm{lb} / i \mathrm{in}^{2}\right)}=8.1 \mathrm{in}^{2}\)
\begin{tabular}{|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Nominal Size \\
In Inches \\
b h
\end{tabular} & Surfaced Size In Inches For Design b h & Area (A)
\[
\begin{gathered}
\mathbf{A}=b h \\
\left(\ln ^{2}\right)
\end{gathered}
\] & Section Modulus (S)
\[
\begin{gathered}
S=\frac{b h^{2}}{6} \\
\left(\ln ^{3}\right)
\end{gathered}
\] & Moment of Inertia (l)
\[
\begin{aligned}
& I=\frac{b^{3}}{12} \\
& \left(\ln { }^{4}\right)
\end{aligned}
\] & Board Feet Per Linear Foot of Piece \\
\hline \(2 \times 2\) & \(1.5 \times 1.5\) & 2.25 & 0.562 & 0.422 & 0.33 \\
\hline \(2 \times 3\) & \(1.5 \times 2.5\) & 3.75 & 1.56 & 1.95 & 0.50 \\
\hline \(2 \times 4\) & \(1.5 \times 3.5\) & 5.25 & 3.06 & 5.36 & 0.67 \\
\hline \(2 \times 5\) & \(1.5 \times 4.5\) & 6.75 & 5.06 & 11.39 & . 83 \\
\hline \(2 \times 6\) & \(1.5 \times 5.5\) & 8.25 & 7.56 & 20.80 & 1.00 \\
\hline \(2 \times 8\) & \(1.5 \times 7.25\) & 10.88 & 13.14 & 47.63 & 1.33 \\
\hline \(2 \times 10\) & \(1.5 \times 9.25\) & 13.88 & 21.39 & - 98.93 & 1.67 \\
\hline \(2 \times 12\) & \(1.5 \times 11.25\) & 16.88 & 31.64 & 177.98 & 2.00 \\
\hline \(2 \times 14\) & \(1.5 \times 13.25\) & 19.88 & 43.89 & 290.78 & 2.33 \\
\hline \(3 \times 3\) & \(2.5 \times 2.5\) & 6.25 & 2.60 & 3.26 & 0.75 \\
\hline \(3 \times 4\) & \(2.5 \times 3.5\) & 8.75 & 5.10 & 8.93 & 1.00 \\
\hline \(3 \times 5\) & \(2.5 \times 4.5\) & 11.25 & 8.44 & 18.98 & 1.25 \\
\hline \(3 \times 6\) & \(2.5 \times 5.5\) & 13.75 & 12.60 & 34.66 & 1.50 \\
\hline \(3 \times 8\) & \(2.5 \times 7.25\) & 18.12 & 21.90 & 79.39 & 2.00 \\
\hline \(3 \times 10\) & \(2.5 \times 9.25\) & 23.12 & 35.65 & 164.89 & 2.50 \\
\hline \(3 \times 12\) & \(2.5 \times 11.25\) & 28.12 & 52.73 & 296.63 & 3.00 \\
\hline \(3 \times 14\) & \(2.5 \times 13.25\) & 33.12 & 73.15 & 484.63 & 3.50 \\
\hline \(3 \times 16\) & \(2.5 \times 15.25\) & 38.12 & 96.90 & 738.87 & 4.00 \\
\hline
\end{tabular}

Allowable deflection is not known, but \(I_{\text {req'd }}\) could be determined from \(\Delta=\frac{5 w l^{4}}{384 E I} \leq \Delta_{\text {limit }}\) then \(I_{\text {req'd }} \geq \frac{5 w l^{4}}{384 E \Delta_{\text {limit }}}\)
From the section property table, a \(2 \times 12\) satisfies Areq'd and \(I_{\text {req'd. }}\) (bending governs)

\section*{Girder}

The distributed load on the girder is the reaction of each joist over the 16 inch spacing plus the self weight of the girder.
Guessing a self weight of \(40 \mathrm{lb} / \mathrm{ft}\left(\approx 32 \mathrm{lb} / \mathrm{ft}^{3} \times 1 \mathrm{ft}^{2}\right)\) :
\(\mathrm{w}=\frac{\mathrm{V}}{\text { spacing }}+\) s.w. \(=\frac{587 \mathrm{lb}}{16 \mathrm{in}} \cdot \frac{12 \mathrm{in}}{f t}+40 \mathrm{lb} / \mathrm{ft}=480 \mathrm{lb} / \mathrm{ft}\)
\(M_{\max }=\frac{w l^{2}}{8}=\frac{(480 \mathrm{lb} / f t)(35 f t)^{2}}{8}=73,500^{l b-f t}\)
Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be CF. The charts provided say that \(\mathrm{C}_{\mathrm{F}}\) has been included in the section modulus. If we didn't have a chart that included \(\mathrm{C}_{\mathrm{F}}\) and we don't know the depth, we could guess - say 18 inches:
\(C_{F}=\left(\frac{12}{d}\right)^{1 / 9}=\left(\frac{12}{18}\right)^{1 / 9}=0.956(<1)\) which would need to be multiplied with all the other adjustment factors by \(F_{b}\) to find \(F_{b}^{\prime}\)
\(\mathrm{S}_{\text {req'd }} \geq \frac{M}{F_{b}^{\prime}}=\frac{73,500^{\mathrm{lb}-f t}}{2400^{\mathrm{lb}} / \mathrm{in}{ }^{2}} \cdot(12 \mathrm{in} / f t)=367.5 \mathrm{in}^{3}\)
No information is available to evaluate shear or deflection. Based on that, try a \(51 / 8 \times 22.5\). It has a smaller area than the \(83 / 4\) section with a big enough adjusted \(S\). (Real \(S=5.125 \times 22.5^{2 / 6}=432.42 \mathrm{in}^{3}, C_{F}=0.932\), \(\mathrm{S}_{\text {adjusted }}=403.2 \mathrm{in}^{3}\) )
Check self weight: \(=\gamma \cdot \mathrm{A}=32 \mathrm{lb} / \mathrm{ft}^{3}\left(115.3 \mathrm{in}^{2}\right)\left(\frac{1 \mathrm{ft}}{12 \mathrm{in}}\right)^{2}=26 \mathrm{lb} / f t\) which is less than what was used.
We could try a smaller section, which would mean calculating a new self weight, then moment, then \(\mathrm{S}_{\text {req'd }}\) and comparing \(\mathrm{S}_{\text {actual }}\) to Sreq'd.

The lower self weight means a lower design moment, but the smaller \(C_{F}\) means a smaller allowed stress, so we might end up with the same section.
\(W_{\text {revised }}=480 \mathrm{lb} / \mathrm{ft}+(26-40 \mathrm{lb} / \mathrm{ft}), \quad \mathrm{M}_{\text {revised }}=71,356 \mathrm{lb}-\mathrm{ft}, \mathrm{S}_{\text {req'd }}\) now \(=356.8 \mathrm{in}^{3}\) and the \(51 / 8 \times 22.5\) is the choice for bending.
Of course, we need to satisfy shear and deflection criteria as well.


\section*{Example 2}

\section*{Example Problem 10.20:}

\section*{Design of Wood Columns(Figure 10.66)}

A 22 '-tall glu-lam column is required to support a roof load (including snow) of 40 kips . Assuming \(8^{3} / 4^{\prime \prime}\) in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.
Select from the following sizes:
\[
\begin{aligned}
& 8^{3} / 4^{\prime \prime} \times 9^{\prime \prime}\left(A=78.75 \mathrm{in} .^{2}\right) \\
& 8^{3} / 4^{\prime \prime} \times 10^{1 / 2^{\prime \prime}}\left(A=91.88 \mathrm{in} .^{2}\right) \\
& 8^{3} / 4^{\prime \prime} \times 12^{\prime \prime}\left(A=105.00 \mathrm{in} .^{2}\right)
\end{aligned}
\]

\section*{Solution:}

Glu-lam column: ( \(\left.F_{c}=1650 \mathrm{psi}, E=1.8 \times 10^{6} \mathrm{psi}\right)\)
Try \(8^{3} / 4^{\prime \prime} \times 10^{1 / 2 \prime} 2^{\prime \prime}\left(A=105.00 \mathrm{in}^{2}{ }^{2}\right)\)
\[
\begin{aligned}
\frac{L_{e}}{d} & =\frac{\left(22^{\prime} \times 12 \mathrm{in} . / \mathrm{ft} .\right)}{8.75 \mathrm{in} .} \\
& =30.2<50(\text { max. slenderness ratio }) \\
F_{c E} & =\frac{0.418 E}{\left(L_{e} / d\right)^{2}}=\frac{0.418\left(1.8 \times 10^{6} \mathrm{lb} . / \mathrm{in.}{ }^{2}\right)}{(30.2)^{2}}=825 \mathrm{psi} \\
F_{c}^{*} & \cong F_{c} C_{D}=(1650 \mathrm{psi}) \times\left(\begin{array}{c}
(\text { (snow) }
\end{array}\right)=1900 \mathrm{psi} \\
\frac{F_{c E}}{F_{c}^{*}} & =\frac{825}{1900}=0.43
\end{aligned}
\]

From Appendix Table 14: \(C_{p}=0.403\)
\[
\begin{aligned}
F_{c}^{\prime} & =F_{c}^{*} C_{p}=\left(1900 \mathrm{lb} . / \mathrm{in} .^{2}\right) \times(0.403)=765 \mathrm{psi} \\
P_{a} & =F_{c}^{\prime} \times A=\left(765 \mathrm{lb} . / \mathrm{in} .^{2}\right) \times\left(91.9 \mathrm{in} .^{2}\right) \\
& =70,300 \mathrm{lb} .>40,000 \mathrm{lb} .
\end{aligned}
\]

Cycle again, trying a smaller, more economical section. Try \(8^{3} / 4^{\prime \prime} \times 9^{\prime \prime}\left(A=78.8\right.\) in. \(\left.^{2}\right)\)
Since the critical dimension is still \(8^{3} / 4^{\prime \prime}\), the values for \(F_{c E}, F_{c}^{*}\), and \(F_{c}^{\prime}\) all remain the same as in trial 1 . The only change that affects the capability of the column is the available crosssectional area.
\[
\begin{aligned}
\therefore P_{a} & =F_{c}^{\prime} \times A=\left(765 \mathrm{lb} . / \mathrm{in} .^{2}\right) \times\left(78.8 \mathrm{in} . .^{2}\right) \\
& =60,300 \mathrm{lb} . \\
P_{a} & =60.3 \mathrm{k}>40 \mathrm{k}
\end{aligned}
\]

Use \(8^{3} / 4 " \times 9\) " glu-lam section.

\section*{Case Study in Timber}
adapted from Simplified Design of Wood Structures, James Ambrose, \(5^{\text {th }}\) ed.

\section*{Building description}

The building is a one-story building intended for commercial occupancy. Figure 16.1 presents a building plan, partial elevation, section and elevation of the perimeter shear walls. Light wood framing (assuming the fire resistance requirements have been met) will be used.

Loads

\section*{Live Loads:}

Roof: \(20 \mathrm{lb} / \mathrm{ft}^{2}(0.96 \mathrm{kPa})\)
Wind: critical at \(20 \mathrm{lb} / \mathrm{ft}^{2}(0.96 \mathrm{kPa})\) on vertical exterior surfaces.

\section*{Dead Loads:}

Roofing \& deck: \(7.5 \mathrm{lb} / \mathrm{ft}^{2}(0.36 \mathrm{kPa})\)
Ceiling joists, ceiling \& fixtures:
\[
6.5 \mathrm{lb} / \mathrm{ft}^{2}(0.31 \mathrm{kPa})
\]

Total: \(14 \mathrm{lb} / \mathrm{ft}^{2}(0.67 \mathrm{kPa})\)

\section*{Materials}

Wood framing of Douglas fir-larch, structural grades No. \(1 \& 2\) having a density of \(32 \mathrm{lb} / \mathrm{ft}^{3}\), and AITC glulam timber.

(d) Elevation: East -West Shear Walls

(b) Partial Elevation




Figure 16.1 Building One, general form.

\section*{Structural Elements/Plan}

If the interior partition walls are arranged as in Figure 16.3a, there are options on the arrangement of the roof structure. We will analyze case 16.3 b consisting of roof deck and rafters, stud walls, continuous (two span) beams, and columns.


\section*{Decking \& Rafters:}

The standard size of plywood or structural deck panel is \(4 \mathrm{ft} \times 8 \mathrm{ft}\). The typical orientation is with the long direction with the face grain perpendicular to the rafters or floor joists. (See cross hatching in Figure 16.3.) Typical joist and rafter spacings are 12 in ., 16 in ., and 24 in . on center. If we use 16 in . on center, the total distributed roof loads (with allowable stress design) with an assumed self weight of \(4 \mathrm{lb} / \mathrm{ft}\) is:
\[
\begin{aligned}
& \mathrm{w}=\left(20 \mathrm{lb} / \mathrm{ft}^{2}+14 \mathrm{lb} / \mathrm{ft}^{2}\right) \cdot 16 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft}+4 \mathrm{lb} / \mathrm{ft}=49.3 \mathrm{lb} / \mathrm{ft} \\
& \mathrm{M}_{\max }=\frac{\mathrm{wL}^{2}}{8}=\frac{49.3^{\mathrm{lb} / \mathrm{tt}}\left(21^{\mathrm{ft}}\right)^{2}}{8}=2718^{\mathrm{lb}=\mathrm{ft}}
\end{aligned}
\]

Tabular allowable stresses for No. 2 Douglas fir-larch, 2 " \(-4 "\) thick and \(2 "\) to \(4 "\) wide are:
\[
F_{b-\text { single }}=875^{\text {psi }}, F_{v}=95^{\text {psi }}, F_{c \perp}=625^{\text {psi }}, F_{c}=1300^{\text {psi }}, E=1,600,000^{\text {psi }}
\]

The load duration for roof loads, \(\mathrm{C}_{\mathrm{D}}=1.25\). The repetitive member factor, \(\mathrm{C}_{\mathrm{r}}=1.15\), applies and the adjusted allowed stress for a fully braced 2 x is:
\[
\mathrm{F}_{\mathrm{b}}^{\prime}=\mathrm{C}_{\mathrm{D}} \mathrm{C}_{\mathrm{r}} \mathrm{~F}_{\mathrm{b}}=(1.25)(1.15)(875 \mathrm{psi})=1258 \mathrm{psi}
\]

The required section modulus is
\[
\mathrm{S}_{\text {reád }} \geq \frac{\mathrm{M}}{F_{\mathrm{b}}^{\prime}}=\frac{2718^{\mathrm{lb}-\mathrm{ft}} \cdot 12^{\mathrm{in} / \mathrm{tt}}}{1258^{\mathrm{si}}}=25.9 \mathrm{in}^{3}
\]

A \(2 \times 12\) will work if the deflection is limited to allowable for the building code. (This tends to govern for floors. Shear stress should also be checked).
\begin{tabular}{|c|c|c|c|c|c|}
\hline SECTIO JOISTS & I PROPE AND BEA & TIES S &  & & \\
\hline  & Surfaced Size In Inches For Design b h & Area (A)
\[
\begin{gathered}
\mathbf{A}=\mathrm{bh} \\
\left(\ln ^{2}\right)
\end{gathered}
\] & Section Modulus (S)
\[
\begin{gathered}
S=\frac{b h^{2}}{6} \\
\left(\ln ^{3}\right)
\end{gathered}
\] & Moment of Inertia (l)
\[
\begin{aligned}
& I=\frac{b h^{3}}{12} \\
& \left(\ln 4^{4}\right)
\end{aligned}
\] & Board Feet Per Linear Foot of Piece \\
\hline \(2 \times 2\) & \(1.5 \times 1.5\) & 2.25 & 0.562 & 0.422 & 0.33 \\
\hline \(2 \times 3\) & \(1.5 \times 2.5\) & 3.75 & 1.56 & 1.95 & 0.50 \\
\hline \(2 \times 4\) & \(1.5 \times 3.5\) & 5.25 & 3.06 & 5.36 & 0.67 \\
\hline \(2 \times 5\) & \(1.5 \times 4.5\) & 6.75 & 5.06 & 11.39 & . 83 \\
\hline \(2 \times 6\) & \(1.5 \times 5.5\) & 8.25 & 7.56 & 20.80 & 1.00 \\
\hline \(2 \times 8\) & \(1.5 \times 7.25\) & 10.88 & 13.14 & 47.63 & 1.33 \\
\hline \(2 \times 10\) & \(1.5 \times 9.25\) & 13.88 & 21.39 & 98.93 & 1.67 \\
\hline \(2 \times 12\) & \(1.5 \times 11.25\) & 16.88 & 31.64 & 177.98 & 2.00 \\
\hline \(2 \times 14\) & \(1.5 \times 13.25\) & 19.88 & 43.89 & 290.78 & 2.33 \\
\hline \(3 \times 3\) & \(2.5 \times 2.5\) & 6.25 & 2.60 & 3.26 & 0.75 \\
\hline \(3 \times 4\) & \(2.5 \times 3.5\) & 8.75 & 5.10 & 8.93 & 1.00 \\
\hline \(3 \times 5\) & \(2.5 \times 4.5\) & 11.25 & 8.44 & 18.98 & 1.25 \\
\hline \(3 \times 6\) & \(2.5 \times 5.5\) & 13.75 & 12.60 & 34.66 & 1.50 \\
\hline \(3 \times 8\) & \(2.5 \times 7.25\) & 18.12 & 21,90 & 79.39 & 2.00 \\
\hline \(3 \times 10\) & \(2.5 \times 9.25\) & 23.12 & 35.65 & 164.89 & 2.50 \\
\hline \(3 \times 12\) & \(2.5 \times 11.25\) & 28.12 & 52.73 & 296.63 & 3.00 \\
\hline \(3 \times 14\) & \(2.5 \times 13.25\) & 33.12 & 73.15 & 484.63 & 3.50 \\
\hline \(3 \times 16\) & \(2.5 \times 15.25\) & 38.12 & 96.90 & 738.87 & 4.00 \\
\hline \multicolumn{6}{|c|}{2} \\
\hline
\end{tabular}

\section*{Continuous Beams:}

The distributed load, including an estimated self weight of \(11 \mathrm{lb} / \mathrm{ft}\) (about a 6 in x 12 in section) of a glulam beam can be found from:
rafter distributed load:
\[
\frac{\gamma \cdot A \cdot \text { trib. width }}{\text { rafter spacing }}=\frac{\left(32 \mathrm{lb} / \mathrm{ft}^{3}\right)\left(16.88 \mathrm{in}^{2}\right)\left(21^{\mathrm{ft}} / 2^{8^{f t}} / 2\right)}{16 \mathrm{in}} \cdot\left(\frac{1 \mathrm{ft}}{12 \mathrm{in}}\right)^{2} \cdot \frac{12 i n}{f t}=40.8^{\mathrm{lb} / \mathrm{ft}}
\]
roof load:
\[
\left(20 \mathrm{lb} / \mathrm{ft}^{2}+14 \mathrm{lb} / \mathrm{ft}^{2}\right) \cdot(21 \mathrm{ft} / 2+8 \mathrm{ft} / 2)=493^{\mathrm{lb} / \mathrm{ft}}
\]
total distributed load:
\[
w=40.8 \mathrm{lb} / \mathrm{ft}+493 \mathrm{lb} / \mathrm{ft}+11 \mathrm{lb} / \mathrm{ft}=545 \mathrm{lb} / \mathrm{ft}
\]

The maximum positive moment is \(0.07 \mathrm{wL}^{2}\) and the maximum negative moment over the support is \(0.125 \mathrm{wL}^{2}\), where L is the length of one span. \(\mathrm{V}_{\max }=0.625 \mathrm{wL}\). (These values come from a beam diagram.)
\[
\begin{aligned}
& \mathrm{M}_{\max }=0.125(545 \mathrm{lb} / \mathrm{ft})(16.67 \mathrm{ft})^{2}=18,931 \mathrm{lb-ft} \\
& \mathrm{~V}_{\max }=0.625(545 \mathrm{lb} / \mathrm{ft})(16.67 \mathrm{ft})=5678 \mathrm{lb} \\
& \mathrm{~F}_{\mathrm{b}}^{\prime}=\mathrm{C}_{\mathrm{D}} \mathrm{~F}_{\mathrm{b}}=(1.25)(2400 \mathrm{psi})=3000 \mathrm{psi} \\
& S_{\text {req }}{ }^{\prime} \geq \frac{M}{F_{b}^{\prime}}=\frac{18931^{\text {lb-ft }}}{3000^{p s i}} \cdot 12 \mathrm{in} / f \mathrm{ft}=75.7 \mathrm{in}^{3}
\end{aligned}
\]

From SECTION PROPERTIES/STANDARD SIZES, the \(51 / 8 " \times 10.5 "\) is adequate, although a \(31 / 8 " \times 13.5 "\) could be evaluated.


\section*{ROOF BEAMS} CONSTRUCTION LOAD

Simple Span Beams
For Preliminary Design Purposes
Lamination thickness: 1.500 in .
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{BEAM SIZE} & \multirow[t]{2}{*}{BEAM
WEIGHT
plf} & \multicolumn{17}{|c|}{BEAM CAPACITY, UNIFORMLOAD w, plf} \\
\hline \begin{tabular}{l}
Width \\
b, in.
\end{tabular} & Depth d, in. & & \[
\begin{array}{|c}
\hline \text { SPAN, ft } \\
8 \\
\hline
\end{array}
\] & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 & 24 \\
\hline \(31 / 8\) & 6 & 4.6 & 586 D & 412 D & 300 D & 225 D & 174 D & 137 D & 109 D & 89 D & - & - & -- & -- & -- & - & - & -- & - \\
\hline \(31 / 8\) & \(71 / 2\) & 5.7 & 916 B & 723 B & 586 D & 440 D & 339 D & 267 D & 214 D & 174 D & 143 D & 119 D & 100 D & -- & -- & -- & -- & -- & - \\
\hline \(31 / 8\) & 9 & 6.8 & 1318 B & 1042 B & 844 B & 697 B & 586 D & 461 D & 369 D & 300 D & 247 D & 206 D & 174 D & 148 D & 127 D & 109 D & 95 D & -- & - \\
\hline \(31 / 8\) & \(101 / 2\) & 8.0 & 1794 B & 1418 B & 1148 B & 949 B & 798 B & 680 B & 586 D & 476 D & 393 D & 327 D & 276 D & 234 D & 201 D & 174 D & 151 D & 132 D & 116 D \\
\hline \(31 / 8\) & 12 & 9.1 & 2344 B & 1852 B & 1500 B & 1240 B & 1042 B & 888 B & 765 B & 667 B & 586 D & 488 D & 412 D & 350 D & 300 D & 259 D & 225 D & 197 D & 174 D \\
\hline \(31 / 8\) & 13 1/2 & 10.3 & 2935 S & 2344 B & 1898 B & 1569 B & 1318 B & 1123 B & 969 B & 844 B & 742 B & 657 B & 586 D & 498 D & 427 D & 369 D & 321 D & 281 D & 247 D \\
\hline \(31 / 8\) & 15 & 11.4 & 3000 * & 2885 S & 2344 B & 1937 B & 1628 B & 1387 B & 1196 B & 1042 B & 916 B & 811 B & 723 B & 649 B & 586 D & 506 D & 440 D & 385 D & 339 D \\
\hline \(31 / 8\) & \(161 / 2\) & 12.5 & 3000* & 3000* & 2836 B & 2344 B & 1969 B & 1678 B & 1447 B & 1260 B & 1108 B & 981 B & 875 B & 786 B & 709 B & 643 B & 586 D & 513 D & 451 D \\
\hline \(31 / 8\) & 18 & 13.7 & 3000* & 3000* & 3000 * & 2789 B & 2344 B & 1997 B & 1722 B & 1500 B & 1318 B & 1168 B & 1042 B & 935 B & 844 B & 765 B & 697 B & 638 B & 583 B \\
\hline \(31 / 8\) & \(191 / 2\) & 14.8 & 3000 * & 3000* & 3000 * & 3000* & 2751 B & 2344 B & 2021 B & 1760 B & 1547 B & 1371 B & 1223 B & 1097 B & 990 B & 898 B & 815 B & 743 B & 679 B \\
\hline \(51 / 8\) & 6 & 7.5 & 961 D & 675 D & 492 D & 370 D & 285 D & 224 D & 179 D & 146 D & - & - & -- & - & -- & - & - & -- & - \\
\hline \(51 / 8\) & \(71 / 2\) & 9.3 & 1501 B & 1186 B & 961 D & 722 D & 556 D & 437 D & 350 D & 285 D & 235 D & 196 D & 165 D & - & -- & - & - & -- & - \\
\hline \(51 / 8\) & 9 & 11.2 & 2162 B & 1708 B & 1384 B & 1144 B & 961 D & 756 D & 605 D & 492 D & 405 D & 338 D & 285 D & 242 D & 208 D & 179 D & 156 D & -- & - \\
\hline \(51 / 8\) & \(101 / 2\) & 13.1 & 2943 B & 2325 B & 1883 B & 1557 B & 1308 B & 1114 B & 961 D & 781 D & 644 D & 537 D & 452 D & 384 D & 330 D & 285 D & 248 D & 217 D & 191 D \\
\hline \(51 / 8\) & 12 & 14.9 & 3844 B & 3037 B & 2460 B & 2033 B & 1708 B & 1456 B & 1255 B & 1093 B & 961 D & 801 D & 675 D & 574 D & 492 D & 425 D & 370 D & 323 D & 285 D \\
\hline \(51 / 8\) & \(131 / 2\) & 16.8 & 4813 S & 3844 B & 3113 B & 2573 B & 2162 B & 1842 B & 1588 B & 1384 B & 1216 B & 1077 B & 961 D & 817 D & 701 D & 605 D & 526 D & 461 D & 405 D \\
\hline \(51 / 8\) & 15 & 18.7 & 5591 S & 4731 S & 3844 B & 3177 B & 2669 B & 2274 B & 1961 B & 1708 B & 1501 B & 1328 B & 1178 B & 1052 B & 944 B & 830 D & 722 D & 632 D & 556 D \\
\hline \(51 / 8\) & \(161 / 2\) & 20.6 & 6000* & 5412 S & 4651 B & 3844 B & 3230 B & 2752 B & 2373 B & 2067 B & 1808 B & 1592 B & 1412 B & 1261 B & 1132 B & 1022 B & 926 B & 841 D & 740 D \\
\hline \(51 / 8\) & 18 & 22.4 & 6000* & 6000* & 5271 S & 4574 B & 3844 B & 3275 B & 2824 B & 2443 B & 2133 B & 1878 B & 1666 B & 1487 B & 1335 B & 1205 B & 1093 B & 996 B & 911 B \\
\hline \(51 / 8\) & \(191 / 2\) & 24.3 & 6000* & 6000* & 5922 S & 5158 S & 4511 B & 3841 B & 3288 B & 2844 B & 2484 B & 2187 B & 1940 B & 1731 B & 1555 B & 1403 B & 1273 B & 1159 B & 1060 B \\
\hline \(51 / 8\) & 21 & 26.2 & 6000 * & 6000* & 6000* & 5740 S & 5065 S & 4422 B & 3785 B & 3274 B & 2859 B & 2518 B & 2233 B & 1993 B & 1790 B & 1615 B & 1465 B & 1334 B & 1220 B \\
\hline \(51 / 8\) & \(221 / 2\) & 28.0 & 6000* & 6000 * & 6000* & 6000* & 5591 S & 4986 S & 4315 B & 3733 B & 3260 B & 2870 B & 2546 B & 2272 B & 2040 B & 1842 B & 1670 B & 1521 B & 1391 B \\
\hline \(51 / 8\) & 24 & 29.9 & 6000* & 6000 * & 6000* & 6000* & 6000 * & 5467 S & 4878 B & 4220 B & 3685 B & 3245 B & 2878 B & 2569 B & 2306 B & 2082 B & 1888 B & 1720 B & 1573 B \\
\hline \(51 / 8\) & \(251 / 2\) & 31.8 & 6000 * & 6000 * & 6000* & 6000 * & 6000 * & 5974 S & 5362 S & 4735 B & 4135 B & 3641 B & 3229 B & 2882 B & 2588 B & 2336 B & 2119 B & 1930 B & 1765 B \\
\hline \(63 / 4\) & 6 & 9.8 & 1266 D & 889 D & 648 D & 487 D & 375 D & 295 D & 236 D & 192 D & - & - & -- & - & - & - & - & - & - \\
\hline \(63 / 4\) & \(71 / 2\) & 12.3 & 1978 B & 1563 B & 1266 D & 951 D & 732 D & 576 D & 461 D & 375 D & 309 D & 258 D & 217 D & - & - & - & - & -- & - \\
\hline \(63 / 4\) & 9 & 14.8 & 2848 B & 2250 B & 1823 B & 1506 B & 1266 D & 995 D & 797 D & 648 D & 534 D & 445 D & 375 D & 319 D & 273 D & 236 D & 205 D & -- & - \\
\hline \(63 / 4\) & \(101 / 2\) & 17.2 & 3876 B & 3063 B & 2481 B & 2050 B & 1723 B & 1468 B & 1266 D & 1029 D & 848 D & 707 D & 595 D & 506 D & 434 D & 375 D & 326 D & 285 D & 251 D \\
\hline \(63 / 4\) & 12 & 19.7 & 5063 B & 4000 B & 3240 B & 2678 B & 2250 B & 1917 B & 1653 B & 1440 B & 1265 B & 1055 D & 889 D & 756 D & 648 D & 560 D & 487 D & 426 D & 375 D \\
\hline \(63 / 4\) & \(131 / 2\) & 22.1 & 6339 S & 5063 B & 4101 B & 3389 B & 2848 B & 2426 B & 2092 B & 1812 B & 1583 B & 1393 B & 1236 B & 1076 D & 923 D & 797 D & 693 D & 607 D & 534 D \\
\hline \(63 / 4\) & 15 & 24.6 & 7364 S & 6231 S & 5063 B & 4184 B & 3516 B & 2990 B & 2559 B & 2214 B & 1933 B & 1702 B & 1510 B & 1348 B & 1210 B & 1092 B & 951 D & 832 D & 732 D \\
\hline \(63 / 4\) & \(161 / 2\) & 27.1 & 8000* & 7128 S & 6126 B & 5063 B & 4239 B & 3583 B & 3067 B & 2653 B & 2317 B & 2040 B & 1809 B & 1615 B & 1450 B & 1309 B & 1187 B & 1081 B & 975 D \\
\hline \(63 / 4\) & 18 & 29.5 & 8000* & 8000 * & 6943 S & 6004 B & 5001 B & 4228 B & 3618 B & 3130 B & 2734 B & 2407 B & 2135 B & 1905 B & 1711 B & 1544 B & 1401 B & 1276 B & 1167 B \\
\hline \(63 / 4\) & 19 1/2 & 32.0 & 8000* & 8000* & 7800 S & 6794 S & 5823 B & 4922 B & 4213 B & 3644 B & 3183 B & 2802 B & 2485 B & 2218 B & 1992 B & 1798 B & 1631 B & 1485 B & 1358 B \\
\hline \(63 / 4\) & 21 & 34.5 & 8000* & 8000* & 8000 * & 7560 S & 6671 S & 5666 B & 4850 B & 4196 B & 3664 B & 3226 B & 2861 B & 2554 B & 2293 B & 2070 B & 1877 B & 1710 B & 1564 B \\
\hline \(63 / 4\) & \(221 / 2\) & 36.9 & 8000* & 8000* & 8000* & 8000* & 7364 S & 6460 B & 5529 B & 4783 B & 4177 B & 3678 B & 3262 B & 2912 B & 2614 B & 2360 B & 2140 B & 1949 B & 1783 B \\
\hline \(63 / 4\) & 24 & 39.4 & 8000* & 8000* & 8000* & 8000* & 8000 * & 7200 S & 6250 B & 5407 B & 4722 B & 4157 B & 3687 B & 3291 B & 2955 B & 2667 B & 2419 B & 2204 B & 2015 B \\
\hline \(63 / 4\) & \(251 / 2\) & 41.8 & 8000* & 8000* & 8000* & 8000* & 8000* & 7869 S & 7013 B & 6067 B & 5298 B & 4665 B & 4137 B & 3693 B & 3316 B & 2993 B & 2715 B & 2473 B & 2261 B \\
\hline \(63 / 4\) & 27 & 44.3 & 8000* & 8000* & 8000* & 8000* & 8000* & 8000* & 7674 S & 6763 B & 5906 B & 5200 B & 4612 B & 4117 B & 3696 B & 3337 B & 3026 B & 2756 B & 2521 B \\
\hline \(63 / 4\) & \(281 / 2\) & 46.8 & 8000* & 8000 * & 8000* & 8000* & 8000* & 8000 * & 8000* & 7495 B & 6545 B & 5763 B & 5111 B & 4562 B & 4096 B & 3697 B & 3353 B & 3054 B & 2793 B \\
\hline
\end{tabular}

TABLE SPECIFICATIONS: This table applies to straight, simply supported glued laminated timber beams under dry
conditions of use. Beams must be laterally supported at the top along the length of the beam and at the top and bottom
at the ends. The load carrying capacities tabulated are for total load including the weight of the member.
BEAM WEIGHT: 35.0 pounds per cubic foot was used to determine beam weight per lineal foot shown in the table.
DESIGN VALUE MODIFICATIONS: The allowable stress in bending , Fb , has been adjusted by the AITC volume factor, CV .
For determination of load carrying capacities governed by shear, loads within a distance "d" (the depth of the beam) from the ends have been neglected. DEFLECTION LIMITS: For roof beams, deflection is limited to span \(/ 180\) for total load.
CONTROLLING VALUES: Values marked with a D are controlled by deflection, B are bending controlled, and S are shear controlled.
SPAN: Span is defined as the length from centerline to centerline of bearing. This span is the length used in standard engineering equations to calculate deflection, bending and shear.
*The values have been limited to reasonable capacities. Engineering calculations may allow for greater capacities.
Feb-2001
While these capacity tables have been prepared in accordance with recognized engineering principles and are based on the most accurate and reliable technical data available, these tables should not be used or relied upon for any general or specific application without competent professional examination and verification of their accuracy, suitability, and applicability by a licensed professional engineer, designer, or architect. AITC MAKES NO REPRESENTATION OR WARRANTY, EXPRESSED OR IMPLIED, THAT THE INFORMATION CONTAINED HEREIN IS SUITABLE FOR ANY GENERAL OR SPECIFIC USE OR IS FREE FROM INFRINGEMENT OF ANY PATENT OR COPYRIGHT. ANY USER OF THIS INFORMATION ASSUMES ALL RISK AND LIABILITY ARISING FROM SUCH USE.

The self weight should be determined to compare to the assumption. Table DF-25 indicates the self weight is \(13 \mathrm{lb} / \mathrm{ft}\), and that size at our span is controlled by deflection (I for \(\Delta=\mathrm{L} / 180\) ), but this chart is for simply supported beams and \(\Delta_{\max }=\frac{5 w L^{4}}{384 E I}\).
The maximum deflection for a two span beam can be found with \(\Delta_{\max }=\frac{w L^{4}}{185 E I}\), which is only \(0.415 x\) the deflection of a simply supported span.

For sawn lumber, a 6x14 would be required from the comparison chart.

Evaluate shear strength:
\[
\begin{aligned}
& \mathrm{F}_{\mathrm{v}}^{\prime}=\mathrm{C}_{\mathrm{D}} \mathrm{~F}_{\mathrm{v}}=(1.25) 240 \mathrm{psi}=300 \mathrm{psi} \\
& f_{v}=\frac{3 V}{2 A}=\frac{3(5678 \mathrm{lb})}{2\left(53.8 \mathrm{in}^{2}\right)}=158 \mathrm{psi}
\end{aligned}
\]
which is less than the allowable of \(300 \mathrm{psi}(\mathrm{OK})\).

\section*{Stud Walls \& Columns:}
\begin{tabular}{|c|c|c|c|c|}
\hline \multicolumn{5}{|l|}{Equivalent Glulam Sections for Dimension Lumber/Timber Beams} \\
\hline \multirow[t]{3}{*}{\begin{tabular}{l}
Sawn \({ }^{4}\) \\
Sections \\
Nominal
\end{tabular}} & \multicolumn{4}{|c|}{Roof Beams \({ }^{\text {1, } 2}\)} \\
\hline & \multicolumn{2}{|l|}{Select Structural} & \multicolumn{2}{|l|}{No. 1} \\
\hline & Douglas & Southern & Douglas & Southern \\
\hline Size & Fir/Larch & Pine & FirlLarch & Pine \\
\hline \(3 \times 8\) & \(31 / 8 \times 6\) & \(3 \times 6{ }^{7 / 8}\) & \(31 / 2 \times 6\) & \(3 \times 51 / 2\) \\
\hline \(3 \times 10\) & \(31 / 8 \times 7^{1 / 2}\) & \(3 \times 81 / 4\) & \(31 / 8 \times 6\) & \(3 \times 67 / 8\) \\
\hline \(3 \times 12\) & 31/8x9 & \(3 \times 95 / 8\) & \(31 / 8 \times 7{ }^{1 / 2}\) & \(3 \times 81 / 4\) \\
\hline \(3 \times 14\) & 31/8x9 & \(3 \times 11\) & \(3^{1 / 8 \times 77^{1 / 2}}\) & \(3 \times 9^{5 / 8}\) \\
\hline \(4 \times 6\) & \(31 / 8 \times 6\) & \(3 \times 6{ }^{7 / 8}\) & \(31 / 8 \times 6\) & \(3 \times 51 / 2\) \\
\hline \(4 \times 8\) & 31/8x \({ }^{1 / 1 / 2}\) & \(3 \times 81 / 4\) & \(31 / \mathrm{x} \times 6\) & \(3 \times 67 / 8\) \\
\hline \(4 \times 10\) & \(31 / 8 \times 9\) & \(3 \times 11\) & \(3^{1 / 8 \times 7^{1 / 2}}\) & \(3 \times 81 / 4\) \\
\hline \(4 \times 12\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 12^{3 / 8}\) & \(31 / \mathrm{sx} 9\) & \(3 \times 95 / 8\) \\
\hline \(4 \times 14\) & \(31 / 8 \times 12\) & \(3 \times 13^{3 / 4}\) & \(31 / 8 \times 10^{1 / 2}\) & \(3 \times 11\) \\
\hline \(4 \times 16\) & \(3^{1 / 8 \times 13^{1 / 2}}\) & \(3 \times 151 / 8\) & \(3^{1 / 8 \times 101 / 2}\) & \(3 \times 12^{3 / 8}\) \\
\hline \(6 \times 8\) & \(51 / 8 \times 71 / 2\) & \(5 \times 67 / 8\) & \(51 / 8 \times 71 / 2\) & \(5 \times 67 / 8\) \\
\hline \(6 \times 10\) & 51/8x9 & \(5 \times 81 / 4\) & \(51 / 8 \times 71 / 2\) & \(5 \times 81 / 4\) \\
\hline \(6 \times 12\) & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 95 / 8\) & \(51 / 8 \times 9\) & \(5 \times 9\) 5/8 \\
\hline \(6 \times 14\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) & \(51 / 8 \times 10^{1 / 2}\) & \(5 \times 11\) \\
\hline \(6 \times 16\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) & \(51 / 8 \times 12\) & \(5 \times 12^{3 / 8}\) \\
\hline \(6 \times 18\) & \(51 / 8 \times 15\) & \(5 \times 151 / 8\) & \(51 / 8 \times 13^{1 / 2}\) & \(5 \times 13^{3 / 4}\) \\
\hline \(6 \times 20\) & \(51 / 8 \times 18\) & \(5 \times 161 / 2\) & \(51 / 8 \times 161 / 2\) & \(5 \times 151 / 8\) \\
\hline
\end{tabular}

Building codes dictate the maximum height for slenderness ( 10 ft typical), and the spacing of wall studs depending on what they support (roof, roof and one floor, roof and two floors). Structural design focuses on shear wall behavior.

The interior column load is:
\[
\mathrm{P}=1.25 \mathrm{wL}=1.25(545 \mathrm{lb} / \mathrm{ft}+2 \mathrm{lb} / \mathrm{ft} \text { of extra beam self weight })(16.67 \mathrm{ft})=11.4^{\mathrm{kips}}
\]

For a 10 ft braced column height, choose a \(6 \times 6\).

TABLE 10.1 Safe Loads for Wood Columns \({ }^{a}\)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{Column Section} & \multicolumn{11}{|c|}{Unbraced Length (ft)} \\
\hline Nominal Size & Area
(in.2) & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 & 22 & 24 & 26 \\
\hline \(4 \times 4\) & 12.25 & 11.1 & 7.28 & 4.94 & 3.50 & 2.63 & & & & & & \\
\hline \(4 \times 6\) & 19.25 & 17.4 & 11.4 & 7.76 & 5.51 & 4.14 & & & & & & \\
\hline \(4 \times 8\) & 25.375 & 22.9 & 15.1 & 10.2 & 7.26 & 6.46 & & & & & & \\
\hline \(6 \times 6\) & 30.25 & 27.6 & 24.8 & 20.9 & 16.9 & 13.4 & 10.7 & 8.71 & 7.17 & 6.53 & & \\
\hline \(6 \times 8\) & 41.25 & 37.6 & 33.9 & 28.5 & 23.1 & 18.3 & 14.6 & 11.9 & 9.78 & 8.91 & & \\
\hline \(6 \times 10\) & 52.25 & 47.6 & 43.0 & 36.1 & 29.2 & 23.1 & 18.5 & 15.0 & 13.4 & 11.3 & & \\
\hline \(8 \times 8\) & 56.25 & 54.0 & 51.5 & 48.1 & 43.5 & 38.0 & 32.3 & 27.4 & 23.1 & 19.7 & 16.9 & 14.6 \\
\hline \(8 \times 10\) & 71.25 & 68.4 & 65.3 & 61.0 & 55.1 & 48.1 & 41.0 & 34.7 & 29.3 & 24.9 & 21.4 & 18.4 \\
\hline \(8 \times 12\) & 86.25 & 82.8 & 79.0 & 73.8 & 66.7 & 58.2 & 49.6 & 42.0 & 35.4 & 30.2 & 26.0 & 22.3 \\
\hline \(10 \times 10\) & 90.25 & 88.4 & 85.9 & 83.0 & 79.0 & 73.6 & 67.0 & 60.0 & 52.9 & 46.4 & 40.4 & 35.5 \\
\hline \(10 \times 12\) & 109.25 & 107 & 104 & 100 & 95.6 & 89.1 & 81.2 & 72.6 & 64.0 & 56.1 & 48.9 & 42.9 \\
\hline \(10 \times 14\) & 128.25 & 126 & 122 & 118 & 112 & 105 & 95.3 & 85.3 & 75.1 & 65.9 & 57.5 & 50.4 \\
\hline \(12 \times 12\) & 132.25 & 130 & 128 & 125 & 122 & 117 & 111 & 104 & 95.6 & 86.9 & 78.3 & 70.2 \\
\hline \(14 \times 14\) & 182.25 & 180 & 178 & 176 & 172 & 168 & 163 & 156 & 148 & 139 & 129 & 119 \\
\hline \(16 \times 16\) & 240.25 & 238 & 236 & 234 & 230 & 226 & 222 & 216 & 208 & 200 & 190 & 179 \\
\hline
\end{tabular}
\({ }^{a}\) Load capacity in kips for solid-sawn sections of No. 1 grade Douglas fir-larch under normal moisture and load duration conditions.

\section*{Wind Design:}

Diaphragms are categorized as flexible or rigid and must resist lateral forces in both transverse and longitudinal directions. A diaphragm is made up of a shear-resisting element (sheathing) and boundary members called chords and collectors (struts or drag struts). The chords are designed to carry the moment in the diaphragm. The collectors are designed to transmit the horizontal reactions to the shear walls.


Figure 15.6 Flanged and webbed beam analogy for a horizontal, wood-framed diaphragm. The structural behavior is often compared to that of a steel I section on its side (Figure 15.6).

Tables in building codes for combinations of plywood grade, common nail size, plywood thickness, how the panels are arrayed and if blocking is used provide allowable shear in pounds per foot.

Consideration of lateral wind loads will be presented, but uplift on the roof must be accounted for with anchorage if the live load exceeds the downward gravity loads.
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{8}{*}{PANEL GRADE} & \multirow[b]{8}{*}{\begin{tabular}{l}
COMMON \\
MAIL SLZE
\end{tabular}} & \multirow[b]{7}{*}{minimum NALL penetration IN FRAMING (Inches)} & \multirow[b]{7}{*}{minimum NOMINAL PANEL THICKNES s (Inches)} & \multirow[b]{7}{*}{Minimum NOMINAL WIDTH OF FRAMING MEMBER (Inches)} & \multicolumn{4}{|c|}{BLOCKED DIAPHRAGMS} & \multicolumn{2}{|l|}{UNBLOCKED DIAPHRAGMS} \\
\hline & & & & & \multicolumn{4}{|l|}{Nail spacing ( n .) at diaphragm boundaries (all cases), at continuous panel edges parallel to load (Cases 3 and 4) and at all panel edges (Cases 5 and 6)} & \multicolumn{2}{|l|}{ at supported edges} \\
\hline & & & & & & & & & & \\
\hline & & & & & 6 & 4 & \(2^{1 / 2}{ }^{2}\) & \(2^{2}\) & & \\
\hline & & & & & & Ing ( l & er pana & & Case 1 (No unblocked edges & All oth \\
\hline & & & & & & & & & or continuous & configurations \\
\hline & & & & & 6 & 6 & 4 & 3 & cinse
losd) & (Cases 2, 5 and 6) \({ }^{\text {a }}\) \\
\hline & & \multicolumn{3}{|c|}{\(\times 25.4\) for mm} & \multicolumn{6}{|c|}{\(\times 0.0146\) for \(\mathrm{N} / \mathrm{mm}\)} \\
\hline \multirow{3}{*}{Structural 1} & 6 d & \(1^{1 / 4}\) & 5/16 & 2
3 & \[
\begin{aligned}
& 185 \\
& 210
\end{aligned}
\] & \[
\begin{aligned}
& 250 \\
& 280
\end{aligned}
\] & \[
\begin{aligned}
& 375 \\
& 420
\end{aligned}
\] & \[
\begin{aligned}
& 420 \\
& 475
\end{aligned}
\] & \[
\begin{aligned}
& 165 \\
& 185
\end{aligned}
\] & \[
\begin{aligned}
& 125 \\
& 140
\end{aligned}
\] \\
\hline & 8d & \(1^{1 / 2}\) & \(3 / 8\) & 2 & \[
\begin{aligned}
& 270 \\
& 300
\end{aligned}
\] & \[
\begin{aligned}
& 360 \\
& 400
\end{aligned}
\] & \[
\begin{aligned}
& 530 \\
& 600
\end{aligned}
\] & \[
\begin{aligned}
& 600 \\
& 675
\end{aligned}
\] & \[
\begin{aligned}
& 240 \\
& 265
\end{aligned}
\] & \[
\begin{aligned}
& 180 \\
& 200
\end{aligned}
\] \\
\hline & \(10 \mathrm{~d}^{3}\) & \(1^{5 / 8}\) & 15/32 & 2 & \[
\begin{aligned}
& 320 \\
& 360
\end{aligned}
\] & \[
\begin{aligned}
& 425 \\
& 480
\end{aligned}
\] & \[
\begin{aligned}
& 640 \\
& 720
\end{aligned}
\] & \[
\begin{aligned}
& 730 \\
& 820
\end{aligned}
\] & \[
\begin{aligned}
& 285 \\
& 320
\end{aligned}
\] & \[
\begin{aligned}
& 215 \\
& 240
\end{aligned}
\] \\
\hline \multirow{7}{*}{C-D, C-C, Sheathing, and other grades covered in UBC Standard 23-2 or 23-3} & \multirow[t]{2}{*}{6d} & \multirow[t]{2}{*}{\(1^{1 / 4}\)} & 5/16 & 2 & \[
\begin{aligned}
& 170 \\
& 190
\end{aligned}
\] & \[
\begin{aligned}
& 225 \\
& 250
\end{aligned}
\] & \[
\begin{aligned}
& 335 \\
& 380
\end{aligned}
\] & \[
\begin{aligned}
& 380 \\
& 430
\end{aligned}
\] & \[
\begin{aligned}
& 150 \\
& 170
\end{aligned}
\] & \[
\begin{aligned}
& 110 \\
& 125
\end{aligned}
\] \\
\hline & & & \(3 / 8\) & 2
3 & \[
\begin{aligned}
& 185 \\
& 210
\end{aligned}
\] & \[
\begin{aligned}
& 250 \\
& 280
\end{aligned}
\] & \[
\begin{aligned}
& 375 \\
& 420
\end{aligned}
\] & \[
\begin{aligned}
& 420 \\
& 475
\end{aligned}
\] & \[
\begin{aligned}
& 165 \\
& 185
\end{aligned}
\] & \[
\begin{aligned}
& 125 \\
& 140
\end{aligned}
\] \\
\hline & \multirow{3}{*}{8d} & \multirow{3}{*}{\(1^{1 / 2}\)} & \(3 / 8\) & 2 & \[
\begin{aligned}
& 240 \\
& 27, j
\end{aligned}
\] & \[
\begin{aligned}
& 320 \\
& 360
\end{aligned}
\] & \[
\begin{aligned}
& 480 \\
& 540
\end{aligned}
\] & \[
\begin{aligned}
& 545 \\
& 610
\end{aligned}
\] & \[
\begin{aligned}
& 215 \\
& 240
\end{aligned}
\] & \[
\begin{aligned}
& 160 \\
& 180
\end{aligned}
\] \\
\hline & & & 7/16 & \(\frac{2}{3}\) & \[
\begin{aligned}
& 255 \\
& 285
\end{aligned}
\] & \[
\begin{aligned}
& 340 \\
& 380
\end{aligned}
\] & \[
\begin{aligned}
& 505 \\
& 570
\end{aligned}
\] & \[
\begin{aligned}
& 575 \\
& 645
\end{aligned}
\] & \[
\frac{230}{255}
\] & \[
\begin{aligned}
& 170 \\
& 190
\end{aligned}
\] \\
\hline & & & 15/32 & 2 & \[
\begin{aligned}
& 270 \\
& 300
\end{aligned}
\] & \[
\begin{aligned}
& 360 \\
& 400
\end{aligned}
\] & \[
\begin{aligned}
& 530 \\
& 600
\end{aligned}
\] & \[
\begin{aligned}
& 600 \\
& 675
\end{aligned}
\] & \[
\begin{aligned}
& 240 \\
& 265
\end{aligned}
\] & \[
\begin{aligned}
& 180 \\
& 200
\end{aligned}
\] \\
\hline & \multirow[t]{2}{*}{\(10 d^{3}\)} & \multirow[t]{2}{*}{\(1^{5 / 8}\)} & 15/32 & 2 & \[
\begin{aligned}
& 290 \\
& 325
\end{aligned}
\] & \[
\begin{aligned}
& 385 \\
& 430
\end{aligned}
\] & \[
\begin{aligned}
& 575 \\
& 650
\end{aligned}
\] & \[
\begin{aligned}
& 655 \\
& 735
\end{aligned}
\] & \[
\begin{aligned}
& 255 \\
& 290
\end{aligned}
\] & \[
\begin{aligned}
& 190 \\
& 215
\end{aligned}
\] \\
\hline & & & 19/32 & 2
3 & \[
\begin{aligned}
& 320 \\
& 360
\end{aligned}
\] & \[
\begin{aligned}
& 425 \\
& 480
\end{aligned}
\] & \[
\begin{aligned}
& 640 \\
& 720
\end{aligned}
\] & \[
\begin{aligned}
& 730 \\
& 820
\end{aligned}
\] & \[
\begin{aligned}
& 285 \\
& 320
\end{aligned}
\] & \[
\begin{aligned}
& 215 \\
& 240
\end{aligned}
\] \\
\hline
\end{tabular}

\footnotetext{
\({ }^{1}\) These values are for short-time loads due to wind or earthquake and must be reduced 25 percent for normal loading. Space nails 12 inches ( 305 mm ) on center along intermediate framing members.
Allowable shear values for nails in framing members of other species set forth in Division III, Part III, shall be calculated for all other grades by multiplying the shear capacities for nails in Structural I by the following factors: 0.82 for species with specific gravity greater than or equal to 0.42 but less than 0.49 , and 0.65 for species with a specific gravity less than 0.42 .
\({ }^{2}\) Framing at adjoining panel edges shall be 3 -inch \((76 \mathrm{~mm})\) nominal or wider and nails shall be staggered where nails are spaced 2 inches ( 51 mm ) or \(2^{1 / 2}\) inches ( 64 mm ) on center.
\({ }^{3}\) Framing at adjoining panel edges shall be 3 -inch ( 76 mm ) nominal or wider and nails shall be staggered where 10d nails having penetration into framing of more than \(1^{3} / 8\) inches ( 41 mm ) are spaced 3 inches ( 76 mm ) or less on center.
}

\section*{North-South}


Figure 16.5 Building One, wall functions and wind pressure development.
The tributary height for the wall and parapet is \(17.5 \mathrm{ft} / 2+2.5 \mathrm{ft}=11.25 \mathrm{ft}\)
The distributed lateral wind load \(=\left(20 \mathrm{lb} / \mathrm{ft}^{2}\right) 11.25 \mathrm{ft}=225 \mathrm{lb} / \mathrm{ft}\)
The total lateral wind load \(=(225 \mathrm{lb} / \mathrm{ft})(100 \mathrm{ft})=22,500 \mathrm{lb}\)


Case 2
parapet cantilevered from roof
(a) Wall funcions for wind

The unit shear (or distributed shear) in the diaphragm \(=11,250 \mathrm{lb} /(50 \mathrm{ft})=225 \mathrm{lb} / \mathrm{ft}\);
so a roof deck can be chosen that has an allowable shear > \(225 \mathrm{lb} / \mathrm{ft}\).

Knowing that \(1 / 2\) in decking is the minimum for a membrane-type roof, we use table \(23-\mathrm{II}-\mathrm{H}\) to select \(15 / 32\) in. sheathing with 2 x framing and 8 d nails at 6 in. at all panel edges and a blocked diaphragm having an allowable shear in pounds per foot of \(270 \mathrm{lb} / \mathrm{ft}\).

The moment of the "deep beam" is used to determine the force in the top and bottom chords as show in Figure 16.6 which is 5.62 kips.

The unit shear in the two shear walls of 21 ft each
\[
=11,250 \mathrm{lb} /(2 \cdot 21 \mathrm{ft})=268 \mathrm{lb} / \mathrm{ft} ;
\]
so a stud wall can be chosen that has an allowable shear > \(268 \mathrm{lb} / \mathrm{ft}\).

Using table 23 -II-I-1, \(3 / 8\) in. plywood sheathing with 6 d nails at 4 in . at all panel edges directly applied to


Figure 16.6 Spanning functions of the roof diaphragm. framing (not over gypsum sheathing) has an allowable shear in pounds per foot of \(300 \mathrm{lb} / \mathrm{ft}\).

TABLE \(23-\mathrm{Il}-1\)-1-ALLOWABLE SHEAR FOR WIND OR SEISMIC FORCES IN POUNDS PER FOOT FOR WOOD STRUCTURAL PANEL
SHEAR WALLS WITH FRAMING OF DOUGLAS FIR-LARCH OR SOUTHERN PINE

\({ }^{1}\) All panel edges backed with 2 -inch ( 51 mm ) nominal or wider framing. Panels installed either horizontally or vertically. Space nails at 6 inches ( 152 mm ) on center along intermediate framing members for \(3 / 8\)-inch ( 9.5 mm ) and \(7 / 16^{-\mathrm{inch}}(11 \mathrm{~mm})\) panels installed on studs spaced 24 inches ( 610 mm ) on center and 12 inches ( 305 mm ) on center for other conditions and panel thicknesses. These values are for short-time loads due to wind or earthquake and must be reduced 25 percent for normal loading.
Allowable shear values for nails in framing members of other species set forth in Division III, Part III, shall be calculated for all other grades by multiplying the shear capacities for nails in Structural I by the following factors: 0.82 for species with specific gravity greater than or equal to 0.42 but less than 0.49 , and 0.65 for species with a specific gravity less than 0.42 .
\({ }^{2}\) Where panels are applied on both faces of a wall and nail spacing is less than 6 inches \((152 \mathrm{~mm})\) on center on either side, panel joints shall be offset to fall on different framing members or framing shall be 3 -inch ( 76 mm ) nominal or thicker and nails on each side shall be staggered.
\({ }^{3}\) Where allowable shear values exceed 350 pounds per foot ( \(5.11 \mathrm{~N} / \mathrm{mm}\) ), foundation sill plates and all framing members receiving edge nailing from abutting panels shall not be less than a single 3 -inch \((76 \mathrm{~mm})\) nominal member. Nails shall be staggered.
\({ }^{4}\) The values for \(3 / 8\)-inch ( 9.5 mm ) and \({ }^{7} / 16\)-inch ( 11 mm ) panels applied direct to framing may be increased to values shown for \(15 / 32\)-inch ( 12 mm ) panels, provided studs are spaced a maximum of 16 inches ( 406 mm ) on center or panels are applied with long dimension across studs.
\({ }^{5}\) Galvanized nails shall be hot-dipped or tumbled.

Wall overturning must be considered from the shear and compared to the resisting moment from gravity loads and proper anchorage must be provided to keep the wall from sliding off the foundation. Referring to Figure 16.7:
\[
\mathrm{V}=11.25 \mathrm{k} / 2=5.625 \mathrm{k}
\]

Roof dead load is determined from a tributary area of half a rafter spacing width, one rafter, and the wall length
\[
\text { roof } \mathrm{DL}=\left(14 \mathrm{lb} / \mathrm{ft}^{2} \cdot 16 \mathrm{in} / 12 \mathrm{in} / \mathrm{ft} / 2+4 \mathrm{lb} / \mathrm{ft}\right) 21 \mathrm{ft}=280 \mathrm{lb}
\]

Wall dead load can be determined with the material weights for stud walls, sheathing, gypsum board and wood shingles:
\[
\text { wall } \mathrm{DL}=\left(2 \mathrm{lb} / \mathrm{ft}^{2}+3 \mathrm{lb} / \mathrm{ft}^{2}+5 \mathrm{lb} / \mathrm{ft}^{2}+2 \mathrm{lb} / \mathrm{ft}^{2}\right)(21 \mathrm{ft})(17 \mathrm{ft})=4.3 \mathrm{k}
\]

overturning moment \(=(5.625 \mathrm{k})(17 \mathrm{ft})=95.6 \mathrm{k}-\mathrm{ft}\)
resisting moment \(=(4.6 \mathrm{k})(21 \mathrm{ft}) / 2=48.4 \mathrm{k}-\mathrm{ft}\)


Figure 16.7


The resisting moment is not enough to compensate for the overturning moment. We like the factor of safety for overturning to be 1.5 , and there is no safety in this case, which means we must provide a tie down in tension (T). The L shape of the corner will help some resisting overturning, as well as the glulam beam reaction.

For equilibrium of moments \((\) positive \(=\) negative \()\)
\[
S F=\frac{M_{\text {resist }}}{M_{\text {overturning }}} \geq 1.5
\]
\[
\mathrm{T}(21 \mathrm{ft})+48.4 \mathrm{k}-\mathrm{ft}=(95.6 \mathrm{k}-\mathrm{ft}) 1.5 ; \quad \mathrm{T}_{\mathrm{req}{ }^{\prime d}}=4.52 \mathrm{k}
\]

The shear must be resisted, and the code minimum bolting usually consists of \(1 / 2\) in. diameter bolts at 1 ft from the wall ends and at a maximum of 6 ft on center for the remainder of the wall length. If design for wind loading allows us to increase the allowable stress by \(1 / 3\), the number of bolts from single shear in a \(2 "\) sill plate parallel to the grain will be:
(1.33)(480 lb/bolt)(n) \(\geq 5,625 \mathrm{lb}\)

TABLE 11.1 Bolt Design Values for Wood Joints with Douglas Fir-Larch (lb/bolt)
\(\mathrm{n} \geq 8.8\) bolts
Use 9 bolts, spaced at 2.375 ft (see next page for description of design value symbols)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{THICKNESS} & \(\sim\) & \multicolumn{6}{|c|}{\multirow[b]{2}{*}{DOUGLAS FIR-LARCH}} \\
\hline \multirow[t]{2}{*}{\[
\frac{z}{\sum} \frac{\frac{\sim}{\|}}{\sum} \sum_{i=1}^{n}
\]} & \multirow[t]{2}{*}{\[
\frac{\omega}{\omega} \sum_{i=1}^{\infty}
\]} & \multirow[t]{2}{*}{} & & & & & & \\
\hline & & & \multicolumn{3}{|r|}{SINGLE SHEAR} & \multicolumn{3}{|l|}{DOUBLE SHEAR} \\
\hline \(\mathrm{t}_{\mathrm{m}}\) & \(t_{s}\) & D & \(\mathrm{Z}_{\|}\) & \(\mathrm{Z}_{\text {si }}\) & \(\mathrm{Z}_{\mathrm{mL}}\) & \(\mathrm{Z}_{1}\) & \(\mathrm{Z}_{\text {s }}\) & \(\mathrm{Z}_{\mathrm{m} \perp}\) \\
\hline inches & inches & inches & lbs. & lbs. & lbs. & lbs. & lbs. & lbs. \\
\hline \multirow{5}{*}{1-1/2} & \multirow{5}{*}{1-1/2} & 1/2 & 480 & 300 & 300 & 1050 & 730 & 470 \\
\hline & & 5/8 & 600 & 360 & 360 & 1310 & 1040 & 530 \\
\hline & & 3/4 & 720 & 420 & 420 & 1580 & 1170 & 590 \\
\hline & & 7/8 & 850 & 470 & 470 & 1840 & 1260 & 630 \\
\hline & & 1 & 970 & 530 & 530 & 2100 & 1350 & 680 \\
\hline
\end{tabular}
\(\mathrm{Z}_{\|}=\)nominal lateral design value for single bolt in connection with all wood members loaded parallel to grain
\(\mathrm{Z}_{\mathrm{s} \perp}=\) nominal lateral design value for single bolt in wood-to-wood connection with main member loaded parallel to grain and side member loaded perpendicular to grain
\(\mathrm{Z}_{\mathrm{m} \perp}=\) nominal lateral design value for single bolt in wood-to-wood connection with main member loaded parallel to grain and side member loaded perpendicular to grain and side member loaded parallel to grain

\section*{East-West}


Figure 16.5 Building One, wall functions and wind pressure development.
The tributary height for the wall and parapet and the distributed lateral wind load are the same as in the North-South direction.

The total lateral wind load \(=(225 \mathrm{lb} / \mathrm{ft})(50 \mathrm{ft})=11,250 \mathrm{lb}\)
The end reactions to the lateral load \(=11,250 \mathrm{lb} / 2=5,625 \mathrm{lb}\)
The unit shear (or distributed shear) in the diaphragm \(=5,625 \mathrm{lb} /(100 \mathrm{ft})=56.25 \mathrm{lb} / \mathrm{ft}\).
It is convenient to use the diaphragm structural panel construction chosen in the NorthSouth direction with a capacity of \(270 \mathrm{lb} / \mathrm{ft}\).

The unit shear (or distributed shear) in the five shear walls of 10.67 ft each:
\(=5,625 \mathrm{lb} /(5 \cdot 10.67 \mathrm{ft})=105 \mathrm{lb} / \mathrm{ft}\).
It is convenient to use the shear wall structural panel construction chosen in the NorthSouth direction with a capacity of \(300 \mathrm{lb} / \mathrm{ft}\).

\section*{Steel Design}
\begin{tabular}{|c|c|}
\hline \multicolumn{2}{|l|}{Notation:} \\
\hline \(a \quad=\) name for width dimension & \(F_{e}=\) elastic critical buckling stress \\
\hline \(A \quad=\) name for area & \(F_{p}=\) allowable bearing stress \\
\hline \(A_{g}=\) gross area, equal to the total area ignoring any holes & \[
\begin{aligned}
& F_{u}=\text { ultimate stress prior to failure } \\
& F_{y}=\text { yield strength }
\end{aligned}
\] \\
\hline \(A_{\text {req'd-adj }}=\) area required at allowable stress when shear is adjusted to include self weight & \[
\begin{array}{ll}
F_{y w} & =\text { yield strength of web material } \\
h & =\text { name for a height } \\
h_{c} & =\text { height of the web of a wide flange }
\end{array}
\] \\
\hline \(A_{w}=\) area of the web of a wide flange section, as is \(A_{\text {web }}\) & \[
H=\begin{array}{ll} 
& \text { steel section } \\
= & \text { shorthand for lateral pressure load }
\end{array}
\] \\
\hline \(A I S C=\) American Institute of Steel Construction & \(I \quad=\) moment of inertia with respect to neutral axis bending \\
\hline \(A S D=\) allowable stress design & \(I_{y} \quad=\) moment of inertia about the y axis \\
\hline \(b \quad=\) name for a (base) width & \(J=\) polar moment of inertia \\
\hline = name for height dimension & \(k \quad=\) distance from outer face of W \\
\hline \(b_{f} \quad=\) width of the flange of a steel beam cross section & flange to the web toe of fillet \(=\) shape factor for plastic design of \\
\hline \(B \quad=\) width of a column base plate & steel beams \\
\hline \(B_{1} \quad=\) factor for determining \(M_{u}\) for combined bending and compression & \[
\begin{aligned}
K \quad= & \text { effective length factor for columns, } \\
& \text { as is } k
\end{aligned}
\] \\
\hline \(=\) largest distance from the neutral axis to the top or bottom edge of a beam. as is \(c_{\text {max }}\) & \[
\begin{array}{ll}
l & =\text { name for length, as is } L \\
& =\text { column base plate design variable } \\
L & =\text { name for length or span length, as is }
\end{array}
\] \\
\hline \(c_{1}=\) coefficient for shear stress for a rectangular bar in torsion & \[
\begin{aligned}
& l \\
= & \text { shorthand for live load }
\end{aligned}
\] \\
\hline \(C_{b} \quad=\) modification factor for moment in ASD \& LRFD steel beam design & \(L_{b} \quad=\) unbraced length of a steel beam in LRFD design \\
\hline \(C_{m}=\) modification factor accounting for combined stress in steel design & \(L_{e} \quad=\) effective length that can buckle for column design, as is \(\ell_{e}\) \\
\hline \(C_{v} \quad=\) web shear coefficient & \(L_{r} \quad=\) shorthand for live roof load \\
\hline \[
\begin{aligned}
d \quad & =\text { name for depth } \\
& =\text { depth of a wide flange section }
\end{aligned}
\] & \(=\) maximum unbraced length of a steel beam in LRFD design for \\
\hline \(D \quad=\) shorthand for dead load & inelastic lateral-torsional buckling \\
\hline \(D L=\) shorthand for dead load & \(L_{p} \quad=\) maximum unbraced length of a \\
\hline \[
\begin{aligned}
E \quad & =\text { shorthand for earthquake load } \\
& =\text { modulus of elasticity }
\end{aligned}
\] & steel beam in LRFD design for full plastic flexural strength \\
\hline \(f_{a}=\) axial stress & \(L L=\) shorthand for live load \\
\hline \(f_{b} \quad=\) bending stress & \(L R F D=\) load and resistance factor design \\
\hline \(f_{p} \quad=\) bearing stress & \(m \quad=\) edge distance for a column base \\
\hline \(f_{v} \quad=\) shear stress & plate \\
\hline \(f_{v-\text { max }}=\) maximum shear stress & \(M=\) internal bending moment \\
\hline \(f_{y} \quad=\) yield stress & \(M_{a}=\) required bending moment (ASD) \\
\hline \(F \quad=\) shorthand for fluid load & \(M_{\text {max }}=\) maximum internal bending moment \\
\hline \(F_{a}=\) allowable axial (compressive) stress & \(M_{\text {max-adj }}=\) maximum bending moment \\
\hline \(F_{b}=\) allowable bending stress & adjusted to include self weight \\
\hline \(F_{c r}=\) flexural buckling stress & \\
\hline
\end{tabular}


\section*{Steel Design}

Structural design standards for steel are established by the Manual of Steel Construction published by the American Institute of Steel Construction, and uses Allowable Stress Design and Load and Factor Resistance Design. The \(13^{\text {th }}\) edition combines both methods in one volume and provides common requirements for analyses and design and requires the application of the same set of specifications.


\section*{Materials}

American Society for Testing Materials (ASTM) is the organization responsible for material and other standards related to manufacturing. Materials meeting their standards are guaranteed to have the published strength and material properties for a designation.

A36 - carbon steel used for plates, angles
A572 - high strength low-alloy used for some beams
A992 - for building framing used for most beams
(A572 Grade 60 has the same properties as A992)
\[
\begin{aligned}
& \mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}, \mathrm{~F}_{\mathrm{u}}=58 \mathrm{ksi}, \mathrm{E}=29,000 \mathrm{ksi} \\
& \mathrm{~F}_{\mathrm{y}}=60 \mathrm{ksi}, \mathrm{~F}_{\mathrm{u}}=75 \mathrm{ksi}, \mathrm{E}=30,000 \mathrm{ksi} \\
& \mathrm{~F}_{\mathrm{y}}=50 \mathrm{ksi}, \mathrm{~F}_{\mathrm{u}}=65 \mathrm{ksi}, \mathrm{E}=30,000 \mathrm{ksi}
\end{aligned}
\]

ASD
\[
R_{a} \leq R_{n} / \Omega
\]
where \(\quad R_{a}=\) required strength (dead or live; force, moment or stress)
\(\mathrm{R}_{\mathrm{n}}=\) nominal strength specified for ASD
\(\Omega=\) safety factor
Factors of Safety are applied to the limit stresses for allowable stress values:
\[
\begin{array}{ll}
\text { bending (braced, } \mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}} \text { ) } & \Omega=1.67 \\
\text { bending (unbraced, } \left.\mathrm{L}_{\mathrm{p}}<\mathrm{L}_{\mathrm{b}} \text { and } \mathrm{L}_{\mathrm{b}}>\mathrm{L}_{\mathrm{r}}\right) & \Omega=1.67 \text { (nominal moment reduces) } \\
\text { shear (beams) } & \Omega=1.67 \\
\text { shear (bolts) } & \Omega=2.00 \text { (tabular nominal strength) } \\
\text { shear (welds) } & \Omega=2.00
\end{array}
\]
- \(\mathrm{L}_{\mathrm{b}}\) is the unbraced length between bracing points, laterally
- \(\quad \mathrm{L}_{\mathrm{p}}\) is the limiting laterally unbraced length for the limit state of yielding
- \(\mathrm{L}_{\mathrm{r}}\) is the limiting laterally unbraced length for the limit state of inelastic lateral-torsional buckling

LRFD
\[
\begin{aligned}
& R_{u} \leq \phi R_{n} \\
& \text { where } \cdots R_{u}=\Sigma \gamma_{i} R_{i} \\
& \phi=\text { resistance factor } \\
& \gamma=\text { load factor for the type of load } \\
& \mathrm{R}=\text { load (dead or live; force, moment or stress) } \\
& \mathrm{R}_{\mathrm{u}}=\text { factored load (moment or stress) } \\
& \mathrm{R}_{\mathrm{n}}=\text { nominal load (ultimate capacity; force, moment or stress) }
\end{aligned}
\]

Nominal strength is defined as the
capacity of a structure or component to resist the effects of loads, as determined by computations using specified material strengths (such as yield strength, \(\mathrm{F}_{\mathrm{y}}\), or ultimate strength, \(\mathrm{F}_{\mathrm{u}}\) ) and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions

\section*{Factored Load Combinations}

The design strength, \(\phi R_{n}\), of each structural element or structural assembly must equal or exceed the design strength based on the ASCE-7 combinations of factored nominal loads:
\[
\begin{aligned}
& 1.4(D+F) \\
& 1.2(D+F)+1.6(L+H)+0.5\left(L_{r} \text { or } S \text { or } R\right) \\
& 1.2 D+1.6\left(L_{r} \text { or } S \text { or } R\right)+(L \text { or } 0.8 W) \\
& 1.2 D+1.6 W+L+0.5\left(L_{r} \text { or } S \text { or } R\right) \\
& 1.2 D+1.0 E+L+0.2 S \\
& 0.9 D+1.6 W+1.6 \mathrm{H} \\
& 0.9 D+1.0 E+1.6 \mathrm{H}
\end{aligned}
\]

\section*{Criteria for Design of Beams}

Allowable normal stress or normal stress from LRFD should not be exceeded:
\[
\begin{gathered}
F_{b} \text { or } \phi F_{n} \geq f_{b}=\frac{M c}{I} \\
\left(M_{a} \leq M_{n} / \Omega \text { or } M_{u} \leq \phi_{b} M_{n}\right)
\end{gathered}
\]

Knowing M and \(\mathrm{F}_{\mathrm{b}}\), the minimum section modulus fitting the limit is:
\[
Z_{r e q^{\prime} d} \geq \frac{M_{a}}{F_{y} \Omega} \quad\left(S_{r e q^{\prime} d} \geq \frac{M}{F_{b}}\right)
\]

Besides strength, we also need to be concerned about serviceability. This involves things like limiting deflections \& cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

\section*{Determining Maximum Bending Moment}

Drawing V and M diagrams will show us the maximum values for design. Computer applications are very helpful.

\section*{Determining Maximum Bending Stress}

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a non-prismatic member, the stress varies with the cross section AND the moment.

\section*{Deflections}

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well.

Elastic curve equations can be superpositioned ONLY if the stresses are in the elastic range. The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.

\section*{Allowable Deflection Limits}

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.
\[
\Delta_{\text {actual }} \leq \Delta_{\text {allowable }}=L / v \text { alue }
\]
\begin{tabular}{|c|l|l|}
\hline \multicolumn{1}{|c|}{ Use } & \multicolumn{1}{|c|}{ LL only } & DL+LL \\
\hline Roof beams: & & \\
\hline Industrial & \(\mathrm{L} / 180\) & \(\mathrm{~L} / 120\) \\
\hline Commercial & & \\
\hline plaster ceiling & \(\mathrm{L} / 240\) & \(\mathrm{~L} / 180\) \\
\hline no plaster & \(\mathrm{L} / 360\) & \(\mathrm{~L} / 240\) \\
\hline Floor beams: & & \\
\hline Ordinary Usage & \(\mathrm{L} / 360\) & \(\mathrm{~L} / 240\) \\
\hline Roof or floor (damageable elements) & \(\mathrm{L} / 480\) \\
\hline
\end{tabular}

\section*{Lateral Buckling}

With compression stresses in the top of a beam, a sudden "popping" or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger \(\mathrm{I}_{\mathrm{y}}\).

\section*{Local Buckling in Steel I Beams- Web Crippling or Flange Buckling}


Concentrated forces on a steel beam can cause the web to buckle (called web crippling). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.


The maximum support load and interior load can be determined from:
\[
\begin{aligned}
& P_{n(\text { max -end })}=(2.5 k+N) F_{y w} t_{w} \\
& P_{n \text { (interior) }}=(5 k+N) F_{y w} t_{w}
\end{aligned}
\]
\[
\text { where } \quad \begin{aligned}
& t_{w}=\text { thickness of the web } \\
& N=\text { bearing length } \\
& k=\text { dimension to fillet found in beam section tables }
\end{aligned}
\]
\[
\phi=1.00(\mathrm{LRFD}) \quad \Omega=1.50(\mathrm{ASD})
\]

\section*{Beam Loads \& Load Tracing}

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the tributary area that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element ad infinitum, to the ground.

\section*{LRFD Bending or Flexure}

For determining the flexural design strength, \(\phi_{b} M_{n}\), for resistance to pure bending (no axial load) in most flexural members where the following conditions exist, a single calculation will suffice:
\[
\Sigma \gamma_{i} R_{i}=M_{u} \leq \phi_{b} M_{n}=0.9 F_{y} Z
\]
\[
\begin{array}{ll}
\text { where } & \mathrm{M}_{\mathrm{u}}=\text { maximum moment from factored loads } \\
& \phi_{\mathrm{b}}=\text { resistance factor for bending }=0.9
\end{array}
\]
\(\mathrm{M}_{\mathrm{n}}=\) nominal moment (ultimate capacity)
\(\mathrm{F}_{\mathrm{y}}=\) yield strength of the steel
\(\mathrm{Z}=\) plastic section modulus

\section*{Plastic Section Modulus}

Plastic behavior is characterized by a yield point and an increase in strain with no increase in stress.

\section*{Internal Moments and Plastic Hinges}


Plastic hinges can develop when all of the material in a cross section sees the yield stress. Because all the material at that section can strain without any additional load, the member segments on either side of the hinge can rotate, possibly causing instability.

For a rectangular section:
Elastic to \(f_{y}\) :
\[
M_{y}=\frac{I}{c} f_{y}=\frac{b h^{2}}{6} f_{y}=\frac{b(2 c)^{2}}{6} f_{y}=\frac{2 b c^{2}}{3} f_{y}
\]

Fully Plastic: \(\quad M_{u l t}\) or \(M_{p}=b c^{2} f_{y}=3 / 2 M_{y}\)


For a non-rectangular section and internal equilibrium at \(\sigma_{y}\), the n.a. will not necessarily be at the centroid. The n.a. occurs where the \(\mathrm{A}_{\text {tension }}=\mathrm{A}_{\text {compression }}\). The reactions occur at the centroids of the tension and compression areas.
\[
A_{\text {tension }}=A_{\text {compression }}
\]



\section*{Shape Factor:}

The ratio of the plastic moment to the elastic moment at yield:
\[
k=M_{p} / M_{y} \quad \begin{aligned}
& \mathrm{k}=3 / 2 \text { for a rectangle } \\
& \mathrm{k} \approx 1.1 \text { for an I beam }
\end{aligned}
\]

Plastic Section Modulus
\[
Z=\frac{M_{p}}{f_{y}} \quad \text { and } \quad k=Z / S
\]

Design for Shear
\[
V_{a} \leq V_{n} / \Omega \text { or } V_{u} \leq \phi_{v} V_{n}
\]

The nominal shear strength is dependent on the cross section shape. Case 1: With a thick or stiff web, the shear stress is resisted by the web of a wide flange shape (with the exception of a handful of W's). Case 2: When the web is not stiff for doubly symmetric shapes, singly symmetric shapes (like channels) (excluding round high strength steel shapes), inelastic web buckling occurs. When the web is very slender, elastic web buckling occurs, reducing the capacity even more:
1. For \(h / t_{w} \leq 2.24 \sqrt{\frac{E}{F_{y}}} \quad V_{n}=0.6 F_{y w} A_{w} \quad \phi_{\mathrm{v}}=1.00(\mathrm{LRFD}) \quad \Omega=1.50(\mathrm{ASD})\)
where \(h\) equals the clear distance between flanges less the fillet or corner radius for rolled shapes
\[
\mathrm{V}_{\mathrm{n}}=\text { nominal shear strength }
\]
\[
\mathrm{F}_{\mathrm{yw}}=\text { yield strength of the steel in the web }
\]
\[
A_{w}=t_{w} d=\text { area of the web }
\]
2. For \(h / t_{w}>2.24 \sqrt{\frac{E}{F_{y}}} \quad V_{n}=0.6 F_{y w} A_{w} C_{v} \quad \phi_{\mathrm{v}}=0.9(\mathrm{LRFD}) \quad \Omega=1.67\) (ASD)
where \(\mathrm{C}_{\mathrm{v}}\) is a reduction factor (1.0 or less by equation)

\section*{Design for Flexure}
\[
M_{a} \leq M_{n} / \Omega \text { or } M_{u} \leq \phi_{b} M_{n} \quad \phi_{\mathrm{b}}=0.90(\mathrm{LRFD}) \quad \Omega=1.67(\mathrm{ASD})
\]

The nominal flexural strength \(\mathrm{M}_{\mathrm{n}}\) is the lowest value obtained according to the limit states of
1. yielding, limited at length \(L_{p}=1.76 r_{y} \sqrt{\frac{E}{F_{y}}}\), where \(\mathrm{r}_{\mathrm{y}}\) is the radius of gyration in \(y\)
2. lateral-torsional buckling limited at length \(L_{r}\)
3. flange local buckling
4. web local buckling

Beam design charts show available moment, \(M_{n} / \Omega\) and \(\phi_{b} M_{n}\), for unbraced length, \(L_{b}\), of the compression flange in one-foot increments from 1 to 50 ft . for values of the bending coefficient \(\mathrm{C}_{\mathrm{b}}=1\). For values of \(1<\mathrm{C}_{\mathrm{b}} \leq 2.3\), the required flexural strength \(\mathrm{M}_{\mathrm{u}}\) can be reduced by dividing it by \(\mathrm{C}_{\mathrm{b}}\). \(\quad \mathrm{C}_{\mathrm{b}}=1\) when the bending moment at any point within an unbraced length is larger than that at both ends of the length. \(\mathrm{C}_{\mathrm{b}}\) of 1 is conservative and permitted to be used in any case.
When the free end is unbraced in a cantilever or overhang, \(\mathrm{C}_{\mathrm{b}}=1\). The full formula is provided below.)

NOTE: the self weight is not included in determination of \(\phi_{b} M_{n}\)

\section*{Compact Sections}

For a laterally braced compact section (one for which the plastic moment can be reached before local buckling) only the limit state of yielding is applicable. For unbraced compact beams and non-compact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable.
Compact sections meet the following criteria: \(\frac{b_{f}}{2 t_{f}} \leq 0.38 \sqrt{\frac{E}{F_{y}}}\) and \(\frac{h_{c}}{t_{w}} \leq 3.76 \sqrt{\frac{E}{F_{y}}}\) where:
\(b_{f}=\) flange width in inches
\(t_{f}=\) flange thickness in inches
\(E=\) modulus of elasticity in ksi
\(F_{y}=\) minimum yield stress in ksi
\(h_{c}=\) height of the web in inches
\(t_{w}=\) web thickness in inches

With lateral-torsional buckling the nominal flexural strength is
\[
M_{n}=C_{b}\left\lfloor M_{p}-\left(M_{p}-0.7 F_{y} S_{x}\right)\left(\frac{L_{b}-L_{p}}{L_{r}-L_{p}}\right)\right\rfloor \leq M_{p}
\]

where \(\mathrm{C}_{\mathrm{b}}\) is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:
\[
C_{b}=\frac{12.5 M_{\max }}{2.5 M_{\max }+3 M_{A}+4 M_{B}+3 M_{C}}
\]
\(\mathrm{M}_{\text {max }}=\) absolute value of the maximum moment in the unbraced beam segment \(\mathrm{M}_{\mathrm{A}}=\) absolute value of the moment at the quarter point of the unbraced beam segment \(M_{B}=\) absolute value of the moment at the center point of the unbraced beam segment \(\mathrm{M}_{\mathrm{C}}=\) absolute value of the moment at the three quarter point of the unbraced beam segment length.

\section*{Available Flexural Strength Plots}

Plots of the available moment for the unbraced length for wide flange sections are useful to find sections to satisfy the design criteria of \(M_{a} \leq M_{n} / \Omega\) or \(M_{u} \leq \phi_{b} M_{n}\). The maximum moment that can be applied on a beam (taking self weight into account), \(\mathrm{M}_{\mathrm{a}}\) or \(\mathrm{M}_{\mathrm{u}}\), can be plotted against the unbraced length, \(\mathrm{L}_{\mathrm{b}}\). The limit \(\mathrm{L}_{\mathrm{p}}\) is indicated by a solid \(\operatorname{dot}(\bullet)\), while \(\mathrm{L}_{\mathrm{r}}\) is indicated by an open dot (O). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used. \(\mathrm{C}_{\mathrm{b}}\), which is a modification factor for non-zero moments at the ends, is 1 for simply supported beams ( 0 moments at the ends). (see figure)


\section*{Design Procedure}

The intent is to find the most light weight member (which is economical) satisfying the section modulus size.
1. Determine the unbraced length to choose the limit state (yielding, lateral torsional buckling or more extreme) and the factor of safety and limiting moments. Determine the material.
2. Draw V \& M , finding \(\mathrm{V}_{\text {max }}\) and \(\mathrm{M}_{\text {max }}\).for unfactored loads (ASD, \(V_{a} \& M_{a}\) ) or from factored loads (LRFD, \(V_{u} \& M_{u}\) )
3. Calculate \(\mathrm{S}_{\text {req'd }}\) or Z when yielding is the limit state. This step is equivalent to determining if \(f_{b}=\frac{M_{\max }}{S} \leq F_{b}, S_{\text {req'd }} \geq \frac{M_{\max }}{F_{b}}=\frac{M_{\max }}{F_{y} / \Omega}\) and \(Z \geq \frac{M_{u}}{\phi_{b} F_{b}}\) to meet the design criteria that
\[
M_{a} \leq M_{n} / \Omega \text { or } M_{u} \leq \phi_{b} M_{n}
\]

If the limit state is something other than yielding, determine the nominal moment, \(\mathrm{M}_{\mathrm{n}}\), or use plots of available moment to unbraced length, \(\mathrm{L}_{\mathrm{b}}\).
4. For steel: use the section charts to find a trial S or Z and remember that the beam self weight (the second number in the section designation) will increase \(S_{\text {req'd. }}\) or \(Z\) The design charts show the lightest section within a grouping of similar S's or Z's.

TABLE 9.1 Load Factor Resistance Design Selection
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Designation} & \multirow[b]{2}{*}{\[
\begin{gathered}
Z_{x} \\
\text { in. }
\end{gathered}
\]} & \multicolumn{4}{|c|}{\(F_{y}=36 \mathrm{ksi}\)} \\
\hline & & \[
\begin{aligned}
& L_{p} \\
& \mathrm{ft}
\end{aligned}
\] & \[
\begin{aligned}
& L_{r} \\
& \mathrm{ft}
\end{aligned}
\] & \[
\begin{gathered}
M_{p} \\
\text { kip-ft }
\end{gathered}
\] & \[
\underset{\text { kip-ft }}{M_{r}}
\] \\
\hline W \(33 \times 141\) & 514 & 10.1 & 30.1 & 1,542 & 971 \\
\hline W \(30 \times 148\) & 500 & 9.50 & 30.6 & 1,500 & 945 \\
\hline W \(24 \times 162\) & 468 & 12.7 & 45.2 & 1,404 & 897 \\
\hline W \(24 \times 146\) & 418 & 12.5 & 42.0 & 1,254 & 804 \\
\hline W \(33 \times 118\) & 415 & 9.67 & 27.8 & 1,245 & 778 \\
\hline W \(30 \times 124\) & 408 & 9.29 & 28.2 & 1,224 & 769 \\
\hline W \(21 \times 147\) & 373 & 12.3 & 46.4 & 1,119 & 713 \\
\hline W \(24 \times 131\) & 370 & 12.4 & 39.3 & 1,110 & 713 \\
\hline W \(18 \times 158\) & 356 & 11.4 & 56.5 & 1,068 & 672 \\
\hline
\end{tabular}
****Determine the "updated" \(V_{\max }\) and \(M_{\max }\) including the beam self weight, and verify that the updated \(S_{\text {req'd }}\) has been met. ******
5. Evaluate horizontal shear using \(\mathrm{V}_{\text {max }}\). This step is equivalent to determining if \(f_{v} \leq F_{v}\) is satisfied to meet the design criteria that \(V_{a} \leq V_{n} / \Omega\) or \(V_{u} \leq \phi_{v} V_{n}\)
For I beams: \(\quad f_{v-\max }=\frac{3 V}{2 A} \approx \frac{V}{A_{w e b}}=\frac{V}{t_{w} d} \quad V_{n}=0.6 F_{y w} A_{w} \quad\) or \(V_{n}=0.6 F_{y w} A_{w} C_{v}\)
Others:
\[
f_{v-\max }=\frac{V Q}{I b}
\]
6. Provide adequate bearing area at supports. This step is equivalent to determining if \(f_{p}=\frac{P}{A} \leq F_{p}\) is satisfied to meet the design criteria that \(P_{a} \leq P_{n} / \Omega\) or \(P_{u} \leq \phi P_{n}\)
7. Evaluate shear due to torsion \(\quad f_{v}=\frac{T \rho}{J}\) or \(\frac{T}{c_{1} a b^{2}} \leq F_{v}\) (circular section or rectangular)
8. Evaluate the deflection to determine if \(\Delta_{\operatorname{maxLL}} \leq \Delta_{L L \text {-allowed }}\) and/or \(\Delta_{\max \text { Total }} \leq \Delta_{\text {Tota-lallowed }}\)
\(* * * *\) note: when \(\Delta_{\text {calculated }}>\Delta_{\text {limit }}, I_{\text {required }}\) can be found with: \(\quad I_{\text {req'd }} \geq \frac{\Delta_{\text {toobig }}}{\Delta_{\text {limit }}} I_{\text {trial }}\)
and \(S_{\text {req'd }}\) will be satisfied for similar self weight \(* * * * *\)

\section*{FOR ANY EVALUATION:}

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

\section*{Load Tables for Uniformly Loaded Joists \& Beams}

Tables exist for the common loading situation of uniformly distributed load. The tables either provide the safe distributed load based on bending and deflection limits, they give the allowable span for specific live and dead loads including live load deflection limits. If the load is not uniform, an equivalent uniform load can be calculated from the maximum moment equation:
\[
M_{\max }=\frac{w_{\text {equivalen }} L^{2}}{8}
\]

If the deflection limit is less, the design live load to check against allowable must be increased, ex.
\[
w_{\text {adjusted }}=w_{\text {ll-have }}\left(\frac{L / 360}{L / 400}\right) \quad \begin{aligned}
& \text { table limit } \\
& \text { wanted }
\end{aligned}
\]

\section*{Criteria for Design of Columns}

If we know the loads, we can select a section that is adequate for strength \& buckling. If we know the length, we can find the limiting load satisfying strength \& buckling.

Design for Compression
American Institute of Steel Construction (AISC) Manual \(13^{\text {th }}\) ed:
\[
\begin{aligned}
& P_{a} \leq P_{n} / \Omega \text { or } P_{u} \leq \phi_{c} P_{n} \quad \text { where } \\
& P_{u}=\Sigma \gamma_{i} P_{i}
\end{aligned}
\]

\(\gamma\) is a load factor
\(P\) is a load type
\(\phi\) is a resistance factor
\(P_{n}\) is the nominal load capacity (strength)
\[
\phi=0.90(\mathrm{LRFD}) \quad \Omega=1.67(\mathrm{ASD})
\]

For compression \(\quad P_{n}=F_{c r} A_{g}\)
where: \(\quad A_{g}\) is the cross section area and \(F_{\text {cr }}\) is the flexural buckling stress

The flexural buckling stress, \(F_{c r}\), is determined as follows:
\[
\begin{aligned}
& \text { when } \frac{K L}{r} \leq 4.71 \sqrt{\frac{E}{F_{y}}} \text { or }\left(F_{e} \geq 0.44 F_{y}\right) \text { : } \\
& \qquad F_{c r}=\left\lfloor 0.658^{\frac{F_{y}}{F_{e}}}\right\rfloor F_{y} \\
& \text { when } \frac{K L}{r}>4.71 \sqrt{\frac{E}{F_{y}}} \text { or }\left(F_{e}<0.44 F_{y}\right) \text { : } \\
& F_{c r}=0.877 F_{e}
\end{aligned}
\]
where \(F_{e}\) is the elastic critical buckling stress: \(\quad F_{e}=\frac{\pi^{2} E}{(K L / r)^{2}}\)

\section*{Design Aids}

Tables exist for the value of the flexural buckling stress based on slenderness ratio. In addition, tables are provided in the AISC Manual for Available Strength in Axial Compression based on the effective length with respect to least radius of gyration, \(r_{y}\). If the critical effective length is about the largest radius of gyration, \(r_{\mathrm{x}}\), it can be turned into an effective length about the y axis with the fraction \(r_{x} / r_{y}\).



\section*{Procedure for Analysis}
1. Calculate \(\mathrm{KL} / \mathrm{r}\) for each axis (if necessary). The largest will govern the buckling load.
2. Find \(\mathrm{F}_{\mathrm{cr}}\) as a function of \(\mathrm{KL} / \mathrm{r}\) from the appropriate equation (above) or table.
3. Compute \(\mathrm{P}_{\mathrm{n}}=\mathrm{F}_{\mathrm{cr}} \cdot \mathrm{A}_{\mathrm{g}}\)
or alternatively compute \(\mathrm{f}_{\mathrm{c}}=\mathrm{P}_{\mathrm{a}} / \mathrm{A}\) or \(\mathrm{P}_{\mathrm{u}} / \mathrm{A}\)
4. Is the design satisfactory?

Is \(\mathrm{P}_{\mathrm{a}} \leq \mathrm{P}_{\mathrm{n}} / \Omega\) or \(\mathrm{P}_{\mathrm{u}} \leq \phi_{\mathrm{c}} \mathrm{P}_{\mathrm{n}}\) ? \(\Rightarrow\) yes, it is; no, it is no good
or Is \(\mathrm{f}_{\mathrm{c}} \leq \mathrm{F}_{\mathrm{cr}} / \Omega\) or \(\phi_{\mathrm{c}} \mathrm{F}_{\mathrm{cr}}\) ? \(\Rightarrow\) yes, it is; no, it is no good

\section*{Procedure for Design}
1. Guess a size by picking a section.
2. Calculate \(\mathrm{KL} / \mathrm{r}\) for each axis (if necessary). The largest will govern the buckling load.
3. Find \(\mathrm{F}_{\mathrm{cr}}\) as a function of \(\mathrm{KL} / \mathrm{r}\) from appropriate equation (above) or table.
4. Compute \(\mathrm{P}_{\mathrm{n}}=\mathrm{F}_{\mathrm{cr}} \cdot \mathrm{A}_{\mathrm{g}}\)
or alternatively compute \(\mathrm{f}_{\mathrm{c}}=\mathrm{P}_{\mathrm{a}} / \mathrm{A}\) or \(\mathrm{P}_{\mathrm{u}} / \mathrm{A}\)
5. Is the design satisfactory?

Is \(\mathrm{P} \leq \mathrm{P}_{\mathrm{n}} / \Omega\) or \(\mathrm{P}_{\mathrm{u}} \leq \phi_{\mathrm{c}} \mathrm{P}_{\mathrm{n}}\) ? yes, it is; no, pick a bigger section and go back to step 2 .
Is \(\mathrm{f}_{\mathrm{c}} \leq \mathrm{F}_{\mathrm{cr}} / \Omega\) or \(\phi_{\mathrm{c}} \mathrm{F}_{\mathrm{cr}}\) ? \(\Rightarrow\) yes, it is; no, pick a bigger section and go back to step 2 .
6. Check design efficiency by calculating percentage of stress used:=
\[
\frac{P_{a}}{P_{n} / \Omega} \cdot 100 \% \text { or } \frac{P_{u}}{\phi_{c} P_{n}} \cdot 100 \%
\]

If value is between \(90-100 \%\), it is efficient.
If values is less than \(90 \%\), pick a smaller section and go back to step 2.

\section*{Columns with Bending (Beam-Columns)}

In order to design an adequate section for allowable stress, we have to start somewhere:
1. Make assumptions about the limiting stress from:
- buckling
- axial stress
- combined stress
1. See if we can find values for \(\underline{r}\) or \(\underline{A}\) or \(\underline{Z}\) (S for ASD)
2. Pick a trial section based on if we think r or A is going to govern the section size.
3. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.
4. Did the section pass the stress test?
- If not, do you increase r or A or S ?
- If so, is the difference really big so that you could decrease r or A or S to make it more efficient (economical)?
5. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

\section*{Design for Combined Compression and Flexure:}

The interaction of compression and bending are included in the form for two conditions based on the size of the required axial force to the available axial strength. This is notated as \(P_{r}\) (either P from ASD or \(\mathrm{P}_{\mathrm{u}}\) from LRFD) for the axial force being supported, and \(P_{c}\) (either \(P_{n} / \Omega\) for ASD or \(\phi_{c} P_{n}\) for LRFD). The increased bending moment due to the \(\mathrm{P}-\Delta\) effect must be determined and used as the moment to resist.

For \(\frac{P_{r}}{P_{c}} \geq 0.2: \quad \frac{P}{P_{n} / \Omega}+\frac{8}{9}\left(\frac{M_{x}}{M_{n x} / \Omega}+\frac{M_{y}}{M_{n y} / \Omega}\right) \leq 1.0 \quad \frac{P_{u}}{\phi_{c} P_{n}}+\frac{8}{9}\left(\frac{M_{u x}}{\phi_{b} M_{n x}}+\frac{M_{u y}}{\phi_{b} M_{n y}}\right) \leq 1.0\)
(ASD)
For \(\frac{P_{r}}{P_{c}}<0.2: \quad \frac{P}{2 P_{n} / \Omega}+\left(\frac{M_{x}}{M_{n x} / \Omega}+\frac{M_{y}}{M_{n y} / \Omega}\right) \leq 1.0 \quad \frac{P_{u}}{2 \phi_{c} P_{n}}+\left(\frac{M_{u x}}{\phi_{b} M_{n x}}+\frac{M_{u y}}{\phi_{b} M_{n y}}\right) \leq 1.0\)
(ASD)
(LRFD)
(LRFD)
where:
for compression \(\quad \phi_{\mathrm{c}}=0.90(\mathrm{LRFD}) \quad \Omega=1.67\) (ASD) for bending \(\quad \phi_{\mathrm{b}}=0.90(\mathrm{LRFD}) \quad \Omega=1.67(\mathrm{ASD})\)
For a braced condition, the moment magnification factor \(B_{l}\) is determined by \(B_{1}=\frac{C_{m}}{1-\left(P_{u} / P_{e 1}\right)} \leq 1.0\) where \(C_{m}\) is a modification factor accounting for end conditions

When not subject to transverse loading between supports in plane of bending:
\(=0.6-0.4\left(\mathrm{M}_{1} / \mathrm{M}_{2}\right)\) where \(\mathrm{M}_{1}\) and \(\mathrm{M}_{2}\) are the end moments and \(\mathrm{M}_{1}<\mathrm{M}_{2} . \mathrm{M}_{1} / \mathrm{M}_{2}\) is positive when the member is bent in reverse curvature (same direction), negative when bent in single curvature.
When there is transverse loading between the two ends of a member:
\(=0.85\), members with restrained (fixed) ends
\(=1.00\), members with unrestrained ends
\(\mathrm{P}_{\mathrm{el}}=\) Euler buckling strength
\[
P_{e 1}=\frac{\pi^{2} E A}{(K l / r)^{2}}
\]

\section*{Criteria for Design of Connections and Tension Members}

Refer to the specific note set.

\section*{Criteria for Design of Column Base Plates}

Column base plates are designed for bearing on the concrete (concrete capacity) and flexure because the column "punches" down the plate and it could bend upward near the edges of the column (shown as \(0.8 b_{f}\) and \(0.95 d\) ). The plate dimensions are B and N and are preferably in full inches with thicknesses in multiples of \(1 / 8\) inches.


LRFD minimum thickness: \(t_{\text {min }}=l \sqrt{\frac{2 P_{u}}{0.9 F_{y} B N}}\)
where \(l\) is the larger of \(m, n\) and \(\lambda n^{\prime}\)
\[
\begin{array}{ll}
m=\frac{N-0.95 d}{2} & n=\frac{B-0.8 b_{f}}{2} \\
n^{\prime}=\frac{\sqrt{d b_{f}}}{4} & \lambda=\frac{2 \sqrt{X}}{(1+\sqrt{1-X})} \leq 1
\end{array}
\]
where X depends on the concrete bearing capacity of \(\phi_{\mathrm{c}} P_{p}\), with
\[
\begin{aligned}
\phi_{\mathrm{c}} & =0.65 \text { and } \mathrm{P}_{\mathrm{p}}=0.85 f^{\prime} \mathrm{c} \mathrm{~A} \\
X & =\frac{4 d b_{f}}{\left(d+b_{f}\right)^{2}} \cdot \frac{P_{u}}{\phi_{c} P_{p}}=\frac{4 d b_{f}}{\left(d+b_{f}\right)^{2}} \cdot \frac{P_{u}}{\phi_{c}\left(0.85 f_{c}^{\prime}\right) B N}
\end{aligned}
\]

\section*{Beam Design Flow Chart}


Listing of \(W\) shapes in Descending Order of \(Z_{x}\) for Beam Design
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{US} \\
\left(\mathrm{in.}{ }^{3}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{US} \\
\left(\text { (in. }{ }^{4}\right) \\
\hline
\end{gathered}
\] & Section & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{6} \mathrm{~mm} .{ }^{4}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{3} \mathrm{~mm} .3\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{US} \\
\left(\mathrm{in.}{ }^{3}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{US} \\
\left(\text { in. }{ }^{4}\right) \\
\hline
\end{gathered}
\] & Section & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{6} \mathrm{~mm} .{ }^{4}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{3} \mathrm{~mm} .3\right) \\
\hline
\end{gathered}
\] \\
\hline 514 & 7450 & W33X141 & 3100 & 8420 & 289 & 3100 & W24X104 & 1290 & 4740 \\
\hline 511 & 5680 & W24X176 & 2360 & 8370 & 287 & 1900 & W14X159 & 791 & 4700 \\
\hline 509 & 7800 & W36X135 & 3250 & 8340 & 283 & 3610 & W30X90 & 1500 & 4640 \\
\hline 500 & 6680 & W30X148 & 2780 & 8190 & 280 & 3000 & W24X103 & 1250 & 4590 \\
\hline 490 & 4330 & W18X211 & 1800 & 8030 & 279 & 2670 & W21X111 & 1110 & 4570 \\
\hline 487 & 3400 & W14X257 & 1420 & 7980 & 278 & 3270 & W27X94 & 1360 & 4560 \\
\hline 481 & 3110 & W12X279 & 1290 & 7880 & 275 & 1650 & W12X170 & 687 & 4510 \\
\hline 476 & 4730 & W21X182 & 1970 & 7800 & 262 & 2190 & W18X119 & 912 & 4290 \\
\hline 468 & 5170 & W24X162 & 2150 & 7670 & 260 & 1710 & W14X145 & 712 & 4260 \\
\hline 467 & 6710 & W33X130 & 2790 & 7650 & 254 & 2700 & W24X94 & 1120 & 4160 \\
\hline 464 & 5660 & W27X146 & 2360 & 7600 & 253 & 2420 & W21X101 & 1010 & 4150 \\
\hline 442 & 3870 & W18X192 & 1610 & 7240 & 244 & 2850 & W27X84 & 1190 & 4000 \\
\hline 437 & 5770 & W30X132 & 2400 & 7160 & 243 & 1430 & W12X152 & 595 & 3980 \\
\hline 436 & 3010 & W14X233 & 1250 & 7140 & 234 & 1530 & W14X132 & 637 & 3830 \\
\hline 432 & 4280 & W21X166 & 1780 & 7080 & 230 & 1910 & W18X106 & 795 & 3770 \\
\hline 428 & 2720 & W12X252 & 1130 & 7010 & 224 & 2370 & W24X84 & 986 & 3670 \\
\hline 418 & 4580 & W24X146 & 1910 & 6850 & 221 & 2070 & W21X93 & 862 & 3620 \\
\hline 415 & 5900 & W33X118 & 2460 & 6800 & 214 & 1240 & W12X136 & 516 & 3510 \\
\hline 408 & 5360 & W30X124 & 2230 & 6690 & 212 & 1380 & W14X120 & 574 & 3470 \\
\hline 398 & 3450 & W18X175 & 1440 & 6520 & 211 & 1750 & W18X97 & 728 & 3460 \\
\hline 395 & 4760 & W27X129 & 1980 & 6470 & 200 & 2100 & W24X76 & 874 & 3280 \\
\hline 390 & 2660 & W14X211 & 1110 & 6390 & 198 & 1490 & W16X100 & 620 & 3240 \\
\hline 386 & 2420 & W12X230 & 1010 & 6330 & 196 & 1830 & W21X83 & 762 & 3210 \\
\hline 378 & 4930 & W30X116 & 2050 & 6190 & 192 & 1240 & W14X109 & 516 & 3150 \\
\hline 373 & 3630 & W21X147 & 1510 & 6110 & 186 & 1530 & W18X86 & 637 & 3050 \\
\hline 370 & 4020 & W24X131 & 1670 & 6060 & 185 & 1070 & W12X120 & 445 & 3050 \\
\hline 356 & 3060 & W18X158 & 1270 & 5830 & 177 & 1830 & W24X68 & 762 & 2900 \\
\hline 355 & 2400 & W14X193 & 999 & 5820 & 174 & 1300 & W16X89 & 541 & 2870 \\
\hline 348 & 2140 & W12X210 & 891 & 5700 & 173 & 1110 & W14X99 & 462 & 2830 \\
\hline 346 & 4470 & W30X108 & 1860 & 5670 & 169 & 1600 & W21X73 & 666 & 2820 \\
\hline 343 & 4080 & W27X114 & 1700 & 5620 & 164 & 933 & W12X106 & 388 & 2690 \\
\hline 333 & 3220 & W21X132 & 1340 & 5460 & 160 & 1330 & W18X76 & 554 & 2670 \\
\hline 327 & 3540 & W24X117 & 1470 & 5360 & 159 & 1480 & W21X68 & 616 & 2620 \\
\hline 322 & 2750 & W18X143 & 1140 & 5280 & 157 & 999 & W14X90 & 416 & 2570 \\
\hline 320 & 2140 & W14X176 & 891 & 5240 & 153 & 1550 & W24X62 & 645 & 2510 \\
\hline 312 & 3990 & W30X99 & 1660 & 5110 & 147 & 1110 & W16X77 & 462 & 2460 \\
\hline 311 & 1890 & W12X190 & 787 & 5100 & 147 & 833 & W12X96 & 347 & 2410 \\
\hline 307 & 2960 & W21X122 & 1230 & 5030 & 146 & 716 & W10X112 & 298 & 2410 \\
\hline 305 & 3620 & W27X102 & 1510 & 5000 & 146 & 1170 & W18X71 & 487 & 2390 \\
\hline 290 & 2460 & W18X130 & 1020 & 4750 & & & & & continued) \\
\hline
\end{tabular}

Listing of W Shapes in Descending order of \(Z_{x}\) for Beam Design (Continued)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline \[
\begin{gathered}
\hline \hline \mathrm{Z}_{\mathrm{x}}-\mathrm{US} \\
\left(\mathrm{in.}{ }^{3}\right)
\end{gathered}
\] & \[
\begin{gathered}
\hline \mathrm{I}_{\mathrm{x}}-\mathrm{US} \\
\left(\text { (in. }{ }^{4}\right. \text { ) } \\
\hline
\end{gathered}
\] & Section & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{6} \mathrm{~mm} .{ }^{4}\right)
\end{gathered}
\] & \[
\begin{gathered}
\hline \mathrm{Z}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{3} \mathrm{~mm} .3\right)
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{US} \\
\left(\text { in. }^{3}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\hline \mathrm{I}_{\mathrm{x}}-\mathrm{US} \\
\left(\mathrm{in.}{ }^{4}\right) \\
\hline
\end{gathered}
\] & Section & \[
\begin{gathered}
\mathrm{I}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{6} \mathrm{~mm} .{ }^{4}\right) \\
\hline
\end{gathered}
\] & \[
\begin{gathered}
\mathrm{Z}_{\mathrm{x}}-\mathrm{SI} \\
\left(10^{3} \mathrm{~mm} .3\right) \\
\hline
\end{gathered}
\] \\
\hline 144 & 1330 & W21X62 & 554 & 2360 & 66.5 & 510 & W18X35 & 212 & 1090 \\
\hline 139 & 881 & W14X82 & 367 & 2280 & 64.0 & 348 & W12X45 & 145 & 1050 \\
\hline 133 & 1350 & W24X55 & 562 & 2200 & 63.5 & 448 & W16X36 & 186 & 1050 \\
\hline 132 & 1070 & W18X65 & 445 & 2180 & 61.5 & 385 & W14X38 & 160 & 1010 \\
\hline 131 & 740 & W12X87 & 308 & 2160 & 59.4 & 228 & W8X58 & 94.9 & 980 \\
\hline 130 & 954 & W16X67 & 397 & 2130 & 57.0 & 307 & W12X40 & 128 & 934 \\
\hline 129 & 623 & W10X100 & 259 & 2130 & 54.7 & 248 & W10X45 & 103 & 900 \\
\hline 129 & 1170 & W21X57 & 487 & 2110 & 54.5 & 340 & W14X34 & 142 & 895 \\
\hline 126 & 1140 & W21X55 & 475 & 2060 & 53.7 & 375 & W16X31 & 156 & 885 \\
\hline 126 & 795 & W14X74 & 331 & 2060 & 51.2 & 285 & W12X35 & 119 & 839 \\
\hline 123 & 984 & W18X60 & 410 & 2020 & 49.0 & 184 & W8X48 & 76.6 & 803 \\
\hline 118 & 662 & W12X79 & 276 & 1950 & 47.2 & 291 & W14X30 & 121 & 775 \\
\hline 115 & 722 & W14X68 & 301 & 1880 & 46.7 & 209 & W10X39 & 87.0 & 767 \\
\hline 113 & 534 & W10X88 & 222 & 1850 & 44.2 & 301 & W16X26 & 125 & 724 \\
\hline 112 & 890 & W18X55 & 370 & 1840 & 43.0 & 238 & W12X30 & 99.1 & 706 \\
\hline 110 & 984 & W21X50 & 410 & 1800 & 40.1 & 245 & W14X26 & 102 & 659 \\
\hline 108 & 597 & W12X72 & 248 & 1770 & 39.7 & 146 & W8X40 & 60.8 & 652 \\
\hline 107 & 959 & W21X48 & 399 & 1750 & 38.5 & 171 & W10X33 & 71.2 & 636 \\
\hline 105 & 758 & W16X57 & 316 & 1720 & 37.1 & 204 & W12X26 & 84.9 & 610 \\
\hline 102 & 640 & W14X61 & 266 & 1670 & 36.6 & 170 & W10X30 & 70.8 & 600 \\
\hline 100 & 800 & W18X50 & 333 & 1660 & 34.7 & 127 & W8X35 & 52.9 & 569 \\
\hline 96.8 & 455 & W10X77 & 189 & 1600 & 33.2 & 199 & W14X22 & 82.8 & 544 \\
\hline 95.5 & 533 & W12X65 & 222 & 1590 & 31.3 & 144 & W10×26 & 59.9 & 513 \\
\hline 95.4 & 843 & W21X44 & 351 & 1560 & 30.4 & 110 & W8x31 & 45.8 & 498 \\
\hline 91.7 & 659 & W16X50 & 274 & 1510 & 29.2 & 156 & W12X22 & 64.9 & 480 \\
\hline 90.6 & 712 & W18X46 & 296 & 1490 & 27.1 & 98.0 & W8X28 & 40.8 & 446 \\
\hline 86.5 & 541 & W14X53 & 225 & 1430 & 26.0 & 118 & W10x22 & 49.1 & 426 \\
\hline 86.4 & 475 & W12X58 & 198 & 1420 & 24.6 & 130 & W12X19 & 54.1 & 405 \\
\hline 85.2 & 394 & W10X68 & 164 & 1400 & 23.1 & 82.7 & W8X24 & 34.4 & 379 \\
\hline 82.1 & 586 & W16X45 & 244 & 1350 & 21.4 & 96.3 & W10x19 & 40.1 & 354 \\
\hline 78.4 & 612 & W18X40 & 255 & 1280 & 20.4 & 75.3 & W8X21 & 31.3 & 334 \\
\hline 78.1 & 484 & W14X48 & 201 & 1280 & 20.1 & 103 & W12x16 & 42.9 & 329 \\
\hline 77.3 & 425 & W12X53 & 177 & 1280 & 18.6 & 81.9 & W10X17 & 34.1 & 306 \\
\hline 74.4 & 341 & W10X60 & 142 & 1220 & 17.3 & 88.6 & W12X14 & 36.9 & 285 \\
\hline 72.2 & 518 & W16X40 & 216 & 1200 & 17.0 & 61.9 & W8X18 & 25.8 & 279 \\
\hline 71.8 & 391 & W12X50 & 163 & 1180 & 15.9 & 68.9 & W10X15 & 28.7 & 262 \\
\hline 69.6 & 272 & W8X67 & 113 & 1150 & 13.6 & 48.0 & W8X15 & 20.0 & 223 \\
\hline 69.4 & 428 & W14X43 & 178 & 1140 & 12.6 & 53.8 & W10X12 & 22.4 & 206 \\
\hline 66.5 & 303 & W10X54 & 126 & 1090 & 11.4 & 39.6 & W8X13 & 16.5 & 187 \\
\hline & & & & & 8.87 & 30.8 & W8X10 & 12.8 & 145 \\
\hline
\end{tabular}

Available Critical Stress, \(\phi_{c} F_{c r}\), for Compression Members, ksi ( \(F_{y}=36 \mathrm{ksi}\) and \(\phi_{c}=0.90\) )
\begin{tabular}{llllllllll}
\hline\(K L / r\) & \(\phi_{c} F_{c r}\) & \(K L / r\) & \(\phi_{c} F_{c r}\) & \(K L / r\) & \(\phi_{c} F_{c r}\) & \(K L / r\) & \(\phi_{c} F_{c r}\) & \(K L / r\) & \(\phi_{c} F_{c r}\) \\
\hline 1 & 32.4 & 41 & 29.7 & 81 & 22.9 & 121 & 15.0 & 161 & 8.72 \\
2 & 32.4 & 42 & 29.5 & 82 & 22.7 & 122 & 14.8 & 162 & 8.61 \\
3 & 32.4 & 43 & 29.4 & 83 & 22.5 & 123 & 14.6 & 163 & 8.50 \\
4 & 32.4 & 44 & 29.3 & 84 & 22.3 & 124 & 14.4 & 164 & 8.40 \\
5 & 32.4 & 45 & 29.1 & 85 & 22.1 & 125 & 14.2 & 165 & 8.30 \\
6 & 32.3 & 46 & 29.0 & 86 & 22.0 & 126 & 14.0 & 166 & 8.20 \\
7 & 32.3 & 47 & 28.8 & 87 & 21.8 & 127 & 13.9 & 167 & 8.10 \\
8 & 32.3 & 48 & 28.7 & 88 & 21.6 & 128 & 13.7 & 168 & 8.00 \\
9 & 32.3 & 49 & 28.6 & 89 & 21.4 & 129 & 13.5 & 169 & 7.91 \\
10 & 32.2 & 50 & 28.4 & 90 & 21.2 & 130 & 13.3 & 170 & 7.82 \\
11 & 32.2 & 51 & 28.3 & 91 & 21.0 & 131 & 13.1 & 171 & 7.73 \\
12 & 32.2 & 52 & 28.1 & 92 & 20.8 & 132 & 12.9 & 172 & 7.64 \\
13 & 32.1 & 53 & 27.9 & 93 & 20.5 & 133 & 12.8 & 173 & 7.55 \\
14 & 32.1 & 54 & 27.8 & 94 & 20.3 & 134 & 12.6 & 174 & 7.46 \\
15 & 32.0 & 55 & 27.6 & 95 & 20.1 & 135 & 12.4 & 175 & 7.38 \\
16 & 32.0 & 56 & 27.5 & 96 & 19.9 & 136 & 12.2 & 176 & 7.29 \\
17 & 31.9 & 57 & 27.3 & 97 & 19.7 & 137 & 12.0 & 177 & 7.21 \\
18 & 31.9 & 58 & 27.1 & 98 & 19.5 & 138 & 11.9 & 178 & 7.13 \\
19 & 31.8 & 59 & 27.0 & 99 & 19.3 & 139 & 11.7 & 179 & 7.05 \\
20 & 31.7 & 60 & 26.8 & 100 & 19.1 & 140 & 11.5 & 180 & 6.97 \\
21 & 31.7 & 61 & 26.6 & 101 & 18.9 & 141 & 11.4 & 181 & 6.90 \\
22 & 31.6 & 62 & 26.5 & 102 & 18.7 & 142 & 11.2 & 182 & 6.82 \\
23 & 31.5 & 63 & 26.3 & 103 & 18.5 & 143 & 11.0 & 183 & 6.75 \\
24 & 31.4 & 64 & 26.1 & 104 & 18.3 & 144 & 10.9 & 184 & 6.67 \\
25 & 31.4 & 65 & 25.9 & 105 & 18.1 & 145 & 10.7 & 185 & 6.60 \\
26 & 31.3 & 66 & 25.8 & 106 & 17.9 & 146 & 10.6 & 186 & 6.53 \\
27 & 31.2 & 67 & 25.6 & 107 & 17.7 & 147 & 10.5 & 187 & 6.46 \\
28 & 31.1 & 68 & 25.4 & 108 & 17.5 & 148 & 10.3 & 188 & 6.39 \\
29 & 31.0 & 69 & 25.2 & 109 & 11.3 & 149 & 10.2 & 189 & 6.32 \\
30 & 30.9 & 70 & 25.0 & 110 & 17.1 & 150 & 10.0 & 190 & 6.26 \\
31 & 30.8 & 71 & 24.8 & 111 & 16.9 & 151 & 9.91 & 191 & 6.19 \\
32 & 30.7 & 72 & 24.7 & 112 & 16.7 & 152 & 9.78 & 192 & 6.13 \\
33 & 30.6 & 73 & 24.5 & 113 & 16.5 & 153 & 9.65 & 193 & 6.06 \\
34 & 30.5 & 74 & 24.3 & 114 & 163 & 154 & 9.53 & 194 & 6.00 \\
35 & 30.4 & 75 & 24.1 & 115 & 16.2 & 155 & 9.40 & 195 & 5.94 \\
36 & 30.3 & 76 & 23.9 & 116 & 16.0 & 156 & 9.28 & 196 & 5.88 \\
37 & 30.1 & 77 & 23.7 & 117 & 15.8 & 157 & 9.17 & 197 & 5.82 \\
38 & 30.0 & 78 & 23.5 & 118 & 15.6 & 158 & 9.05 & 198 & 5.76 \\
39 & 29.9 & 79 & 23.3 & 119 & 15.4 & 159 & 8.94 & 199 & 5.70 \\
40 & 29.8 & 80 & 23.1 & 120 & 15.2 & 160 & 8.82 & 200 & 5.65 \\
\hline & & & & & & & & & \\
\hline
\end{tabular}

Available Critical Stress, \(\phi_{c} F_{c r}\), for Compression Members, ksi ( \(F_{y}=50\) ksi and \(\phi_{c}=0.90\) )
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline KL/r & \(\phi_{c} F_{c r}\) & KL/r & \(\phi_{c} F_{c r}\) & KL/r & \(\phi_{c} F_{c r}\) & KL/r & \(\phi_{c} F_{c r}\) & KL/r & \(\phi_{c} F_{c r}\) \\
\hline 1 & 45.0 & 41 & 39.8 & 81 & 27.9 & 121 & 15.4 & 161 & 8.72 \\
\hline 2 & 45.0 & 42 & 39.6 & 82 & 27.5 & 122 & 15.2 & 162 & 8.61 \\
\hline 3 & 45.0 & 43 & 39.3 & 83 & 27.2 & 123 & 14.9 & 163 & 8.50 \\
\hline 4 & 44.9 & 44 & 39.1 & 84 & 26.9 & 124 & 14.7 & 164 & 8.40 \\
\hline 5 & 44.9 & 45 & 38.8 & 85 & 26.5 & 125 & 14.5 & 165 & 8.30 \\
\hline 6 & 44.9 & 46 & 38.5 & 86 & 26.2 & 126 & 14.2 & 166 & 8.20 \\
\hline 7 & 44.8 & 47 & 38.3 & 87 & 25.9 & 127 & 14.0 & 167 & 8.10 \\
\hline 8 & 44.8 & 48 & 38.0 & 88 & 25.5 & 128 & 13.8 & 168 & 8.00 \\
\hline 9 & 44.7 & 49 & 37.8 & 89 & 25.2 & 129 & 13.6 & 169 & 7.91 \\
\hline 10 & 44.7 & 50 & 37.5 & 90 & 24.9 & 130 & 13.4 & 170 & 7.82 \\
\hline 11 & 44.6 & 51 & 37.2 & 91 & 24.6 & 131 & 13.2 & 171 & 7.73 \\
\hline 12 & 44.5 & 52 & 36.9 & 92 & 24.2 & 132 & 13.0 & 172 & 7.64 \\
\hline 13 & 44.4 & 53 & 36.6 & 93 & 23.9 & 133 & 12.8 & 173 & 7.55 \\
\hline 14 & 44.4 & 54 & 36.4 & 94 & 23.6 & 134 & 12.6 & 174 & 7.46 \\
\hline 15 & 44.3 & 55 & 36.1 & 95 & 23.3 & 135 & 12.4 & 175 & 7.38 \\
\hline 16 & 44.2 & 56 & 35.8 & 96 & 22.9 & 136 & 12.2 & 176 & 7.29 \\
\hline 17 & 44.1 & 57 & 35.5 & 97 & 22.6 & 137 & 12.0 & 177 & 7.21 \\
\hline 18 & 43.9 & 58 & 35.2 & 98 & 22.3 & 138 & 11.9 & 178 & 7.13 \\
\hline 19 & 43.8 & 59 & 34.9 & 99 & 22.0 & 139 & 11.7 & 179 & 7.05 \\
\hline 20 & 43.7 & 60 & 34.6 & 100 & 21.7 & 140 & 11.5 & 180 & 6.97 \\
\hline 21 & 43.6 & 61 & 34.3 & 101 & 21.3 & 141 & 11.4 & 181 & 6.90 \\
\hline 22 & 43.4 & 62 & 34.0 & 102 & 21.0 & 142 & 11.2 & 182 & 6.82 \\
\hline 23 & 43.3 & 63 & 33.7 & 103 & 20.7 & 143 & 11.0 & 183 & 6.75 \\
\hline 24 & 43.1 & 64 & 33.4 & 104 & 20.4 & 144 & 10.9 & 184 & 6.67 \\
\hline 25 & 43.0 & 65 & 33.0 & 105 & 20.1 & 145 & 10.7 & 185 & 6.60 \\
\hline 26 & 42.8 & 66 & 32.7 & 106 & 19.8 & 146 & 10.6 & 186 & 6.53 \\
\hline 27 & 42.7 & 67 & 32.4 & 107 & 19.5 & 147 & 10.5 & 187 & 6.46 \\
\hline 28 & 42.5 & 68 & 32.1 & 108 & 19.2 & 148 & 10.3 & 188 & 6.39 \\
\hline 29 & 42.3 & 69 & 31.8 & 109 & 18.9 & 149 & 10.2 & 189 & 6.32 \\
\hline 30 & 42.1 & 70 & 31.4 & 110 & 18.6 & 150 & 10.0 & 190 & 6.26 \\
\hline 31 & 41.9 & 71 & 31.1 & 111 & 18.3 & 151 & 9.91 & 191 & 6.19 \\
\hline 32 & 41.8 & 72 & 30.8 & 112 & 18.0 & 152 & 9.78 & 192 & 6.13 \\
\hline 33 & 41.6 & 73 & 30.5 & 113 & 17.7 & 153 & 9.65 & 193 & 6.06 \\
\hline 34 & 41.4 & 74 & 30.2 & 114 & 17.4 & 154 & 9.53 & 194 & 6.00 \\
\hline 35 & 41.1 & 75 & 29.8 & 115 & 17.1 & 155 & 9.40 & 195 & 5.94 \\
\hline 36 & 40.9 & 76 & 29.5 & 116 & 16.8 & 156 & 9.28 & 196 & 5.88 \\
\hline 37 & 40.7 & 77 & 29.2 & 117 & 16.5 & 157 & 9.17 & 197 & 5.82 \\
\hline 38 & 40.5 & 78 & 28.8 & 118 & 16.2 & 158 & 9.05 & 198 & 5.76 \\
\hline 39 & 40.3 & 79 & 28.5 & 119 & 16.0 & 159 & 8.94 & 199 & 5.70 \\
\hline 40 & 40.0 & 80 & 28.2 & 120 & 15.7 & 160 & 8.82 & 200 & 5.65 \\
\hline
\end{tabular}

\title{
STANDARD SPECIFICATIONS FOR OPEN WEB STEEL JOISTS, K-SERIES
}

Adopted by the Steel Joist Institute November 4, 1985
Revised to November 10, 2003 - Effective March 01, 2005

\section*{SECTION 1. SCOPE}

This specification covers the design, manufacture and use of Open Web Steel Joists, K-Series. Load and Resistance Factor Design (LRFD) and Allowable Strength Design (ASD) are included in this specification.

SECTION 2.
DEFINITION

The term "Open Web Steel Joists K-Series," as used herein, refers to open web, parallel chord, load-carrying members suitable for the direct support of floors and roof decks in buildings, utilizing hot-rolled or cold-formed steel, including coldformed steel whose yield strength* has been attained by cold working. K-Series Joists shall be designed in accordance with this specification to support the uniformly distributed loads given in the Standard Load Tables for Open Web Steel Joists, K-Series, attached hereto.
The KCS Joist is a K-Series Joist which is provided to address the problem faced by specifying professionals when trying to select joists to support uniform plus concentrated loads or other non-uniform loads.
The design of chord sections for K-Series Joists shall be based on a yield strength of \(50 \mathrm{ksi}(345 \mathrm{MPa})\). The design of web sections for K-Series Joists shall be based on a yield strength of either \(36 \mathrm{ksi}(250 \mathrm{MPa})\) or \(50 \mathrm{ksi}(345 \mathrm{MPa})\). Steel used for K-Series Joists chord or web sections shall have a minimum yield strength determined in accordance with one of the procedures specified in Section 3.2, which is equal to the yield strength assumed in the design.
* The term "Yield Strength" as used herein shall designate the yield level of a material as determined by the applicable method outlined in paragraph 13.1 "Yield Point", and in paragraph 13.2 "Yield Strength", of ASTM A370, Standard Test Methods and Definitions for Mechanical Testing of Steel Products, or as specified in paragraph 3.2 of this specification.

\footnotetext{
Standard Specifications and Load Tables, Open Web Steel Joists, K-Series,
}

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SECTION 3.
MATERIALS

\subsection*{3.1 STEEL}

The steel used in the manufacture of chord and web sections shall conform to one of the following ASTM Specifications:
- Carbon Structural Steel, ASTM A36/A36M.
- High-Strength, Low-Alloy Structural Steel, ASTM A242/A242M.
- High-Strength Carbon-Manganese Steel of Structural Quality, ASTM A529/A529M, Grade 50.
- High-Strength Low-Alloy Columbium-Vanadium Structural Steel, ASTM A572/A572M, Grade 42 and 50.
- High-Strength Low-Alloy Structural Steel with 50 ksi (345 MPa) Minimum Yield Point to 4 inches ( 100 mm ) Thick, ASTM A588/A588M.
- Steel, Sheet and Strip, High-Strength, Low-Alloy, HotRolled and Cold-Rolled, with Improved Corrosion Resistance, ASTM A606.
- Steel, Sheet, Cold-Rolled, Carbon, Structural, HighStrength Low-Alloy and High-Strength Low-Alloy with Improved Formability, ASTM A1008/A1008M
- Steel, Sheet and Strip, Hot-Rolled, Carbon, Structural, High-Strength Low-Alloy and High-Strength Low-Alloy with Improved Formability, ASTM A1011/A1011M
or shall be of suitable quality ordered or produced to other than the listed specifications, provided that such material in the state used for final assembly and manufacture is weldable and is proved by tests performed by the producer or manufacturer to have the properties specified in Section 3.2.

\subsection*{3.2 MECHANICAL PROPERTIES}

The yield strength used as a basis for the design stresses prescribed in Section 4 shall be either \(36 \mathrm{ksi}(250 \mathrm{MPa})\) or \(50 \mathrm{ksi}(345 \mathrm{MPa})\). Evidence that the steel furnished meets or exceeds the design yield strength shall, if requested, be provided in the form of an affidavit or by witnessed or certified test reports.
For material used without consideration of increase in yield strength resulting from cold forming, the specimens shall be taken from as-rolled material. In the case of material, the mechanical properties of which conform to the requirements of one of the listed specifications, the test specimens and procedures shall conform to those of such specifications and to ASTM A370.

In the case of material, the mechanical properties of which do not conform to the requirements of one of the listed specifications, the test specimens and procedures shall conform to the applicable requirements of ASTM A370, and the specimens shall exhibit a yield strength equal to or exceeding the design yield strength and an elongation of not less than (a) 20 percent in 2 inches ( 51 millimeters) for sheet and strip, or (b) 18 percent in 8 inches ( 203 millimeters) for plates, shapes and bars with adjustments for thickness for plates, shapes and bars as prescribed in ASTM A36/A36M, A242/A242M, A529/A529M, A572/A572M, A588/A588M, whichever specification is applicable on the basis of design yield strength.
The number of tests shall be as prescribed in ASTM A6/A6M for plates, shapes, and bars; and ASTM A606, A1008/A1008M and A1011/A1011M for sheet and strip.
If as-formed strength is utilized, the test reports shall show the results of tests performed on full section specimens in accordance with the provisions of the AISI North American Specifications for the Design of Cold-Formed Steel Structural Members. They shall also indicate compliance with these provisions and with the following additional requirements:
a) The yield strength calculated from the test data shall equal or exceed the design yield strength.
b) Where tension tests are made for acceptance and control purposes, the tensile strength shall be at least 6 percent greater than the yield strength of the section.
c) Where compression tests are used for acceptance and control purposes, the specimen shall withstand a gross shortening of 2 percent of its original length without cracking. The length of the specimen shall be not greater than 20 times the least radius of gyration.
d) If any test specimen fails to pass the requirements of the subparagraphs (a), (b), or (c) above, as applicable, two retests shall be made of specimens from the same lot. Failure of one of the retest specimens to meet such requirements shall be the cause for rejection of the lot represented by the specimens.

\subsection*{3.3 PAINT}

The standard shop paint is intended to protect the steel for only a short period of exposure in ordinary atmospheric conditions and shall be considered an impermanent and provisional coating.
When specified, the standard shop paint shall conform to one of the following:
a) Steel Structures Painting Council Specification, SSPC No. 15.
b) Or, shall be a shop paint which meets the minimum performance requirements of the above listed specification.

\section*{SECTION 4.}

\section*{DESIGN AND} MANUFACTURE

\subsection*{4.1 METHOD}

Joists shall be designed in accordance with these specifications as simply supported, uniformly loaded trusses supporting a floor or roof deck so constructed as to brace the top chord of the joists against lateral buckling. Where any applicable design feature is not specifically covered herein, the design shall be in accordance with the following specifications:
a) Where the steel used consists of hot-rolled shapes, bars or plates, use the American Institute of Steel Construction, Specification for Structural Steel Buildings.
b) For members that are cold-formed from sheet or strip steel, use the American Iron and Steel Institute, North American Specification for the Design of Cold-Formed Steel Structural Members.

\section*{Design Basis:}

Designs shall be made according to the provisions in this Specification for either Load and Resistance Factor Design (LRFD) or for Allowable Strength Design (ASD).

\section*{Load Combinations:}

\section*{LRFD:}

When load combinations are not specified to the joist manufacturer, the required stress shall be computed for the factored loads based on the factors and load combinations as follows:
\[
\begin{aligned}
& 1.4 D \\
& 1.2 D+1.6\left(L, \text { or } L_{r} \text {, or } S \text {, or } R\right)
\end{aligned}
\]

\section*{ASD:}

When load combinations are not specified to the joist manufacturer, the required stress shall be computed based on the load combinations as follows:
D
\[
D+\left(L, \text { or } L_{r} \text {, or } S \text {, or } R\right)
\]

Where:
\[
\begin{aligned}
& D=\text { dead load due to the weight of the structural elements } \\
& \quad \text { and the permanent features of the structure } \\
& L=\text { live load due to occupancy and movable equipment } \\
& L_{r}=\text { roof live load } \\
& S=\text { snow load } \\
& R=\text { load due to initial rainwater or ice exclusive of the } \\
& \\
& \quad \text { ponding contribution }
\end{aligned}
\]

When special loads are specified and the specifying professional does not provide the load combinations, the provisions of ASCE 7, "Minimum Design Loads for Buildings and Other Structures" shall be used for LRFD and ASD load combinations.

\subsection*{4.2 DESIGN AND ALLOWABLE STRESSES}

\section*{Design Using Load and Resistance Factor Design (LRFD)}

Joists shall have their components so proportioned that the required stresses, \(f_{u}\), shall not exceed \(\phi F_{n}\) where,
\begin{tabular}{lll}
\(\mathrm{f}_{\mathrm{u}}\) & \(=\) required stress & \(\mathrm{ksi}(\mathrm{MPa})\) \\
\(\mathrm{F}_{\mathrm{n}}\) & \(=\) nominal stress & \(\mathrm{ksi}(\mathrm{MPa})\) \\
\(\phi\) & \(=\) resistance factor & \\
\(\phi \mathrm{F}_{\mathrm{n}}\) & \(=\) design stress &
\end{tabular}

\section*{Design Using Allowable Strength Design (ASD)}

Joists shall have their components so proportioned that the required stresses, \(f\), shall not exceed \(F_{n} / \Omega\) where,
\begin{tabular}{llll}
f & \(=\) & required stress & ksi \((\mathrm{MPa})\) \\
\(\mathrm{F}_{\mathrm{n}}=\) & nominal stress & ksi (MPa) \\
\(\Omega\) & \(=\) & safety factor & \\
\(\mathrm{F}_{n} / \Omega=\) & allowable stress &
\end{tabular}

\section*{Stresses:}
(a) Tension: \(\phi_{\mathrm{t}}=0.90\) (LRFD) \(\Omega=1.67\) (ASD)

For Chords: \(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}(345 \mathrm{MPa})\)
For Webs: \(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}(345 \mathrm{MPa})\), or \(\mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}(250 \mathrm{MPa})\)
Design Stress \(=0.9 \mathrm{~F}_{\mathrm{y}}(\) LRFD \()\)
Allowable Stress \(=0.6 \mathrm{~F}_{\mathrm{y}}\) (ASD)
(b) Compression: \(\phi_{\mathrm{c}}=0.90\) (LRFD) \(\Omega_{\mathrm{c}}=1.67\) (ASD)

For members with \(\ell / \mathrm{r} \leq 4.71 \sqrt{\mathrm{E} / \mathrm{QF}_{\mathrm{y}}}\)
\[
\begin{equation*}
F_{c r}=Q\left[0.658^{\left(\frac{Q F_{y}}{F_{\mathrm{e}}}\right)}\right] F_{\mathrm{y}} \tag{4.2-3}
\end{equation*}
\]

For members with \(\ell / r>4.71 \sqrt{E / Q F_{y}}\)
\[
\begin{equation*}
\mathrm{F}_{\mathrm{cr}}=0.877 \mathrm{~F}_{\mathrm{e}} \tag{4.2-4}
\end{equation*}
\]

Where \(\mathrm{F}_{\mathrm{e}}=\) Elastic buckling stress determined in accordance with Equation 4.2-5.
\[
\begin{equation*}
\mathrm{F}_{\mathrm{e}}=\frac{\pi^{2} \mathrm{E}}{(\ell / \mathrm{r})^{2}} \tag{4.2-5}
\end{equation*}
\]

For hot-rolled sections, "Q" is the full reduction factor for slender compression elements.
Design Stress \(=0.9 \mathrm{~F}_{\mathrm{cr}}\) (LRFD)
Allowable Stress \(=0.6 \mathrm{~F}_{\mathrm{cr}}\) (ASD)
In the above equations, \(\ell\) is taken as the distance in inches (millimeters) between panel points for the chord members
and the appropriate length for web members, and \(r\) is the corresponding least radius of gyration of the member or any component thereof. \(E\) is equal to \(29,000 \mathrm{ksi}(200,000\) MPa ).
Use \(1.2 \ell / r_{x}\) for a crimped, first primary compression web member when a moment-resistant weld group is not used for this member; where \(r_{x}=\) member radius of gyration in the plane of the joist.
For cold-formed sections the method of calculating the nominal column strength is given in the AISI, North American Specification for the Design of Cold-Formed Steel Structural Members.
(c) Bending: \(\phi_{\mathrm{b}}=0.90\) (LRFD) \(\Omega_{\mathrm{b}}=1.67\) (ASD)

Bending calculations are to be based on using the elastic section modulus.

For chords and web members other than solid rounds:
\(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}(345 \mathrm{MPa})\)
Design Stress \(=0.9 \mathrm{~F}_{\mathrm{y}}(\mathrm{LRFD})\)
Allowable Stress \(=0.6 \mathrm{~F}_{\mathrm{y}}\) (ASD)
For web members of solid round cross section:
\(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}(345 \mathrm{MPa})\), or \(\mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}(250 \mathrm{MPa})\)
Design Stress \(=1.45 \mathrm{~F}_{\mathrm{y}}(\mathrm{LRFD})\)
Allowable Stress \(=0.95 \mathrm{~F}_{\mathrm{y}}(\mathrm{ASD})\)
For bearing plates:
\(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}(345 \mathrm{MPa})\), or \(\mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}(250 \mathrm{MPa})\)
Design Stress \(=1.35 \mathrm{~F}_{\mathrm{y}}(\) LRFD \()\)
Allowable Stress \(=0.90 \mathrm{~F}_{\mathrm{y}}\) (ASD)

\subsection*{4.3 MAXIMUM SLENDERNESS RATIOS}

The slenderness ratio, \(\ell /\) r, where \(\ell\) is as used in Section 4.2 (b) and \(r\) is the corresponding least radius of gyration, shall not exceed the following:

Top chord interior panels
Top chord end panels 120
Compression members other than top chord. . . . . . . . . 200
Tension members ........................................... 240

\subsection*{4.4 MEMBERS}

\section*{(a) Chords}

The bottom chord shall be designed as an axially loaded tension member.

The radius of gyration of the top chord about its vertical axis shall not be less than \(\ell / 145\) where \(\ell\) is the spacing in inches (millimeters) between lines of bridging as specified in Section 5.4(c).

The top chord shall be considered as stayed laterally by the floor slab or roof deck when attachments are in accordance with the requirements of Section 5.8(e) of these specifications.

The top chord shall be designed for only axial compressive stress when the panel length, \(\ell\), does not exceed 24 inches ( 609 mm ). When the panel length exceeds 24 inches ( 609 mm ), the top chord shall be designed as a continuous member subject to combined axial and bending stresses and shall be so proportioned that:

\section*{For LRFD:}
at the panel point:
\[
\begin{equation*}
\mathrm{f}_{\mathrm{au}}+\mathrm{f}_{\mathrm{bu}} \leq 0.9 \mathrm{~F}_{\mathrm{y}} \tag{4.4-1}
\end{equation*}
\]
at the mid panel: for \(\frac{f_{a u}}{\phi_{\mathrm{c}} F_{c r}} \geq 0.2\),
\[
\begin{equation*}
\frac{f_{\mathrm{au}}}{\phi_{\mathrm{c}} F_{\mathrm{cr}}}+\frac{8}{9}\left[\frac{\mathrm{C}_{\mathrm{m}} \mathrm{f}_{\mathrm{bu}}}{\left[1-\left(\frac{f_{\mathrm{au}}}{\phi_{\mathrm{c}} F_{\mathrm{e}}}\right)\right] Q \phi_{\mathrm{b}} F_{\mathrm{y}}}\right] \leq 1.0 \tag{4.4-2}
\end{equation*}
\]
for \(\frac{f_{a u}}{\phi_{\mathrm{c}} \mathrm{F}_{\mathrm{cr}}}<0.2\),
\[
\begin{equation*}
\left(\frac{\mathrm{f}_{\mathrm{au}}}{2 \phi_{\mathrm{c}} \mathrm{~F}_{\mathrm{cr}}}\right)+\left[\frac{\mathrm{C}_{\mathrm{m}} \mathrm{f}_{\mathrm{bu}}}{\left[1-\left(\frac{\mathrm{f}_{\mathrm{au}}}{\phi_{\mathrm{c}} \mathrm{~F}_{\mathrm{e}}}\right)\right] \mathrm{Q}_{\mathrm{b}} \mathrm{~F}_{\mathrm{y}}}\right] \leq 1.0 \tag{4.4-3}
\end{equation*}
\]
\(\mathrm{f}_{\mathrm{au}}=\mathrm{P}_{\mathrm{u}} / \mathrm{A}=\) Required compressive stress, ksi (MPa)
\(P_{u}=\) Required axial strength using LRFD load combinations, kips ( N )
\(f_{b u}=M_{u} / S=\) Required bending stress at the location under consideration, ksi (MPa)
\(M_{u}=\) Required flexural strength using LRFD load combinations, kip-in. (N-mm)
\(\mathrm{S}=\) Elastic Section Modulus, in. \({ }^{3}\left(\mathrm{~mm}^{3}\right)\)
\(\mathrm{F}_{\mathrm{cr}}=\) Nominal axial compressive stress in ksi (MPa) based on \(\ell l \mathrm{r}\) as defined in Section 4.2(b),
\(\mathrm{C}_{\mathrm{m}}=1-0.3 \mathrm{f}_{\mathrm{au}} / \phi \mathrm{F}_{\mathrm{e}}\) for end panels
\(C_{m}=1-0.4 \mathrm{f}_{\mathrm{au}} / \phi \mathrm{F}_{\mathrm{e}}\) for interior panels
\(\mathrm{F}_{\mathrm{y}}=\) Specified minimum yield strength, ksi (MPa)
\(\mathrm{F}_{\mathrm{e}}=\frac{\pi^{2} \mathrm{E}}{\left(\ell / \mathrm{r}_{\mathrm{x}}\right)^{2}}\), ksi (MPa)
Where \(\ell\) is the panel length, in inches (millimeters), as defined in Section 4.2(b) and \(r_{x}\) is the radius of gyration about the axis of bending.
\(\mathrm{Q}=\) Form factor defined in Section 4.2(b)
\(\mathrm{A}=\) Area of the top chord, \(\mathrm{in} .^{2}\left(\mathrm{~mm}^{2}\right)\)

\section*{For ASD:}
at the panel point:
\[
\begin{equation*}
\mathrm{f}_{\mathrm{a}}+\mathrm{f}_{\mathrm{b}} \leq 0.6 \mathrm{~F}_{\mathrm{y}} \tag{4.4-4}
\end{equation*}
\]
at the mid panel: for \(\frac{f_{a}}{F_{a}} \geq 0.2\),
\[
\begin{gather*}
\frac{f_{a}}{F_{a}}+\frac{8}{9}\left[\frac{C_{m} f_{b}}{\left[1-\left(\frac{1.67 f_{a}}{F_{e}}\right)\right] Q F_{b}}\right] \leq 1.0  \tag{4.4-5}\\
\text { for } \frac{f_{a}}{F_{a}}<0.2, \\
\left(\frac{f_{a}}{2 F_{a}}\right)+\left[\frac{C_{m} f_{b}}{\left[1-\left(\frac{1.67 f_{a}}{F_{e}}\right)\right] Q F_{b}}\right] \leq 1.0 \tag{4.4-6}
\end{gather*}
\]
\(\mathrm{f}_{\mathrm{a}}=\mathrm{P} / \mathrm{A}=\) Required compressive stress, ksi (MPa)
\(P=\) Required axial strength using ASD load combinations, kips (N)
\(f_{b}=M / S=\) Required bending stress at the location under consideration, ksi (MPa)
\(M=\) Required flexural strength using ASD load combinations, kip-in. (N-mm)
\(\mathrm{S}=\) Elastic Section Modulus, in. \({ }^{3}\left(\mathrm{~mm}^{3}\right)\)
\(\mathrm{F}_{\mathrm{a}}=\) Allowable axial compressive stress based on \(\ell / \mathrm{r}\) as defined in Section 4.2(b), ksi (MPa)
\(\mathrm{F}_{\mathrm{b}}=\) Allowable bending stress; \(0.6 \mathrm{~F}_{\mathrm{y}}\), ksi (MPa)
\(\mathrm{C}_{\mathrm{m}}=1-0.50 \mathrm{f}_{\mathrm{a}} / \mathrm{F}_{\mathrm{e}}\) for end panels
\(C_{m}=1-0.67 \mathrm{f}_{\mathrm{a}} / \mathrm{F}_{\mathrm{e}}\) for interior panels
(b) Web

The vertical shears to be used in the design of the web members shall be determined from full uniform loading, but such vertical shears shall be not less than 25 percent of the end reaction. Due consideration shall be given to the effect of eccentricity. The effect of combined axial compression and bending may be investigated using the provisions of Section 4.4(a), letting \(C_{m}=0.4\) when bending due to eccentricity produces reversed curvature.
Interior vertical web members used in modified Warren type web systems shall be designed to resist the gravity loads supported by the member plus an additional axial load of \(1 / 2\) of 1.0 percent of the top chord axial force.
(c) Extended Ends

The magnitude and location of the loads to be supported, deflection requirements, and proper bracing of extended
top chords or full depth cantilever ends shall be clearly indicated on the structural drawings.

\subsection*{4.5 CONNECTIONS}

\section*{(a) Methods}

Joist connections and splices shall be made by attaching the members to one another by arc or resistance welding or other accredited methods.
(1) Welded Connections
a) Selected welds shall be inspected visually by the manufacturer. Prior to this inspection, weld slag shall be removed.
b) Cracks are not acceptable and shall be repaired.
c) Thorough fusion shall exist between weld and base metal for the required design length of the weld; such fusion shall be verified by visual inspection.
d) Unfilled weld craters shall not be included in the design length of the weld.
e) Undercut shall not exceed \(1 / 16\) inch (2 millimeters) for welds oriented parallel to the principal stress.
f) The sum of surface (piping) porosity diameters shall not exceed \(1 / 16\) inch ( 2 millimeters) in any 1 inch ( 25 millimeters) of design weld length.
g) Weld spatter that does not interfere with paint coverage is acceptable.
(2) Welding Program

Manufacturers shall have a program for establishing weld procedures and operator qualification, and for weld sampling and testing. (See Technical Digest \#8 Welding of Open Web Steel Joists.)
(3) Weld Inspection by Outside Agencies (See Section 5.12 of these specifications)

The agency shall arrange for visual inspection to determine that welds meet the acceptance standards of Section 4.5(a)(1) above. Ultrasonic, X-Ray, and magnetic particle testing are inappropriate for joists due to the configurations of the components and welds.
(b) Strength
(1) Joint Connections - Joint connections shall be capable of withstanding forces due to an ultimate load equal to at least 1.35 times the LRFD, or 2.0 times the ASD load shown in the applicable Standard Load Table.
(2) Shop Splices - Splices may occur at any point in chord or web members. Members containing a butt weld splice shall develop an ultimate tensile force of at least \(57 \mathrm{ksi}(393 \mathrm{MPa})\) times the full design area of the chord or web. The term "member" shall be defined as all component parts comprising the chord or web, at the point of the splice.

\section*{(c) Eccentricity}

Members connected at a joint shall have their centroidal axes meet at a point if practical. Otherwise, due consideration shall be given to the effect of eccentricity. In no case shall eccentricity of any web member at a joint exceed \(3 / 4\) of the over-all dimension, measured in the plane of the web, of the largest member connected. The eccentricity of any web member shall be the perpendicular distance from the centroidal axis of that web member to the point on the centroidal axis of the chord which is vertically above or below the intersection of the centroidal axes of the web members forming the joint. Ends of joists shall be proportioned to resist bending produced by eccentricity at the support.

\subsection*{4.6 CAMBER}

Joists shall have approximate camber in accordance with the following:

\section*{TABLE 4.6-1}
\begin{tabular}{rrrr}
\multicolumn{2}{c}{ Top Chord Length } & \multicolumn{2}{c}{ Approximate Camber } \\
\(20^{\prime}-0^{\prime \prime}\) & \((6096 \mathrm{~mm})\) & \(1 / 4^{\prime \prime}\) & \((6 \mathrm{~mm})\) \\
\(30^{\prime}-0^{\prime \prime}\) & \((9144 \mathrm{~mm})\) & \(3 / 8^{\prime \prime}\) & \((10 \mathrm{~mm})\) \\
\(40^{\prime}-0^{\prime \prime}\) & \((12192 \mathrm{~mm})\) & \(5 / 8^{\prime \prime}\) & \((16 \mathrm{~mm})\) \\
\(50^{\prime}-0^{\prime \prime}\) & \((15240 \mathrm{~mm})\) & \(1{ }^{\prime \prime}\) & \((25 \mathrm{~mm})\) \\
\(60^{\prime}-0^{\prime \prime}\) & \((18288 \mathrm{~mm})\) & \(11 / 2^{\prime \prime}\) & \((38 \mathrm{~mm})\)
\end{tabular}

The specifying professional shall give consideration to coordinating joist camber with adjacent framing.

\subsection*{4.7 VERIFICATION OF DESIGN AND MANUFACTURE}

\section*{(a) Design Calculations}

Companies manufacturing K-Series Joists shall submit design data to the Steel Joist Institute (or an independent agency approved by the Steel Joist Institute) for verification of compliance with the SJI Specifications. Design data shall be submitted in detail and in the format specified by the Institute.
(b) Tests of Chord and Web Members

Each manufacturer shall, at the time of design review by the Steel Joist Institute or other independent agency, verify by tests that the design, in accordance with Sections 4.1 through 4.5 of this specification, will provide the theoretical strength of critical members. Such tests shall be evaluated considering the actual yield strength of the members of the test joists.

Material tests for determining mechanical properties of component members shall be conducted.
(c) Tests of Joints and Connections

Each manufacturer shall verify by shear tests on representative joints of typical joists that connections will meet the provision of Section 4.5(b). Chord and web members may be reinforced for such tests.

\section*{(d) In-Plant Inspections}

Each manufacturer shall verify their ability to manufacture K-Series Joists through periodic In-Plant Inspections. Inspections shall be performed by an independent agency approved by the Steel Joist Institute. The frequency, manner of inspection, and manner of reporting shall be determined by the Steel Joist Institute. The plant inspections are not a guarantee of the quality of any specific joists; this responsibility lies fully and solely with the individual manufacturer.

\section*{SECTION 5. APPLICATION}

\subsection*{5.1 USAGE}

These specifications shall apply to any type of structure where floors and roofs are to be supported directly by steel joists installed as hereinafter specified. Where joists are used other than on simple spans under uniformly distributed loading as prescribed in Section 4.1, they shall be investigated and modified if necessary to limit the required stresses to those listed in Section 4.2.
CAUTION: If a rigid connection of the bottom chord is to be made to the column or other support, it shall be made only after the application of the dead loads. The joist is then no longer simply supported, and the system must be investigated for continuous frame action by the specifying professional.
The designed detail of a rigid type connection and moment plates shall be shown on the structural drawings by the specifying professional. The moment plates shall be furnished by other than the joist manufacturer.

\subsection*{5.2 SPAN}

The span of a joist shall not exceed 24 times its depth.

\subsection*{5.3 END SUPPORTS}

\section*{(a) Masonry and Concrete}

K-Series Joists supported by masonry or concrete are to bear on steel bearing plates and shall be designed as steel bearing. Due consideration of the end reactions and all other vertical or lateral forces shall be taken by the specifying professional in the design of the steel bearing plate and the masonry or concrete. The ends of K-Series Joists shall extend a distance of not less than 4 inches ( 102 millimeters) over the masonry or concrete support and be anchored to the steel bearing plate. The plate shall be located not more than \(1 / 2\) inch ( 13 millimeters) from the face of the wall and shall be not less than 6 inches ( 152 millimeters) wide perpendicular to the length of the joist. The plate is to be designed by the specifying professional and shall be furnished by other than the joist manufacturer.

Where it is deemed necessary to bear less than 4 inches (102 millimeters) over the masonry or concrete support, special consideration is to be given to the design of the
steel bearing plate and the masonry or concrete by the specifying professional. The joists must bear a minimum of \(21 / 2\) inches ( 64 millimeters) on the steel bearing plate.
(b) Steel

Due consideration of the end reactions and all other vertical and lateral forces shall be taken by the specifying professional in the design of the steel support. The ends of K-Series Joists shall extend a distance of not less than \(21 / 2\) inches (64 millimeters) over the steel supports.

\subsection*{5.4 BRIDGING}

Top and bottom chord bridging is required and shall consist of one or both of the following types.
(a) Horizontal

Horizontal bridging shall consist of continuous horizontal steel members. Attachments to the joist chords shall be made by welding or mechanical means and shall be capable of resisting a nominal (unfactored) horizontal force of not less than 700 pounds ( 3114 Newtons).
The ratio of unbraced length to least radius of gyration, \(\ell / r\), of the bridging member shall not exceed 300 , where \(\ell\) is the distance in inches (millimeters) between attachments and \(r\) is the least radius of gyration of the bridging member.
(b) Diagonal

Diagonal bridging shall consist of cross-bracing with a \(\ell / \mathrm{r}\) ratio of not more than 200, where \(\ell\) is the distance in inches (millimeters) between connections and \(r\) is the least radius of gyration of the bracing member. Where crossbracing members are connected at their point of intersection, the \(\ell\) distance shall be taken as the distance in inches (millimeters) between connections at the point of intersection of the bracing members and the connections to the chord of the joists. Connections to the chords of steel joists shall be made by positive mechanical means or by welding.

\section*{(c) Quantity and Spacing}

The number of rows of top chord bridging shall not be less than as shown in Bridging Tables 5.4-1 and the spacing shall meet the requirements of Section 4.4(a). The number of rows of bottom chord bridging, including bridging required per Section 5.11, shall not be less than the number of top chord rows. Rows of bottom chord bridging are permitted to be spaced independently of rows of top chord bridging. The spacing of rows of bottom chord bridging shall meet the slenderness requirement of Section 4.3 and any specified strength requirements.

\section*{(d) Bottom Chord Bearing Joists}

Where bottom chord bearing joists are utilized, a row of diagonal bridging shall be provided near the support(s). This bridging shall be installed and anchored before the hoisting cable(s) is released.


TABLE 5.4-1

\section*{NUMBER OF ROWS OF TOP CHORD BRIDGING**}

Refer to the K-Series Load Table and Specification Section 6 for required bolted diagonal bridging.
Distances are Joist Span lengths in feet - See "Definition of Span" preceding Load Table.
\begin{tabular}{|c|c|c|c|c|c|}
\hline *Section Number & \begin{tabular}{l}
One \\
Row
\end{tabular} & Two Rows & Three Rows & Four Rows & Five Rows \\
\hline \#1 & Up thru 16 & Over 16 thru 24 & Over 24 thru 28 & & \\
\hline \#2 & Up thru 17 & Over 17 thru 25 & Over 25 thru 32 & & \\
\hline \#3 & Up thru 18 & Over 18 thru 28 & Over 28 thru 38 & Over 38 thru 40 & \\
\hline \#4 & Up thru 19 & Over 19 thru 28 & Over 28 thru 38 & Over 38 thru 48 & \\
\hline \#5 & Up thru 19 & Over 19 thru 29 & Over 29 thru 39 & Over 39 thru 50 & Over 50 thru 52 \\
\hline \#6 & Up thru 19 & Over 19 thru 29 & Over 29 thru 39 & Over 39 thru 51 & Over 51 thru 56 \\
\hline \#7 & Up thru 20 & Over 20 thru 33 & Over 33 thru 45 & Over 45 thru 58 & Over 58 thru 60 \\
\hline \#8 & Up thru 20 & Over 20 thru 33 & Over 33 thru 45 & Over 45 thru 58 & Over 58 thru 60 \\
\hline \#9 & Up thru 20 & Over 20 thru 33 & Over 33 thru 46 & Over 46 thru 59 & Over 59 thru 60 \\
\hline \#10 & Up thru 20 & Over 20 thru 37 & Over 37 thru 51 & Over 51 thru 60 & \\
\hline \#11 & Up thru 20 & Over 20 thru 38 & Over 38 thru 53 & Over 53 thru 60 & \\
\hline \#12 & Up thru 20 & Over 20 thru 39 & Over 39 thru 53 & Over 53 thru 60 & \\
\hline
\end{tabular}
* Last digit(s) of joist designation shown in Load Table
** See Section 5.11 for additional bridging required for uplift design.


\subsection*{5.5 INSTALLATION OF BRIDGING}

Bridging shall support the top and bottom chords against lateral movement during the construction period and shall hold the steel joists in the approximate position as shown on the joist placement plans.
The ends of all bridging lines terminating at walls or beams shall be anchored thereto.

\subsection*{5.6 END ANCHORAGE}

\section*{(a) Masonry and Concrete}

Ends of K-Series Joists resting on steel bearing plates on masonry or structural concrete shall be attached thereto with a minimum of two \(1 / 8\) inch ( 3 millimeters) fillet welds 1 inch ( 25 millimeters) long, or with two 1/2 inch ( 13 millimeters) ASTM A307 bolts, or the equivalent.
(b) Steel

Ends of K-Series Joists resting on steel supports shall be attached thereto with a minimum of two \(1 / 8\) inch ( 3 millimeters) fillet welds 1 inch ( 25 millimeters) long, or with two \(1 / 2\) inch ( 13 millimeters) ASTM A307 bolts, or the equivalent. When K-Series Joists are used to provide lateral stability to the supporting member, the final connection shall be made by welding or as designated by the specifying professional.
(c) Uplift

Where uplift forces are a design consideration, roof joists shall be anchored to resist such forces (Refer to Section 5.11 Uplift).

\subsection*{5.7 JOIST SPACING}

Joists shall be spaced so that the loading on each joist does not exceed the design load (LRFD or ASD) for the particular joist designation and span as shown in the applicable load tables.

\subsection*{5.8 FLOOR AND ROOF DECKS}

\section*{(a) Material}

Floor and roof decks may consist of cast-in-place or precast concrete or gypsum, formed steel, wood, or other suitable material capable of supporting the required load at the specified joist spacing.
(b) Thickness

Cast-in-place slabs shall be not less than 2 inches (51 millimeters) thick.
(c) Centering

Centering for cast-in-place slabs may be ribbed metal lath, corrugated steel sheets, paper-backed welded wire fabric, removable centering or any other suitable material capable of supporting the slab at the designated joist spacing. Centering shall not cause lateral displacement or damage to the top chord of joists during installation or removal of the centering or placing of the concrete.

\section*{(d) Bearing}

Slabs or decks shall bear uniformly along the top chords of the joists.
(e) Attachments

The spacing for slab or deck attachments along the joist top chord shall not exceed 36 inches ( 914 millimeters), and shall be capable of resisting a nominal (unfactored) lateral force of not less than 300 pounds ( 1335 Newtons), i.e., 100 plf ( \(1.46 \mathrm{kN} / \mathrm{m}\) ).

\section*{(f) Wood Nailers}

Where wood nailers are used, such nailers in conjunction with deck or slab shall be attached to the top chords of the joists in conformance with Section 5.8(e).

\section*{(g) Joist With Standing Seam Roofing}

The stiffness and strength of standing-seam roof clips varies from one manufacturer to another. Therefore, some roof systems cannot be counted on to provide lateral stability to the joists which support the roof. Sufficient stability must be provided to brace the joists laterally under the full design load. The compression chord must resist the chord axial design force in the plane of the joist (i.e., \(x-x\) axis buckling) and out of the plane of the joist (i.e., \(y\)-y axis buckling). Out-of-plane strength may be achieved by adjusting the bridging spacing and/or increasing the compression chord area, the joist depth, and the \(y\)-axis radius of gyration. The effective slenderness ratio in the \(y\)-direction equals \(0.94 \mathrm{~L} / r_{y}\); where L is the bridging spacing in inches (millimeters). The maximum bridging spacing may not exceed that specified in Section 5.4(c).
Horizontal bridging members attached to the compression chords and their anchorage's must be designed for a compressive axial force of 0.0025 nP , where n is the number of joists between end anchors and \(P\) is the chord design force in kips (Newtons). The attachment force between the horizontal bridging member and the compression chord is 0.005 P . Horizontal bridging attached to the tension chords shall be proportioned so that the slenderness ratio between attachments does not exceed 300 . Diagonal bridging shall be proportioned so that the slenderness ratio between attachments does not exceed 200.


\subsection*{5.9 DEFLECTION}

The deflection due to the design nominal live load shall not exceed the following:

Floors: 1/360 of span.
Roofs: \(1 / 360\) of span where a plaster ceiling is attached or suspended.
\(1 / 240\) of span for all other cases.
The specifying professional shall give consideration to the effects of deflection and vibration* in the selection of joists.
* For further reference, refer to Steel Joist Institute Technical Digest \#5, "Vibration of Steel Joist-Concrete Slab Floors" and the Institute's Computer Vibration Program.

\subsection*{5.10 PONDING*}

The ponding investigation shall be performed by the specifying professional.
* For further reference, refer to Steel Joist Institute Technical Digest \#3, "Structural Design of Steel Joist Roofs to Resist Ponding Loads" and AISC Specifications.

\subsection*{5.11 UPLIFT}

Where uplift forces due to wind are a design requirement, these forces must be indicated on the contract drawings in terms of NET uplift in pounds per square foot (Pascals). The contract documents shall indicate if the net uplift is based upon LRFD or ASD. When these forces are specified, they must be considered in the design of joists and/or bridging. A single line of bottom chord bridging must be provided near the first bottom chord panel points whenever uplift due to wind forces is a design consideration.*
* For further reference, refer to Steel Joist Institute Technical Digest \#6, "Structural Design of Steel Joist Roofs to Resist Uplift Loads".

\subsection*{5.12 INSPECTION}

Joists shall be inspected by the manufacturer before shipment to verify compliance of materials and workmanship with the requirements of these specifications. If the purchaser wishes an inspection of the steel joists by someone other than the manufacturer's own inspectors, they may reserve the right to do so in their "Invitation to Bid" or the accompanying "Job Specifications".
Arrangements shall be made with the manufacturer for such inspection of the joists at the manufacturing shop by the purchaser's inspectors at purchaser's expense.

\subsection*{5.13 PARALLEL CHORD SLOPED JOISTS}

The span of a parallel chord sloped joist shall be defined by the length along the slope. Minimum depth, load-carrying capacity, and bridging requirements shall be determined by the sloped definition of span. The Standard Load Table capacity shall be the component normal to the joist.

\section*{SECTION 6.* ERECTION STABILITY AND HANDLING}

When it is necessary for the erector to climb on the joists, extreme caution must be exercised since unbridged joists may exhibit some degree of instability under the erector's weight.

\section*{(a) Stability Requirements}
1) Before an employee is allowed on the steel joist: BOTH ends of joists at columns (or joists designated as column joists) shall be attached to its supports. For all other joists a minimum of one end shall be attached before the employee is allowed on the joist. The attachment shall be in accordance with Section 5.6 - End Anchorage.
When a bolted seat connection is used for erection purposes, as a minimum, the bolts must be snug tightened. The snug tight condition is defined as the tightness that exists when all plies of a joint are in firm contact. This may be attained by a few impacts of an impact wrench or the full effort of an employee using an ordinary spud wrench.
2) On steel joists that do not require erection bridging as shown by the unshaded area of the Load Tables, only one employee shall be allowed on the steel joist unless all bridging is installed and anchored.
* For a thorough coverage of this topic, refer to SJI Technical Digest \#9, "Handling and Erection of Steel Joists and Joist Girders".
3) Where the span of the steel joist is within the Red shaded area of the Load Table, the following shall apply:
a) The row of bridging nearest the mid span of the steel joists shall be bolted diagonal erection bridging; and
b) Hoisting cables shall not be released until this bolted diagonal erection bridging is installed and anchored, unless an alternate method of stabilizing the joist has been provided; and
c) No more than one employee shall be allowed on these spans until all other bridging is installed and anchored.
4) When permanent bridging terminus points cannot be used during erection, additional temporary bridging terminus points are required to provide stability.
5) In the case of bottom chord bearing joists, the ends of the joist must be restrained laterally per Section 5.4(d).
6) After the joist is straightened and plumbed, and all bridging is completely installed and anchored, the ends of the joists shall be fully connected to the supports in accordance with Section 5.6 End Anchorage.

\section*{DEFINITION OF SPAN}
(U. S. Customary Units)


NOTES: 1) DESIGN LENGTH = SPAN - 0.33 FT.
2) BEARING LENGTH FOR STEEL SUPPORTS SHALL NOT BE LESS THAN 2 1/2 INCHES; FOR MASONRY AND CONCRETE NOT LESS THAN 4 INCHES.
3) PARALLEL CHORD JOISTS INSTALLED TO A SLOPE GREATER THAN 1/2 INCH PER FOOT SHALL USE SPAN DEFINED BY THE LENGTH ALONG THE SLOPE.

\section*{ACCESSORIES AND DETAILS}

\section*{FABRICATION}
- Depth
2.5 in
- Maximum Length

10 ft
- Minimum Length

3 ft
- Contact your local Vulcraft plant for sloped or pitched seat information.

\subsection*{2.5K SERIES SIMPLE SPAN INFORMATION}
\begin{tabular}{|c|c|c|c|}
\hline 2.5K TYPE & \(\mathbf{2 . 5 K 1}\) & \(\mathbf{2 . 5 K 2}\) & \(\mathbf{2 . 5 K 3}\) \\
\hline \(\mathrm{S} \mathrm{in}^{3}\) & 0.62 & 0.84 & 1.2 \\
\hline \(\mathrm{I} \mathrm{in}^{4}\) & 0.78 & 1.1 & 1.5 \\
\hline \(\mathrm{WT} \mathrm{lbs} / \mathrm{ft}\) & 3.0 & 4.2 & 6.4 \\
\hline
\end{tabular}


NOTE: 2.5K SERIES NOT U.L. APPROVED.


NOTE: 2.5K SERIES NOT U.L. APPROVED.

\section*{LRFD}

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{11}{|l|}{LOAD TABLE FOR LOOSE OUTRIGGERS} \\
\hline \multirow[b]{3}{*}{OUTRIGGER TYPE} & \multicolumn{10}{|l|}{TOTAL ALLOWABLE LOAD FOR UNSUPPORTED CANTILEVER PLF*} \\
\hline & \multicolumn{10}{|c|}{SPAN ft-in} \\
\hline & 2'-0" & 2'-6" & 3'-0" & 3'-6" & 4'-0" & 4'-6" & 5'-0" & 5'-6" & 6'-0" & 6'-6" \\
\hline 2.5K1 & 825 & 749 & 519 & 381 & 293 & 231 & 188 & 155 & - & - \\
\hline 2.5K2 & 825 & 825 & 698 & 512 & 392 & 311 & 251 & 207 & 174 & - \\
\hline 2.5K3 & 825 & 825 & 825 & 740 & 566 & 447 & 362 & 299 & 252 & 215 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{11}{|l|}{LOAD TABLE FOR LOOSE OUTRIGGERS} \\
\hline \multirow[b]{3}{*}{OUTRIGGER TYPE} & \multicolumn{10}{|l|}{TOTAL ALLOWABLE LOAD FOR UNSUPPORTED CANTILEVER PLF*} \\
\hline & \multicolumn{10}{|c|}{SPAN ft-in} \\
\hline & 2'-0" & 2'-6" & 3'-0" & 3'-6" & 4'-0' & 4'-6" & 5'-0" & 5'-6" & 6'-0" & 6'-6" \\
\hline 2.5K1 & 550 & 499 & 346 & 254 & 195 & 154 & 125 & 103 & - & - \\
\hline 2.5K2 & 550 & 550 & 465 & 341 & 261 & 207 & 167 & 138 & 116 & - \\
\hline 2.5K3 & 550 & 550 & 550 & 493 & 377 & 298 & 241 & 199 & 168 & 143 \\
\hline
\end{tabular}
*Serviceability requirements must be checked by the specifying professional.

\section*{K SERIES OPEN WEB STEEL JOISTS}


ANCHORAGE TO STEEL
SEE SJI SPECIFICATION 5.3 (b) AND 5.6


CEILING EXTENSION


ANCHORAGE TO MASONARY
SEE SJI SPECIFICATION 5.3 (a) AND 5.6


BOTTOM CHORD STRUT


BOLTED CONNECTION* TYPICALLY REQUIRED AT COLUMNS


HEADERS
Note: If header does not bear at a Joist Panel Point add extra web in field as shown. EW or Panel Point by Vulcraft

\section*{MAXIMUM DUCT OPENING SIZES (K SERIES)*}
\begin{tabular}{|c|c|c|c|}
\hline \begin{tabular}{c} 
JOIST \\
DEPTH
\end{tabular} & ROUND & SQUARE & RECTANGLE \\
\hline 8 inches & 5 inches & \(4 \times 4\) inches & \(3 \times 8\) inches \\
\hline 10 inches & 6 inches & \(5 \times 5\) inches & \(3 \times 8\) inches \\
\hline 12 inches & 7 inches & \(6 \times 6\) inches & \(4 \times 9\) inches \\
\hline 14 inches & 8 inches & \(6 \times 6\) inches & \(5 \times 9\) inches \\
\hline 16 inches & 9 inches & \(71 / 2 \times 71 / 2\) inches & \(6 \times 10\) inches \\
\hline 18 inches & 11 inches & \(8 \times 8\) inches & \(7 \times 11\) inches \\
\hline 20 inches & 11 inches & \(9 \times 9\) inches & \(7 \times 12\) inches \\
\hline 22 inches & 12 inches & \(91 / 2 \times 91 / 2\) inches & \(8 \times 12\) inches \\
\hline 24 inches & 13 inches & \(10 \times 10\) inches & \(8 \times 13\) inches \\
\hline 26 inches & \(151 / 2\) inches & \(12 \times 12\) inches & \(9 \times 18\) inches \\
\hline 28 inches & 16 inches & \(13 \times 13\) inches & \(9 \times 18\) inches \\
\hline 30 inches & 17 inches & \(14 \times 14\) inches & \(10 \times 18\) inches \\
\hline \multicolumn{5}{|c|}{\({ }^{*}\) FOR LH SERIES CONSULT wITH VULCRAFT } \\
\hline
\end{tabular}

SEE SJI SPECIFICATION - SECTION 6. FOR HANDLING AND ERECTION OF KSERIES OPEN WEB STEEL JOISTS AND SJI TECHNICAL DIGEST NO. 9.

SPECIFYING PROFESSIONAL MUST INDICATE ON STRUCTURAL DRAWINGS SIZE AND LOCATION OF ANY DUCT THAT IS TO PASS THRU JOIST.

\section*{ACCESSORIES AND DETAILS}

K SERIES OPEN WEB STEEL JOISTS


HORIZONTAL BRIDGING
SEE SJI SPECIFICATION 5.5 AND 6.


BRIDGING ANCHORS
SEE SJI SPECIFICATION 5.5 AND 6.

NOTE: DO NOT WELD BRIDGING TO JOIST WEB MEMBERS.
DO NOT HANG ANY MECHANICAL, ELECTRICAL, ETC. FROM BRIDGING.


WELDED CROSS BRIDGING
SEE SJI SPECIFICATION 5.5 AND 6. HORIZONTAL BRIDGING SHALL BE USED IN SPACE ADJACENT TO THE WALL TO ALLOW FOR PROPER DEFLECTION OF THE JOIST NEAREST THE WALL.

FULL DEPTH CANTILEVER END
SEE SJI SPECIFICATION 5.4 (d) AND 5.5 FOR BRIDGING REQUIREMENTS.



BOLTED CROSS BRIDGING
SEE SJI SPECIFICATION 5.5 AND 6.
(a) Horizontal Bridging units shall be used in the space adjacent to the wall to allow for proper deflection of the joist nearest the wall.
(b) For required bolt size refer to bridging table on page 136. NOTE: Clip configuration may vary from that shown.


SQUARE END
SEE SJI SPECIFICATION 5.4 (d) AND 5.5 FOR BRIDGING REQUIREMENTS.


DEEP BEARINGS CONFIGURATION MAY VARY

\section*{STANDARD TYPES}

Longspan steel joists can be furnished with either underslung or square ends, with parallel chords or with single or double pitched top chords to provide sufficient slope for roof drainage.

The Longspan joist designation is determined by its nominal depth at the center of the span, except for offset double pitched joists, where the depth should be given at the ridge. A part of the designation should be either the section number or the total design load over the design live load (TL/LL given in plf).
All pitched joists will be cambered in addition to the pitch unless specified otherwise.


\section*{CAMBER}

Non-Standard Types: The design professional shall provide on the structural drawings the amount of camber desired in inches. If camber is not specified, Vulcraft will use the camber values for LH and DLH joists based on top chord length.
Standard Types: The camber listed in the table will be fabricated into the joists unless the design professional specifically states otherwise on the structural drawings.

\section*{NON-STANDARD TYPES}

The following joists can also be supplied by Vulcraft, however, THE DISTRICT SALES OFFICE OR MANUFACTURING FACILITY NEAREST YOU SHOULD BE CONTACTED FOR ANY LIMITATIONS IN DEPTH OR LENGTH.

**Contact Vulcraft for minimum depth at ends.
CAMBER FOR STANDARD TYPES
LH \&DLH series joists shall have camber in accordance with the following table:***
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{2}{|l|}{\begin{tabular}{l}
Top Chord \\
Length
\end{tabular}} & \multicolumn{2}{|r|}{\begin{tabular}{l}
Approx. \\
Camber
\end{tabular}} \\
\hline 20'-0" & (6096 mm) & 1/4" & (6mm) \\
\hline \(30 \cdot 0\) & (9144 mm) & 3/8" & (10 mm) \\
\hline \(40^{\prime}-0\) & ( 12192 mm ) & 5/8" & (16 mm) \\
\hline \(50 ' 0\) - & (15240 mm) & \(1{ }^{\prime \prime}\) & ( 25 mm ) \\
\hline 60'-0" & ( 18288 mm ) & 11/2" & (38 mm) \\
\hline \(70^{\prime}-0\) & (21336 mm) & 2 " & (51 mm) \\
\hline \(80^{\prime}-0\) & (24384 mm) & \(23 / 4 "\) & (70 mm) \\
\hline \(90 \cdot 0\) & (27432 mm) & \(31 / 2{ }^{\prime \prime}\) & (89 mm) \\
\hline 100'-0" & ( 30480 mm ) & \(41 / 4 "\) & (108 mm) \\
\hline 110'-0" & (33528 mm) & \(5{ }^{\prime \prime}\) & (127 mm) \\
\hline 120'-0" & (36576 mm) & \(6{ }^{\prime \prime}\) & (152 mm) \\
\hline 130'-0" & (39621 mm) & \(7{ }^{7}\) & (178 mm) \\
\hline 140'-0" & (42672 mm) & 8" & (203 mm) \\
\hline 144'-0" & ( 43890 mm ) & \(81 / 2^{\prime \prime}\) & (216 mm) \\
\hline
\end{tabular}
*** NOTE: If full camber is not desired near walls or other structural members please note on the structural drawings.

\section*{LH \& DLH SERIES LONGSPAN STEEL JOISTS}


ANCHORAGE TO STEEL SEE SJI SPECIFICATION 104.4 (b) AND 104.7 (b)


CEILING EXTENSION


ANCHORAGE TO MASONRY SEE SJI SPECIFICATION 104.4 (a) AND 104.7 (a)


BOTTOM CHORD EXTENSION
*If bottom chord extension is to be bolted or welded the specifiying professional must provide axial loads on structural drawings.


BOLTED CONNECTION See Note (c) Typically required at columns


TOP CHORD EXTENSION See Note (a)

NOTE: Configurations may vary from that shown.


SQUARE END
See SJI Specification 104.5 (f).
Cross bridging required at end of bottom bearing joist.


HORIZONTAL BRIDGING
For the proper use of horizontal bridging refer to sections 104.5(a) and 105.

NOTE: Do not weld bridging to web members. Do not hang any mechanical, electrical, etc. from bridging.


\section*{CROSS BRIDGING}
(a) Horizontal Bridging units shall be used in the space adjacent to the wall to allow for proper deflection of the joist nearest the wall.
(b) For required bolt size refer to bridging table on page 136. NOTE: Clip configuration may vary from that shown.

LH \& DLH SERIES OPEN WEB STEEL JOISTS SLOPED SEAT REQUIREMENTS LOW END

HIGH END
\begin{tabular}{|c|c|c|c|}
\hline \multirow[t]{2}{*}{} &  & SLOPE RATE & \begin{tabular}{l}
HIGH END MINIMUM **SEAT DEPTH \\
d
\end{tabular} \\
\hline &  & 1/4:12
3/8:12
1/2:12
1:12
1/2:12 & \[
\begin{gathered}
61 / 2^{\prime \prime} \\
61 / 2^{\prime \prime} \\
61 / 2^{\prime \prime} \\
61 / 2^{\prime \prime} \\
7^{\prime \prime}
\end{gathered}
\] \\
\hline  &  & \[
\begin{gathered}
2: 12 \\
21 / 2: 12 \\
3: 12 \\
31 / 2: 12 \\
4: 12 \\
41 / 2: 12 \\
5: 12 \\
\hline
\end{gathered}
\] & \(7^{\prime \prime}\)
\(71 / 2^{\prime \prime}\)
\(71 / 2^{\prime \prime}\)
\(8^{\prime \prime}\)
\(81 / 2^{\prime \prime}\)
\(81 / 2^{\prime \prime}\)
\(91 / 2^{\prime \prime}\) \\
\hline  &  & 6:12 \& OV & R SEE BELOW \\
\hline
\end{tabular}
* \(71 / 2\) " at 18 and 19 chord section numbers. Consult Vulcraft for information when TCX's are present.
** Add \(21 / 2^{2}\) to seat depths at 18 and 19 chord section numbers.

\section*{NOTES:}
(1) Depths shown are the minimums required for fabrication of sloped bearing seats.
(2) \(d=5 / 8+5 / \cos \theta+6 \tan \theta\)
(3) Clearance must be checked at outer edge of support as shown in detail B. Increase bearing depth as required to permit passage of 5 " deep extension.
(4) If extension depth greater than 5 " is required (see detail \(B\) and \(D\) ) increase bearing depths accordingly.

\section*{VULCRAFT LH \& DLH SERIES / GENERAL INFORMATION}

\section*{HIGH STRENGTH}

\section*{ECONOMICAL}

DESIGN - Vulcraft LH \& DLH Series long span steel joists are designed in accordance with the specifications of the Steel Joist Institute.

\section*{ROOF SPANS TO 144'-0}

\section*{FLOOR SPANS TO 120'-0}

PAINT - Vulcraft joists receive a shop-coat of rust inhibitive primer whose performance characteristics conform to those of the Steel Joist Institute specification 102.4.

SPECIFICATIONS see page 50.

ACCESSORIES see page 45.
LH \& DLH SERIES DETAILS
BASE LENGTH = CLEAR SPAN \(+1^{\prime}-0^{\prime \prime}\)

1/2" BEARING DEPTH FOR
18 \& 19 SECTION NO.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{6}{|c|}{MAXIMUM JOIST SPACING FOR DIAGONAL BRIDGING} \\
\hline \multicolumn{6}{|c|}{BRIDGING ANGLE SIZE-EQUAL LEG ANGLES} \\
\hline JOIST DEPTH & \[
\begin{gathered}
1 \times 7 / 64 \\
(25 \mathrm{~mm} \times 3 \mathrm{~mm}) \\
r=.20^{\prime \prime}
\end{gathered}
\] & \[
\begin{gathered}
1-1 / 4 \times 7 / 64 \\
(32 \mathrm{~mm} \times 3 \mathrm{~mm}) \\
r=.25^{\prime \prime}
\end{gathered}
\] & \[
\begin{gathered}
1-1 / 2 \times 7 / 64 \\
(38 \mathrm{~mm} \times 3 \mathrm{~mm})
\end{gathered}
\]
\[
r=.30^{\prime \prime}
\] & \[
\begin{gathered}
1-3 / 4 \times 7 / 64 \\
(45 \mathrm{~mm} \times 3 \mathrm{~mm})
\end{gathered}
\]
\[
r=.35^{\prime \prime}
\] & \[
\begin{gathered}
2 \times 1 / 8 \\
(51 \mathrm{~mm} \times 3 \mathrm{~mm}) \\
\mathrm{r}=.40^{\prime \prime}
\end{gathered}
\] \\
\hline 32 & 6'-1"(1854mm) & 7'-10"(2387mm) & \(9^{\prime}-7{ }^{\text {" }}\) (2921mm) & 11'-4"(3454mm) & 13'-0"(3962mm) \\
\hline 36 & & \(7{ }^{\prime}-9\) "(2362mm) & 9'6"(2895 mm) & \(11^{\prime}-3\) " 3429 mm ) & \(12^{\prime}-11^{\prime \prime}(3973 \mathrm{~mm})\) \\
\hline 40 & & \(7^{\prime}-7^{\prime \prime}(2311 \mathrm{~mm})\) & \(9^{\prime}-5\) " \((2870 \mathrm{~mm})\) & 11'-2"(3403mm) & 12'-10"(3911mm) \\
\hline 44 & & \(7^{7}-5\) "(2260mm) & \(9^{\prime}-3\) " \((2819 \mathrm{~mm})\) & \(11^{\prime}-0{ }^{\prime \prime}(3352 \mathrm{~mm})\) & \(12^{1}-9\) "(3886mm) \\
\hline 48 & & 7'-3"(2209mm) & 9'-2"(2794 mm) & 10'-11"(3327mm) & 12'-8"(3860mm) \\
\hline 52 & & & \(9^{\prime}-0\) "(2743 mm) & \(10^{\prime}-9\) "(3276mm) & \(12^{\prime}-7\) "(3835mm) \\
\hline 56 & & & 8'-10"(2692 mm) & \(10^{\prime}-8{ }^{\prime \prime}(3251 \mathrm{~mm})\) & 12'-5"(3784mm) \\
\hline 60 & & & \(8^{\prime}-7{ }^{\text {" }}\) (2616 mm) & \(10^{\prime}-6{ }^{\prime \prime}(3200 \mathrm{~mm})\) & 12'-4"(3759mm) \\
\hline 64 & & & \(8^{1}-5^{\prime \prime}(2565 \mathrm{~mm})\) & \(10^{\prime}-4{ }^{\prime \prime}(3149 \mathrm{~mm})\) & 12'-2"(3708mm) \\
\hline 68 & & & 8'-2"(2489 mm) & 10'-2"(3098mm) & \(12^{\prime}-0\) "(3657mm) \\
\hline 72 & & & \(8^{\prime}-0^{\prime \prime}(2438 \mathrm{~mm})\) & 10'-0'(3048mm) & 11'-10"(3606mm) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{7}{|c|}{ MAXIMUM JOIST SPACING FOR HORIZONTAL BRIDGING } \\
SPANS OVER 60' REQUIRE BOLTED DIAGONAL BRIDGING
\end{tabular}
*REFER TO THE LAST DIGITS OF JOIST DESIGNATION CONNECTION TO JOIST MUST RESIST FORCES LISTED IN TABLE 104.5.1.
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{LH \& DLH TABLE
MINIMUM BEARING LENGTHS} \\
\hline Joist Type & On Masonry & \[
\begin{gathered}
\text { On } \\
\text { Concrete }
\end{gathered}
\] & On Steel \\
\hline LH 02 thru 17 DLH 10 thru 19 & \(6^{\prime \prime}\) & \(6^{\prime \prime}\) & \(4 "\) \\
\hline \multicolumn{4}{|l|}{MINIMUM BEARING PLATE WIDTHS} \\
\hline LH 02 thru LH 12 DLH 10 thru DLH 12 & \(9 "\) & \(9 "\) & \\
\hline LH 13 thru LH 17 DLH 13 thru DLH 19 & \(12^{\prime \prime}\) & \(12^{\prime \prime}\) & \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline \multirow[t]{2}{*}{SECTION NUMBER*} & & \multicolumn{2}{|l|}{HORIZONTAL BRACING FORCE**} \\
\hline & \[
\begin{gathered}
\hline \text { MAX. SPACING } \\
\text { OF LINES OF } \\
\text { BRIDGING }
\end{gathered}
\] & lbs. & (N) \\
\hline 02, 03, 04 & 11'-0" (3352mm) & 400 & (1779) \\
\hline 05-06 & 12'-0" (3657mm) & 500 & (2224) \\
\hline 07-08 & 13'-0" (3962mm) & 650 & (2891) \\
\hline 09-10 & 14'-0" ( 4267 mm ) & 800 & (3558) \\
\hline 11-12 & 16'0" ( 4876 mm ) & 1000 & (4448) \\
\hline 13-14 & 16'-0" (4876mm) & 1200 & (5337) \\
\hline 15-16 & 21'-0" (6400mm) & 1600 & (7117) \\
\hline 17 & 21'-0' (6400mm) & 1800 & (8006) \\
\hline 18-19 & 26'0" (7924mm) & 2000 & (8896) \\
\hline \begin{tabular}{l}
NUMBER OF \\
*LAST TWO DIG \\
**NOMINAL BR
\end{tabular} & ES OF BRIDGING BAS TS OF JOIST DESIGNA CING FORCE IS UNFAC & D ON CL ION. tored. & R SPAN. \\
\hline MIN. A307 & BOLT REQ'D FOR & CONN & ECTION \\
\hline & SECTION & A307 & BOLT \\
\hline SERIES & NUMBER* & DIAM & TER \\
\hline LH/DLH & 2-12 & 3/8" (9 & mm) \\
\hline LH/DLH & 13-17 & 1/2" (1 & 2mm) \\
\hline DLH & 18 \& 19 & 5/8" (1 & mm) \\
\hline
\end{tabular}
*LAST TWO DIGITS OF JOIST DESIGNATION.

NOTES:1. Special designed LH and DLH can be supplied in longer lengths. See SLH Series Page 73.
2. Additional bridging may be required when joists support standing seam roof decks. The specifying professional should require that the joist manufacturer check the system and provide bridging as required to adequately brace the joists against lateral movement. For bridging requirements due to uplift pressures refer to sect. 104.12.

\section*{Examples:}

Steel

\section*{Example 1 (AISC Design Examples vV13.0)}

\section*{Example F.1-1a W-Shape Flexural Member Design in StrongAxis Bending, Continuously Braced.}

\section*{Given:}

Select an ASTM A992 W-shape beam with a simple span of 35 feet. Limit the member to a maximum nominal depth of 18 in . Limit the live load deflection to \(L / 360\). The nominal loads are a uniform dead load of \(0.45 \mathrm{kip} / \mathrm{ft}\) and a uniform live load of \(0.75 \mathrm{kip} / \mathrm{ft}\). Assume the beam is continuously braced.


Beam Loading \& Bracing Diagram
(full lateral support)

\section*{Solution:}

\section*{Material Properties:}
\[
\text { ASTM A992 } \quad F_{y}=50 \mathrm{ksi} \quad F_{u}=65 \mathrm{ksi} \quad \text { Manual }
\]

Calculate the required flexural strength
\begin{tabular}{|c|c|}
\hline LRFD & ASD \\
\hline \[
\begin{aligned}
w_{u} & =1.2(0.450 \mathrm{kip} / \mathrm{ft})+1.6(0.750 \mathrm{kip} / \mathrm{ft}) \\
& =1.74 \mathrm{kip} / \mathrm{ft}
\end{aligned}
\] & \[
\begin{aligned}
w_{a} & =0.450 \mathrm{kip} / \mathrm{ft}+0.750 \mathrm{kip} / \mathrm{ft} \\
& =1.20 \mathrm{kip} / \mathrm{ft}
\end{aligned}
\] \\
\hline \[
M_{u}=\frac{1.74 \mathrm{kip} / \mathrm{ft}(35.0 \mathrm{ft})^{2}}{8}=266 \mathrm{kip}-\mathrm{ft}
\] & \[
M_{a}=\frac{1.20 \mathrm{kip} / \mathrm{ft}(35.0 \mathrm{ft})^{2}}{8}=184 \mathrm{kip}-\mathrm{ft}
\] \\
\hline
\end{tabular}

Calculate the required moment of inertia for live-load deflection criterion of L/360
\[
\begin{aligned}
& \Delta_{\max }=\frac{L}{360}=\frac{35.0 \mathrm{ft}(12 \mathrm{in} . / \mathrm{ft})}{360}=1.17 \mathrm{in} . \\
& I_{x(r e q d)}=\frac{5 w l^{4}}{384 E \Delta_{\max }}=\frac{5(0.750 \mathrm{kip} / \mathrm{ft})(35.0 \mathrm{ft})^{4}(12 \mathrm{in} . / \mathrm{ft})^{3}}{384(29,000 \mathrm{ksi})(1.17 \mathrm{in} .)}=748 \mathrm{in} .{ }^{4}
\end{aligned}
\]

Manual
Table 3-23
Diagram 1

Manual Table 3-2

Manual Table 3-2

Example 1 (continued)


\section*{Example F.1-2a W-Shape Flexural Member Design in Strong-Axis Bending, Braced at Third Points}

\section*{Given:}

Verify the strength of the W18 \(\times 50\) beam selected in Example F.1-1a if the beam is braced at the ends and third points rather than continuously braced.


Beam Loading \& Bracing Diagram (bracing at ends and third points)

\section*{Solution:}

Required flexural strength at midspan from Example F.1-1a
\begin{tabular}{|l|l|}
\hline \multicolumn{1}{|c|}{ LRFD } & \multicolumn{1}{c|}{ ASD } \\
\hline\(M_{u}=266\) kip- ft & \(M_{a}=184\) kip- ft \\
\hline
\end{tabular}

\section*{Example 1 (continued)}
\(L_{b}=\frac{35.0 \mathrm{ft}}{3}=11.7 \mathrm{ft}\)

By inspection, the middle segment will govern. For a uniformly loaded beam braced at the ends and third points, \(C_{b}=1.01\) in the middle segment. Conservatively neglect this small adjustment in this case.

Manual
Table 3-1

Obtain the available strength from Table 3-10
Enter Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft . Obtain the available strength from the appropriate vertical scale to the left.
\begin{tabular}{|c|c|}
\hline LRFD & ASD \\
\hline\(\phi_{\mathrm{b}} M_{n} \approx 302 \mathrm{kip}-\mathrm{ft}>266 \mathrm{kip}-\mathrm{ft} \quad\) o.k. & \(\frac{M_{n}}{\Omega_{b}} \approx 201 \mathrm{kip}-\mathrm{ft}>184 \mathrm{kip}-\mathrm{ft} \quad\) o.k. \\
\hline
\end{tabular}

\footnotetext{
Manual
Table 3-10
}


For F1-1a, the unbraced length is zero. There is no zero on the chart, so the far left is used starting at the moment required of \(266 \mathrm{k}-\mathrm{ft}\). When a W 18 is not encountered with a greater moment capacity going up on the page, going to the right will intersect with a W18 line.

For F1-2a, the unbraced length is 11.7 ft . The same procedure applies, starting at a moment required of 266 k - ft . If no match is close to the

\section*{Example 2 (LRFD)}

\section*{U.S. CUSTOMARY UNITS AND (METRIC UNITS)}

Factored Load diagram per ASCE 7 2.3.2(3) \(\quad 1.2 \mathrm{D}+1.6 \mathrm{~S}\)


Joist manufacturer to design joist to support factored loads as shown.
Joist Supplier to design joist to support loads as shown above.
\[
\begin{aligned}
& \text { Total Load }=\frac{256}{2}(8)+(288+72) 30+600+960 \\
& +360=13,744 \mathrm{lbs} . \\
& R_{\mathrm{L}}=\frac{256(8)}{2}\left[\frac{30-8 / 3}{30}\right]+\frac{(288+72)(30)}{2}+600\left[\frac{9}{30}\right] \\
& +960\left[\frac{7}{30}\right]+660\left[\frac{3}{30}\right]=
\end{aligned}
\]
\(\mathrm{R}_{\mathrm{L}}=6773 \mathrm{bs}\).
\(\mathrm{R}_{\mathrm{R}}=6971 \mathrm{lbs}\).
\[
\text { Assume } R_{R}=\frac{W_{01}(\mathrm{~L})}{2}, \quad \mathrm{~W}_{\mathrm{e1}}=\frac{2(6971)}{30}=465 \mathrm{lbs} / \mathrm{ft}
\]

Point of Max. Mom. \(=\) Point of Zero Shear \((\mathrm{V})=\mathrm{L}_{1}\)
(dist. from rt. end of Jst)
\(\mathrm{V}=\) Zero \(=6971-(360+600+960)-(288+72)\left(\mathrm{L}_{1}\right)\)
\(L_{1}=14.03 \mathrm{ft}\).
\(M @ L_{1}=6971(14.03)-360(11.03)-\)
\[
960(7.03)-600(5.03)-\frac{(288+72)(14.03)^{2}}{2}
\]
\(M=48,634 \mathrm{ft} . \mathrm{lbs}\).
Assume \(\mathrm{M}=\frac{\mathrm{W}_{\mathrm{ea} 2}(\mathrm{~L})^{2}}{8}, \mathrm{~W}_{\mathrm{e} 2}=\frac{8(48.634)}{(30)^{2}}=432.31 \mathrm{bs} . / \mathrm{ft}\).
Using \(W_{e 1}=465 \mathrm{LB} / \mathrm{tt}\). @ SPAN \(=30^{\prime}\),
and \(\mathrm{D}=18^{\prime \prime}\)
Select 18 K 7 for total load (502) and live load (180) and call it: 18K9SP

\section*{(c) Special Considerations}

The specifying professional shall indicate on the construction documents special considerations including:
a) Profiles for non-standard joist and Joist Girder configurations (Standard joist and Joist Girder configurations are as indicated in the Steel Joist Institute Standard Specifications Load Tables \& Weight Tables of latest adoption).
b) Oversized or other non-standard web openings
c) Extended ends
d) Deflection criteria for live and total loads for non-SJI standard joists
e) Non-SJI standard bridging

LRFD
STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based on a 50 ksi Maximum Yield Strength Loads shown in Pounds per Linear Foot (plf)
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Joist \\
Designation
\end{tabular} & 18K3 & 18K4 & 18K5 & 18K6 & 18K7 & 18K9 & 18K10 \\
\hline Depth (In.) & 18 & 18 & 18 & 18 & 18 & 18 & 18 \\
\hline Approx. Wt. (lbs./ft.) & 6.6 & 7.2 & 7.7 & 8.5 & 9 & 10.2 & 11.7 \\
\hline \[
\begin{gathered}
\text { Span (ft.) } \\
\downarrow \\
18
\end{gathered}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 550 \\
& \hline
\end{aligned}
\] \\
\hline 19 & \[
\begin{aligned}
& 771 \\
& 494 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 523
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 523
\end{aligned}
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& 523
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& 523
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\] & \[
\begin{aligned}
& 825 \\
& 523
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 523
\end{aligned}
\] \\
\hline 20 & \[
\begin{aligned}
& 694 \\
& 423
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 490
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 490
\end{aligned}
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\] & \[
\begin{aligned}
& 825 \\
& 490
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 490
\end{aligned}
\] \\
\hline 21 & \[
\begin{aligned}
& 630 \\
& 364
\end{aligned}
\] & \[
\begin{aligned}
& 759 \\
& 426
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 460
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 460
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& 825 \\
& 460
\end{aligned}
\] \\
\hline 22 & \[
\begin{aligned}
& 573 \\
& 316 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 690 \\
& 370 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 777 \\
& 414 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 438 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 438 \\
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& 825 \\
& 438 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 438 \\
& \hline
\end{aligned}
\] \\
\hline 23 & \[
\begin{aligned}
& 523 \\
& 276 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 630 \\
& 323 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 709 \\
& 362 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 774 \\
& 393 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 418 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 418 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 418 \\
& \hline
\end{aligned}
\] \\
\hline 24 & \[
\begin{aligned}
& 480 \\
& 242 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 577 \\
& 284 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
651 \\
318 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 709 \\
& 345 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 789 \\
& 382 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 396 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 396 \\
& \hline
\end{aligned}
\] \\
\hline 25 & \[
\begin{array}{r}
441 \\
214 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 532 \\
& 250 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 600 \\
& 281 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 652 \\
& 305
\end{aligned}
\] & \[
\begin{aligned}
& 727 \\
& 337 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 377 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 377 \\
& \hline
\end{aligned}
\] \\
\hline 26 & \[
\begin{aligned}
& 408 \\
& 190
\end{aligned}
\] & \[
\begin{aligned}
& 492 \\
& 222
\end{aligned}
\] & \[
\begin{aligned}
& 553 \\
& 249
\end{aligned}
\] & \[
603
\] & \[
\begin{aligned}
& 672 \\
& 299
\end{aligned}
\] & \[
\begin{aligned}
& 807 \\
& 354
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 361
\end{aligned}
\] \\
\hline 27 & \[
\begin{aligned}
& 378 \\
& 169 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 454 \\
& 198 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
513 \\
222 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 558 \\
& 241 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 622 \\
& 267 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 747 \\
& 315 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 347 \\
& \hline
\end{aligned}
\] \\
\hline 28 & \[
\begin{aligned}
& 351 \\
& 151 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 423 \\
& 177 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
\hline 477 \\
199 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 519 \\
& 216 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 577 \\
& 239 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 694 \\
& 282
\end{aligned}
\] & \[
\begin{aligned}
& 822 \\
& 331
\end{aligned}
\] \\
\hline 29 & \[
\begin{aligned}
& 327 \\
& 136
\end{aligned}
\] & \[
\begin{aligned}
& 394 \\
& 159 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 444 \\
& 179
\end{aligned}
\] & \[
\begin{aligned}
& 483 \\
& 194 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 538 \\
& 215 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 646 \\
& 254 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 766 \\
& 298 \\
& \hline
\end{aligned}
\] \\
\hline 30 & \[
\begin{aligned}
& 304 \\
& 123
\end{aligned}
\] & \[
\begin{aligned}
& 367 \\
& 144
\end{aligned}
\] & \[
\begin{aligned}
& 414 \\
& 161
\end{aligned}
\] & \[
\begin{aligned}
& 451 \\
& 175
\end{aligned}
\] & \[
\begin{aligned}
& 502 \\
& 194
\end{aligned}
\] & \[
\begin{aligned}
& 603 \\
& 229
\end{aligned}
\] & \[
\begin{aligned}
& 715 \\
& 269
\end{aligned}
\] \\
\hline 31 & \[
\begin{aligned}
& 285 \\
& 111
\end{aligned}
\] & \[
\begin{aligned}
& 343 \\
& 130 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 387 \\
& 146
\end{aligned}
\] & \[
\begin{aligned}
& 421 \\
& 158
\end{aligned}
\] & \[
\begin{aligned}
& 469 \\
& 175 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 564 \\
& 207
\end{aligned}
\] & \[
\begin{aligned}
& 669 \\
& 243 \\
& \hline
\end{aligned}
\] \\
\hline
\end{tabular}
- Top values are total factored distributed load from strength and deflection criteria.
- Values below in gray are for live load deflection limit (unfactored).

\section*{Example 3 (AISC Design Examples vV13.0)}

\section*{Example E.1b W-Shape Column Design with Intermediate Bracing}

\section*{Given:}


Solution:
Calculate the required strength
\begin{tabular}{|c|c|}
\hline LRFD & ASD \\
\hline\(P_{u}=1.2(140 \mathrm{kips})+1.6(420 \mathrm{kips})=840 \mathrm{kips}\) & \(P_{a}=140 \mathrm{kips}+420 \mathrm{kips}=560 \mathrm{kips}\) \\
\hline
\end{tabular}

Select a column using Manual Table 4-1.
For a pinned-pinned condition, \(K=1.0\)
Since the unbraced lengths differ in the two axes, select the member using the y-y axis then verify the strength in the \(x-x\) axis.

Enter Table 4-1 with a y-y axis effective length, \(K L_{y}\), of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a \(\mathrm{W} 14 \times 90\). A 15 ft long \(\mathrm{W} 14 \times 90\) provides an available strength in the \(y\)-y direction of
\begin{tabular}{|l|l|}
\hline \multicolumn{1}{|c|}{ LRFD } & \multicolumn{1}{|c|}{ ASD } \\
\hline\(\phi P_{n}=1000 \mathrm{kips}\) & \(P_{n} / \Omega=667 \mathrm{kips}\) \\
\hline
\end{tabular}

The \(r_{x} / r_{y}\) ratio for this column, shown at the bottom of Manual Table 4-1, is 1.66 . The equivalent \(y\)-y axis effective length for strong axis buckling is computed as
\(K L=\frac{30.0 \mathrm{ft}}{1.66}=18 \mathrm{ft}\)
From the table, the available strength of a W \(14 \times 90\) with an effective length of 18 ft is
\begin{tabular}{|ll|ll|}
\hline \multicolumn{2}{|c|}{ LRFD } & \multicolumn{1}{c|}{ ASD } \\
\hline\(\phi_{c} P_{n}=928 \mathrm{kips}>840 \mathrm{kips}\) & o.k. & \(P_{n} / \Omega_{c}=618 \mathrm{kips}>560 \mathrm{kips}\) & o.k. \\
\hline
\end{tabular}

Manual Table 4-1

The available compression strength is governed by the \(x-x\) axis flexural buckling limit state.

Example 3 (continued)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{\(F_{y}=50 \mathrm{ksi}\)} & \multicolumn{9}{|r|}{Table 4-1 (continued) Available Strength in Axial Compression, kips w Shapes} & & \multicolumn{2}{|l|}{14} \\
\hline \multicolumn{2}{|c|}{Shape} & \multicolumn{12}{|c|}{W14×} \\
\hline \multicolumn{2}{|c|}{Wt/ft} & \multicolumn{2}{|l|}{145} & \multicolumn{2}{|l|}{132} & \multicolumn{2}{|l|}{120} & \multicolumn{2}{|l|}{109} & \multicolumn{2}{|c|}{99} & \multicolumn{2}{|c|}{90} \\
\hline \multicolumn{2}{|c|}{\multirow[t]{2}{*}{Design}} & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{C} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{C} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) \\
\hline & & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD & ASD & LRFD \\
\hline \multirow{26}{*}{} & 0 & 1280 & 1920 & 1160 & - 1740 & 1060 & 1590 & 959 & 1440 & 872 & 1310 & 792 & 1190 \\
\hline & 6 & 1250 & 1870 & 1130 & 1700 & 1030 & 1550 & 934 & 1400 & 849 & 1280 & 771 & 1160 \\
\hline & 7 & 1240 & 1860 & 1120 & 1680 & 1020 & 1530 & 924 & 1390 & 840 & 1260 & 763 & 1150 \\
\hline & 8 & 1220 & 1840 & 1110 & 1660 & 1010 & 1510 & 914 & 1370 & 831 & 1250 & 754 & 1130 \\
\hline & 9 & 1210 & 1820 & 1090 & 1640 & 995 & 1500 & 902 & 1360 & 820 & 1230 & 745 & 1120 \\
\hline & 10 & 1200 & 1800 & 1080 & 1620 & 981 & 1470 & 889 & 1340 & 808 & 1210 & 734 & 1100 \\
\hline & 11 & 1180 & 1770 & 1060 & 1590 & 965 & 1450 & 875 & 1320 & 795 & 1200 & 722 & 1090 \\
\hline & 12 & 1160 & 1740 & 1040 & 1570 & 949 & 1430 & 860 & 1290 & 781 & 1170 & 709 & 1070 \\
\hline & 13 & 1140 & 1720 & 1020 & 1540 & 931 & 1400 & 844 & 1270 & 767 & 1150 & 696 & 1050 \\
\hline & 14 & 1120 & 1690 & 1000 & 1510 & 912 & 1370 & 827 & 1240 & 751 & 1130 & 682 & 1020 \\
\hline & 15 & 1100 & 1650 & 982 & 1480 & 893 & 1340 & 809 & 1220 & 734 & 1100 & 667 & 1000 \\
\hline & 16 & 1080 & 1620 & 959 & 1440 & 872 & 1310 & 790 & 1190 & 717 & 1080 & 651 & 978 \\
\hline & 17 & 1050 & 1580 & 936 & 1410 & 851 & 1280 & 771 & 1160 & 699 & 1050 & 635 & 954 \\
\hline & 18 & 1030 & 1550 & 912 & 1370 & 829 & 1250 & 751 & 1130 & 681 & 1020 & 618 & 928 \\
\hline & 19 & 1000 & 1510 & 887 & 1330 & 806 & 1210 & 730 & 1100 & 662 & 995 & 600 & 902 \\
\hline & 20 & 979 & 1470 & 862 & 1300 & 783 & 1180 & 709 & 1070 & 642 & 966 & 583 & 876 \\
\hline & 22 & 926 & 1390 & 809 & 1220 & 735 & 1100 & 665 & 1000 & 602 & 906 & 546 & 821 \\
\hline & 24 & 871 & 1310 & 756 & 1140 & 685 & 1030 & 620 & 932 & 562 & 844 & 509 & 765 \\
\hline & 26 & 815 & 1230 & 702 & 1050 & 636 & 956 & 575 & 864 & 520 & 782 & 471 & 708 \\
\hline & 28 & 759 & 1140 & 647 & 973 & 586 & 881 & 530 & 797 & 479 & 720 & 434 & 652 \\
\hline & 30 & 702 & 1060 & 594 & 892 & 537 & 807 & 485 & 730 & 438 & 659 & 397 & 596 \\
\hline & 32 & 647 & 972 & 541 & 814 & 489 & 735 & 442 & 664 & 399 & 599 & 361 & 542 \\
\hline & 34 & 592 & 890 & 491 & 738 & 443 & 666 & 400 & 601 & 360 & 542 & 326 & 490 \\
\hline & 36 & 540 & 811 & 441 & 663 & 398 & 598 & 359 & 540 & 323 & 486 & 292 & 439 \\
\hline & 38 & 489 & 734 & 396 & 595 & 357 & 537 & 322 & 484 & 290 & 436 & 262 & 394 \\
\hline & 40 & 441 & 663 & 358 & 537 & 322 & 484 & 291 & 437 & 262 & 393 & 236 & 355 \\
\hline \multicolumn{14}{|c|}{Properties} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{6}{*}{}} & 191 & 287 & 175 & 263 & 151 & 227 & 128 & 191 & 111 & 167 & 95.9 & 144 \\
\hline & & 22.7 & 34.0 & 21.5 & 32.3 & 19.7 & 29.5 & 17.5 & 26.3 & 16.2 & 24.3 & 14.7 & 22.0 \\
\hline & & 477 & 717 & 407 & 612 & 312 & 468 & 220 & 330 & 173 & 260 & 129 & 194 \\
\hline & & 222 & 334 & 199 & 298 & 165 & 249 & 138 & 208 & 114 & 171 & 94.3 & 142 \\
\hline & & \multicolumn{2}{|r|}{\multirow[t]{2}{*}{\[
\begin{aligned}
& 14.1 \\
& 61.7 \\
& \hline
\end{aligned}
\]}} & \multicolumn{2}{|r|}{\multirow[t]{2}{*}{\[
\begin{aligned}
& 13.3 \\
& 56.0
\end{aligned}
\]}} & \multicolumn{2}{|r|}{\multirow[t]{2}{*}{\[
\begin{aligned}
& 13.2 \\
& 52.0
\end{aligned}
\]}} & \multicolumn{2}{|r|}{13.2} & \multicolumn{2}{|r|}{13.5} & \multicolumn{2}{|r|}{15.2} \\
\hline & & & & & & & & \multicolumn{2}{|r|}{48.4} & \multicolumn{2}{|r|}{45.3} & \multicolumn{2}{|r|}{42.6} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{7}{*}{}} & \multicolumn{2}{|l|}{} & \multicolumn{2}{|l|}{38.8} & \multicolumn{2}{|r|}{35.3} & \multicolumn{2}{|c|}{32.0} & \multicolumn{2}{|r|}{29.1} & \multicolumn{2}{|r|}{26.5} \\
\hline & & \multicolumn{2}{|l|}{\[
\begin{gathered}
42.7 \\
1710
\end{gathered}
\]} & \multicolumn{2}{|l|}{1530} & \multicolumn{2}{|l|}{1380} & \multicolumn{2}{|l|}{1240} & \multicolumn{2}{|l|}{1110} & \multicolumn{2}{|l|}{999} \\
\hline & & \multicolumn{2}{|l|}{\[
677
\]} & \multicolumn{2}{|l|}{548} & \multicolumn{2}{|l|}{495} & \multicolumn{2}{|l|}{447} & \multicolumn{2}{|l|}{402} & \multicolumn{2}{|l|}{362} \\
\hline & & & 3.98 & \multicolumn{2}{|r|}{3.76} & \multicolumn{2}{|r|}{3.74} & \multicolumn{2}{|r|}{3.73} & \multicolumn{2}{|r|}{3.71} & \multicolumn{2}{|r|}{3.70} \\
\hline & & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{48900}} & & 1. 67 & & 1.67 & & . 67 & & 1.66 & & 1.66 \\
\hline & & & & \multicolumn{2}{|l|}{43800} & \multicolumn{2}{|l|}{\[
39500
\]} & \multicolumn{2}{|l|}{35500} & \multicolumn{2}{|l|}{31800} & \multicolumn{2}{|l|}{28600} \\
\hline & & \multicolumn{2}{|l|}{19400} & \multicolumn{2}{|l|}{\[
15700
\]} & \multicolumn{2}{|l|}{\[
14200
\]} & \multicolumn{2}{|l|}{12800} & \multicolumn{2}{|l|}{11500} & \multicolumn{2}{|l|}{10400} \\
\hline \multicolumn{2}{|c|}{ASD} & \multicolumn{2}{|l|}{LRFD} & \multicolumn{10}{|l|}{\multirow[t]{2}{*}{}} \\
\hline \multicolumn{2}{|c|}{\(\Omega_{c}=1.67\)} & \multicolumn{2}{|l|}{\(\phi_{C}=0.90\)} & & & & & & & & & & \\
\hline
\end{tabular}

\section*{Example 4 (LRFD)}

Investigate the accepatbility of a W16 x 67 used as a beam-column under the unfactored loading shown in the figure. It is A992 steel ( \(\mathrm{F}_{\mathrm{y}}=50 \mathrm{ksi}\) ). Assume \(25 \%\) of the load is dead load with \(75 \%\) live load.

\section*{SOLUTION:}

DESIGN LOADS (shown on figure):
Axial load \(=1.2(0.25)(350 \mathrm{k})+1.6(0.75)(350 \mathrm{k})=525 \mathrm{k}\)
Moment at joint \(=1.2(0.25)(60 \mathrm{k}\)-tt \()+1.6(0.75)(60 \mathrm{k}\)-tt \()=90^{k-t t}\)
Determine column capacity and fraction to choose the appropriate interaction equation:

\[
\begin{aligned}
& \frac{k L}{r_{x}}=\frac{15 \mathrm{ft}\left(12 \mathrm{in} /{ }_{f t}\right)}{6.96 \mathrm{in}}=25.9 \text { and } \frac{k L}{r_{y}}=\frac{15 \mathrm{ft}(12 \mathrm{in} / f t}{2.46 \mathrm{in}}=73 \text { (governs) } \\
& P_{c}=\phi_{c} P_{n}=\phi_{c} F_{c r} A_{g}=(30.5 \mathrm{ksi}) 19.7 \mathrm{in}^{2}=600.85 \mathrm{k} \\
& \frac{P_{r}}{P_{c}}=\frac{525 \mathrm{k}}{600.85 \mathrm{k}}=0.87>0.2 \text { so use } \frac{P_{u}}{\phi_{c} P_{n}}+\frac{8}{9}\left(\frac{M_{u x}}{\phi_{b} M_{n x}}+\frac{M_{u y}}{\phi_{b} M_{n y}}\right) \leq 1.0
\end{aligned}
\]

There is no bending about the \(y\) axis, so that term will not have any values.
Determine the bending moment capacity in the \(x\) direction:
The unbraced length to use the full plastic moment \(\left(L_{p}\right)\) is listed as 8.69 ft , and we are over that so of we don't want to determine it from formula, we can find the beam in the Available Moment vs. Unbraced Length tables. The value of \(\phi \mathrm{M}_{\mathrm{n}}\) at \(L_{b}=15 \mathrm{ft}\) is 422 k - ft .
Determine the magnification factor when \(\mathrm{M}_{1}=0, \mathrm{M}_{2}=90 \mathrm{k}\)-ft:

\[
\begin{aligned}
& C_{m}=0.6-0.4 \frac{M_{1}}{M_{2}}=0.6-\frac{0^{k-f t}}{90^{k-f t}}=0.6 \leq 1.0 \quad P_{e 1}=\frac{\pi^{2} E A}{(\mathrm{Kl} / \mathrm{r})^{2}}=\frac{\pi^{2}\left(30 \times 10^{3} \mathrm{ksi}\right) 19.7 \mathrm{in}^{2}}{(25.9)^{2}}=8,695.4 \mathrm{k} \\
& B_{1}=\frac{C_{m}}{1-\left(P_{u} / P_{e 1}\right)}=\frac{0.6}{1-(525 k / 8695.4 \mathrm{k})}=0.64 \geq 1.0 \quad \text { USE } 1.0 \quad \mathrm{M}_{\mathrm{u}}=(1) 90 \mathrm{k}-\mathrm{ft}
\end{aligned}
\]

Finally, determine the interaction value:
\(\frac{P_{u}}{\phi_{c} P_{n}}+\frac{8}{9}\left(\frac{M_{u x}}{\phi_{b} M_{n x}}+\frac{M_{u y}}{\phi_{b} M_{n y}}\right)=0.87+\frac{8}{9}\left(\frac{90^{k-f t}}{422^{k-f t}}\right)=1.06 \not \pm 1.0\)

This is NOT OK. (and outside error tolerance). The section should be larger.

\section*{Case Study in Steel}
adapted from Structural Design Guide, Hoffman, Gouwens, Gustafson \& Rice., \(2^{\text {nd }}\) ed.

\section*{Building description}

The building is a one-story steel structure, typical of an office building. The figure shows that it has three 30 ft . bays in the short direction and a large number of bays in the long direction. Some options for the structural system include fully restrained with rigid connections and fixed column bases, simple framing with "pinned" connections and column bases requiring bracing against sideway, and simple framing with continuous beams and shear connections, pinned column bases and bracing against sidesway. This last situation is the one we'll evaluate as shown in Figure 2.5(c).


\section*{Loads}

\section*{Live Loads:}

Snow on Roof: \(30 \mathrm{lb} / \mathrm{ft}^{2}(1.44 \mathrm{kPa})\)
Wind: \(20 \mathrm{lb} / \mathrm{ft}^{2}(0.96 \mathrm{kPa})\)

\section*{Dead Loads:}

Roofing: \(8 \mathrm{lb} / \mathrm{ft}^{2}(0.38 \mathrm{kPa})\)
Estimated decking: \(3 \mathrm{lb} / \mathrm{ft}^{2}(0.141\)
Ceiling: \(7 \mathrm{lb} / \mathrm{ft}^{2}(0.34 \mathrm{kPa})\)
Total: \(18 \mathrm{lb} / \mathrm{ft}^{2}(0.86 \mathrm{kPa})\)

\section*{Materials}

A36 steel for the connection angles : ( \(\mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}, \mathrm{F}_{\mathrm{u}}=58 \mathrm{ksi}\) ) and A992 ( for the beams and columns \(\left(\mathrm{F}_{\mathrm{y}}=50\right.\)


K series open web joists and roof de

\section*{Decking:}

Decking selection is typically allowable stress design. Tables will give allowable total uniform load (taking self weight into account) based on stresses and deflection criteria for typical spans and how many spans are


Figure 2.5(c) Type SF - cantilever-suspended span system, braced against sidesway supported. The table (and description) for a Vulcraft 1.0 E deck is provided.

Areas in gray are governed by live load roof deflection.
The total load with snow and roofing \(=30 \mathrm{psf}+8 \mathrm{psf}=38 \mathrm{psf}\).
VERTICAL LOADS FOR TYPE 1.0E
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{3}{*}{No. of Spans} & \multirow[b]{3}{*}{\begin{tabular}{l}
Deck \\
Type
\end{tabular}} & \multirow[t]{3}{*}{Max. SDI Const Span} & \multicolumn{11}{|c|}{Allowable Total (Dead + Live) Uniform Load (PSF)} \\
\hline & & & \multicolumn{11}{|c|}{Span (ft.-in.) C. to C. of Support} \\
\hline & & & 2'-6 & 3'-0 & 3'-6 & 4'-0 & 4'-6 & 5'-0 & 5'-6 & 6'-0 & 6'-6 & 7'-0 & 7'-6 \\
\hline \multirow{4}{*}{1} & E26 & 2'-10 & 178 & 107 & 71 & 51 & 39 & 31 & 26 & 22 & 20 & 18 & 16 \\
\hline & E24 & 3'-5 & 249 & 148 & 97 & 68 & 51 & 40 & 32 & 27 & 24 & 21 & 19 \\
\hline & E22 & 3'-10 & 316 & 187 & 122 & 85 & 63 & 48 & 39 & 32 & 27 & 24 & 21 \\
\hline & E20 & 4'-2 & 379 & 224 & 145 & 100 & 73 & 56 & 45 & 37 & 31 & 27 & 24 \\
\hline \multirow{4}{*}{2} & E26 & 3'-4 & 273 & 189 & 139 & 107 & 81 & 62 & 49 & 40 & 34 & 29 & 25 \\
\hline & E24 & 4'-0 & 396 & 275 & 202 & 153 & 111 & 83 & 65 & 52 & 43 & 37 & 32 \\
\hline & E22 & 4'-6 & 515 & 357 & 263 & 190 & 137 & 102 & 79 & 63 & 52 & 44 & 37 \\
\hline & E20 & 5'-0 & 634 & 440 & 323 & 227 & 162 & 121 & 94 & 74 & 61 & 51 & 43 \\
\hline \multirow{4}{*}{3} & E26 & 3'-4 & 310 & 198 & 128 & 89 & 66 & 51 & 40 & 33 & 28 & 25 & 22 \\
\hline & E24 & 4'-0 & 469 & 276 & 177 & 122 & 89 & 67 & 53 & 43 & 36 & 31 & 27 \\
\hline & E22 & 4'-6 & 588 & 344 & 221 & 151 & 109 & 82 & 64 & 52 & 43 & 36 & 31 \\
\hline & E20 & 5'-0 & 707 & 413 & 264 & 180 & 129 & 97 & 75 & 60 & 50 & 42 & 36 \\
\hline
\end{tabular}

Notes: 1. Load tables are calculated using sectional properties based on the steel design thickness shown in the
Steel Deck Institute (SDI) Design Manual.
2. Loads shown in the shaded areas are governed by the live load deflection not in excess of \(1 / 240\) of the span. A dead load of 10 PSF has been included.

\section*{Open Web Joists:}

Maximum Sheet Length 42'-0 Extra Charge for Lengths Under 6'-0
 design or LRFD resistance for flexure (not for deflection). The total factored distributed load for joists at 6 ft on center will be:
\[
\begin{aligned}
\mathrm{w}_{\text {total }} & =\left(1.2 \times 18 \mathrm{lb} / \mathrm{ft}^{2}+1.6 \times 30 \mathrm{lb} / \mathrm{ft}^{2}\right)(6 \mathrm{ft})+1.2(8 \mathrm{lb} / \mathrm{ft} \text { estimated }) \\
& =427.2 \mathrm{lb} / \mathrm{ft} \quad\left(\text { with } 1.2 D+1.6\left(L \text {, or } L_{r}, \text { or } S, \text { or } R\right) \text { by catalogue }\right) \\
\mathrm{w}_{\text {live }} & =30 \mathrm{lb} / \mathrm{ft}^{2}(6 \mathrm{ft})=180 \mathrm{lb} / \mathrm{ft}
\end{aligned}
\]

STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based on a 50 ksi Maximum Yield Strength - Loads Shown in Pounds per Linear Foot (plf)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline Joist Designation & 18K3 & 18K4 & 18K5 & 18K6 & 18K7 & 18K9 & 18K10 & 20K3 & 20K4 & 20K5 & 20K6 & 20K7 & 20K9 & 20K10 & 22K4 & 22K5 & 22K6 & 22K7 & 22K9 & 22K10 & 22K11 \\
\hline Depth (In.) & 18 & 18 & 18 & 18 & 18 & 18 & 18 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 22 & 22 & 22 & 22 & 22 & 22 & 22 \\
\hline Approx. Wt. (lbs./ft.) & 6.6 & 7.2 & 7.7 & 8.5 & 9 & 10.2 & 11.7 & 6.7 & 7.6 & 8.2 & 8.9 & 9.3 & 10.8 & 12.2 & 8 & 8.8 & 9.2 & 9.7 & 11.3 & 12.6 & 13.8 \\
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\begin{aligned}
& 453 \\
& 219 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{l|}
\hline 511 \\
245 \\
\hline
\end{array}
\] & \[
\begin{array}{r}
556 \\
266 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& \hline 619 \\
& 295 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 745 \\
& 349
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 385
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 385 \\
& \hline
\end{aligned}
\] \\
\hline 31 & \[
\begin{aligned}
& 285 \\
& 111 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 343 \\
& 130
\end{aligned}
\] & \[
\begin{aligned}
& 387 \\
& 146
\end{aligned}
\] & \[
\begin{aligned}
& 421 \\
& 158
\end{aligned}
\] & \[
\begin{aligned}
& 469 \\
& 175
\end{aligned}
\] & \[
\begin{aligned}
& 564 \\
& 207
\end{aligned}
\] & \[
\begin{aligned}
& 669 \\
& 243 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
318 \\
138 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 384 \\
& 162 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
433 \\
182 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 471 \\
& 198 \\
& \hline
\end{aligned}
\] & \[
\begin{array}{r}
525 \\
219 \\
\hline
\end{array}
\] & \[
\begin{array}{r}
631 \\
259 \\
\hline
\end{array}
\] & \[
\begin{aligned}
& 748 \\
& 304
\end{aligned}
\] & \[
\begin{aligned}
& 424 \\
& 198 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 478 \\
& 222 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 520 \\
& 241 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 580 \\
& 267
\end{aligned}
\] & \[
\begin{aligned}
& 697 \\
& 316 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 369 \\
& \hline
\end{aligned}
\] & \[
\begin{aligned}
& 825 \\
& 369 \\
& \hline
\end{aligned}
\] \\
\hline
\end{tabular}

Deflection will limit the selection, and the most lightweight choice is the 22 K 4 which weighs approximately \(8 \mathrm{lb} / \mathrm{ft}\). Special provisions for bridging are required for the shaded area lengths and sections.

\section*{Continuous Beams:}

LRFD design is required for the remaining structural steel for the combinations of load involving Dead, Snow and Wind. The bracing must be designed to resist the lateral wind load.

The load values are:
for \(\mathrm{D}: \mathrm{w}_{\mathrm{D}}=18 \mathrm{lb} / \mathrm{ft}^{2} \cdot 30 \mathrm{ft}+(8 \mathrm{lb} / \mathrm{ft} \cdot 30 \mathrm{ft}) / 6 \mathrm{ft}=580 \mathrm{lb} / \mathrm{ft}\)
for \(\mathrm{S}: \mathrm{w}_{\mathrm{S}}=30 \mathrm{lb} / \mathrm{ft}^{2} \cdot 30 \mathrm{ft}=900 \mathrm{lb} / \mathrm{ft}\)
for \(\mathrm{W}: \mathrm{w}_{\mathrm{W}}=20 \mathrm{lb} / \mathrm{ft}^{2} \cdot 30 \mathrm{ft}=600 \mathrm{lb} / \mathrm{ft}\) (up or down) and laterally \(\mathrm{V}=600 \mathrm{lb} / \mathrm{ft}(15 \mathrm{ft} / 2)=4500 \mathrm{lb}\)

These DO NOT consider self weight of the beam.

The applicable combinations for the tributary width of 30 ft . are:
\(1.4 D \quad \mathrm{w}_{\mathrm{u}}=1.4(580 \mathrm{lb} / \mathrm{ft})=812 \mathrm{lb} / \mathrm{ft}\)
\(1.2 D+1.6 L+0.5\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\[
\mathrm{w}_{\mathrm{u}}=1.2(580 \mathrm{lb} / \mathrm{ft})+0.5(900 \mathrm{lb} / \mathrm{ft})=1146 \mathrm{lb} / \mathrm{ft}
\]
\(1.2 D+1.6\left(L_{r}\right.\) or \(S\) or \(\left.R\right)+(L\) or \(0.8 W)\)
\(\mathrm{w}_{\mathrm{u}}=1.2(580 \mathrm{lb} / \mathrm{ft})+1.6(900 \mathrm{lb} / \mathrm{ft})+0.8(600 \mathrm{lb} / \mathrm{ft})=\underline{2616 \mathrm{lb} / \mathrm{ft}}\)
\(1.2 D+1.6 W+L+0.5\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\(\mathrm{w}_{\mathrm{u}}=1.2(580 \mathrm{lb} / \mathrm{ft})+1.6(600 \mathrm{lb} / \mathrm{ft})+0.5(900 \mathrm{lb} / \mathrm{ft})=2106 \mathrm{lb} / \mathrm{ft}\)
\(1.2 D+1.0 E+L+0.25 S\)
\(\mathrm{w}_{\mathrm{u}}=1.2(580 \mathrm{lb} / \mathrm{ft})+0.25(900 \mathrm{lb} / \mathrm{ft})=921 \mathrm{lb} / \mathrm{ft}\)
\(0.9 D+1.6 W+1.6 H \quad \mathrm{w}_{\mathrm{u}}=0.9(580 \mathrm{lb} / \mathrm{ft})+1.6(-600 \mathrm{lb} / \mathrm{ft})[\) uplift \(]=-438 \mathrm{lb} / \mathrm{ft}(u p)\)
\(L, R, L_{r}, E \& H\) don't exist for our case.

For the largest load case, the shear \(\&\) bending moment diagrams are:


For the beams, we know that the maximum unbraced length is 6 ft . For the middle 6 feet of the end span, the moment is nearly uniform, so \(\mathrm{C}_{\mathrm{b}}=1\) is acceptable \(\left(\mathrm{C}_{\mathrm{b}}=1.08\right.\) for constant moment \()\). For the interior span, \(\mathrm{C}_{\mathrm{b}}\) is nearly 1 as well.


Choosing a W18x35 ( \(\left.\mathrm{M}_{\mathrm{u}}=229 \mathrm{k}-\mathrm{ft}\right)\) for the end beams, and a W12x30 \(\left(\mathrm{M}_{\mathrm{u}}=158 \mathrm{k}-\mathrm{ft}\right)\) for the interior beam, the self weight can be included in the total weight. The diagrams change to:


Check beam shear: \(\quad V_{u} \leq \phi_{v} V_{n}=1.0\left(0.6 F_{y w} A_{w}\right)\)
\[
\begin{aligned}
& \text { Exterior } \mathrm{V}_{\mathrm{u}}=34.67 \mathrm{k} \leq 1.0(0.6)(50 \mathrm{ksi})(17.1 \mathrm{in} .)(0.3 \mathrm{in} .)=153.9 \mathrm{k} \quad \mathrm{OK} \\
& \mathrm{~W} 18 \times 35: \mathrm{d}=17.7 \mathrm{in} ., \mathrm{t}_{\mathrm{w}}=0.3 \mathrm{in} ., \mathrm{I}_{\mathrm{x}}=510 \mathrm{in} .{ }^{4} \\
& \text { Interior } \mathrm{V}_{\mathrm{u}}=45.05 \mathrm{k} \leq 1.0(0.6)(50 \mathrm{ksi})(12.3 \mathrm{in} .)(0.26 \mathrm{in} .)=95.94 \mathrm{k} \mathrm{OK} \\
& \quad \mathrm{~W} 12 \times 30: \mathrm{d}=12.3 \mathrm{in} ., \mathrm{t}_{\mathrm{w}}=0.26 \mathrm{in} ., \mathrm{I}_{\mathrm{x}}=238 \mathrm{in} .{ }^{4}
\end{aligned}
\]

Check deflection (NO LOAD FACTORS) for total and live load (gravity and snow).

Exterior Beam: worst deflection is from no live load on the center span:


Maximum \(\Delta_{\text {total }}=2.20 \mathrm{in}\).
\[
\text { Is } \Delta_{\text {total }} \leq \mathrm{L} / 240=360 \mathrm{in} . / 240=1.5 \mathrm{in} . ? \quad \text { NO GOOD }
\]

We need an \(\mathrm{I} \geq(2.20 \mathrm{in} . / 1.5 \mathrm{in}).\left(510 \mathrm{in} .{ }^{4}\right)=748 \mathrm{in} .{ }^{4}\)

Maximum \(\Delta_{\text {live }}=1.86\) in.
\[
\text { Is } \Delta_{\text {live }} \leq \mathrm{L} / 360=360 \text { in. } / 360=1.0 \text { in.? NO GOOD }
\]

We need an \(\mathrm{I} \geq(1.86 \mathrm{in} . / 1.0 \mathrm{in}).\left(510 \mathrm{in} .{ }^{4}\right)=949 \mathrm{in}^{4}{ }^{4}\)
The W21x48 looks promising, but it has a note that it exceeds the compact limit for flexure.

Choose a W21 x \(50\left(\mathrm{I}_{\mathrm{x}}=984 \mathrm{in} .{ }^{4}\right)\) (because the W21x48 would require extra work!)

Now, \(\Delta_{\text {live }}=1.07\) in., which is reasonable close.


Interior Beam: worst deflection is from load on all spans:


Maximum \(\Delta_{\text {total }}(\) at midspan \()=1.31 \mathrm{in}\).
\[
\text { Is } \Delta_{\text {total }} \leq \mathrm{L} / 240=360 \mathrm{in} . / 240=1.5 \mathrm{in} . ? \quad \text { OK }
\]

Maximum \(\Delta_{\text {live }}(\) at midspan \()=0.94\) in.
\[
\text { Is } \Delta_{\text {live }} \leq \mathrm{L} / 360=360 \mathrm{in} . / 360=1.0 \mathrm{in} . ? \quad \text { OK }
\]

\section*{Columns:}

The load in the interior columns: \(\mathrm{P}_{\mathrm{u}}=85 \mathrm{k}\) (sum of the shears). This column will see minimal eccentricity from the difference in shear and half the column depth as the moment arm.

The load in the exterior columns: \(\mathrm{P}_{\mathrm{u}}=35 \mathrm{k}\). These columns will see some eccentricity from the beam shear connections. We can determine this by using half the column depth as the eccentricity distance.

The effective length of the columns is 15 ft (no intermediate bracing). Table \(4-1\) shows design strength in kips for W8 shapes (the smallest). The lightest section at 15 feet has a capacity of 230 k; much greater than what we need even with eccentricity.

The exterior column connection moment (unmagnified) when the W8x31 depth \(=8.0\) in
\[
(35 k)(8.0 \mathrm{in} / 2)\left(\frac{1 f t}{12 \mathrm{in}}\right)==11.7^{\mathrm{k}-\mathrm{ft}}
\]

The capacity of a W8x31 with an unbraced length of 15 ft (from another beam chart \()=114^{\mathrm{k}-\mathrm{ft}}\).

For \(\frac{P_{r}}{P_{c}}<0.2: \quad \frac{P_{u}}{2 \phi_{c} P_{n}}+\left(\frac{M_{u x}}{\phi_{b} M_{n x}}+\frac{M_{u y}}{\phi_{b} M_{n y}}\right) \leq 1.0\)

\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|c|}{Shape} & \multicolumn{4}{|l|}{W8×} \\
\hline \multicolumn{2}{|c|}{Wt/ft} & \multicolumn{2}{|r|}{35} & \multicolumn{2}{|l|}{31} \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{Design}} & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) & \(P_{n} / \Omega_{c}\) & \(\phi_{c} P_{n}\) \\
\hline & & ASD & LRFD & ASD & LRFD \\
\hline \multirow{23}{*}{} & 0 & 308 & 463 & 273 & 411 \\
\hline & 6 & 281 & 423 & 249 & 374 \\
\hline & 7 & 272 & 409 & 241 & 362 \\
\hline & 8 & 262 & 394 & 232 & 348 \\
\hline & 9 & 251 & 377 & 222 & 333 \\
\hline & 10 & 239 & 359 & 211 & 317 \\
\hline & 11 & 226 & 340 & 200 & 301 \\
\hline & 12 & 213 & 321 & 189 & 283 \\
\hline & 13 & 200 & 301 & 177 & 266 \\
\hline & 14 & 187 & 281 & 165 & 248 \\
\hline & 15 & 174 & 261 & 153 & 230 \\
\hline & 16 & 160 & 241 & 141 & 212 \\
\hline & 17 & 147 & 221 & 130 & 195 \\
\hline & 18 & 135 & 203 & 118 & 178 \\
\hline & 19 & 123 & 184 & 108 & 162 \\
\hline & 20 & 111 & 166 & 97.2 & 146 \\
\hline & 22 & 91.5 & 138 & 80.3 & 121 \\
\hline & 24 & 76.9 & 116 & 67.5 & 101 \\
\hline & 26 & 65.5 & 98.5 & 57.5 & 86.5 \\
\hline & 28 & 56.5 & 84.9 & 49.6 & 74.5 \\
\hline & 30 & 49.2 & 74.0 & 43.2 & 64.9 \\
\hline & 32 & 43.3 & 65.0 & 38.0 & 57.1 \\
\hline & 34 & & & & \\
\hline
\end{tabular}
\[
\frac{35 k}{230 k}=0.15<0.2: \quad \frac{35 k}{2(230 k)}+\left(\frac{11.7^{k-f t}}{114^{k-f t}}\right)=0.179 \leq 1.0
\]
so OK for eccentric loading of the beam-column (but we knew that).

\section*{Beam Shear Splice Connection:}

For this all-bolted single-plate shear splice, \(\mathrm{R}_{\mathrm{u}}=35 \mathrm{k}\)
\[
\begin{aligned}
& \mathrm{W} 21 \times 50: \mathrm{d}=20.8 \mathrm{in} ., \mathrm{t}_{\mathrm{w}}=0.38 \mathrm{in} . \\
& \mathrm{W} 12 \times 30: \mathrm{d}=12.3 \mathrm{in} ., \mathrm{t}_{\mathrm{w}}=0.26 \mathrm{in} .
\end{aligned}
\]

The plate material is A 36 with \(\mathrm{F}_{\mathrm{y}}=36 \mathrm{ksi}\) and \(\mathrm{F}_{\mathrm{u}}=58 \mathrm{ksi}\). We need to check that we can fit a plate within the fillets and provide enough


For \(3 / 4\) in. diameter A325-N bolts and standard holes without a concern for deformation of the holes, the capacity per bolt is:
\[
\begin{array}{ll}
\text { shear: } \quad & : R_{u} \leq \phi_{v} R_{n} \phi=0.75, R_{n}=F_{n} A_{b}, \text { where } \mathrm{F}_{\mathrm{n}}=54 \mathrm{ksi} \\
& 35 k \leq n(0.75)(54 k s i)\left[\frac{\pi(0.75 \text { in })^{2}}{4}\right] \\
& \text { so } \mathrm{n} \geq 1.96 . \text { Use } 2 \text { bolts }(1 @ 3 \mathrm{in} .+2 @ 1.25 \approx 5.5 \mathrm{in} .<10.125 \mathrm{in.})
\end{array}
\]
bearing for 2 rows of bolts: depends on thickness of thinnest web ( \(\mathrm{t}=0.26 \mathrm{in}\).) and the connected material
\[
R_{u} \leq \phi R_{n} \quad \phi=0.75, R_{n}=1.5 L_{c} t F_{u} \leq 3.0 d t F_{u}
\]
\(\mathrm{L}_{\mathrm{c}}=1.75 \mathrm{in}\). from the vertical edge of the beam to the edge of a hole
\[
\begin{aligned}
35 k \leq 2^{\text {bolts }} & {[0.75(1.5)(1.75 \text { in })(0.26 \text { in })(65 \mathrm{ksi})=38.0 \mathrm{k}} \\
\leq & 2^{\text {bolts }}[0.75(3)(0.75 \text { in })(0.26 \text { in })(65 \mathrm{ksi})=57.0 \mathrm{k} \mathrm{OK}
\end{aligned}
\]

If the spacing between the holes across the splice is 4 in ., the eccentricity, \(e_{x}\), is 2 inches. We need to find C , which represents the number of bolts that are effective in resisting the eccentric shear force.


Fig. 10-22. Eccentricity in a symmetrical shear splice.
\(r_{n}\) is the nominal shear per bolt:

\(C_{\text {min }}=\frac{35 k}{\left.0.75(54 k s i)^{(0.75 i n}\right)^{2} \pi / 4}=1.95 \quad\) (which we found as \(n\) )
C off the table is 2.54 bolts which is more than the minimum of 1.95 (which is why we have 2 ). OK.

If the plate is \(3 / 8 \mathrm{in}\). thick \(\times 8 \mathrm{in}\). wide \(\times 9 \mathrm{in}\). tall, check bolt bearing on plate:
\[
\begin{aligned}
& \phi R_{n}=2.4 d t F_{u} \text { (per bolt) } \\
& 2 \text { bolts }[2.4(0.75 \mathrm{in} .)(0.375 \mathrm{in} .)(58 \mathrm{ksi})=78.3 \mathrm{k}>35 \mathrm{k} \text { OK }
\end{aligned}
\]

Check flexure of the plate:
design moment: \(\quad M_{u}=\frac{R_{u} e}{2}=\frac{35 k \times 4 i n}{2}=70.0 \mathrm{k}-\mathrm{in}\)
yielding capacity: \(\quad \phi M_{n}=\phi F_{y} S_{x} \quad \phi=0.9\) (5.5 in. tall section, 3/8 in. thick) \(0.9(36 k s i)\left\lfloor\frac{0.375 \operatorname{in}(5.5 i n)^{2}}{6}\right\rfloor=61.25 \mathrm{k}\)-in \(>70.0 \mathrm{k}\)-in NOT OK with 6 in. tall, \(\phi M_{n}=72.9 \mathrm{k}-\mathrm{in}\)
rupture
\[
\phi M_{n}=\phi F_{u} S_{\text {net }} \quad \phi=0.75
\]
\[
S_{n e t}=I_{n e t} / c \text { and can be looked up or calculated }=1.74 \mathrm{in}^{3}
\]
\[
0.75(58 \mathrm{ksi})\left(1.74 \mathrm{in}^{3}\right)=75.7 \mathrm{k}-\mathrm{in}>70.0 \mathrm{k}-\mathrm{in} \quad \mathrm{OK}
\]

Check shear yielding of the plate: \(R_{u} \leq \phi R_{n} \quad \phi=1.00 \quad R_{n}=0.6 F_{y} A_{g}\)
\[
(1.00)[0.6(36 \mathrm{ksi})(6 \mathrm{in} .)(0.375 \mathrm{in} .)]=48.6 \mathrm{k}>35 \mathrm{k} \quad \mathrm{OK}
\]


Check shear rupture of the plate: \(R_{u} \leq \phi R_{n} \quad \phi=0.75 \quad R_{n}=0.6 F_{u} A_{n v}\) for \(3 / 4 "\) diameter bolts, the effective hole width is \((0.75+1 / 8)=0.875\) in.: (0.75)[0.6(58 ksi)(6 in. \(-2 \times 0.875 \mathrm{in}).(0.375 \mathrm{in}).]=41.6 \mathrm{k}>35 \mathrm{k} \quad\) OK

Check block shear rupture of the plate: \(\quad R_{u} \leq \phi R_{n} \quad \phi=0.75\)
\[
R_{n}=0.6 F_{u} A_{n v}+U_{b s} F_{u} A_{n t} \leq 0.6 F_{y} A_{g v}+U_{b s} F_{u} A_{n t}
\]
with \(U_{b s}=0.5\) when the tensile stress is non-uniform. (The tensile stress switches direction across the splice.) (and assuming 2 in . of width to the center of the bolt hole)
\[
\begin{aligned}
& R_{n}=0.60(58 k s i)(0.375 \text { in })\left[1.5 \text { in }+3 \text { in }-1.5^{\text {holes }}(0.875)\right]+ \\
& 0.5(58 \mathrm{ksi})(0.375 \mathrm{in})(2 \mathrm{in}-0.875 \mathrm{in} / 2)=58.6 .9 k \\
& \leq 0.6(36 k s i)(0.375 i n)(1.5 i n+3 i n)+0.5(36 k s i)(0.375 i n)(2 i n-0.875 i n / 2)=47.0 k \\
& 35 \mathrm{k}<0.75(47.0 \mathrm{k})=35.2 \mathrm{k} \quad \mathrm{OK} \\
& \text { Column base plates are designed for bearing on the concrete } \\
& \text { (concrete capacity) and plastic hinge development from flexure } \\
& \text { because the column "punches" down the plate and it could bend } \\
& \text { upward near the edges of the column (shown as } 0.8 b_{f} \text { and } 0.95 d \text { ). } \\
& \text { The plate dimensions are B and } \mathrm{N} \text {. The concrete has a } \\
& \text { compressive strength, } f^{\prime}{ }_{c}=3 \mathrm{ksi} \text {. }
\end{aligned}
\]

For \(\mathrm{W} 8 \times 31: \mathrm{d}=8.0 \mathrm{in}\)., \(\mathrm{b}_{\mathrm{f}}=8.0 \mathrm{in}\)., and if we provide width to put in bolt holes, we could use a 12 in . by12 in. plate (allowing about 2 inches each side). We will look at the interior column load of 85 k .
minimum thickness: \(t_{\text {min }}=l \sqrt{\frac{2 P_{u}}{0.9 F_{y} B N}}\)
where \(l\) is the larger of \(m, n\) and \(\lambda n^{\prime}\)
\[
\begin{aligned}
& m=(N-0.95 d) / 2=(12 \mathrm{in} .-0.95 \times 8.0 \mathrm{in} .) / 2=2.2 \mathrm{in} . \\
& n=\left(B-0.8 b_{f}\right) / 2=(12 \mathrm{in.}-0.8 \times 8.0 \mathrm{in} .) / 2=2.8 \mathrm{in} . \\
& n^{\prime}=\frac{\sqrt{d b_{f}}}{4}=\frac{\sqrt{8.0 \mathrm{in} \cdot 8.0 \mathrm{in}}}{4}=2.0 \mathrm{in} .
\end{aligned}
\]
\(\lambda\) is derived from a term \(X\) which takes the bounding area of the column, the perimeter, the axial force, and the concrete compressive strength into account:
\[
\begin{aligned}
& X=\frac{4 d b_{f}}{\left(d+b_{f}\right)^{2}} \cdot \frac{P_{u}}{\phi_{c} P_{p}}=\frac{4 d b_{f}}{\left(d+b_{f}\right)^{2}} \cdot \frac{P_{u}}{\phi_{c}\left(0.85 f_{c}^{\prime}\right) B N}=\frac{4 \cdot 8.0 \mathrm{in} \cdot 8.0 \mathrm{in}}{(8.0 \mathrm{in}+8.0 \mathrm{in})^{2}} \cdot \frac{85 \mathrm{k}}{0.6(0.85 \cdot 3 \mathrm{ksi}) 12 \mathrm{in} \cdot 12 \mathrm{in}} \\
& =0.386 \\
& \lambda=\frac{2 \sqrt{X}}{(1+\sqrt{1-X)}} \leq 1=\frac{2 \sqrt{0.386}}{(1+\sqrt{1-0.386)}}=0.697 \text { so } \lambda n^{\prime}=(0.697)(2.0 \mathrm{in} .)=1.39 \mathrm{in} . \\
& t_{p}=l \sqrt{\frac{2 P_{u}}{0.9 F_{y} B N}}=(2.8 \mathrm{in}) \sqrt{\frac{2 \cdot 85 k}{0.9(36 \mathrm{ksi})(12 \mathrm{in})(12 \mathrm{in})}}=0.534 \mathrm{in} .
\end{aligned}
\]

Use a 9/16 in. thick plate.
The anchor bolts must also be able to resist lateral shear. There also is friction between the steel and concrete to help. The International Building Code provided specifications for minimum edge distances and anchorage.

\section*{Continuous Beam Over Interior Column:}

BEAM OVER COLUMN (WITH CONTINUITY)
The design for this connection will involve a bearing plate at the top of the column, with a minimum number of bolts through the beam flanges to the plate. Because there will be high local compression, stiffener plates for the web will need to be added (refer to a plate girder design). Flexure with a reduced cross section area of the flanges should be checked.

\section*{Masonry Design}

\section*{Notation:}
\(A \quad=\) name for area
\(A_{n} \quad=\) net area, equal to the gross area subtracting any reinforcement
\(A_{n v}=\) net shear area of masonry
\(A_{s}=\) area of steel reinforcement in masonry design
\(A_{s t} \quad=\) area of steel reinforcement in masonry column design
\(A_{v} \quad=\) area of concrete shear stirrup reinforcement
\(A C I=\) American Concrete Institute
\(A S C E=\) American Society of Civil Engineers
\(b \quad=\) width, often cross-sectional
\(=\) total width of material at a horizontal section
\(C_{m} \quad=\) compression force in the masonry for masonry design
\(C M U=\) shorthand for concrete masonry unit
\(d \quad=\) effective depth from the top of a reinforced masonry beam to the centroid of the tensile steel
\(D \quad=\) shorthand for dead load
\(e \quad=\) eccentric distance of application of a force \((P)\) from the centroid of a cross section
\(E \quad=\) shorthand for earthquake load
\(E_{m} \quad=\) modulus of elasticity of masonry
\(E_{s} \quad=\) modulus of elasticity of steel
\(f_{a}=\) axial stress
\(f_{b}=\) bending stress
\(f_{m}=\) calculated compressive stress in masonry
\(f_{m}^{\prime}=\) masonry design compressive stress
\(f_{s} \quad=\) stress in the steel reinforcement for masonry design
\(f_{v} \quad=\) shear stress
\(F_{a}=\) allowable axial stress
\(F_{b}=\) allowable bending stress
\(F_{s} \quad=\) allowable tensile stress in reinforcement for masonry design
\(F_{t} \quad=\) allowable tensile stress
\(F_{v}=\) allowable shear stress
\(F_{v m}=\) allowable shear stress of the masonry
\begin{tabular}{|c|c|}
\hline \(F_{v s}\) & \(=\) allowable shear stress of the shear reinforcement \\
\hline \(h\) & \(=\) name for height \\
\hline & \(=\) effective height of a wall or column \\
\hline \(I_{n}\) & moment of inertia of the net section \\
\hline \(j\) & \(=\) multiplier by effective depth of masonry section for moment arm, jd \\
\hline \(k\) & \(=\) multiplier by effective depth of masonry section for neutral axis, kd \\
\hline K & \(=\) type of masonry mortar \\
\hline \(L\) & = shorthand for live load \\
\hline M & internal bending moment \\
\hline & type of masonry mortar \\
\hline \(M_{m}\) & \(=\) moment capacity of a reinforced masonry beam governed by steel stress \\
\hline \(M_{s}\) & \(=\) moment capacity of a reinforced masonry beam governed by masonry stress \\
\hline & Masonry Structural Joint Council \\
\hline & \(=\) modulus of elasticity transformation coefficient for steel to masonry \\
\hline & \(=\) shorthand for neutral axis (N.A.) \\
\hline & = type of masonry mortar \\
\hline & = National Concrete Masonry \\
\hline \(O\) & = type of masonry mortar \\
\hline \(P\) & = name for axial force vector \\
\hline \(P_{a}\) & allowable axial load in columns \\
\hline \(P_{e}\) & = critical (Euler) buckling load \\
\hline \(Q\) & = first area moment about a neutral axis \\
\hline \(r\) & \(=\) radius of gyration \\
\hline \(s\) & \(=\) spacing of stirrups in reinforced masonry \\
\hline \(S\) & \begin{tabular}{l}
\(=\) type of masonry mortar \\
\(=\) section modulus
\end{tabular} \\
\hline \(t\) & \(=\) name for thickness \\
\hline \(T_{s}\) & \(=\) tension force in the steel reinforcement for masonry design \\
\hline TMS & \(=\) The Masonry Society \\
\hline V & = internal shear force \\
\hline & = shorthand for wind load \\
\hline
\end{tabular}


\section*{Masonry Design}

Structural design standards for reinforced masonry are established by the Masonry Standards Joint Committee consisting of ACI, ASCE and The Masonry Society (TMS), and presents allowable stress design as well as limit state (strength) design.

\section*{Materials}

Masonry mortars are mixtures of water, masonry cement, lime, and sand. The strengths are categorized by letter designations (from MaSoNwOrK).
\begin{tabular}{|c|c|}
\hline Designation & strength range \\
\hline M & 2500 psi \\
\hline S & 1800 psi \\
\hline N & 750 psi \\
\hline O & 350 psi \\
\hline K & 75 psi \\
\hline
\end{tabular}
\(f^{\prime} \mathrm{m}=\) masonry prism test compressive strength
Deformed reinforcing bars come in grades \(40,50 \& 60\) (for 40 ksi, 50 ksi and 60 ksi yield strengths). Sizes are given nominally as \# of \(1 / 8\) ".

Clay and concrete masonry units are porous, and their durability with respect to weathering is an important consideration. The amount of water in the mortar is important as well as the absorption capacity of the units for good bond; both for strength and for weatherproofing. Because of the moisture and tendency for shrinkage and swelling, it is critical to provide control joints for expansion and contraction.

\section*{Masonry Walls}

Masonry walls can be reinforced or unreinforced, grouted or ungrouted, single wythe or cavity, prestressed or not. Cavity walls will require ties to force the two walls separated by the cavity to act as one.

From centuries of practice, the height to thickness ratio is limited because of slenderness (h/t < 25 or 35 depending on code). Most walls will see bending from wind or eccentricity along with bearing (combined stresses).

\section*{Allowable Stresses}
- If tension stresses result, the allowable tensile strength for unreinforced walls must not be exceeded. These are relatively low ( \(40-70 \mathrm{psi}\) ) and are shown in Table 2.2.3.2.
- If compression stresses result, the allowable strength (in bending) for unreinforced masonry \(\mathrm{F}_{\mathrm{b}}=1 / 3 f_{\mathrm{m}}\)
- If compression stresses result, the allowable strength (in bending) for reinforced masonry \(\mathrm{F}_{\mathrm{b}}=0.45 f^{\prime} \mathrm{m}\)
- Shear stress in unreinforced masonry cannot exceed \(\mathrm{F}_{\mathrm{v}}=1.5 \sqrt{f_{m}^{\prime}} \leq 120 \mathrm{psi}\).
- Shear stress in reinforced masonry for \(\mathrm{M} /(\mathrm{Vd}) \leq 0.25\) cannot exceed \(\mathrm{F}_{\mathrm{v}}=3.0 \sqrt{f_{m}^{\prime}}\)
- Shear stress in reinforced masonry for \(\mathrm{M} /(\mathrm{Vd}) \geq 1.0\) cannot exceed \(\mathrm{F}_{\mathrm{v}}=2.0 \sqrt{f_{m}^{\prime}}\)
- Allowable tensile stress, \(\mathrm{F}_{\mathrm{s}}\), in grades \(40 \& 50\) steel is 20 ksi , grade 60 is 32 ksi , and wire joint reinforcement is 30 ksi ..
where \(f^{\prime}{ }_{m}=\) specified compressive strength of masonry
Table 2.2.3.2 - Allowable flexural tensile stresses for clay and concrete masonry, psi (kPa) ( \(\mathrm{F}_{\mathrm{t}}\) )
\begin{tabular}{|l|c|c|c|c|}
\hline \multirow{2}{*}{\begin{tabular}{c} 
Direction of flexural tensile \\
stress and masonry type
\end{tabular}} & \multicolumn{4}{|c|}{ Mortar types } \\
\cline { 2 - 5 } & \multicolumn{4}{|c|}{\begin{tabular}{c} 
Portland cement/lime or \\
mortar cement (PCL)
\end{tabular}} \\
\cline { 2 - 5 } & M or S & \multicolumn{2}{|c|}{\begin{tabular}{c} 
Masonry cement or air entrained \\
portland cement/lime
\end{tabular}} \\
\hline \begin{tabular}{l} 
Normal to bed joints \\
Solid units \\
Hollow units' \\
Ungrouted \\
Fully grouted
\end{tabular} & \(53(366)\) & \(40(276)\) & \(32(221)\) & \(20(138)\) \\
\hline \begin{tabular}{l} 
Parallel to bed joints in running \\
bond
\end{tabular} & \(33(228)\) & \(25(172)\) & \(20(138)\) & \(12(83)\) \\
\begin{tabular}{l} 
Solid units \\
Hollow units \\
Ungrouted and partially \\
grouted
\end{tabular} & \(66(593)\) & \(84(579)\) & \(81(559)\) & \(77(531)\) \\
\(\quad\)\begin{tabular}{l} 
Fully grouted
\end{tabular} & \(106(731)\) & \(80(552)\) & \(64(441)\) & \(40(276)\) \\
\hline \begin{tabular}{l} 
Parallel to bed joints in masonry \\
not laid in running bond \\
Continuous grout section \\
parallel to bed joints
\end{tabular} & \(106(731)\) & \(50(345)\) & \(40(276)\) & \(25(172)\) \\
\begin{tabular}{l} 
Other
\end{tabular} & \(80(552)\) & \(64(441)\) & \(40(276)\) \\
\hline
\end{tabular}

1 For partially grouted masonry, allowable stresses shall be determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

\section*{Loads on Lintels in Masonry Walls}

Arching action is present in masonry walls when there is an opening and sufficient wall width on either side of the opening to resist the arch thrust. A lintel is required to support the weight of the wall material above the opening. When arching action is present, the weight that must be supported can be determined from a 45 degree angle. This area may be a triangle, or trapezoid if the wall height above the lintel is less than half the opening width. The distributed load is calculated as height x wall thickness x specific weight of the masonry.


When there are concentrated loads on the wall, the load can be distributed to a width at the lintel height based on a 60 degree angle.

\section*{Reinforced Masonry Members}

For stress analysis in masonry flexural members
- the strain is linear
 DESIGN BEAM FOR ENTIRE WALL AREA.

- the compressive stress in the masonry is linear
- the tensile stress in the steel is not at yield
- any masonry in tension is assumed to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

\section*{Load Combinations}

D
\(D+L\)
\(D+0.75\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\(D+0.75 L+0.75\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\(D+(0.6 \mathrm{~W}\) or \(0.7 E)\)
\(D+0.75 L+0.75(0.6 W)+0.75\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\(D+0.75 L+0.75(0.6 W)+0.75\left(L_{r}\right.\) or \(S\) or \(\left.R\right)\)
\(0.6 D+0.6 W\)
\(0.6 D+0.7 E\)


\section*{Internal Equilibrium}
\[
\rho=\frac{A_{s}}{b d} \quad \Sigma F=0: \quad A_{s} f_{s}=f_{m} b \frac{k d}{2}
\]
\(\mathrm{C}_{\mathrm{m}}=\) compression in masonry \(=\) stress x area \(=f_{m} \frac{b(k d)}{2}\)
\(\mathrm{T}_{\mathrm{s}}=\) tension in steel \(=\) stress x area \(=A_{s} f_{s}\)
\(C_{m}=T_{s}\) and \(\bullet M_{m}=T_{s}(d-k d / 3)=T_{s}(j d)\) and \(M_{s}=C_{m}(j d)\)
```

where $\quad f_{m}=$ stress in mortar at extreme fiber
$\mathrm{kd}=$ height to neutral axis
$b=$ width of section
$\mathrm{f}_{\mathrm{s}}=$ stress in steel at d
$\mathrm{A}_{\mathrm{s}}=$ area of steel reinforcement
$\mathrm{d}=$ depth to n.a. of reinforcement
$\mathrm{j}=(1-\mathrm{k} / 3)$

```

For flexure design:
\[
\begin{aligned}
& M \leq M_{m} \text { or } M_{s} \\
& \text { so, } M_{m}=T(j d)=0.5 f_{m} b d^{2} j k \text { and } M_{s}=C(j d)=\rho b d^{2} j f_{s}
\end{aligned}
\]

The design is adequate when \(f_{b} \leq F_{b}\) in the masonry and \(f_{s} \leq F_{s}\) in the steel.

\section*{Shear Strength}

Shear stress is determined by \(f_{v}=\mathrm{V} / \mathrm{A}_{\mathrm{nv}}\) where \(\mathrm{A}_{\mathrm{nv}}\) is net shear area. Shear strength is determined from the shear capacity of the masonry and the stirrups: \(F_{v}=F_{v m}+F_{v s}\). Stirrup spacings are limited to \(\mathrm{d} / 2\) but not to exceed 48 in .
where:
\[
\begin{aligned}
& F_{v m}=\frac{1}{2}\left[\left(4.0-1.75\left(\frac{M}{V d}\right)\right) \sqrt{f_{m}^{\prime}}\right]+0.25 \frac{P}{A_{n}} \quad \text { where } \mathrm{M} /(\mathrm{Vd}) \text { is positive and cannot exceed } 1.0 \\
& F_{v s}=0.5\left(\frac{A_{v} F_{s} d}{A_{n v} s}\right) \quad\left(F_{v}=3.0 \sqrt{f_{m}^{\prime}} \text { when } \mathrm{M} /(\mathrm{Vd}) \geq 0.25\right) \\
& \left(F_{v}=2.0 \sqrt{f_{m}^{\prime}} \text { when } \mathrm{M}(\mathrm{Vd}) \geq 1.0 .\right) \text { Values can be linearly interpolated. }
\end{aligned}
\]

Table B. 2 BALANCED SECTION PROPERTIES FOR RECTANGULAR MASONRY SECTIONS WITH TENSION REINFORCEMENT
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline \multirow{3}{*}{Reinforcement} & \multirow[b]{2}{*}{\[
\begin{gathered}
f_{m}^{\prime} \\
(\mathrm{psi})
\end{gathered}
\]} & \multirow[t]{2}{*}{Modular Ratio
\[
n=E_{s} / E_{m}
\]} & \multirow[b]{2}{*}{\[
\begin{gathered}
F_{b}=f_{m} / 3 \\
(\mathrm{psi})
\end{gathered}
\]} & \multicolumn{4}{|c|}{Balanced Section Properties} \\
\hline & & & & \(k\) & \(j\) & K & \(p=A_{s} / b d\) \\
\hline & \multicolumn{7}{|l|}{With Special Inspection-Full Code Values} \\
\hline \(\% \stackrel{\rightharpoonup}{\sim}\) & 1350 & 22 & 450 & 0.333 & 0.889 & 66.6 & 0.00375 \\
\hline \(\bigcirc\) & 1500 & 20 & 500 & 0.333 & 0.889 & 74.0 & 0.00416 \\
\hline \% & 2000 & 15 & 667 & 0.333 & 0.889 & 89.7 & 0.00556 \\
\hline - 4 & 4000 & 7.5 & 1333 & 0.333 & 0.889 & 197.0 & 0.01111 \\
\hline \(8 \stackrel{\rightharpoonup}{\square}\) & 1350 & 22 & 450 & 0.273 & 0.909 & 55.8 & 0.00256 \\
\hline \(\div 8\) & 1500 & 20 & 500 & 0.273 & 0.909 & 62.0 & 0.00284 \\
\hline \% 11 & 2000 & 15 & 667 & 0.273 & 0.909 & 82.7 & 0.00379 \\
\hline く & 4000 & 7.5 & 1333 & 0.273 & 0.909 & 165.4 & 0.00758 \\
\hline
\end{tabular}

The amount of steel reinforcement is limited. Too much reinforcement, or over-reinforced will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be under reinforced.

The reinforcement ratio is a fraction: \(\rho=\frac{A_{s}}{b d}\) and must be less than \(\rho_{b}\) where the balanced reinforcement ratio is a function of steel strength and masonry strength.

\section*{Flexure Design of Reinforcement}

One method is to choose a reinforcement ratio, find steel area, check stresses and moment:
1. find \(\rho_{b}\) and assume a value of \(\rho<\rho_{b}\)
2. find \(\mathrm{k}, \mathrm{j}\) and calculate \(b d^{2}=\frac{M}{\rho j F_{s}}\) where \(\mathrm{F}_{\mathrm{s}}\) is allowed stress in steel.

Choose nice b \& d values.
3. find \(A_{s}=\frac{M}{F_{s} j d}\)
3. check design for \(M<M_{s}=A_{S} F_{S}(j d)\)
4. check masonry flexural stress against allowable: \(f_{m}=\frac{M}{0.5 b d^{2} j k}<F_{b}\)

\section*{Load and Resistance Factor Design}

The design methodology is similar to reinforced concrete ultimate strength design. It is useful with high shear values and for seismic design. The limiting masonry strength is \(0.80 f^{\prime}\) m.

\section*{Force-Moment Interaction}


Combined stresses and the reduction of axial load with moment is similar to that for reinforced concrete column design as shown in the interaction diagram:

Reinforcement is typically placed in the center of walls. Grouting is placed in hollows with reinforcing, while other hollows may be empty. Stirrups are avoided.

Biaxial bending can occur in columns and stresses must satisfy:
\[
\frac{f_{a}}{F_{a}}+\frac{f_{b}}{F_{b}} \leq 1
\]


When maximum moment occurs somewhere other than at the end of the column or wall, a "virtual" eccentricity can be determined from \(e=M / P\).

\section*{Masonry Columns}

Columns are classified as having \(\mathrm{b} / \mathrm{t}<3\) and \(\mathrm{h} / \mathrm{t}>4\). Slender columns have a minimum side dimension of 8 " and must have \(\mathrm{h} / \mathrm{t} \leq 25\). They must be designed with an eccentricity of \(10 \%\) of the side dimension, and satisfy the interaction relationship of \(\frac{f_{a}}{F_{a}}+\frac{f_{b}}{F_{b}} \leq 1\), the tensile stress cannot exceed the allowable: \(f_{b}-f_{a} \leq F_{t}\) and the compressive stress exceed allowable for reinforced masonry: \(f_{a}+f_{b} \leq F_{b}\) provided \(f_{a} \leq F_{a}\).

For purely axial loading, the capacity \(\mathrm{P}_{\mathrm{a}}\) depends on the slenderness ratio of \(\mathrm{h} / \mathrm{r}\) :
unreinforced
\[
\begin{array}{ll}
P_{a}=\left[0.25 f_{m}^{\prime} A_{n}\left[1-\left(\frac{h}{140 r}\right)^{2}\right]\right. & \text { for } \mathrm{h} / \mathrm{r} \leq 99 \\
P_{a}=\left[0.25 f_{m}^{\prime} A_{n}\right]\left(\frac{70 r}{h}\right)^{2} & \text { for } \mathrm{h} / \mathrm{r}>99
\end{array}
\]
reinforced
\[
\begin{array}{ll}
P_{a}=\left[0.25 f_{m}^{\prime} A_{n}+0.65 A_{s t} F_{s}\right]\left[1-\left(\frac{h}{140 r}\right)^{2}\right] & \text { for } \mathrm{h} / \mathrm{r} \leq 99 \\
P_{a}=\left[0.25 f_{m}^{\prime} A_{n}+0.65 A_{s t} F_{s}\right]\left(\frac{70 r}{h}\right)^{2} & \text { for } \mathrm{h} / \mathrm{r}>99
\end{array}
\]
where
\[
\begin{aligned}
& \mathrm{h}=\text { effective length } \\
& \mathrm{r}=\text { least radius of gyration } \\
& \mathrm{A}_{\mathrm{n}}=\text { net area of masonry } \\
& \mathrm{A}_{\mathrm{st}}=\text { area of steel reinforcement } \\
& f_{m}^{\prime}=\text { specified masonry compressive strength } \\
& \mathrm{F}_{\mathrm{s}}=\text { allowed compressive strength of reinforcement }
\end{aligned}
\]


The least radius of gyration can be found with \(\sqrt{\frac{I}{A}}\) for a rectangle with side dimensions of \(\mathrm{b} \& \mathrm{~d}\) as:
\[
r=\sqrt{\frac{\frac{d b^{3}}{12}}{b d}}=\sqrt{\frac{b^{2}}{12}}=\frac{b}{\sqrt{12}}
\]
where \(b\) is the smaller of the two side dimensions.

\section*{Examples:}

Masonry
Example 1
Determine the maximum lateral force, H (by wind), as per MSJC.

8" CMU wall
Type S - PCL mortar
solidly grouted \(\mathbf{f}_{\mathbf{m}}=\mathbf{3 0 0 0} \mathbf{~ p s i}\)
Grade 60 stee
\#4 @ 32" Grade 60 steel

2 - \#8's each end of wall


Modular size (actual)

Case A: neglect all reinforcement
flexure
( \(\mathrm{M}=\mathrm{Hx} 8^{\prime}\) moment arm)
\[
S_{g}=\frac{7.63 \times 80^{2}}{6}=8139 \mathrm{in}^{3}
\]
shear

\section*{Case A: neglect all reinforcement}

Case B: consider vertical reinf., neglect horizontal reinf.
Case \(C\) : consider vertical and horizontal reinf.
Case D: design horizontal reinforcement for max. shear
shear
\(-120+\frac{96 \times H}{8139}=0 \quad H=10,174 \mathrm{lbs} .=10.2 \mathrm{kips}\)
\[
\begin{array}{cl}
\boldsymbol{F}_{v}=1.5 \sqrt{f_{m}^{\prime}}=1.5 \sqrt{3000}=82.2 \mathrm{psi} & f_{v}=\frac{V Q}{I_{n} b}=\frac{3 V}{2 \mathrm{~A}} \text { (solid rectangle) } \\
\boldsymbol{V}_{\max }=\frac{\mathbf{2}}{\mathbf{3}} \boldsymbol{F}_{v} \boldsymbol{b} \boldsymbol{t}=\frac{2}{\mathbf{3}}(82.2 \mathrm{psi})(7.63 \times 80)=33.4 \mathrm{kips} & \\
\text { (wall area) }
\end{array}
\]

Case B: consider only vertical reinforcement
Flexure: neglecting \(\mathbf{f}_{\mathbf{a}} \quad\) (allowed stress for grade 60 steel)

Shear
\[
\begin{aligned}
& \boldsymbol{M} / \boldsymbol{V} \boldsymbol{d}=\frac{\mathbf{8 . \boldsymbol { 0 } ^ { \prime }}}{\mathbf{6 . 0}}=\mathbf{1 . 3 3}>\boldsymbol{I} \\
& \boldsymbol{f o r} \frac{\boldsymbol{M}}{\boldsymbol{V} \boldsymbol{d}}>\boldsymbol{1} \quad F_{v m a x}=2 \sqrt{f_{m}^{\prime}}=2.0 \sqrt{3000}=109.5 \mathrm{psi} \quad f_{v}=\frac{V}{A_{n v}} \\
& F_{v m}=\frac{1}{2}\left[\left(4.0-1.75\left(\frac{M}{V d}\right)\right) \sqrt{f_{m}^{\prime}}\right\rfloor+0.25\left(\frac{P}{A_{n}}\right)=\frac{1}{2}[(4.0-1.75(1.33)) \sqrt{3000}]+0.25(120 \mathrm{psi})=75.8 \mathrm{psi} \\
& V_{\text {max }}=A_{n v} F_{v}=\left(7.63^{\prime \prime}\right)(80 \prime)(75.8 \mathrm{psi}) / 1000=46.3 \mathrm{kps}
\end{aligned}
\]
(actual width of 8 " nominal CMU block)

\section*{Case C: consider all reinforcement}

\section*{Flexure: same as case B}

\section*{Shear}
\[
\begin{aligned}
& F_{v m}=75.8 p s i \\
& F_{v s}=0.5\left(\frac{A_{v} F_{s} d}{A_{n} s}\right)=0.5\left(\frac{\left(0.20 \mathrm{in}^{2}\right)(32 \mathrm{ksi})(72 \mathrm{in})}{(7.63 \mathrm{in})(80 \mathrm{in})(32 \mathrm{in})}\right) 1000 \mathrm{lb} / \mathrm{k}=11.8 \mathrm{psi} \\
& F_{v}=87.6 \mathrm{psi} \\
& V_{\max }=87.6 \mathrm{psi}(7.63 \mathrm{in})(80 \mathrm{in}) / 1000=53.5 \mathrm{kips} \quad f_{v}=\frac{V}{A_{n v}}
\end{aligned}
\]

\section*{Case D: design horizontal reinforcement for maximum shear strength}
\[
\begin{aligned}
& F_{v s}=F_{v \max }-F_{v m}=109.5 \mathrm{psi}-75.8 \mathrm{psi}=33.7 \mathrm{psi} \\
& s=0.5\left(\frac{A_{v} F_{s} d}{A_{n} F_{V S}}\right)=0.5\left(\frac{\left(0.20 \mathrm{in}^{2}\right)(32 \mathrm{ksi})(72 \mathrm{in})}{(7.63 \mathrm{in})(80 \mathrm{in})(33.7 \mathrm{psi})}\right) 1000 \mathrm{lb} / \mathrm{k}=.11 .2 \mathrm{in} . .
\end{aligned}
\]
using \#4 rebars ( \(A_{v}=0.20\) in \(^{2}\) ) use \#4@8 in. horizontal

\section*{Example 2}

A 12 in. nominal solid brick column, 16 ft high, is built with brick, M mortar, and Grade 40 reinforcement. There are 4 - \#4 bars with \#2 ties at 8 in. on center. The column must carry an axial load of 63 kips . Check if the column design is adequate. \(f_{\mathrm{m}}^{\prime}=5,300 \mathrm{psi}\).

\section*{SOLUTION:}

Find the allowable axial load, \(\mathrm{P}_{\mathrm{a}}\) : which depends on \(\mathrm{h} / \mathrm{r}\)
\(\mathrm{r}=\sqrt{I / A}=\sqrt{d b^{3} / 12 b d}=b / \sqrt{12}=11.5\) in \(\times 0.289=3.3\) in (where b is the smallest dimension)
so \(\mathrm{b} / \mathrm{r}=16 \mathrm{ft} \times 12 \mathrm{in} / \mathrm{ft} / 3.3 \mathrm{in}=58<99\)
\[
\left.P_{a}=\left[0.25 f_{m}^{\prime} A_{n}+0.65 A_{s t} F_{s}\right] 1-\left(\frac{h}{140 r}\right)^{2} \right\rvert\,
\]
\(\mathrm{A}_{\mathrm{s}}=4\left(0.20 \mathrm{in}^{2}\right)=0.8 \mathrm{in} .^{2}\)
\(\mathrm{A}_{\mathrm{n}}=11.5 \mathrm{in} \times 11.5 \mathrm{in}-0.8 \mathrm{in}^{2}=131.5 \mathrm{in}^{2}\)
\(\mathrm{F}_{\mathrm{s}}=20 \mathrm{ksi}\),
\[
P_{a}=\left[0.25(5.3 k s i) 131.5 \mathrm{sin}^{2}+0.65\left(0.8 \mathrm{in}^{2}\right) 20 \mathrm{ksi}\right]\left[1-\left(\frac{16 f t(12 \mathrm{in} / \mathrm{ft})}{140(3.3 \mathrm{in})}\right)^{2}\right\rfloor=152.7 \mathrm{psi}
\]

Find the bending stress, \(\mathrm{f}_{\mathrm{b}}\) :
\(f_{b}=M / S, M=P e\), where \(e=0.1(11.5 \mathrm{in})=1.2 \mathrm{in}\).
\[
\mathrm{f}_{\mathrm{b}}=63 \mathrm{k}(1000 \mathrm{lb} / \mathrm{k})(1.2 \mathrm{in}) /\left(11.5 \times 11.5^{2} / 6\right) \mathrm{in}^{3}=298.2 \mathrm{psi}
\]

Is \(\frac{f_{a}}{F_{a}}+\frac{f_{b}}{F_{b}} \leq 1\) or equivalently \(\frac{P}{P_{a}}+\frac{f_{b}}{F_{b}} \leq 1\)
\(\mathrm{F}_{\mathrm{b}}=0.45 \mathrm{f}_{\mathrm{m}}=0.45(5,300 \mathrm{psi})=2385 \mathrm{psi}\)
\[
\frac{63 k}{152.7 k}+\frac{298.2 p s i}{2387 p s i}=0.54<1 \quad \mathrm{OK}
\]

Example 3
Determine the maximum transverse wind load, w, per MSJC.


2-wythe hollow-clay tile with Type \(S\) Portland cement lime mortar \(\mathbf{f}^{\prime}{ }_{\mathrm{m}}=\mathbf{4 5 0 0} \mathbf{~ p s i}\), special inspection provided
for a 1-foot width of wall:
\(A=144\) in \(^{2} ; ~ S=b^{2} / 6=288\) in \(^{3} ; \mathbf{r}=0.289 \mathrm{t}=3.47\) '
(considering solid section through mortar joint)
\[
I=b t^{3} / 12=1728 \text { in }^{4} ; r=\sqrt{I / A}=\sqrt{1728 / 144}=3.464 i n
\]
( \(b\) is the 1 ft width of wall and \(t\) is the thickness)

\section*{Case " \(A\) " with wind}

Weak section has been assumed to be through mortar bed joint. This assumes that unit strength will be at least twice that of the mortar (ratio of mortar area to clay area).
\[
\begin{aligned}
& \text { at midheight of wall : } M=\frac{P e}{2}+\frac{w h^{2}}{8} \\
& M=10 \mathrm{kip} \times \frac{3 \mathrm{in} .}{2}+\frac{w(15)^{2}}{8} \times 12 \frac{\mathrm{in} .}{\mathrm{ft} .} \\
& M=338 w+15.0 \\
& \text { where } w=k s f \text { and } M=k i p-i n \\
& \text { tension criterion : }-\frac{\boldsymbol{P}}{\boldsymbol{A}}+\frac{\boldsymbol{M}}{\boldsymbol{S}}=\boldsymbol{F}_{\boldsymbol{t}}=53 \text { psi (Table 2.2.3.2) } \\
& -\frac{\mathbf{1 0} \boldsymbol{k i p}}{\mathbf{1 4 4} \boldsymbol{\text { in}}^{2}}+\frac{\mathbf{3 3 8} \boldsymbol{w}+\mathbf{1 5 . 0}}{\mathbf{2 8 8} \boldsymbol{\text { in}}^{\mathbf{3}}}=0.053 p s i \quad w=60.0 \mathrm{psf}
\end{aligned}
\]

Note: assume \(F_{t}\) for solid units since mortar bed is full with respect to tension normal to bed joint.

for large \(P\) and small \(w\) : critical location is at top of wall: \(\quad M=P e\) for small \(P\) and large w: critical location is near midheight: \(M=P e / 2+w h^{\mathbf{2}} / 8\)

\section*{Case "A" with wind}
compression criterion :
\(\frac{\boldsymbol{f}_{\boldsymbol{a}}}{\boldsymbol{F}_{\boldsymbol{a}}}+\frac{\boldsymbol{f}_{\boldsymbol{b}}}{\boldsymbol{F}_{\boldsymbol{b}}}<1.0\)
\(\boldsymbol{M}=\mathbf{3 3 8} \times 0.060 \mathrm{ksf}+15.0=35.3 \mathrm{kip}-\mathrm{in}\)
\(\boldsymbol{f}_{\boldsymbol{a}}=\frac{\boldsymbol{P}}{\boldsymbol{A}}=\frac{\mathbf{1 0}}{\mathbf{1 4 4}}=\mathbf{0 . 0 6 9} \boldsymbol{k s i} \quad \boldsymbol{f}_{\boldsymbol{b}}=\frac{\boldsymbol{M}}{\boldsymbol{S}}=\frac{\mathbf{3 5 . 3}}{\mathbf{2 8 8}}=\mathbf{0 . 1 2 3} \boldsymbol{k s i} \quad F_{b}=0.33 f_{m}^{\prime}=0.33(4500 \mathrm{psi})=1500 \mathrm{psi}\)
\(\frac{h^{\prime}}{r}=\frac{15 \times 12}{3.47}=51.8 \quad F_{a}=0.25 f_{m}^{\prime}\left[1-\left(\frac{h^{\prime}}{140 r}\right)^{2}\right]=0.216 f_{m}^{\prime}=970 p s i\)
\(=0.25(4500 \mathrm{psi})\left\lfloor 1-\left(\frac{15 \cdot 12 \mathrm{in}}{140 \cdot 3.47 \mathrm{in}}\right)^{2}\right\rfloor=970 \mathrm{psi}\)
(psi)
\(\frac{69}{970}+\frac{123}{1500}=0.071+0.082=0.153<1.0\) ok.
Case " \(B\) " without wind
at top of wall : \(\quad M=P e=30 \mathrm{kip}-\mathrm{in}\).
tension criterion: \(\quad-\frac{\boldsymbol{P}}{\boldsymbol{A}}+\frac{\boldsymbol{M}}{\boldsymbol{S}}=\boldsymbol{F}_{\mathbf{t}}=53 \mathrm{psi}\)
\[
\begin{aligned}
& -\frac{10 \boldsymbol{k i p}}{144 \boldsymbol{i n}^{2}}+\frac{30 \boldsymbol{k i p}-\boldsymbol{i n}}{288 \boldsymbol{i n}^{3}} \leq 0.053 k s i \text { ? } \\
& -0.0694 \boldsymbol{k s i}+\mathbf{0 . 0 1 0 4} \boldsymbol{k s i}=\mathbf{0 . 0 3 4 8} \boldsymbol{k s i}<10.053 \text { ksi ok }
\end{aligned}
\]

\author{
from Building Structures, \(2^{\text {nd }}\) ed., Ambrose, 1993
}

\section*{CHAPTER THIRTY-NINE}

\section*{General Considerations}

Chapter 39 summarizes the general issues involved in foundation design, the properties and behavioral characteristics of foundation materials of significance for design work, and the problems of establishing useful design data and criteria.

\subsection*{39.1. BASIC PROBLEMS IN FOUNDATION DESIGN}

The design of the foundation for a building cannot be separated from the overall problems of the building structure and the building and site designs in general. Nevertheless, it is useful to consider the specific aspects of the foundation design that must be dealt with.

\section*{Site Exploration}

For purposes of the foundation design, as well as for the building and site development in general, it is necessary to know the actual site conditions. This investigation usually consists of two parts: determination of the ground surface conditions, and of the subsurface conditions. The surface conditions are determined by a site survey that establishes the three-dimensional geometry of the surface and the location of various objects and features on the site. Where they exist, the location of buried objects such as sewer lines, underground power and telephone lines, and so on, may also be shown on the site survey.

Unless they are known from previous explorations, the subsurface conditions must be determined by penetrating the surface to obtain samples of materials at various levels below the surface. Inspection and testing of these samples in the field, and possibly in a testing lab, is used to identify the materials and to establish a general description of the subsurface conditions.

\section*{Site Design}

Site design consists of positioning the building on the site and the general development, or redevelopment, of the site contours and features. The building must be both horizontally and vertically located. Recontouring the site may involve both taking away existing materials (called cutting) and building up to a new surface with materials brought in or borrowed from other locations on the site (called filling).

Development of controlled site drainage for water runoff is an important part of the site design.

\section*{Selection of Foundation Type}

The first formal part of the foundation design is the determination of the type of foundation system to be used. This decision cannot normally be made until the surface and subsurface conditions are known in some detail and the general size, shape, and location of the building are determined. In some cases it may be necessary to proceed with an approximate design of several possible foundation schemes so that the results can be compared.

\section*{Design of Foundation Elements}

With the building and site designs reasonably established, the site conditions known, and the type of foundation determined, work can proceed to the detailed design of individual structural elements of the foundation system.

\section*{Construction Planning}

In many cases the construction of the foundation requires a lot of careful planning. Some of the possible problems include conditions requiring dewatering the site during construction, bracing the sides of the excavation, underpinning adjacent properties or buildings, excavating difficult objects such as large tree roots or existing constructions, and working with difficult soils such as wet clays, quick sands or silts, soils with many large boulders, and so on. The feasibility of dealing with these problems, primarily in terms of cost and delays, may influence the foundation design as well as the positioning of the building on the site and the general site development.

\section*{Inspection and Testing}

During the design and construction of the foundation there are several times when it may be necessary to perform inspection or testing. Whether done by the designer or by others, the results of the inspections and tests will be used to influence design decisions or to verify the adequacy of the completed designs or construction. The need for this work will depend on the size of the building, the type of construction, the specific subsurface conditions, the type
of foundation system, and the various problems encountered during construction. Some of the ordinary inspections or tests are as follows:

Preliminary Site Investigation. The preliminary investigation usually consists of a site survey and some minimal subsurface investigation prior to the construction and often prior to the final design of the foundation. For major projects or difficult subsurface conditions it is usually necessary to have this information even before the preliminary site design and building design can be done.
Detailed Site Design. In some cases it is necessary to have additional information prior to the final design or the construction of the foundation. In some instances it is possible to incorporate this investigation with the early stages of the foundation work, with any necessary design adjustments made as the work progresses.
Inspection and Testing during Construction. At a bare minimum the completed excavation should be visually inspected prior to any construction to verify that the actual conditions encountered are those assumed for the design. In some cases the site conditions, the type of foundation, or the nature of the building may require extensive and continuous inspection and testing throughout the foundation construction process. Inspections by both the designer and the permit-granting agency may be required.
Inspection and Testing after Construction. In some cases it may be necessary to perform inspection and testing after the foundation construction is complete. This is usually required where progressive soil deformation is anticipated over time or with seasonal changes.

\section*{Remedial Alterations}

For various reasons it is often necessary to modify the foundation in some way from the original design. This is best done prior to construction, of course, but must sometimes be done as repair or renovation. The remedial measures may be obvious and simple to accomplish, or may require the best efforts of the most-qualified experts. Some of the situations that may require remedial alterations are:

Unanticipated Subsurface Conditions. Where the site conditions are very nonuniform or the preliminary investigations sketchy, or for other reasons, it may be necessary to modify the design due to actual encountered conditions.
Unanticipated Construction Problems. Weather conditions, unusual excavation problems, unavoidable delays, and a host of other possibilities may necessitate expedient change of the design.
Construction Errors. Foundation construction is usually done under the crudest and sloppiest of working conditions. Great accuracy and perfection is not to be expected. Overexcavation, mislocation of elements, er-
rors in dimensions, omission of details, and so on, are common.
Inadequate Performance of the Foundation. During construction, or even at some time after completion of the building, there may be evidence of excessive settlement, uneven settlement, horizontal shifting, tilting, or other forms of foundation failure.

\subsection*{39.2. SOIL CONSIDERATIONS RELATED TO FOUNDATION DESIGN}

The principal properties and behavior characteristics of soils that are of direct concern in foundation design are the following:

Strength. For bearing-type foundations the main concern is resistance to vertical compression. Resistance to horizontal pressure and to friction are of concern when foundations must resist the force of wind, earthquakes, or retained earth.
Strain Resistance. Deformation of soil under stress is of concern in designing for limitations of the movements of foundations, such as the vertical settlement of bearing foundations.
Stability. Frost action, fluctuations in water content, seismic shock, organic decomposition, and disturbance during construction are some of the things that may produce changes in physical properties of soils. The degree of sensitivity of the soil to these actions is called its relative stability.

\section*{Properties Affecting Construction Activity}

A number of possible factors may affect construction activity, including the following:

The relative ease of excavation.
Ease of and possible effects of site dewatering during construction.
Feasibility of using excavated materials as fill material.
Ability of the soil to stand on a vertical side of an excavation.
Effects of construction activity-notably the movement of workers and equipment-on unstable soils.

\section*{Miscellaneous Conditions}

In specific situations various factors may affect the foundation design or the problems to be dealt with during construction. Some examples are the following:

Location of the water table, affecting soil strength or stability, need for waterproofing basements, requirement for dewatering during construction, and so on.
Nonuniform soil conditions on the site, such as soil strata that are not horizontal, strips or pockets of poor soil, and so on.

Local frost conditions, affecting the depth required for bearing foundations and possible heave and settlement of exterior pavements.
Deep excavation or dewatering operations, possibly affecting the stability of adjacent properties, buildings, streets, and so on.

All of these concerns must be anticipated and dealt with in designing buildings and in planning and estimating construction costs. Persons charged with responsibility for design and planning foundation construction must have some understanding of the characteristics of ordinary soils so that they can translate information about site conditions into usable data. The discussions that follow deal with the basic nature of soils of various types, the behavior and design considerations of various foundation elements and systems, and the means for obtaining and using information about specific site conditions.

\subsection*{39.3. FOUNDATION DESIGN CRITERIA}

For the design of ordinary bearing-type foundations several structural properties of a soil must be established. The principal values are the following:

Allowable Bearing Pressure. This is the maximum permissible value for vertical compression stress at the contact surface of bearing elements. It is typically quoted in units of pounds or kips per square foot of contact surface.
Compressibility. This is the predicted amount of volumetric consolidation that determines the amount of set-
tlement of the foundation. Quantification is usually done in terms of the actual dimension of vertical settlement predicted for the foundation.
Active Lateral Pressure. This is the horizontal pressure exerted against retaining structures, visualized in its simplest form as an equivalent fluid pressure. Quantification is in terms of a density for the equivalent fluid given in actual unit weight value or as a percentage of the soil unit weight.
Passive Lateral Pressure. This is the horizontal resistance offered by the soil to forces against the soil mass. It is also visualized as varying linearly with depth in the manner of a fluid pressure. Quantification is usually in terms of a specific pressure increase per unit of depth.
Friction Resistance. This is the resistance to sliding along the contact bearing face of a footing. For cohesionless soils it is usually given as a friction coefficient to be multiplied by the compression force. For clays it is given as a specific value in pounds per square foot to be multiplied by the contact area.

Whenever possible, stress limits should be established as the result of a thorough investigation and the recommendations of a qualified soils engineer. Most building codes allow for the use of presumptive values for design. These are average values, on the conservative side usually, that may be used for soils identified by groupings used by the codes. Reprints of portions of the \(U B C, 1991\) edition, and the Building Code of the City of Los Angeles, 1976 edition, are given in Appendix D; both contain presumptive values for design. Soil types are identified only rather broadly in the \(U B C\), whereas the Los Angeles code uses what is essentially the unified system in establishing allowable bearing pressures.
from Foundation Analysis and Design, \(5^{\text {th }}\) ed., Bowles, 1996

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\section*{4-14 BEARING CAPACITY BASED ON BUILDING CODES (PRESUMPTIVE PRESSURE)}

In many cities the local building code stipulates values of allowable soil pressure to use whel designing foundations. These values are usually based on years of experience, althoughin some cases they are simply used from the building code of another city. Values such as the: are also found in engineering and building-construction handbooks. These arbitrary values soil pressure are often termed presumptive pressures. Most building codes now stipulate the other soil pressures may be acceptable if laboratory testing and engineering consideration can justify the use of alternative values. Presumptive pressures are based on a visual so: classification.

Table 4-8 indicates representative values of building code pressures. These values ar: primarily for illustrative purposes, since it is generally conceded that in all but minor corstruction projects some soil exploration should be undertaken. Major drawbacks to the used presumptive soil pressures are that they do not reflect the depth of footing, size of footing location of water table, or potential settlements.

TABLE 4-8
Presumptive bearing capacities from indicated building codes, \(\mathbf{k P a}\)
Soil descriptions vary widely between codes. The following represents author's interpretations.
\begin{tabular}{|c|c|c|c|c|}
\hline Soil description & Chicago, 1995 & \[
\begin{gathered}
\text { Natl. Board } \\
\text { of Fire } \\
\text { Underwriters, } \\
1976
\end{gathered}
\] & \[
\begin{gathered}
\text { BOCA,* } \\
1993
\end{gathered}
\] & \begin{tabular}{l}
Uniform \\
Bldg. Code, \(1991 \dagger\)
\end{tabular} \\
\hline Clay, very soft & 25 & & & \\
\hline Clay, soft & 75 & 100 & 100 & 100 \\
\hline Clay, ordinary & 125 & & & \\
\hline Clay, medium stiff & 175 & 100 & & 100 \\
\hline Clay, stiff & 210 & & 140 & \\
\hline Clay, hard & 300 & & & \\
\hline Sand, compact and clean & 240 & T & 140 & 200 \\
\hline Sand, compact and silty & 100 & & & \\
\hline Inorganic silt, compact & 125 & & & \\
\hline Sand, loose and fine & & 140 & 140 & 210 \\
\hline Sand, loose and coarse, or sand-gravel mixture, or compact and fine & & \[
\begin{gathered}
\text { to } \\
400
\end{gathered}
\] & 240 & 300 \\
\hline Gravel, loose and compact coarse sand & 300 & & 240 & 300 \\
\hline Sand-gravel, compact & & \(\underline{1}\) & 240 & 300 \\
\hline Hardpan, cemented sand, cemented gravel & 600 & 950 & 340 & \\
\hline Soft rock & & & & \\
\hline Sedimentary layered rock (hard shale, sandstone, siltstone) & & & 6000 & 1400 \\
\hline Bedrock & 9600 & 9600 & 6000 & 9600 \\
\hline
\end{tabular}

\footnotetext{
Note: Values converted from psf to kPa and rounded.
*Building Officials and Code Administrators International, Inc.
\(\dagger\) Author interpretation.
}

\section*{Foundation Design - Structure}
\begin{tabular}{|c|c|}
\hline Not & \\
\hline & \[
\begin{aligned}
& =\text { equivalent square column size in } \\
& \text { spread footing design } \\
& =\text { depth of the effective compression } \\
& \text { block in a concrete beam }
\end{aligned}
\] \\
\hline A & \(=\) name for area \\
\hline \(A_{g}\) & \(=\) gross area, equal to the total area ignoring any reinforcement \\
\hline & \(=\) area required to satisfy allowable stress \\
\hline \(A_{s}\) & \(=\) area of steel reinforcement in concrete design \\
\hline \(A_{1}\) & \(=\) area of column in spread footing design \\
\hline \(A_{2}\) & \(=\) projected bearing area of column load in spread footing design \\
\hline \(b\) & \begin{tabular}{l}
\(=\) width of retaining wall stem at ba \\
\(=\) rectangular column dimension in concrete footing design \\
\(=\) width, often cross-sectional
\end{tabular} \\
\hline \(b_{f}\) & \(=\) width of the flange of a steel or cross section \\
\hline \(b_{o}\) & \(=\) perimeter length for two-way shear in concrete footing design \\
\hline B & \[
\begin{aligned}
&= \text { spread footing or retaining wall } \\
& \text { base dimension in concrete design } \\
&= \text { dimension of a steel base plate for } \\
& \text { concrete footing design }
\end{aligned}
\] \\
\hline \(B_{s}\) & \(=\) width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design \\
\hline c & \(=\) rectangular column dimension in concrete footing design \\
\hline C & \(=\) dimension of a steel base plate for concrete footing design \\
\hline \(d\) & \[
\begin{aligned}
& =\text { effective depth from the top of a } \\
& \text { reinforced concrete member to the } \\
& \text { centroid of the tensile steel } \\
& =\text { name for diameter }
\end{aligned}
\] \\
\hline & ar diameter of a reinforcing bar \\
\hline \(d_{f}\) & \[
\begin{aligned}
= & \text { depth of a steel column flange } \\
& \text { (wide flange section) }
\end{aligned}
\] \\
\hline & \(=\) eccentric distance of application of a force ( P ) from the centroid of a cross section \\
\hline & = symbol for \\
\hline
\end{tabular}
\(f_{c}^{\prime}=\) concrete design compressive stress
\(f_{y} \quad=\) yield stress or strength
\(F_{\text {horizontal-resisting }}=\) total force resisting horizontal sliding
\(F_{\text {sliding }}=\) total sliding force
\(F_{X}=\) force in the \(x\) direction
\(h_{f} \quad=\) height of a concrete spread footing
\(H \quad=\) height of retaining wall
\(H_{A} \quad\) = horizontal force due to active soil pressure
\(l_{d} \quad=\) development length for reinforcing steel
\(l_{d c}=\) development length for column
\(l_{s} \quad=\) lap splice length in concrete design
\(L \quad=\) name for length or span length
\(L_{m} \quad=\) projected length for bending in concrete footing design
\(L^{\prime} \quad=\) length of the one-way shear area in concrete footing design
\(M \quad=\) moment due to a force
\(M_{n} \quad=\) nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete flexure design
\(M_{\text {overturning }}=\) total overturning moment
\(M_{\text {resisting }}=\) total moment resisting overturning about a point
\(M_{u}=\) maximum moment from factored loads for LRFD beam design
\(N \quad=\) name for normal force to a surface
\(o \quad=\) point of overturning of a retaining wall, commonly at the "toe"
\(p_{A} \quad=\) active soil pressure
\(P \quad=\) name for axial force vector
\(P_{\text {dowels }}=\) nominal capacity of dowels from concrete column to footing in concrete design
\(P_{D}=\) dead load axial force
\(P_{L} \quad=\) live load axial force
\(P_{n} \quad=\) nominal column or bearing load capacity in concrete design
\(P_{u} \quad=\) factored axial force
\(q_{\text {allowable }}=\) allowable soil bearing stress in allowable stress design
\(q_{\text {net }}=\) net allowed soil bearing pressure
\begin{tabular}{|c|c|c|c|}
\hline \(q_{u}\) & = factored soil bearing capacity in concrete footing design from load factors & \[
\begin{aligned}
& W \\
& \bar{y}
\end{aligned}
\] & \begin{tabular}{l}
\(=\) name for force due to weight \\
\(=\) the distance in the y direction from a reference axis to the centroid of a
\end{tabular} \\
\hline \(R\) & \(=\) name for reaction force vector & & shape \\
\hline SF & \begin{tabular}{l}
\(=\) shorthand for factor of safety \\
\(=\) thickness of retaining wall stem at top
\end{tabular} & \(\beta_{c}\) & \(=\) ratio of long side to short side of the column in concrete footing design \\
\hline \(T\) & \(=\) name for tension force vector & \(\phi\) & \(=\) resistance factor \\
\hline \(V_{n}\) & \(=\) nominal shear capacity & \(\gamma_{c}\) & \(=\) density or unit weight of concrete \\
\hline \(V_{u 1}\) & = maximum one-way shear from factored loads for LRFD beam design & \(\gamma_{s}\) & \[
\begin{aligned}
& =\text { density or unit weight of soil } \\
& =\text { reinforcement ratio in concrete }
\end{aligned}
\] \\
\hline \(V_{u 2}\) & \(=\) maximum two-way shear from factored loads for LRFD beam design & \(v_{c}\) & \begin{tabular}{l}
beam design \(=\mathrm{A}_{\mathrm{s}} / \mathrm{bd}\) \\
\(=\) shear strength in concrete design
\end{tabular} \\
\hline
\end{tabular}

\section*{Foundation Materials}

Typical foundation materials include:
- plain concrete
- reinforced concrete
- steel
- wood
- composites, ie. steel tubing filled with concrete

\section*{Foundation Design}

\section*{Generalized Design Steps}

Design of foundations with variable conditions and variable types of foundation structures will be different, but there are steps that are typical to every design, including:
1. Calculate loads from structure, surcharge, active \& passive pressures, etc.
2. Characterize soil - hire a firm to conduct soil tests and produce a report that includes soil material properties
3. Determine footing location and depth - shallow footings are less expensive, but the variability of the soil from the geotechnical report will drive choices
4. Evaluate soil bearing capacity - the factor of safety is considered here
5. Determine footing size - these calculations are based on working loads and the allowable soil pressure
6. Calculate contact pressure and check stability
7. Estimate settlements
8. Design the footing structure - design for the material based on applicable structural design codes which may use allowable stress design, LRFD or limit state design (concrete).

\section*{Shallow Foundation Types}

Considered simple and cost effective because little soil is removed or disturbed.

Spread footing - A single column bears on a square or rectangular pad to distribute the load over a bigger area.

Wall footing - A continuous wall bears on a wide pad to distribute the load..


Eccentric footing - A spread or wall footing that also must resist a moment in addition to the axial column load.

Combined footing - Multiple columns (typically two) bear on a rectangular or trapezoidal shaped footing.
Unsymmetrical footing - A footing with a shape that does not


Figure 5.1 Spread footing thapes and dinensions. evenly distribute bearing pressure from column loads and moments. It typically involves a hole or a non-rectangular shape influenced by a boundary or property line.

Strap footing - A combined footing consisting of two spread footings with a beam or strap connecting the slabs. The purpose of this is to limit differential settlements.
Mat foundation - A slab that supports multiple columns. The mat can be stiffened with a grid or grade beams. It is typically used when the soil capacity is very low.

\section*{Deep Foundation Types}

Considerable material and excavation is required, increasing cost and effort.
Retaining Walls - A wall that retains soil or other materials, and must resist sliding and

(c) Counterfort wall

(d) Buttress wall overturning. Can have counterforts, buttresses or keys.

Basement Walls - A wall that encloses a basement space, typically next to a floor slab, and that may be restrained at the top by a floor slab.

Piles - Next choice when spread footings or mats won't work, piles are used to distribute loads by end bearing to strong soil or friction to low strength soils. Can be used to resist uplift, a moment causing overturning, or to compact soils. Also useful when used in combination to control settlements of mats or slabs.

Drilled Piers - Soil is removed to the shape of the pier and concrete is added.

Caissons -Water and possibly wet soil is held back or excavated while the footing is constructed or dropped into place.


\section*{Loads and Stresses}

Bearing loads must be distributed to the soil materials, but because of their variability and the stiffness of the footing pad, the resulting stress, or soil pressure, is not necessarily uniform. But we assume it is for design because dealing with the complexity isn't worth the time or effort.

The increase in weight when replacing soil with concrete is called the overburden. Overburden may also be the result of adding additional soil to the top of the excavation for a retaining wall. It is extra uniformly distributed load that is considered by reducing the allowable soil pressure (instead of increasing the loads), resulting in a net allowable soil pressure, \(q_{\text {net }}\) :
\[
q_{\text {net }}=q_{\text {allowable }}-h_{f}\left(\gamma_{c}-\gamma_{s}\right)
\]

In order to design the footing size, the actual stress \(\mathrm{P} / \mathrm{A}\) must be less than or equal to the allowable pressure:
\[
\frac{P}{A} \leq q_{n e t}
\]


\section*{Design Stresses}

The result of a uniform pressure on the underside of a footing is identical to a distributed load on a slab over a column when looked at upside down. The footing slab must resist bending, one-way shear and two-way shear (punching).

one-way shear

two-way shear

\section*{Stresses with Eccentric Loading}

Combined axial and bending stresses increase the pressure on one edge or corner of a footing. We assume again a linear distribution based on a constant relationship to settling. If the pressure combination is in tension, this effectively means the contact is gone between soil and footing and the pressure is really zero. To avoid zero pressure, the eccentricity must stay within the kern. The maximum pressure must not exceed the net allowable soil pressure.


Overturning is considered in design such that the resisting moment from the soil pressure (equivalent force at load centroid) is greater than the overturning moment, M, by a factor of safety of at least 1.5
\[
S F=\frac{M_{\text {resist }}}{M_{\text {overturning }}} \geq 1.5
\]
where
\[
\begin{aligned}
& \mathrm{M}_{\text {resist }}=\text { average resultant soil pressure } \times \text { width } \times \text { location of load centroid with respect to } \\
& \text { column centroid } \\
& \mathrm{M}_{\text {overturning }}=\mathrm{P} \times \mathrm{e}
\end{aligned}
\]

\section*{Combined Footings}

The design of combined footing requires that the centroid of the area be as close as possible to the resultant of the two column loads for uniform pressure and settling.

\section*{Retaining Walls}

The design of retaining walls must consider overturning, settlement, sliding and bearing pressure. The water in the
 retained soil can significantly affect the loading and the active pressure of the soil. The lateral force acting at a height of \(\mathrm{H} / 3\) is determined from the active pressure, \(p_{A}\), (in force/cubic area) \(\quad H_{A}=\frac{p_{A} H^{2}}{2}\) as:


Overturning is considered the same as for eccentric footings:
\[
S F=\frac{M_{\text {resist }}}{M_{\text {overturning }}} \geq 1.5-2
\]
where
\(\mathrm{M}_{\mathrm{resist}}=\) summation of moments about " o " to resist rotation, typically including the moment due to the weight of the stem and base and the moment due to the passive pressure.
\(\mathrm{M}_{\text {overturning }}=\) moment due to the active pressure about " o ".


Sliding must also be avoided:
\[
S F=\frac{F_{\text {horizontal-resist }}}{F_{\text {sliding }}} \geq 1.25-2
\]

where
\(\mathrm{F}_{\text {horizontal-resist }}=\) summation of forces to resist sliding, typically including the force from the passive pressure and friction ( \(\mathrm{F}=\mu \cdot \mathrm{N}\) where.\(\mu\) is a constant for the materials in contact and N is the normal force to the ground acting down and is shown as R ). \(\mathrm{F}_{\text {sliding }}=\) sliding force as a result of active pressure.

For sizing, some rule of thumbs are:
- footing size, B
- reinforced concrete, \(B \approx 2 / 5-2 / 3\) wall height \((\mathrm{H})\)
- footing thickness, \(\mathrm{h}_{\mathrm{f}} \approx 1 / 12-1 / 8\) footing size (B)
- base of stem, \(\mathrm{b} \approx 1 / 10-1 / 12\) wall height \(\left(\mathrm{H}^{+} \mathrm{h}_{\mathrm{f}}\right)\)
- top of stem, \(\mathrm{t} \geq 12\) inches


\section*{Design of Isolated Square and Rectangular Footings (ACI 318-02)}

NOTE: This procedure assumes that the footing is concentrically loaded and carries no moment so that the soil pressure may be assumed to be uniformly distributed on the base.
1) Find service dead and live column loads:
\(\mathrm{P}_{\mathrm{D}}=\) Service dead load from column
\(\mathrm{P}_{\mathrm{L}}=\) Service live load from column
\(\mathrm{P}=\mathrm{P}_{\mathrm{D}}+\mathrm{P}_{\mathrm{L}}\) (typically - see ACI 9.2)
2) Find design (factored) column load, Pu :
\(\mathrm{P}_{\mathrm{U}}=1.2 \mathrm{P}_{\mathrm{D}}+1.6 \mathrm{P}_{\mathrm{L}}\)
3) Find an approximate footing depth, \(\mathrm{h}_{\mathrm{f}}\)

\(h_{f}=d+4^{\prime \prime}\) and is usually in multiples of 2,4 or 6 inches.
a) For rectangular columns \(\quad 4 d^{2}+2(b+c) d=\frac{P_{u}}{\phi v_{c}}\)
b) For round columns
\[
d^{2}+a d=\frac{P_{u}}{\phi v_{c}} \quad a=\sqrt{\frac{\pi d^{2}}{4}}
\]
where: \(a\) is the equivalent square column size
\[
\begin{aligned}
& v_{c}=4 \sqrt{f_{c}^{\prime}} \text { for two-way shear } \\
& \phi=0.75 \text { for shear }
\end{aligned}
\]
4) Find net allowable soil pressure, \(q_{\text {net }}\) :

By neglecting the weight of any additional top soil added, the net allowable soil pressure takes into account the change in weight when soil is removed and replaced
 by concrete: \(q_{\text {net }}=q_{\text {allowable }}-h_{f}\left(\gamma_{c}-\gamma_{s}\right)\) where \(\gamma_{c}\) is the unit weight of concrete (typically \(150 \mathrm{lb} / \mathrm{ft}^{3}\) ) and \(\gamma_{s}\) is the unit weight of the displaced soil
5) Find required area of footing base and establish length and width:
\[
A_{\text {req }}=\frac{P}{q_{\text {net }}}
\]

For square footings choose \(B \geq \sqrt{A_{\text {req }}}\)
For rectangular footings choose \(B \times L \geq A_{\text {req }}\)


\section*{6) Check transfer of load from column to footing: ACI 15.8}
a) Find load transferred by bearing on concrete in column: ACI 10.17
basic: \(\phi P_{n}=\phi 0.85 f_{c}^{\prime} A_{1}\) where \(\phi=0.65\) and \(A_{1}\) is the area of the column with confinement: \(\phi P_{n}=\phi 0.85 f_{c}^{\prime} A_{1} \sqrt{\frac{A_{2}}{A_{1}}}\) where \(\sqrt{\frac{A_{2}}{A_{1}}}\) cannot exceed 2. IF the column concrete strength is lower than the footing, calculate \(\phi P_{n}\) for the column too.
b) Find load to be transferred by dowels:
\(\phi P_{\text {dowels }}=P_{u}-\phi P_{n}\)
IF \(\phi P_{n} \geq P_{u}\) only nominal dowels are required.
c) Find required area of dowels and choose bars
 Req. dowel \(A_{s}=\frac{\phi P_{\text {dowels }}}{\phi f_{y}}\) where \(\phi=0.65\) and \(f_{y}\) is the reinforcement grade
Choose dowels to satisfy the required area and nominal requirements:
i) Minimum of 4 bars
ii) Minimum \(A_{s}=0.005 A_{g}\) ACI 15.8.2.1
where \(A_{g}\) is the gross column area
iii) 4-\#5 bars
 \(l_{d c}=\frac{0.02 f_{y} d_{b}}{\sqrt{f_{c}^{\prime}}}\) but not less than \(0.0003 f_{y} d_{b}\) or \(8 "\) where \(d_{b}\) is the bar diameter

NOTE: The footing must be deep enough to accept \(l_{d c}\). Hooks are not considered effective in compression and are only used to support dowels during construction.
e) Find length of lapped splices of dowels with column bars: ACI 12.16 \(l_{s}\) is the largest of:
i) larger of \(l_{d c}\) or \(0.0005 f_{y} d_{b}\left(f_{y}\right.\) of grade 60 or less) of smaller bar \(\left(0.0009 f_{y}-24\right) d_{b}\left(f_{y}\right.\) over grade 60\()\)
ii) \(\quad l_{d c}\) of larger bar
iii) not less than 12 "


See ACI 12.17.2 for possible reduction in \(l_{s}\)
7) Check two-way (slab) shear:
a) Find dimensions of loaded area:
i) For concrete columns, the area coincides with the column area, if rectangular, or equivalent square area if circular (see 3)b))
ii) For steel columns an equivalent loaded area whose boundaries are halfway between the faces of the steel column and the edges of the steel base plate is used: ACI 15.4.2c.

\[
b=b_{f}+\frac{\left(B-b_{f}\right)}{2} \text { where } b_{f} \text { is the width of }
\]
column flange and \(B\) is base plate side
\[
c=d_{f}+\frac{\left(C-d_{f}\right)}{2} \text { where } d_{f} \text { is the depth of column flange and } C \text { is base plate side }
\]
b) Find shear perimeter: ACI 11.12.1.2

Shear perimeter is located at a distance of \(d / 2\) outside boundaries of loaded area and length is \(b_{o}=2(c+d)+2(b+d)\)
(average \(\mathrm{d}=\mathrm{h}_{\mathrm{f}}-3 \mathrm{in}\). cover -1 assumed bar diameter)
c) Find factored net soil pressure, \(q_{u}\) :
\(q_{u}=\frac{P_{u}}{B^{2}}\) or \(\frac{P_{u}}{B \times L}\)
d) Find total shear force for two-way shear, \(V_{u 2}\) :
\(V_{u 2}=P_{u}-q_{u}(c+d)(b+d)\)
e) Compare \(V_{u 2}\) to two-way capacity, \(\phi V_{n}\) :

\(V_{u 2} \leq \phi\left(2+\frac{4}{\beta_{c}}\right) \sqrt{f_{c}^{\prime}} b_{o} d \leq \phi 4 \sqrt{f_{c}^{\prime}} b_{o} d \quad\) ACI 11.12.2.1
where \(\phi=0.75\) and \(\beta_{c}\) is the ratio of long side to
short side of the column
NOTE: This should be acceptable because the initial footing size was chosen on the basis of two-way shear limiting. If it is not acceptable, increase \(h_{f}\) and repeat steps starting at b).


\section*{8) Check one-way (beam) shear:}

The critical section for one-way shear extends across the width of the footing at a distance \(d\) from the face of the loaded area (see 7)a) for loaded area). The footing is treated as a cantilevered beam. ACI 11.12.1.1
a) Find projection, \(L^{\prime}\) :
i) For square footing: \(L^{\prime}=\frac{B}{2}-(d+b / 2)\) where b is the smaller dim. of the loaded area
ii) For rectangular footings:

\(L^{\prime}=\frac{L}{2}-(d+\bullet / 2)\) where \(\bullet\) is the dim. parallel to the long side of the footing
b) Find total shear force on critical section, \(V_{u 1}\) :
\(V_{u 1}=B L^{\prime} q_{u}\)
c) Compare \(V_{u 1}\) to one-way capacity, \(\phi V_{n}\) :
\(V_{u 1} \leq \phi 2 \sqrt{f_{c}^{\prime}} B d\) ACI 11.12.3.1 where \(\phi=0.75\)
NOTE: If it is not acceptable, increase \(h_{f}\).

9) Check for bending stress and design reinforcement:

Square footings may be designed for moment in one direction and the same reinforcing used in the other direction. For rectangular footings the moment and reinforcing must be calculated separately in each direction. The critical section for moment extends across the width of the footing at the face of the loaded area. ACI 15.4.1, 15.4.2.
a) Find projection, \(L_{m}\) :
\(L_{m}=\frac{B}{2}-\frac{\bullet}{2}\) where \(\bullet\) is the smaller dim. of column for a square
footing. For a rectangular footing, use the value perpendicular to the critical section.
b) Find total moment, \(\mathrm{M}_{\mathrm{u}}\), on critical section:
\[
M_{u}=q_{u} \frac{B L_{m}^{2}}{2} \quad \text { (find both ways for a rectangular footing) }
\]

c) Find required \(\mathrm{A}_{\mathrm{s}}\) :
\(R_{n}=\frac{M_{n}}{b d^{2}}=\frac{M_{u}}{\phi b d^{2}}\), where \(\phi=0.9\), and \(\rho\) can be found
found from Figure 3.8.1 of Wang \& Salmon.
or:
i) guess \(a\)
ii) \(A_{s}=\frac{0.85 f_{c}^{\prime} b a}{f_{y}}\)
iii) solve for \(a=2\left(d-\frac{M_{u}}{\phi A_{s} f_{y}}\right)\)
iv) repeat from ii) until a converges, solve for \(A_{s}\)


Minimum \(A_{s}\)
\[
\begin{array}{ll}
=0.0018 \mathrm{bh} & \text { Grade } 60 \text { for temperature and shrinkage control } \\
=0.002 \mathrm{bh} & \text { Grade } 40 \text { or } 50
\end{array}
\]

ACI 10.5.4 specifies the requirements of \(\mathbf{7 . 1 2}\) must be met, and max. spacing of 18 "
d) Choose bars:

For square footings use the same size and number of bars uniformly spaced in each direction (ACI 15.4.3). Note that required \(\mathrm{A}_{\mathrm{s}}\) must be furnished in each direction.
For rectangular footings bars in long direction should be uniformly spaced. In the short direction bars should be distributed as follows (ACI 15.4.4 ):
i) In a band of width \(B_{s}\) centered on column:
\[
\# \text { bars }=\frac{2}{L / B+1} \cdot(\# \text { bars in } B) \quad(\text { integer })
\]
ii) Remaining bars in short direction should be uniformly spaced in outer portions of footing.

e) Check development length:

Find required development length, \(l_{d}\), in tension from handout or from equations in ACI 12.2. \(l_{d}\) must be less than \(\left(L_{m}-2\right.\) ") (end cover). If not possible, use more bars of smaller diameter.

\section*{Examples:}

\section*{Foundations}

\section*{Example 1}

For the 16 in. thick 8.5 ft . square reinforced concrete footing carrying 150 kips dead load and 100 kips live load on a 24 in . square column, determine if the footing thickness is adequate for 4000 psi . A 3 in . cover is required with concrete in contact with soil. Also determine the moment for reinforced concrete design.


\section*{SOLUTION:}
1. Find design soil pressure: \(q_{u}=\frac{P_{u}}{A}\)
\[
\begin{aligned}
& \mathrm{P}_{\mathrm{u}}=1.2 \mathrm{D}+1.6 \mathrm{~L}=1.2(150 \mathrm{k})+1.6(100 \mathrm{k})=340 \mathrm{k} \\
& q_{u}=\frac{340 \mathrm{k}}{(8.5 \mathrm{ft})^{2}}=4.71 \mathrm{k} / \mathrm{ft}^{2}
\end{aligned}
\]
2. Evaluate one-way shear at d away from column face ( \(\mathrm{Is} \mathrm{V}_{\mathrm{u}}<\phi \mathrm{V}_{\mathrm{c}}\) ?)
\(d=h_{f}-\) c.c. - distance bar intersection
presuming \#8 bars:

\(d=16\) in. -3 in. (soil exposure) -1 in. \(x(1\) layer of \(\# 8 ' s)=12\) in.
\(V_{u}=\) total shear \(=q_{u}\) (edge area)
\(V_{u}\) on a 1 ft strip \(=q_{u}(\) edge distance \()(1 \mathrm{ft})\)
\(V_{u}=4.71 \mathrm{k} / \mathrm{ft}^{2}[(8.5 \mathrm{ft}-2 \mathrm{ft}) / 2-(12 \mathrm{in}).(1 \mathrm{ft} / 12 \mathrm{in})].(1 \mathrm{ft})=10.6 \mathrm{k}\)
\(\phi \mathrm{V}_{\mathrm{c}}=\) one-way shear resistance \(=\phi 2 \sqrt{f_{c}^{\prime}}\) bd
for \(a\) one foot strip, \(b=12\) in.
\[
\phi \mathrm{V}_{\mathrm{c}}=0.75(2 \sqrt{4000} \mathrm{psi})(12 \mathrm{in} .)(12 \mathrm{in} .)=13.7 \mathrm{k}>10.6 \mathrm{k} \text { OK }
\]
3. Evaluate two-way shear at \(\mathrm{d} / 2\) away from column face (Is \(\mathrm{V}_{\mathrm{u}}<\phi \mathrm{V}_{\mathrm{c}}\) ?)
\(b_{0}=\) perimeter \(=4(24 \mathrm{in} .+12 \mathrm{in})=.4(36 \mathrm{in})=.144 \mathrm{in}\)
\(V_{u}=\) total shear on area outside perimeter \(=P_{u}-q_{u}\) (punch area)
\(V_{u}=340 \mathrm{k}-\left(4.71 \mathrm{k} / \mathrm{ft}^{2}\right)(36 \mathrm{in} .)^{2}(1 \mathrm{ft} / 12 \mathrm{in} .)^{2}=297.6 \mathrm{kips}\)

\(\phi V_{c}=\) two-way shear resistance \(=\phi 4 \sqrt{f_{c}^{\prime}} b_{0} d=0.75(4 \sqrt{4000} \mathrm{psi})(144 \mathrm{in}).(12 \mathrm{in})=.327.9 \mathrm{k}>297.6 \mathrm{k}\) OK
4. Design for bending at column face
\(M_{u}=w_{u} L^{2} / 2\) for a cantilever. \(L=(8.5 \mathrm{ft}-2 \mathrm{ft}) / 2=3.25 \mathrm{ft}\), and \(w_{u}\) for a 1 ft strip \(=q_{u}(1 \mathrm{ft})\)
\(M_{u}=4.71 \mathrm{ksi}(1 \mathrm{ft})(3.25 \mathrm{ft})^{2} / 2=24.9 \mathrm{k}-\mathrm{ft}(\) per ft of width \()\)
To complete the reinforcement design, use \(b=12\) in. and trial \(d=12\) in., choose \(\rho\), determine \(A_{s}\), find if \(\phi M_{n}>M_{u} \ldots \ldots\)

\section*{Example 2}

Determine the depth required for the group of 4 friction piles having 12 in. diameters if the column load is 100 kips and the frictional resistance is \(400 \mathrm{lbs} / \mathrm{ft}^{2}\).

SOLUTION:
The downward load is resisted by a friction force. Friction is determined by multiplying the friction resistance (a stress) by the area: \(F=f A_{S K I N}\)

The area of n cylinders is: \(A_{\text {SKIN }}=n\left(2 \pi \frac{d}{2} L\right)\)


Our solution is to set \(\mathrm{P} \leq \mathrm{F}\) and solve for length:
\[
\begin{aligned}
& 100 \mathrm{k} \leq 400 \mathrm{lb} / \mathrm{ft}^{2}\left(4^{\text {piles }}\right)(2 \pi)\left(\frac{12 \mathrm{in}}{2}\right) L \cdot\left(\frac{1 f t}{12 \mathrm{in}}\right) \cdot\left(\frac{1 \mathrm{k}}{1000 \mathrm{lb}}\right) \\
& L \geq 19.9^{\mathrm{ft}} / \mathrm{pile}
\end{aligned}
\]

\section*{Example 3}


The area of n cylinders is: \(A_{T I P}=\pi \frac{d^{2}}{4}\)
Our solution is to set \(P \leq F\) and solve for length:
\[
300 k \leq 600 \mathrm{lb} / \mathrm{ft}^{2} 2 \pi\left(\frac{36 \mathrm{in}}{2}\right) L \cdot\left(\frac{1 f t}{12 i n}\right) \cdot\left(\frac{1 k}{1000 \mathrm{lb}}\right)+8000 \mathrm{lb} / \mathrm{ft}^{2} \pi \frac{(36 \mathrm{in})^{2}}{4} \cdot\left(\frac{1 f t}{12 \mathrm{in}}\right)^{2} \cdot\left(\frac{1 \mathrm{k}}{1000 \mathrm{lb}}\right)
\]
\(L \geq 43.1 \mathrm{ft}\)

\section*{Example 4}

Determine the factor of safety for overturning and sliding on the 15 ft retaining wall, 16 in . wide stem, 10 ft base, 16 in . heigh base, when the equivalent fluid pressure is \(30 \mathrm{lb} / \mathrm{ft}^{3}\), the weight of the stem of the footing is 4 kips , the weight of the pad is 5 kips , the passive pressure is ignored for this design, and the friction coefficient for sliding is 0.58 . The center of the stem is located 3' from the toe.

SOLUTION:


This is a statics problem, there is no design of materials involved. Overturning is determined by moments from acting forces and the moment from the resisting force. Sliding is determined by the acting horizontal forces and the resisting sliding force which is determined by mulitplying a friction coefficient based on the materials in contact, \(\mu\), by a normal force, \(\mathrm{N}: F=\mu N\)

Find all unknown forces and draw the free body diagram with the weights at the centers of gravity of the stem and base:

The horizontal fluid (equivalent) pressure is a triangularly distributed load with the maximum distributed load equal to the density of water multiplied by the height: \(w_{h}=\gamma H\).
\[
w_{h}=\left(30 \mathrm{lb} / \mathrm{ft}^{3}\right)(15 \mathrm{ft})(1 \mathrm{ft} \text { strip })=450 \mathrm{lb} / \mathrm{tt}
\]


The horizontal force, \(\mathrm{P}_{\mathrm{H}}=w \mathrm{~L} / 2\) acts at a distance of \(1 / 3\) the height from the "fat end" of the triangle is
\[
P_{H}=(450 \mathrm{lb} / \mathrm{ft}) \frac{15 \mathrm{ft}}{2} \cdot\left(\frac{1 \mathrm{k}}{1000 \mathrm{lb}}\right)=3.375 \mathrm{k}
\]

The vertical force from the maximum distributed pressure, \(\mathrm{Pv}_{\mathrm{v}}=\mathrm{wL}\) over the right side of the base (in the middle of 6.33 ft ) is:
\[
P_{V}=(450 \mathrm{lb} / f t)\left[10 \mathrm{ft}-3 \mathrm{ft}-\frac{16 \mathrm{in}}{2} \cdot\left(\frac{1 \mathrm{ft}}{12 \mathrm{in}}\right)\right] \cdot\left(\frac{1 \mathrm{k}}{1000 \mathrm{lb}}\right)=2.85 \mathrm{k}
\]

The total downward loads must be resisted by the normal force acting "up":
\[
\begin{aligned}
& N=4 k+2.85 k+5 k=11.85 k \\
& F=(0.58)(11.85 k)=6.9 k
\end{aligned}
\]

Overturning requirement:
\[
S F=\frac{M_{\text {resist }}}{M_{\text {overturning }}} \geq 1.5-2
\]

The total resisting moment will be from those moments counterclockwise about 0:
\[
\begin{aligned}
& \quad M_{\text {resisting }}=4 \mathrm{k}(3 \mathrm{ft})+5 \mathrm{k}(5 \mathrm{ft})+2.85 \mathrm{k}(10 \mathrm{ft}-6.33 \mathrm{ft} / 2) \\
& =56.5 \mathrm{k}-\mathrm{ft}
\end{aligned}
\]

The overturning moment is only from the horizontal fluid force (clockwise):
\[
\begin{aligned}
& \text { Moverturning }=3.375 \mathrm{k}(5 \mathrm{ft}+16 \mathrm{in} .(1 \mathrm{ft} / 12 \mathrm{in} .))=21.4 \mathrm{k}-\mathrm{ft} \\
& S F=\frac{56.5^{k-f t}}{21.4^{k-f t}}=2.64 \geq 1.5 \quad \text { OK }
\end{aligned}
\]

Sliding requirement:
\[
S F=\frac{F_{\text {horizontat resist }}}{F_{\text {sliding }}} \geq 1.25-2
\]

The total resisting force will be from those opposite the hydraulic force (to the right):
\[
\text { Fresisting }=6.9 \mathrm{k}
\]

The sliding force is only from the horizontal fluid force (to the left):
\[
F_{\text {sliding }}=3.375 \mathrm{k}
\]
\[
S F=\frac{6.9 k}{3.375 k}=2.04 \geq 1.25 \quad \mathrm{OK}
\]

\section*{Supervision Practices International Building Code (2003)}

TABLE 1704.3
REQUIRED VERIFICATION AND INSPECTION OF STEEL CONSTRUCTION
\begin{tabular}{|c|c|c|c|c|}
\hline VERIFICATION AND INSPECTION & CONTINUOUS & PERIODIC & REFERENCED STANDARD \({ }^{\text {a }}\) & IBC REFEREMCE \\
\hline \multicolumn{5}{|l|}{1. Material verification of high-strength bolts, nuts and washers:} \\
\hline a. Identification markings to conform to ASTM standards specified in the approved construction documents. & - & X & Applicable ASTM material specifications; AISC 335, Section A3.4; AISC LRFD, Section A3.3 & - \\
\hline b. Manufacturer's certificate of compliance required. & - & X & - & - \\
\hline \multicolumn{5}{|l|}{2. Inspection of high-strength bolting:} \\
\hline a. Bearing-type connections. & - & X & \multirow{2}{*}{AISC LRFD Section M2.5} & \multirow[b]{2}{*}{1704.3.3} \\
\hline b. Slip-critical connections. & X & X & & \\
\hline \multicolumn{5}{|l|}{3. Material verification of structural steel:} \\
\hline a. Identification markings to conform to ASTM standards specified in the approved construction documents. & - & - & ASTM A 6 orASTM A 568 & \multirow[t]{2}{*}{1708.4} \\
\hline b. Manufacturers' certified mill test reports. & - & - & ASTM A 6 or ASTM A 568 & \\
\hline \multicolumn{5}{|l|}{4. Material verification of weld filler materials:} \\
\hline a. Identification markings to conform to AWS specification in the approved construction documents. & - & - & AISC, ASD, Section A3.6; AISC LRFD, Section A3.5 & - \\
\hline b. Manufacturer's certificate of compliance required. & - & - & - & - \\
\hline 5. Inspection of welding: a. Structural steel: & - & - & & \\
\hline 1) Complete and partial penetration groove welds. & X & - & \multirow{4}{*}{AWS D1.1} & \multirow{4}{*}{1704.3 .1} \\
\hline 2) Multipass fillet welds. & X & - & & \\
\hline 3) Single-pass fillet welds \(>5 / 11^{\prime \prime}\) & X & - & & \\
\hline 4) Single-pass fillet welds \(\leq 5 / 16^{\prime \prime}\) & - & X & & \\
\hline 5) Floor and deck welds. & - & X & AWS D1. 3 & - \\
\hline b. Reinforcing steel: & - & - & & \\
\hline 1) Verification of weldability of reinforcing steel other than ASTM A 706. & - & X & & \\
\hline 2) Reinforcing steel-resisting flexural and axial forces in intermediate and special moment frames, and boundary elements of special reinforced concrete shear walls and shear reinforcement. & X & - & AWS D1. 4 ACI 318: 3.5.2 & 1903.5 .2 \\
\hline 3) Shear reinforcement. & X & - & & \\
\hline 4) Other reinforcing steel. & - & X & & \\
\hline \begin{tabular}{l}
6. Inspection of steel frame joint details for compliance with approved construction documents: \\
a. Details such as bracing and stiffening. \\
b. Member locations. \\
c. Application of joint details at each connection.
\end{tabular} & - & X & - & 1704.3.2 \\
\hline
\end{tabular}

For SI: 1 inch \(=25.4 \mathrm{~mm}\).
a. Where applicable, see also Section 1707.1, Special inspection for seismic resistance.

TABLE 1704.4
REQUIRED VERIFICATION AND INSPECTION OF CONCRETE CONSTRUCTION
\begin{tabular}{|c|c|c|c|c|}
\hline VERIFICATION AND INSPECTION & CONTINUOUS & PERIODIC & REFERENCED STANDARD \({ }^{\text {a }}\) & IBC REFERENCE \\
\hline 1. Inspection of reinforcing steel, including prestressing tendons, and placement. & - & X & ACI 318: 3.5, 7.1-7.7 & \[
\begin{aligned}
& \text { 1903.5, 1907.1, } \\
& \text { 1907.7, 1914.4 }
\end{aligned}
\] \\
\hline 2. Inspection of reinforcing steel welding in accordance with Table 1704.3, Item 5B. & - & - & \[
\begin{gathered}
\text { AWS D1. } 4 \\
\text { ACI 318: } 3.5 .2
\end{gathered}
\] & 1903.5.2 \\
\hline 3. Inspect bolts to be installed in concrete prior to and during placement of concrete where allowable loads have been increased. & X & - & - & 1912.5 \\
\hline 4. Verifying use of required design mix. & - & X & ACI 318: Ch. 4, 5.2-5.4 & \[
\begin{gathered}
1904,1905.2-1905.4 \\
1914.2,1914.3 \\
\hline
\end{gathered}
\] \\
\hline 5. At the time fresh concrete is sampled to fabricate specimens for strength tests, perform slump and air content tests, and determine the temperature of the concrete. & X & - & ASTM C 172 ASTM C 31 ACI 318: 5.6, 5.8 & 1905.6, 1914.10 \\
\hline 6. Inspection of concrete and shotcrete placement for proper application techniques. & X & - & ACI 318: 5.9, 5.10 & \[
\begin{gathered}
1905.9,1905.10 \\
1914.6,1914.7,1914.8
\end{gathered}
\] \\
\hline 7. Inspection for maintenance of specified curing temperature and techniques. & - & X & ACI 318: 5.11-5.13 & \[
\begin{gathered}
\text { 1905.11, } 1905.13, \\
1914.9
\end{gathered}
\] \\
\hline \begin{tabular}{l}
8. Inspection of prestressed concrete: \\
a. Application of prestressing forces. \\
b. Grouting of bonded prestressing tendons in the seismic-force-resisting system.
\end{tabular} & \[
\begin{aligned}
& \mathrm{X} \\
& \mathrm{X}
\end{aligned}
\] & - & \[
\begin{gathered}
\text { ACI 318: } 18.20 \\
\text { ACI 318: } 18.18 .4
\end{gathered}
\] & - \\
\hline 9. Erection of precast concrete members. & - & X & ACI 318: Ch. 16 & - \\
\hline 10. Verification of in-situ concrete strength, prior to stressing of tendons in posttensioned concrete and prior to removal of shores and forms from beams and structural slabs. & - & X & ACI 318: 6.2 & 1906.2 \\
\hline
\end{tabular}

For SI: 1 inch \(=25.4 \mathrm{~mm}\).
2 Where applicable, see also Section 1707.1, Special inspection for seismic resistance.

TABLE 1704.5.1
LEVEL 1 SPECIAL INSPECTION
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{INSPECTION TASK} & \multicolumn{2}{|l|}{FREQUENCY OF INSPECTION} & \multicolumn{3}{|c|}{REFERENCE FOR CRITERIA} \\
\hline & Continuous during task listed & Periodically during task listed & IBC section & ACI 530/ASCE 5/TMS 402 \({ }^{\text {a }}\) & \[
\begin{gathered}
\text { ACI 530.1/ASCE } \\
6 / \mathrm{TMS} 602^{\mathrm{a}}
\end{gathered}
\] \\
\hline \multicolumn{6}{|l|}{1. As masonry construction begins, the following shall be verified to ensure compliance:} \\
\hline a. Proportions of site-prepared mortar. & \multirow{3}{*}{-} & X & \multirow{3}{*}{-} & \multirow{3}{*}{-} & Art. 2.6A \\
\hline b. Construction of mortar joints. & & X & & & Art. 3.3B \\
\hline c. Location of reinforcement and connectors. & & X & & & Art. 3.4, 3.6A \\
\hline d. Prestressing technique. & - & X & - & - & Art. 3.6B \\
\hline e. Grade and size of prestressing tendons and anchorages. & - & X & - & - & \[
\begin{gathered}
\text { Art. } 2.4 \mathrm{~B}, \\
2.4 \mathrm{H}
\end{gathered}
\] \\
\hline \multicolumn{6}{|l|}{2. The inspection program shall verify:} \\
\hline a. Size and location of structural elements. & - & X & - & - & Art. 3.3G \\
\hline b. Type, size and location of anchors, including other details of anchorage of masonry to structural members, frames or other construction. & - & X & - & \[
\begin{gathered}
\text { Sec. 1.2.2(e), } \\
2.1 .4,3.1 .6
\end{gathered}
\] & - \\
\hline c. Specified size, grade and type of reinforcement. & - & X & - & Sec. 1.12 & Art. 2.4, 3.4 \\
\hline d. Welding of reinforcing bars. & X & - & - & \[
\begin{gathered}
\text { Sec. 2.1.10.6.2, } \\
3 \cdot 2 \cdot 3 \cdot 4(\mathrm{~b})
\end{gathered}
\] & - \\
\hline e. Protection of masonry during cold weather (temperature below \(40^{\circ} \mathrm{F}\) ) or hot weather (temperature above \(90^{\circ} \mathrm{F}\) ). & - & X & \[
\begin{gathered}
\text { Sec. 2104.3, } \\
2104.4
\end{gathered}
\] & - & \[
\begin{gathered}
\text { Art. 1.8C, } \\
\text { 1.8D }
\end{gathered}
\] \\
\hline f. Application and measurement of prestressing force. & - & X & - & - & Art. 3.6B \\
\hline \multicolumn{6}{|l|}{3. Prior to grouting, the following shall be verified to ensure compliance:} \\
\hline a. Grout space is clean. & \multirow{4}{*}{-} & X & \multirow{4}{*}{-} & - & Art. 3.2D \\
\hline b. Placement of reinforcement and connectors and prestressing tendons and anchorages. & & X & & Sec. 1.12 & Art. 3.4 \\
\hline c. Proportions of site-prepared grout and prestressing grout for bonded tendons. & & X & & - & Art. 2.6B \\
\hline d. Construction of mortar joints. & & X & & - & Art. 3.3B \\
\hline 4. Grout placement shall be verified to ensure compliance with code and construction document provisions. & X & - & - & - & Art 3.5 \\
\hline a. Grouting of prestressing bonded tendons. & X & - & - & - & Art. 3.6C \\
\hline 5. Preparation of any required grout specimens, mortar specimens and/or prisms shall be observed. & X & - & \[
\begin{gathered}
\text { Sec. 2105.2.2, } \\
2105.3 \\
\hline
\end{gathered}
\] & - & Art. 1.4 \\
\hline 6. Compliance with required inspection provisions of the construction documents and the approved submittals shall be verified. & - & X & - & - & Art. 1.5 \\
\hline
\end{tabular}

ForSI: \({ }^{\circ} \mathrm{C}=\left({ }^{\circ} \mathrm{F}-32\right) / 1.8\).
4 The specific standards referenced are those listed in Chapter 35.

TABLE 1704.5.3
LEVEL 2 SPECIAL INSPECTION
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{INSPECTION TASK} & \multicolumn{2}{|l|}{FREQUENCY OF INSPECTION} & \multicolumn{3}{|c|}{REFERENCE FOR CRITERIA} \\
\hline & Continuous during task listed & Periodically during task listed & \[
\begin{gathered}
\text { IBC } \\
\text { section } \\
\hline
\end{gathered}
\] & ACI 530/ ASCE 5/
TMS \(402^{a}\) & ACI 530.1/ ASCE \(6 /\)
TMS \(602^{a}\) \\
\hline \multicolumn{6}{|l|}{1. From the beginning of masonry construction, the following shall be verified to ensure compliance:} \\
\hline a. Proportions of site-prepared mortar, grout and prestressing grout for bonded tendons. & - & X & - & - & Art. 2.6A \\
\hline b. Placement of masonry units and construction of mortar joints. & - & X & - & - & Art. 3.3B \\
\hline c. Placement of reinforcement, connectors and prestressing tendons and anchorages. & - & X & - & Sec. 1.12 & \[
\begin{gathered}
\text { Art. 3.4, } \\
3.6 \mathrm{~A}
\end{gathered}
\] \\
\hline d. Grout space prior to grouting. & X & - & - & - & Art. 3.2D \\
\hline e. Placement of grout. & X & - & - & - & Art. 3.5 \\
\hline f. Placement of prestressing grout. & X & - & - & - & Art. 3.6C \\
\hline \multicolumn{6}{|l|}{2. The inspection program shall verify:} \\
\hline a. Size and location of structural elements. & - & X & - & - & Art. 3.3G \\
\hline b. Type, size and location of anchors, including other details of anchorage of masonry to structural members, frames or other construction. & X & - & - & \[
\begin{aligned}
& \text { Sec. 1.2.2(e), } \\
& \text { 2.1.4, 3.1.6 }
\end{aligned}
\] & - \\
\hline c. Specified size, grade and type of reinforcement. & & X & - & Sec. 1.12 & Art. 2.4, 3.4 \\
\hline d. Welding of reinforcment. & X & - & - & \[
\begin{gathered}
\text { Sec. 2.1.10.6.2, } \\
3.2 .3 .4(\mathrm{~b}) \\
\hline
\end{gathered}
\] & - \\
\hline e. Protection of masonry during cold weather (temperature below \(40^{\circ} \mathrm{F}\) ) or hot weather (temperature above \(90^{\circ} \mathrm{F}\) ). & - & X & \[
\begin{gathered}
\text { Sec. 2104.3, } \\
2104.4
\end{gathered}
\] & - & \[
\begin{aligned}
& \text { Art. 1.8C, } \\
& \text { 1.8D }
\end{aligned}
\] \\
\hline f. Application and measurement of prestressing force. & X & - & - & - & Art. 3.6B \\
\hline 3. Preparation of any required grout specimens, mortar specimens and/or prisms shall be observed. & X & - & \[
\begin{gathered}
\text { Sec. 2105.2.2, } \\
2105.3 \\
\hline
\end{gathered}
\] & - & Art. 1.4 \\
\hline 4. Compliance with required inspection provisions of the construction documents and the approved submittals shall be verified. & - & X & - & - & Art. 1.5 \\
\hline
\end{tabular}

For SI: \(\quad{ }^{\circ} \mathrm{C}=\left({ }^{\circ} \mathrm{F}-32\right) / 1.8\).
a. The specific standards referenced are those listed in Chapter 35.
TIMBER CONSTRUCTION
MANUAL
FOURTH EDITION
1994
AMERICAN INSTITUTE OF TIMBER
CONSTRUCTION
Englewood, Colorado

\footnotetext{
5
New York • Chichester • Brisbane - Toronto • Singapore
}
after the half arches are in position. When trusses and arches are assembled on the ground at the site, they should be assembled on level blocking to permit concompression joints should be brought into full bearing and compression plates installed where specified.

Before erection, the assembly should be checked for prescribed overall dimensions, prescribed camber, and accuracy of anchorage conncctions. Erection should be planned and executed such a way that the close fir and neat ap pearance of joints and the structure as a whole will not be impaired.

Anchor bolts should be checked prior to the start of erection. Before erection begins, all supports and anchors should be complete, accessible, and free of obstructions. The weights and balance points of the structural timber framing should be determined before lifting begins so that proper equipment and lifting methods may be employed. When long members or timber trusses of long span are raised from a flat to a vertical position preparatory to lifting, stresses entirely different from the normal design stress may be introduced. The magnitude and distribution of these stresses will vary, depending on such factors as the weight, dimensions, and type of member. A competent rigger should consider these factors in decould be located. 1.6.3.2 Bracing

All framing must be true and plumbed. Permanent bracing is bracing so designed and installed as to form an integral part of the final structure. Erection bracing is bracing installed to hold the framing in a safe position until sufficient permanent bracing is in place to provide full stability. Proper and adequate temporary erection bracing is introduced whenever necessary to take care of all loads to which the structure may be subjected during erection, including equipment and its operation. This bracing is left in place as long as may be required for safety. Part or all of the permanent bracing may also act as erection bracing. Erection bracing serves to plumb the framing during erection and gives it adequate stability to receive purlins, joists, and roofing materials. It may include sway bracing, guy ropes, tieing off framing nearest to end walls, steel tie rods with turnbuckle takeups, struts, shoes, and similar items. As erection progresses, bracing is securely fastened in place to take care of all dead load, erection stresses, and normal weather
conditions. Excessive concentrated construction loads, such as bundles of sheathing, piles of purlins, roofing, or other materials, should be avoided. 1.6.3.3 Final Alignment

Final tightening of alignment bolts should not be completed until the structure has been properly aligned. 1.6.3.4 Removal of Temporary Bracing

Temporary erection bracing should be removed only after diaphragms and
permanent bracing are installed, the structure has been properly aligned, and
connections

\title{
US Historical Structure Examples \\ from Historical Building Construction, Donald Friedman, 1995.
}

1835 Obadiah Parker House, New York, Parker designer, demolished. House walls were monolithic concrete, probably with natural lime cement.

1841 [Old] Merchants' Exchange, 55 Wall Street at William Street, New York, Isiah Rogers architect, heavily modified 1907, landmarked. Monolithic allmasonry construction.

1853 New York Crystal Palace, George Carstensen and Charles Gildemeister architects, burned 1858. Castiron columns, wrought-iron arch ribs and truss girders, arched trusses, and all-glass curtain wall, portal bracing for lateral load.

1854 Harper \& Brothers Building, 331 Pearl Street on Franklin Square, New York, John Corlies architect, James Bogardus engineer, James L. Jackson foundry builders, demolished 1925. Brick jack arch floor topped with concrete spanned between wrought-iron floor beams, supported by cast-iron and wrought-iron bowstring truss girders and cast-iron columns. The floor beams were the first lot of wrought-iron beams rolled in the United States, by the Trenton Iron Works. One castiron facade, brick side walls and rear.

1860 United States Warehousing Company grain elevator, Brooklyn, George Johnson engineer, Architectural Iron Works builders, demolished. Castiron frame supporting brick curtain wall, six stories high, cross-braced in all bays by wrought-iron rods.

1860- Watervliet Arsenal, Watervliet, New York, Daniel Badger designer, Architectural Iron Works building, standing. First all-iron building, 100 feet by 196 feet, cast-iron walls and columns, wrought-iron roof trusses, cast-iron girders with wrought-iron tension rods.

1871 Grand Central Depot, \(42^{\text {nd }}\) Street and Park Avenue, New York, John B. Snook architect, Isaac C. Buckhout engineer, Wilhelm Hildenbrand engineer for train shed, Architectural Iron Works builders, altered 1989, demolished 1913. First balloon shed in United States: wrought-iron truss barrel vault for train shed, covered with glass and galvanized iron.

1871 William E. Ward House, Port Chester, Rober Mook architect, Ward designer and builder, standing. First reinforce-concrete building in the United States, entire building monolithic, beams reinforced with wrought-iron I's connected for shear transfer, \(31 / 2\)-inchthick floors reinforced with rods, \(21 / 2\)-inch-thick partitions reinforced with rods, hollow cylinder columns reinforced with hoops.

1875 Tribune Building, Park Row at Nassau Street, New York, Richard Morris Hunt architect, demolished 1966. Probably highest bearing-wall building in New York at 260 feet high. First tower-type building downtown. Wrought-iron beam floors.

1883 Statue of Liberty, Bedloe's Island, Frederic Auguste Bartholdi architectural designer, Gustave Eifflel engineer, Keystone Bridge Company and D. H. King Contracting builders, landmarked. First full-braced frame in New York, first use of steel columns in the United States, early use of concrete in foundation.

1885 Home Insurance Building \({ }^{1}\), Chicago, William LeBarron Jenny, architect and engineer, demolished. Main building 138 feet high, 180 feet with 2 additional floors added later. Construction halted temporarily when building officials were concerned that the building weight was only one-third that of a comparable masonry structure.

1891 Monadnock Building \({ }^{1} 53\) West Jackson Blvd, Chicago, Burnham \& Root architects (north), Holabird \& Roche architects (south), standing. 197 feet high. The northern half is the last Chicago skyscraper built using load-bearing masonry wall construction with walls of six feet thick at the base.

1892 Manhattan Life Insurance Building, 64-68 Broadway, New York, Kimball \& Thompson architects, C. O. Brown engineer, demolished. 67 feet by 119 feet, main building 254 feet high, tower 348 feet high. Tallest building in New York when built, first caisson use on a building anywhere (caissons were used in bridge and tunnel construction as early as 1850s in Europe, 1870s in the United States), fifteen caissons 55 feet below grade, 35 feet below open excavation, cantilevered built-up girders in foundations.

1895 American Surety Building, 96-100 Broadway, New York, Bruce Price architect, standing. First complete skeleton frame in New York, twenty stories and 303 feet high, 85 feet by 85 feet, Z-bar columns, wind braced with rods, caissons to rock 72 feet below curb elevation.

1899 Carson, Pirie, Scott and Company Building \({ }^{1}\), 1 South State Street, Chicago, Louis Sullivan, architect, standing. Steel structure allowed for increased window area.

\footnotetext{
\({ }^{1}\) Wickipedia: http://en.wikipedia.org/
}

1903 Flatiron Building, 175 Fifth Avenue, New York, D. H. Burnham \& Company architects, Corydon Purdy engineer, landmarked. Steel frame with portal bracing and knees at all wind girders, tallest building in the city when completed.

1907 Monolith Building, 45 West \(34^{\text {th }}\) Street, New York, Howells and Stokes architects, standing. The first tall, reinforce-concrete building in the city, had limestone veneer for three floors at base, but exposed concrete above to full twelve-story height, stone veneer later replaced by stucco.

1909 [Old] New York Times Building, \(42^{\text {nd }}\) Street and Broadway, Eidlitz \& McKenzie architects, standing altered. First tall building in the country to be designed using live-load reduction on its columns.

1913 Woolworth Building, 233 Broadway, New York, Case Gilbert architect, Gunvald Aus Company structural engineers, landmarked. Fifty-five stories, 760 feet, 6 inches high, tallest in the city when completed, caissons to rock, with moment-resisting portal frame, all-terra-cotta facade, facade rigidly connected to steel structure, no expansion joints provided, facade restoration required in mid-1980s, designed by Ehrnkrantz Group, over 20,000 panels had to be replaced with fiberglass-reinforced polymer concrete, approximately 100,000 reanchored.

1920 Electric Welding Company of America factory, Brooklyn, T. Leonard McBean engineer. Early use of structural welding, Brooklyn Department of Buildings required a full-scale load test before allowing construction.

1930 Chrysler Building \({ }^{2}\), \(42^{\text {nd }}\) Street and Lexington Avenue, New York, William Van Allen, architect, standing. Briefly, the world's tallest building at 1047 feet to spire prior to the Empire State Building. Steel construction (riveted) with central core and steel-clad roof.

1931 Starrett-Lehigh Building, Eleventh Avenue and \(26^{\text {th }}\) Street, New York, R. G. Cory, W. M. Cory, and Yasuo Matsui architects, Purdy and Henderson engineers, landmarked. Flat slab concrete floors, on concrete columns with mushroom capitals at 21 feet on center above third floor, steel columns below; slabs are cantilevered to support curtain wall, nineteen stories high.

1931 Empire State Building, 350 Fifth Avenue, New York, Shreve, Lamb and Harmon architects, H. G. Balcom and Associates engineers, landmarked. Eighty-

\footnotetext{
\({ }^{2}\) Wickipedia: http://en.wikipedia.org/
}
five stories and 1239 feet high, tallest building in the city when completed, full moment connection wind bracing, early use of aluminum cladding for top tower (dirigible mooring mast), ornament, and spandrel panels, early use of stainless-steel cladding in window edging.

1935 Hayden Planetarium, New York, Trowbridge and Livingston architects, Weiskopf \& Pickworth engineers, standing. Early concrete shell dome, 3 inches thick, supporting projection screen.

1950 Secretariat Building of United Nations, near \(42^{\text {nd }}\) Street and First Avenue, New York, International Committee and Wallace Harrison architects, standing. First tall, glass curtain wall in New York.

1951 Lake Shore Drive Apartments \({ }^{3}\), 860-880 Lake Shore Drive, Chicago, Ludwig Mies van der Rohe, architect, standing. Steel frame with lateral resistance in the exterior (non-curtain) walls from steel plate welded to the frame.

1956425 Park Avenue, New York, Kahn \& Jacobs architects, Charles Meyer engineer, standing. Height 375 feet, "one of the tallest to be built to date with bolted connections," 150,000 field bolts up to \(11 / 8\) inches diameter x 7 -inch grip in size; 200,000 shop rivets. Early use of two-man bolt crews.

1957 Seagram Building, 375 Park Avenue, New York, Ludwig Mies van der Rohe, Philip Johnson, and Kahn \& Jacobs architects, Severud-Elstad-Kreuger engineers, landmarked. At thirty-eight stories and 520 feet high, tallest building using high-strength bolts when built. Shop connections riveted; unfinished bolts used for beam-to-girder connections.

1958 [Former] Union Carbide Building, 270 Park Avenue, New York, standing. At fifty-two stories and more than 700 feet high, tallest bolted frame when built.

1959 Kips Bay Plaza, \(30^{\text {th }}\) Street to \(33^{\text {rd }}\) Street, First Avenue to Second Avenue, New York, I. M. Pei \& Partners and S. J. Kessler architects, August Komendant engineer, standing. Early exposed-concrete apartment houses, using load-bearing exterior walls of Vierendeel truss type.

1960 Western Electric Building, Fulton Street and Broadway, New York, Purdy \& Henderson engineers, standing. At thirty-one stories, tallest steel frame with welded connections in the eastern half of the country when built.

\footnotetext{
\({ }^{3}\) Emporis Buildings: http://www.emporis.com/
}

1961 Chase Manhattan Building, Cedar Street and Nassau Street, New York, Skidmore, Owings \& Merrill architects, Weiskopf \& Pickworth engineers, standing. First glass curtain wall building over 800 feet high, sixty stories, largest building using solely interior bracing, steel rails and mullions mounted to structural frame.

1964 New York State Pavilion, Flushing Meadows Park, Queens, Philip Johnson and Richard Foster architects, Lev Zetlin engineer, standing empty. Early use of slip-forming to create freestanding concrete columns; roof is a bicycle-wheel cable truss.

1964 Marina City Towers \({ }^{4}\), 300 North State Street, Chicago, Bertrand Goldberg, architect. Tallest reinforced concrete structures built at the time with 61 floors. Central load-bearing core with column and beam construction.

1965 CBS Building, 51 West \(52^{\text {nd }}\) Street, New York, Eero Saarinen architect, standing. Early concrete tube and core structure, thirty-nine stories and 491 feet high.

1968 Madison Square Garden, Seventh Avenue and \(33^{\text {rd }}\) Street, New York, Charles Luckman Associates architects, Severud Associates engineers, standing. 425-foot-diameter bicycle-wheel cable truss roof.

1969 John Hancock Building \({ }^{4}\), 875 N. Michigan Avenue, Chicago, Fazlur Khah (Skidmore, Owings \& Merrill) designer and engineer, standing. 1500 feet tall with external cross bracing to resist lateral loads as the predominant architectural feature for the tubular design.

1972 Transamerica Pyramid \({ }^{4}\), 600 Montgomery Street, San Francisco, William Pereira architect, standing. 853 feet tall and was the tallest skyscraper west of the Mississippi River from 1972-1974. Constructed of reinforced concrete, it has a tapering shape from base to tip with two vertical "wings" at the upper stories.

1973 Willis (Sears) Tower \({ }^{4}\), 233 South Wacker Drive, Chicago, Bruce Graham architect, Skidmore Owings and Merrill, engineers, standing. 1721 feet high to spire, and the world's tallest building from 1973-2004. The design incorporates nine steel-unit square tubes in a 3 tube by 3 tube arrangement, with each tube having the footprint of 75 feet by 75 feet. This building was the first with this design.

\footnotetext{
\({ }^{4}\) Wickipedia: http://en.wikipedia.org/
}

1974 Avon Building \({ }^{5}\), 9 W. \(57^{\text {th }}\) Street, New York, Skidmore, Owens and Merrill architects, standing. Lateral forces are resisted by a sloping base in the street direction (setback requirements), and by exposed, inset cross bracing in the narrow direction.

1975 Water Tower Place \({ }^{4,6}\), 845 North Michigan Avenue, Chicago, Loebl, Schlossman, Dart \& Hackl and C.F. Murphy Associates architects, standing. Framed tube constructed of high-performance concrete. It was the world's tallest concrete structure from 1975-1990.

1976 World Trade Center, near Church and Fulton streets, New York, Minoru Yamasaki and Emery Roth architects, Worthington, Skilling, Helle and Jackson engineers, tragically demolished. Early structural tube wind frame, of Vierendeel truss type, early pressureequalized curtain wall.

1977 Citigroup (Citicorp) Center \({ }^{4}\), 601 Lexington Avenue, New York, Stubbins Associates, Emery Roth \& Sons architects, William LeMessurier engineer, standing. 72 feet long cantilever spans achieved with 114 feet high stilt-like columns that accommodated an existing church at the corner of the site. The system was not adequately designed for lateral loads when the connections were changed from welds to bolts, and was reinforced in 1978.

\footnotetext{
\({ }^{5}\) Skidmore, Owings and Merrill: SOM.com
\({ }^{6}\) Emporis Buildings: http://www.emporis.com
}```


[^0]:    ${ }^{1}$ The same $E$ from Sections 1.4 and 12.4 is used for both Sections 2.3.2 and 2.4.1. Refer to the Chapter 11 Commentary for the Seismic Provisions.

[^1]:    ${ }^{2}$ The same $E$ from Sections 1.4 and 12.4 is used for both Sections 2.3.2 and 2.4.1. Refer to the Chapter 11 Commentary for the Seismic Provisions.

[^2]:    User Note: For LRFD designs, the load combinations in SEI/ASCE 7, Section 2.3 apply. For ASD designs, the load combinations in SEI/ASCE 7, Section

[^3]:    * For wide-module joists only.

    1 inch $=25.4$ millimeters

[^4]:    This publication is intended for the use of professionals competent to evaluate the significance and limitations of its contents and who will accept responsibility for the application of the material it contains. The Concrete Reinforcing Steel Institute reports the foregoing material as a matter of information and, therefore, disclaims any and all responsibility for application of the stated principles or for the accuracy of the sources other than material developed by the Institute.

[^5]:    
    Classification of Building Enclosure Condition

    | Partially-Enclosed Building | Enclosed Building |
    | :---: | :---: |
    | Buildings meeting one of the following: <br> - All buildings with intentional openings in the exterior envelop exceeding the lesser of $4 \mathrm{ft}^{2}$ or 1 percent of the total projected wall or roof area on any building side, or <br> - Buildings within the wind-borne debris region with conventional exterior glazing (unprotected from debris impact) exceeding the above opening amounts | All buildings not classified as 'partially enclosed' including: <br> - Buildings not within the windborne debris region, and <br> - Buildings within the wind-borne debris region with glazing protection or impact resistant glazing in accordance with ASCE 7 or the local governing building code. |

    A4.2 Roof System Uplift Loads. Wind pressures from Table A3 shall be applied to the horizontal projected area (HPA) of a roof system to determine uplift loads tributary to structural
    elements, assemblies, and connections that experience loads from multiple roof surfaces.

    A4.3 Components and Cladding Loads. Table A4 shall be used to
    determine inward (positive) and outward (negative) acting wind loads tributary to wall and roof components, cladding, and related connections. Design wind pressures shall be applied the component, cladding, or connection under consideration.
    effects of wind pressure, it shall be factored as follows for Allowable and Resistance Factor or Strength Design (LRFD) methods:
    where W is wind load effect due to
    accordance with Section A4 and D is dead load effect due to estimated actual dead load. Load effects include stresses in or forces applied to structural members, connections,
    or systems.
    Other load combinations and design load effects shall be considered in accordance with ASCE 7, Chapter 2.
    A4.1 Lateral Force Resisting System Loads. Wind pressures from Table A2 shall be applied to building roof and wall vertical projected areas (VPA) corresponding to each of four
    elevations of the building to determine maximum lateral wind forces (shear) tributary to horizontal diaphragms, shear walls, and related connections.
    

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    ## RESIDENTIAL BUILDING LOADS

    Table B1 Notes:

    1. Building enclosure condition affects internal pressures experienced within the building. Because internal pressure acts inward or outward on all exterior building surfaces simultaneously, the net effect on lateral building loads is zero. Therefore, building enclosure condition does not affect determination of lateral building loads in Section A4.1.
    2. Open buildings are not addressed; refer to ASCE 7 for appropriate wind loads. Open buildings have openings in each wall which exceed 80 percent of the wall
    area.

    | TABLE B2 <br> Lateral Wind Loads for Application to Vertical Projected Wall and Roof Area [Exposure B, Mean Roof Height 30 feet] |  |  |  |  |  |  |
    | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
    | Basic Wind Speed (mph) | Design Wind Pressure (psf) |  |  |  |  |  |
    |  | For Roof VPA by Roof Slope |  |  | For Wall VPA by Roof Slope |  |  |
    |  | $\begin{gathered} \leq 20^{\circ} \\ (4: 12) \end{gathered}$ | $\begin{gathered} 25^{\circ} \\ (5.6: 12) \end{gathered}$ | $\begin{aligned} & \geq 30^{\circ} \\ & (7: 12) \end{aligned}$ | $\begin{aligned} & \leq 10^{\circ} \\ & (2: 12) \end{aligned}$ | $\begin{gathered} 20^{\circ} \\ (4: 12) \end{gathered}$ | $\begin{gathered} \geq 30^{\circ} \\ \mathbf{7 : 1 2} \end{gathered}$ |
    | 85 | 0 | 2.4 | 7.7 | 10.2 | 12.5 | 11.2 |
    | 90 | 0 | 2.7 | 8.6 | 11.4 | 14.0 | 12.6 |
    | 100 | 0 | 3.3 | 10.7 | 14.0 | 17.3 | 15.5 |
    | 110 | 0 | 4.0 | 12.9 | 17.0 | 20.9 | 18.8 |
    | 120 | 0 | 4.8 | 15.4 | 20.2 | 25.3 | 22.4 |
    | 130 | 0 | 5.6 | 18.0 | 23.7 | 29.2 | 26.3 |
    | 140 | 0 | 6.5 | 20.9 | 27.5 | 33.6 | 30.5 |
    | 150 | 0 | 7.4 | 24.0 | 31.6 | 38.9 | 35.0 |

    Table B2 Notes:

    1. Table applies to wind exposure category B (urban, suburban, or wooded terrain). For exposure category C (open or coastal exposure), multiply table values by 1.4 .
    2. Table applies to a mean roof height of 30 feet. For other mean roof heights from 15 feet to 40 feet, multiply table values by the following factor: $f_{h}=$
    0.0087 (h) +0.74 where $h$ is the mean roof height in feet.
    3. Interpolation between reported wind speeds and roof slopes shall be permitted. For roof slopes greater than $45^{\circ}(12: 12)$, use wall VPA value.
    4. Extrapolation to wind speeds other than shown shall be permitted by multiplying tabulated values by the ratio of squared wind speeds. For example, a wall VPA pressure of 20.9 psf at 110 mph from the table can be used to determine a pressure for a 170 mph wind speed by multiplying as follows: $(20.9 \mathrm{psf}) \times(170 / 110)^{2}=49.9 \mathrm{psf}$.
    RESIDENTIAL BUILDING LOADS 47
    Table B4 Notes：
     For exposure category C（open or coastal exposure），multiply table values by 1．4． 2．Table applies to enclosed buildings．For partially－enclosed buildings，multiply table values by 1.25 ．
    3．Table applies to a mean roof height of 30 feet or less．For mean roof heights greater than 30 feet and not exceeding
    following factor：$f_{h}=0.0087$（h）+0.74 where $h$ is the mean roof height in feet．
    4．Interpolation between reported wind speeds shall be permitted．Extrapolation of tabulated pressures to wind speeds other than shown shall be done in accordance with note 4 of Table A2．
    5．Non－air permeable claddings（siding and roofing）do not allow venting of air either through the siding or through cavities behind the cladding that lead to vent openings on the same face of the building．Most claddings are air－permeable to wall behind the cladding is relatively non－air permeable．For vinyl cladding， ASTM Standard D3679 permits a 50 percent reduction in wind load for this reason．Similarly，claddings such as brick veneer（with weeps and vent space） and hardboard lap siding have been reported to experience cladding wind load reductions of 30 percent or more．Wind loads on roofing，such as asphalt shingles， have been reported to experience wind load reductions of as much as 25 percent． Refer to the cladding manufacturer for an appropriate air－perme dynamic nature of wind pressures（e．g．，fluttering）and its potential effect（e．g．，fatigue）on some cladding systems and related connections．
     as on the upper surface．If an＂open soffit＂is used，the roof overhang pressure should also apply to the roof sheathing（if sheathed）or the roofing（if not
    sheathed underneath）． 7－02，ASCE，Reston，VA． 2002.
     Structural Loads for One－and Two－Family Dwellings，U．S．Dept．of Housing and Urban Development，Washington，DC． 2001.

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    Fact Sheet 229-96

    ## The "100-Year Flood"

    

    ## Flood designations are based on statistical averages, not on the number of years between big floods.

    The estimates are only as good as the available data. Flood designations are updated as more data are collected or when the conditions change in a river basin.

    ## BIG FLOODS COULD HAPPEN AGAIN IN WASHINGTON DURING ANY YEAR

    Rivers across the Nation seem to be rising to record flood levels almost every year. In Washington, more
    than one 100-year flood has happened on a few rivers in just the past several years. How can 100-year floods happen so often?

    ## WHY DON'T THESE FLOODS HAPPEN EVERY 100 YEARS?

    The term "100-year flood" is misleading because it leads people to believe that it happens only once every 100 years. The truth is that an uncommonly big flood can happen any year. The term "100-year flood" is really a statistical designation, and there is a $\mathbf{1 - i n - 1 0 0}$ chance that a flood this size will happen during any year. Perhaps a better term would be the "1-in-100 chance flood."

    The actual number of years between floods of any given size varies a lot. Big floods happen irregularly because the climate naturally varies over many years. We sometimes get big floods in successive or nearly successive years with several very wet years in a row.

    ## HOW ARE FLOODS DESIGNATED?

    Scientists collect data and study past floods to get a minimum of 10 years of information about the river; a longer record provides a better estimate of the "1-in-100 chance flood." Scientists use statistics and observe how frequently different sizes of floods occurred, and the average number of years between them, to determine the probability that a flood of any given size will be equalled or exceeded during any year.

    ## MANY FLOOD DESIGNATIONS WILL CHANGE OVER TIME

    As more data are collected, or when a river basin is altered in a way that affects the flow of water in the river, scientists re-evaluate the frequency of flooding. Dams and urban development are examples of some man-made changes in a basin that affect floods.

    ## THE USGS COLLECTS ESSENTIAL DATA FOR UNDERSTANDING FLOODS

    Scientists at the USGS measure streamflow in rivers across the State during every major flood. After flood waters recede, the USGS may be funded to locate and survey "high-water marks" where debris and mud lines indicate the highest extent of flood waters. These post-flood surveys are used to estimate maximum flows at sites that could not be reached during the floods and also to map the areas covered by the floods.

    ```
    Streamflow data that have been collected since 1975 on the
    Chehalis River near Doty indicate that the estimated
    streamflow of "1-in-100 chance flood" is higher than it was 20
    years ago.
    The eariler flood designation was accurate on the basis of the data that were available at the time; more large floods happened after 1975 than from 1940-1975.
    The change in the flood designation after 20 years of additional data collection highlights the importance of continued river monitoring.
    Annual peak flow data for 1995 and 1996 are provisional and may change.
    ```

    
    (Larger Version, 182K GIF)
    

    Rapid urban development in the Mercer Creek Basin since 1977 has increased the estimated magnitude of the "1-in-100 chance flood" at Bellevue, Wash.
    

    The completion of Howard Hanson Dam on the Green River has decreased the magnitude of the "1-in-100 chance flood" at Auburn, Wash. since 1961.

    ## DO YOU LIVE ON THE FLOODPLAIN?

    The areas affected by past floods have been mapped by the Federal Emergency Management Agency and many other government agencies. Because of continuing changes in river channels and land use in many basins, the maps may not reflect current information for your area. Inquire at your City or County Building or Planning Department.

    If you live on the designated floodplain, the chances are about 1 in 2 that you will experience a flood during your lifetime. Prepare for a flood as you would for any natural disaster, and make evacuation plans for your family.

    ## FLOODS WILL CONTINUE TO HAPPEN

    Although we can lessen effects of some floods, they are part of the natural cycle of every river and benefit instream habitats by moving material downstream and renewing streambeds. As floods get bigger and spread farther, flood waters slow and deposit sediment on the floodplain. This natural process created valuable farmlands in river valleys of the Pacific Northwest over thousands of years.

    ## Glossary of Flood Terms

    A flood is any relatively high streamflow that overtops the natural or artificial banks of a river.

    Discharge is another term for streamflow; it is the measured volume of water that moves past a point in the river in a given amount of time. Discharge is usually expressed in cubic feet per second.

    The floodplain is the relatively flat lowland that borders a river, usually dry but subject to flooding. Floodplain soils actually are former flood deposits.

    The average number of years between floods of a certain size is the recurrence interval or return period. The actual number of years between floods of any given size varies a lot because of the naturally changing climate.

    A hydrograph is a graph that shows changes in discharge or river stage over time. The time scale may be in minutes, hours, days, months,

    One cubic foot per second (cfs) is about 450 gallons per minute. The average discharge of the Columbia ~ River in September at The Dalles, Oregon, is about 120,000 cfs, which would fill the Seattle Kingdome in less than 10 minutes. The average discharge of the Puyallup River in September is about $1,700 \mathrm{cfs}$ at Puyallup, Wash.
    years, or decades.

    The river stage is the height of the water in the river, measured relative to an arbitrary fixed point.

    ## --Karen Dinicola

    from U.S. Department of the Interior, U.S. Geological Survey, Fact Sheet FS-229-96

    ## For more information contact any of the following:

    The U.S. Geological Survey has served the public and Federal, State, and local goverments since 1879 by collecting, analyzing, and publishing detailed information about the Nation's mineral, land, and water resources. The USGS has been studying the water resources of Washington State since the turn of the century. This information is in a variety of map, book, electronic, and other formats and is available by contacting:
    U.S. Geological Survey

    1201 Pacific Ave., \#600
    Tacoma, WA 98402
    (253) 428-3600

    Fax: (253) 428-3614
    Email:dc_wa@usgs.gov

    Selected data and interpretive reports are available on the USGS Washington "home page" on the World Wide Web at http://wa.water.usgs.gov/

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    URL: http://pubs.water.usgs.gov/FS-229-96

    ## Examples:

    Wind Loading
    Example 1
    Given the structure with three shear walls and rigid roof diaphragm, determine the horizontal shear distributed to the walls (and piers) with a static wind pressure and the overturning moment on each wall. The basic wind speed for the College Station area is within $90-100 \mathrm{mph}$ from ASCE-7. (Use 100 mph )

    Wind Pressure:
    Flat roof ( $0^{\circ}$ )
    Zone A: $10 \%$ of $5 \mathrm{~m}=0.5 \mathrm{~m}$
    Zone C ( $\sim 10 \mathrm{ft}$ height)
    for simplicity use C
    $\mathrm{p}_{\mathrm{s} 30}=10.5 \mathrm{psf}$ x $0.0479 \mathrm{kN} / \mathrm{m}^{2} / \mathrm{psf}$ $=0.5 \mathrm{kN} / \mathrm{m}^{2}(\mathrm{KPa})$
    

    Simplified Design Wind Pressure, $\mathbf{p}_{\mathbf{s} 30}$ (psf) (Expc

    | Basic Wind Speed (mph) | Roof Angle (degrees) |  |  |  |  |  | Zones |  |
    | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
    |  |  |  | Horizontal Pressures |  |  |  | Ver |  |
    |  |  |  | A | B | C | D | E |  |
    | 100 | 0 to $5^{\circ}$ | 1 | 15.9 | -8.2 | 10.5 | -4.9 | -19.1 | -1 |
    |  | $10^{\circ}$ | 1 | 17.9 | -7.4 | 11.9 | -4.3 | -19.1 | -1 |
    |  | $15^{\circ}$ | 1 | 19.9 | -6.6 | 13.3 | -3.8 | -19.1 | -1 |
    |  | $20^{\circ}$ | 1 | 22.0 | -5.8 | 14.6 | -3.2 | -19.1 | -1 |
    |  | $25^{\circ}$ | 1 | 19.9 | 3.2 | 14.4 | 3.3 | -8.8 | -1 |
    |  |  | 2 | -- | --- | --. | -- | -3.4 | - |
    |  | 30 to 45 | 1 | 17.8 | 12.2 | 14.2 | 9.8 | 1.4 | -1 |
    |  |  | 2 | 17.8 | 12.2 | 14.2 | 9.8 | 6.9 | - |

    SOLUTION:
    The wind pressure needs to be turned into a static force by multiplying by the tributary area, which is half way from other "support" to "support" or top of parapet:

    The force along the wide length, $P_{1}$, can be evenly resisted (split) by the end shear walls because they are the same size and stiffness.

    In the long direction, the force $P_{2}$ must be resisted by the piers on one side only. The force should be distributed to each pier based on their stiffnesses (a function of h/L), but the calculation is laborious. This example splits the force proportionally by length.
    

    Would this want to twist? A torsional moment will result if the center of rigidity, which is the resulting location of the moments of the wall rigidities, does not coincide with the center of mass determined from the moments of the wall weight. There is, in effect, an eccentricity.
    

    The overturning moments from the lateral forces at the top of the walls and piers about their bases (or toe) can be calculated.
    

    ## Example 2

    EXAMPLE 9.7 Header Acting as a Chord
    

    Figure 9.9 The header over an opening in a wall may be used as horizontal diaphragm chord.

    Over the window the header serves as the chord. It must be capable of resisting the maximum chord force in addition to gravity loads. The maximum chord force is

    $$
    T=C=\frac{\max . M}{\mathrm{~b}}
    $$

    The connection of the header to the wall must be designed for the chord force at that point:

    $$
    T_{1}=C_{1}=\frac{M_{1}}{b}
    $$

    nOTE: For simplicity, the examples in this book determine the chord forces using the dimension $b$ as the width of the building. Theoretically $b$ is the dimension between the centroids of the diaphragm chords, and the designer may choose to use this smaller, more conservative dimension.
    $\frac{\text { Latest Quake }}{\underline{\text { Info }}} \frac{\text { General Quake }}{\underline{\text { Info }}} \quad \underline{\text { Hazards \& }} \quad \underline{\text { Earthquake }} \quad \underline{\text { Special }} \quad \underline{\text { Features }} \quad \underline{\text { Additional }}$

    ## Reducing Earthquake Losses Throughout the United States

    ## Building Safer Structures

    In this century, major earthquakes in the United States have damaged or destroyed numerous buildings, bridges, and other structures. By monitoring how structures respond to earthquakes and applying the knowledge gained, scientists and engineers are improving the ability of structures to survive major earthquakes. Many lives and millions of dollars have already been saved by this ongoing research.
    

    On October 17, 1989, the magnitude 7.1 Loma Prieta earthquake struck the Santa Cruz Mountains in central California. Sixty miles away, in downtown San Francisco, the occupants of the Transamerica Pyramid were unnerved as the 49-story office building shook for more than a minute. U.S. Geological Survey (USGS) instruments, installed years earlier, showed that the top floor swayed more than 1 foot from side to side. However, no one was seriously injured, and the Transamerica Pyramid was not damaged. This famous San Francisco landmark had been designed to withstand even greater earthquake stresses, and that design worked as planned during the earthquake.
    (Click on image for a full size version - 128K)
    Earthquakes are a widespread hazard in the United States. Colors show
    magnitudes of historical earthquakes: red, 7 or greater; orange, 5.5 to 7 ;
    yellow, 4.5 to 5.5. The U.S. Geological Survey operates instruments in many
    structures in the seismically active areas shown. These instruments measure
    how structures respond to earthquake shaking.
    Designing and building large structures is always a challenge, and that challenge is compounded when they are built in earthquake-prone areas. More than 60 deaths and about $\$ 6$ billion in property damage resulted from the Loma Prieta earthquake. As earth scientists learn more about ground motion during earthquakes and structural engineers use this information to design stronger buildings, such loss of life and property can be reduced.

    To design structures that can withstand earthquakes, engineers must understand the stresses caused by shaking. To this end, scientists and engineers place instruments in structures and nearby on the ground to measure how the structures respond during an earthquake to the motion of the ground beneath. Every time a strong earthquake occurs, the new information gathered enables engineers to refine and improve structural designs and building codes. In 1984 the magnitude 6.2 Morgan Hill, California, earthquake shook the West Valley College campus, 20 miles away. Instruments in the college gymnasium showed that its roof was so flexible that in a stronger or closer earthquake the building might be severely damaged, threatening the safety of occupants. At that time, these flexible roof designs were permitted by the Uniform Building Code (a set of standards used in many states). Many industrial facilities nationwide were built with such roofs.
    (Click on image for a full size version -82 K )
    

    Seismic records (upper right) obtained during the 1984 Morgan Hill, California, earthquake led to an improvement in the Uniform Building Code (a set of standards used in many states). The center of the gym roof shook sideways three to four times as much as the edges. The Code has since been revised to reduce the flexibility of such large-span roof systems and thereby improve their seismic resistance.

    Building codes provide the first line of defense against future earthquake damage and help to ensure public safety. Records of building response to earthquakes, especially those from structures that failed or were damaged, have led to many revisions and improvements in building codes. In 1991, as a direct result of what was learned about the West Valley College gymnasium roof, the Uniform Building Code was revised. It now recommends that such roofs be made less flexible and therefore better able to withstand large nearby earthquakes.

    Earth scientists began recording earthquakes about 1880, but it was not until the 1940's that instruments were installed in buildings to measure their response to earthquakes. The number of instruments installed in strucures increased in the 1950's and 1960's. The first abundant data on the response of structures came from the devastating 1971 San Fernando, California, earthquake, which yielded several dozen records. These records were primitive by today's standards. The first records from instruments sophisticated enough to measure twisting of a building were obtained during the 1979 Imperial Valley, California, earthquake.

    Today there are instruments installed in hospitals, bridges, dams, aqueducts, and other structures throughout the earthquake-prone areas of the United States, including Illinois, South Carolina, New York, Tennessee, Idaho, California, Washington, Alaska, and Hawaii. Both the California Division of Mines and Geology (CDMG) and the USGS operate instruments in California. The USGS also operates instruments in the other seismically active regions of the nation.
    
    (Click on images for full size versions - 192K, 238K, 98K, 114K)
    

    USGS scientists have installed instruments in a variety of structures across the United States to monitor their behavior during earthquakes. Examples shown include a dam, a bridge supporting a large aqueduct, a highway overpass, and a Veterans hospital.

    The majority of deaths and injuries from earthquakes are caused by the damage or collapse of buildings and other structures. These losses can be reduced through documenting and understanding how structures respond to earthquakes. Gaining such knowledge requires a long-term commitment because large devastating earthquakes occur at irregular and often long intervals. Recording instruments must be in place and waiting, ready to capture the response to the next temblor whenever it occurs. The new information acquired by these instruments can then be used to better design earthquake-resistant structures. In this way, earth scientists and engineers help reduce loss of life and property in future earthquakes.

    ## Mehmet Celebi, Robert A. Page, and Linda Seekins

    ## COOPERATING AGENCIES, COMPANIES, AND INSTITUTIONS

    California Department of Transportation
    California Division of Mines and Geology
    City of Los Angeles
    General Services Administration
    Metropolitan Water District of Southern California
    Oregon Department of Highways
    U.S. Army Corps of Engineers
    U.S. Department of Energy
    U.S. Department of Veterans Affairs

    Washington Department of Highways
    Washington Department of Natural Resources
    Private building owners
    For more information contact:
    Earthquake Information Hotline (415) 329-4085
    U.S. Geological Survey, MS 977

    345 Middlefield Road, Menlo Park, CA 94025
    USGS Menlo Park Earthquakes Home Page
    U.S. Geological Survey Fact Sheet-167-95 1995

    Web page by Will Prescott - 1996 April 9

    # Chapter 1: The Nature of Ground Motion and its Effect on Buildings 

    ## GEOLOGIC BACKGROUND

    According to the now generally accepted theory of Plate Tectonics, the earth's crust is divided into several major plates, some 50 miles ( 80 km ) thick, that move slowly and continuously over the interior of the earth.

    Earthquakes are initiated when, due to slowly accumulating pressure, the ground slips abruptly along a geological fault plane on or near a plate boundary. The resulting waves of vibration within the earth create ground motion at the surface which begins to vibrate in a very complex manner. This, in turn, induces forces within buildings that are determined by the precise nature of the ground motion and the construction characteristics of the building.

    The point where the fault first slips is termed the "focus" or "hypocenter." A theoretical point on the earth's surface directly above the focus is termed the "epicenter." (Figure 1.1) The epicenter for the January 17, 1994 Los Angeles earthquake was located in the city of Northridge in the San Fermando Valley.

    The initial break in the fault moves rapidly along the line of the fault, and the distance of this movement largely determines the intensity of ground shaking. Thus the 1906 San Francisco earthquake ruptured along some 250 miles ( 400 km ) of the San Andreas fault. The Loma Prieta, California earthquake of 1989 was unusual since no surface faulting occured, although a broad area of ground cracking indicated a wide distribution of strain. The fault rupture moved upward to within about 6 km of the ground surface area and then spread approximately 20 km along the fault to each side of the initial rupture. (Figure 1.2)

    ## GROUND FAILURE

    ## Surface Faulting

    Slippage along a fault line deep in the earth's surface may eventually result in surface faulting, the crack or split on the earth's surface that provides the layperson's vision of earthquakes. Surface faulting may result in large earth movements: in the 1992 Landers earthquake east of Los Angeles, the earth offset as much as 18 feet at the surface. A building located across a surface fault, no matter how well designed, is almost certain to suffer very severe damage. (Figure 1.3) However, the large disturbance of the ground near a fault is generally quite narrow in width on either side of the fault: in Landers the maximum width of severely disturbed ground was only about 40 meters. Moreover, the probability that buildings will straddle a surface fault is very low
    

    Figure 1.1: Earthquake location
    

    Figure 1.2: The Loma Prieta fault rupture, 1989

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    compared to the likelihood of significant ground motion. So, in seismic design, we design against the vibrations caused by fault slippage and try to ensure that buildings are not built over fault zones.

    ## Landslides, Liquefaction and Subsidence

    The energy released by an earthquake can also trigger ground failure in the form of landslides, liquefaction and subsidence which can have devastating effects on a structure. Even well-built structures, designed to withstand earthquake forces, if built on an unstable site or in the path of a landslide, can fall victim.

    The Alaskan earthquake of 1964 provides examples of structures with the inherent strength to withstand ground shaking that were devastated as a result of the instability of the sites they were built on. (Figure 1.4) While an architect and contractor could take pride in the performance of their buildings on Turnagain Heights or on 4th Street in Anchorage, the decision to build on geologically unstable sites produced catastrophic results. (Figure 1.5) Avoidance of sites with a potential for liquefaction, landslides or subsidence represents the best design approach.

    Ground shaking can also trigger subsidence and liquefaction in soils that are unconsolidated and/or saturated with water. When sandy, water saturated soils are shaken, the bearing capacity of,the soil is reduced as the soil liquefies and flows laterally and vertically. Liquefied soils can produce volcano-like sandboils at the ground surface or flow laterally if the soil is not contained. The ground surface and structures built on shallow foundations can subside several feet or be torn apart as spreading occurs. Dramatic examples of liquefaction from recent earthquakes illustrate again, that even well built structures are vulnerable if adequate attention is not paid to site conditions and foundation design. (Figures 1.6 and 1.7)
    

    Figure 1.6: Liquefaction, Niigata, Japan, 1964

    ## GROUND MOTION

    While ground failure can be an important consequence of any earthquake, the primary effect buildings are designed to resist is ground motion. During an earthquake, wave vibrations emanate from the line of fault rupture and so approach the building from a given direction. The waves begin like ripples in a still pond when a pebble is thrown into it, but the seismic waves rapidly become more complex.

    There are four main wave types, of which "body" waves, within the earth, are the most important for design purposes. (Figure 1.8) First to arrive at the surface is the $P$ or primary wave. In this wave the ground is successively pushed and pulled along the wave front. The effect is of a sharp punch - it feels as if a truck has hit the building. The P wave is followed by the $S$, secondary or shear wave, which is a lateral motion, back and forth (but sideways to the wave front).

    The nature of the waves and their interactions are such that actual movement at the ground will be random: predominantly horizontal, often with considerable directional emphasis, but sometimes with a considerable vertical component. The actual horizontal ground displacement is small, only inches even in a large earthquake, except in the immediate area of the fault rupture where displacements of several feet may occur.

    ## THE MEASUREMENT OF GROUND MOTION

    Measurement of ground motion is important for design purposes because it provides the basis for determining forces, and assessing the relative seismic hazard at different locations.

    Earthquake motion is recorded by a seismograph, an instrument that records the movement, over time, of a freely supported pendulum within a frame: the instrument may be placed on the ground or within a structure.

    In modern seismographs, pendulum movement is converted into electronic signals on tape. Strong-motion seismographs, called accelerometers, are designed to directly record nearby rather than distant ground movement, and they produce a record called an accelerogram. Instruments are normally placed so as to measure movements along the two horizontal axes as well as one vertical. Three measures are of major interest: acceleration, velocity, and displacement.

    ## Acceleration, Velocity, Displacement

    Acceleration is the rate of change of velocity: when multiplied by mass it results in the inertial force that the building must resist. This is a key measure, and forms the basis of the estimation of earthquake forces on buildings: Newton's Second Law of Motion states in essence, that an inertial force, $F$, equals mass $(M)$ multiplied by the acceleration ( $A$ ).
    

    Figure 1.7: Sand boil in a lettuce field, Watsonville, 1989
    

    Figure 1.8: " $P$ " and " $S$ " waves
    

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    Acceleration is commonly measured in " $g$ ' $s$ " - the acceleration of a free falling body due to the earth's gravity (approx. $32 \mathrm{ft} / \mathrm{sec} / \mathrm{sec}$., or $980 \mathrm{~cm} / \mathrm{sec} / \mathrm{sec}$, or 1.0 g .). Velocity, measured in inches or centimeters per second, refers to the rate of ground motion at any time. Displacement, measured in inches or centimeters, refers to the distance a particle is removed from its "at rest" position. (Figure 1.9)

    The level of acceleration generally taken as sufficient to produce some damage to weak construction is 0.10 g . The lower limit of acceleration perceptible to people is set by observation and experiment at approximately 0.001 g or $1 \mathrm{~cm} /$ $\mathrm{sec}^{2}$; at around 0.20 g and above most people will have difficulty keeping their footing and sickness symptoms may be induced. An earthquake causing acceleration approaching 0.5 g on the ground is very high. On upper floors of buildings, maximum accelerations will often be higher, depending on the degree to which the mass and form of the building act to damp the vibratory effects. A figure of 1.00 g , or $100 \%$ of gravity, may be reached, for a fraction of a second. To design for 1.00 g is diagrammatically equivalent, in a static sense, to designing a building that projects horizontally from a vertical surface. (Figure 1.10) When the behavior of real buildings is observed, several factors modify this diagrammatic equivalence, and structures that could never cantilever from a vertical surface can briefly withstand 1.0 g earthquake shaking.

    Acceleration is the measure commonly used to indicate the possible destructive power of an earthquake in relation to a building. A more significant measure is that of acceleration combined with duration, which takes into account the impact of earthquake forces over time. In general, a number of cycles of moderate acceleration, sustained over time, can be much more difficult for a building to withstand than a single peak of much higher value. Seismic instrumentation also measures the duration of strong ground motion, which generally relates to the length of the fault break.

    Typically the extreme vibration will occupy only a few seconds; both the 1989 Loma Prieta and 1994 Northridge earthquakes lasted only a little over ten seconds, yet they caused much destruction. In 1906, in San Francisco, the severe shaking lasted about 45 seconds; in Alaska in 1964 the severe earthquake motion lasted for over 3 minutes.

    Two earthquake measurement systems are in common use and neither, for various reasons, is really satisfactory from the building design viewpoint.

    ## Magnitude: The Size of the Wave

    Earthquake magnitude is the first measure: it is expressed as Richter magnitude based on the scale devised by Professor Charles Richter of the California Institute of Technology in 1935. Richter's scale is based on the maximum amplitude of certain seismic waves recorded on a standard seismograph at a distance of 100 kilometers from the earthquake epicenter. The scale, however, tells nothing about duration, which may be of great significance in causing damage, nor does it tell anything about frequency content which, in its relationship to the building period, as discussed later, is also of great signifi-
    cance in determining damage. Because the instrument is unlikely to be exactly 100 km from the source, Richter developed a method to allow for the diminishing of wave amplitude (or "attenuation") with increased distance, just as the light of a star appears dimmer with distance. (Figure 1.11)

    Because the size of earthquakes varies enormously, the graphic range of wave amplitude measured on seismographs is compressed by using, as a scale, the logarithm to base ten of the recorded wave amplitude. Hence, each unit of Richter magnitude indicates a 10 times increase in wave amplitude. But the energy increase represented by each unit of scale is estimated by seismologists as approximately 31 times. Since Richter magnitude is a measured quantity, the scale is open-ended, but seismologists believe that a Richter magnitude of about 9 represents the largest possible earthquake. A given earthquake will have only one Richter magnitude, though differences in recording result in some argument as to what this will be.*

    ## Intensity: The Amount of Damage

    To provide information directly related to local shaking and building damage, intensity scales are used. These scales are based on subjective observation of the effects of the earthquake on buildings, ground and people. In the United States the most commonly used scale is the Modified Mercalli (MM) originally developed in Europe in 1902, and modified in 1931 to fit construction conditions then prevalent in California and other parts of the United States.

    As a result the MM scale is somewhat dated, with no references to common modern construction systems. This is not much of a disadvantage because earthquake damage is most likely to be concentrated in older buildings, often of the very type that the scale describes. (Figure 1.12) The MM Scale is a twelve point scale, $I-$ XII. The descriptions for MM I are, in abbreviated form, "Not felt. Marginal and long-period effects of large earthquakes." For MM XII the descriptor reads, "Damage nearly total. Large rock masses displaced. Lines of sight and level distorted. Objects thrown into the air." Because earthquake effects vary depending on distance from the epicenter, nature of the ground, and magnitude, an earthquake will have many MM values. The MM scale has been correlated with ground acceleration. For example, MM VII corresponds to a peak acceleration between approximately 0.1 g and 0.29 g .

    ## THE EFFECTS OF GROUND MOTION

    ## Inertial Forces

    While the effects of ground failure can be extremely severe, the most common and widespread cause of earthquake damage is ground shaking. Seismically induced shaking affects buildings in three primary ways: inertial forces, period and resonance, and torsion. Shaking causes damage by internally generated inertial forces generated by vibration of the building's mass.


    

    Figure 1.11: Richter magnitude
    

    Figure 1.12: Damage to an older masonry building

    ## Chapter 2: Site Issues

    ## SITING OF A STRUCTURE - WHERE DOES THE SITE BEGIN?

    From a seismic design standpoint, the "site" is the region within which a structure will be built; and while it is critical that a structure not be built across an active fault trace, it is equally important that siting and design decisions address the potential for increased intensity and duration of ground shaking, accessibility, survival of life lines and potentially hazardous adjacent land uses. Thus, seismic design is not limited to an analysis of the factors within the confines of the site boundary; it extends to a broad environmental analysis of regional and community vulnerability.

    ## SEISMIC RISK AS A SITING CRITERIA

    The factors that impact site vulnerability include proximity to active earthquake faults; susceptibility of the site to ground shaking; the potential for ground failure, including subsidence, lateral spreading, liquefaction, and landslides; adjacent structures and land uses that could pose a threat during or after an earthquakes; and, the potential for inundation resulting from tsunami or dam failure.

    From a site and urban planning standpoint, however, concern should not be limited to the identification on the site of a fault or potential fault rupture, but to the broader impact of ground shaking and geologic failures that could occur in the region. The failure of the regional transportation network, disruption of power or water supply or the isolation of building as a result of ground failure, can be as devastating to a business as actual structural damage.

    Therefore, seismic risks from beyond the building site property line must be considered as design criteria for a structure. These criteria address the relative desirability or risk of an individual site, that is, is one site safer for a particular use than another site; and what factors beyond the site boundary, such as adjacent land uses, geologic stability of adjacent land, or the survivability of lifelines or access, could impact site development?

    ## ACTIVE EARTHQUAKE FAULTS

    If a structure is built over an active fault trace, it should be designed to accommodate displacement or fault offset. (Figure 2.1)

    This is both a challenging and costly effort, with no guarantee of success. The mapping of active faults has been a focus of geologists and urban planners for
    

    Figure 2.1: San Andreas fault in Central California
    

    Figure 2.2: The section of Fairmont Hospital in San Leandro, California, built across a fault trace, was removed.
    several decades. It has been a critical element increasing our understanding of regional seismicity: the frequency of seismic activity, the magnitude of previous seismic events, and the potential for future seismic activity. The fault maps indicate where active surface faulting is identified and where future offset potential exists. Where identified, designers should provide a setback from identified faults for new construction.

    In many areas, development is limited or prohibited within defined zones adjacent to active faults. Programs to map fault zones and limit new construction within established zones have proven successful in reducing earthquake risks to new construction. Unfortunately, earthquake fault traces were often ignored when land was subdivided and developed, presenting a costly dilemma to owners of existing structures in a fault zone.

    Where existing structures are built across fault lines, their structural performance, occupancy and continued use should be reviewed to evaluate the risk they pose. Those sections of structures built across a trace can be removed or occupancy types and loads can be reduced to reduce risk exposure. (Figure 2.2)

    ## IMPACT OF REGIONAL GEOLOGY ON SITE PERFORMANCE

    The geology of a region plays a significant role in determining the potential for shaking and ground failure damage. In relatively old geological regions, such as the eastern and midwestern United States where weathering and erosion have leveled the terrain and laid deep deposits of unconsolidated soils, violent ground shaking resulting from fault rupture thousands of feet below the earth's surface can extend for thousands of square miles. Deep soils can amplify ground shaking intensity, extend duration of violent shaking and limit attenuation of shaking; resulting in greater damage over a larger area than would result in younger or bedrock regions.

    For example, in the central United States, the violent shaking of the New Madrid, Missouri earthquakes of 1811 and 1812 extended across the midwest and was felt as far away as the eastern seaboard. The earthquakes were felt over 2,000,000 square miles! In contrast, the 1906 San Francisco earthquake, estimated to have released 30 times more energy, was felt over only 375,000 square miles. It impacted a much smaller area because the regional geology in California limited propagation and increased attenuation of shaking. In both examples, one without surface manifestations of faulting, and the other with visible surface faulting, regional geology rather than presence of a surface fault determined the extent of potential damage. (Figure 2.3)

    While not building across an earthquake fault is certainly a good rule, building adjacent to a fault may not pose as great a risk as one would expect. A number of recent earthquakes have emphasized that regional and local geology and the lack of attenuation of ground shaking are often more important than proximity to the earthquake's epicenter in determining the impact of an earthquake. The 1985 Mexican earthquake occurred on the coast of Mexico between Acapulco and Ixtapa. Damage close to the epicenter in the coastal resort areas was minor.

    However, 250 miles away in Mexico City, the damage to midrise concrete structures was severe, resulting in several thousand deaths. Again in 1989, the Loma Prieta earthquake, centered in the Santa Cruz Mountains resulted in the deaths of more than 40 persons on the Cypress Viaduct, 60 miles north of Santa Cruz in Oakland. In both cases, the most violent ground shaking did not occur at the epicenter of the earthquakes, but a significant distance away as a result of the propagation of the ground waves, the geology of the region and local soil conditions. Understanding the regional and local geology can tell the designer a great deal about the relative risk of an individual site.

    ## REGIONAL DAMAGE AND ITS IMPACT ON A SITE

    Continued function and operation of a building depends on more than merely the performance of the structure. Damage to lifeline systems providing water, sewer, power, transportation and communication services can isolate a structure, cease operations or production, and leave the structure vulnerable to secondary hazards of fire and hazardous material releases. For buildings containing functions where power, water and/or communications is vital for continued operations or safety, analysis should address the vulnerability of regional lifelines serving the site. If access to the site or to regional transportation networks is critical for ongoing operations or for reaching and maintain-, ing market deliveries, the designer should review the vulnerability of the regional transportation system. (Figure 2.4) While these issues cannot be addressed in building design, their identification for the clients will provide a basis for their understanding of the strengths and limits of a specific site, and for determining the need for back-up facilities, water and power sources, and communication systems that may prove critical to safety and post-earthquake response, recovery, and continued business operations.

    Regional damage, well beyond the property line, can result in isolation of a facility from resources, market or employees, dislocation, and severe economic disruption, even without damage to the structure.
    

    Figure 2.3: Comparison of isoseismals of large U.S. earthquakes

    Figure 2.4: Ground failure occurred at this highway, overstressing column/slab connections.
    

    ## THE ARCHITECT'S ROLE IN SITE SELECTION AND EVALUATION

    Only occasionally is the architect responsible for site selection. Most often, the architect is provided with a site by a client unaware of its vulnerability to seismic forces. The more traditional site analysis would include relevant information on zoning and planning restrictions on the site. A "seismic site analysis" should include an evaluation of local site conditions, adjacent hazards and regional geology, to assist the architect in briefing the client on the expected performance of the selected site, the survivability of transportation and access to the site, and the vulnerability of lifelines serving the site. This data can provide valuable insights for the client and design team in establishing design parameters and in defining expected seismic performance of the structure.

    If, however, the architect participates in site selection, desired structural building performance and post earthquake function can be measured against expected site performance, life line survival and site access in determining the most appropriate location.

    Figure 2.5: Building adjacencies can have major impacts on performance during earthquakes. A large number of structures suffered pounding damage during the 1985 Mexico City event, leading in many cases to partial or full ollapse.

    In either situation, a site analysis should include an assessment of the environment beyond the property line and include adjacent structures and site conditions that could "spill over" onto your site. (Figure 2.5) A complete analysis should address the issues identified in the Site Analysis Checklist.
    

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    ## Site Analysis Checklist

    - Is there an active fault on or adjacent to the site?

    Will the site geology increase ground shaking?
    Does the site contain unconsolidated natural or man-made fills?

    - Is the site geology stable?Is the site susceptible to liquefaction?Are adjacent up-slope and down-slope environments stable?
    - Are post-earthquake access and egress secure?
    - Are transportation, communication and utility lifelines vulnerable to disruption and failure?
    - Are there adjacent land uses that could be hazardous after an earthquake?
    - Are hazardous materials stored or used in the vicinity?
    - Are building setbacks adequate to prevent battering from adjacent structures?
    - Are adjacent structures collapse hazards? Would they collapse onto your site or would their failure otherwise impact the functions of your structure?
    - Is the site subject to inundation from tsunami? Seiche? Dam failure flooding?
    - Are there areas of the site that should be left undeveloped due to:
    - Landslide potential?
    - Inundation potential?
    - High potential for liquefaction?
    - Expected surface faulting?
    - More violent or longer duration ground shaking expected?
    - Areas necessary to provide separation from adjacent uses or structures?
    - Is there adequate space on the site for a safe and "defensible" area of refuge from hazards for building occupants?
    - Does the site plan increase potential for earthquake-induced landslides by:
    - Cutting unstable slopes?
    - Increasing surface runoff?
    - Increasing soil water content?


    ## Chapter 5: The Basics Of Seismic Codes

    ## BUILDING CODES AND SEISMIC PROVISIONS

    The first seismic building code to be developed in the United States was the seismic portion of the Uniform Building Code (UBC) published by the International Conference of Building Officials (ICBO) in California. The seismic provisions of the UBC were developed on a volunteer basis by the Structural Engineers Association of California (SEAOC). Currently, in addition to the UBC, the following are important seismic codes in use:

    - BOCA National Building Code
    - SBCCI Standard Building Code
    - GSA (Federal Buildings)
    - Tri-services (Department of Defense-Military)
    - Title 24, California (Hospitals and Schools)
    - Veterans Administration (Veterans Hospitals)
    - State Historic Building Code (SHBC) [California]
    - City of Los Angeles, Section 88 (Existing URM Buildings)
    - Uniform Code for Building Conservation (UCBC)

    Most of the codes listed above have the stated goal of maintaining life safety; only Title 24 (California) has a higher performance goal of damage control to maintain post-earthquake function in hospitals. The last three listings, which relate to existing buildings, permit lower design force levels than those required for new buildings. (Figure 5.1)

    Starting in the mid-1970s the Federal Government initiated a research program to develop a state-of-the-art approach to a seismic code that would have nationwide applicability. This effort resulted in the 1978 publication of the ATC-3 document (named after the Applied Technology Council, the nonprofit engineering research group that developed it). Subsequently, the document has undergone several revisions and is now known as the National Earthquake Hazards Reduction Program: Recommended Provisions for the Development of Seismic Regulations for New Buildings or the NEHRP Provisions. Published by the Building Seismic Safety Council in Washington, and updated on a 3-year basis, the NEHRP Provisions document is not a code, but a technical resource document to assist in code development. In format, language and content, however, the document is very similar to a seismic code.
    

    Figure 5.I: Advances in building code seismic provisions are intended to ensure life.safety and prevent the types of failure and collapse that occur in pre-code buildings.

    | SUMMARY OF BUILDING CODE SEISMIC DESIGN CONCEPTS |  |  |  |  |
    | :---: | :---: | :---: | :---: | :---: |
    |  | Uniform Building Code(1991) |  | NEHRP Provisions(1991) |  |
    | Goal | Life Safety |  | Life Safety |  |
    | Seismic Load | Base Shear V ( $\mathrm{F}=\mathrm{MA}$ concept) |  | Base Shear V ( $\mathrm{F}=\mathrm{MA}$ Concept) |  |
    |  | $\mathrm{V}=\frac{\mathrm{ZICW}}{\mathrm{R}_{\mathrm{w}}}$ |  | $\mathrm{V}=\mathrm{C}_{s} \mathrm{~W}$ |  |
    |  | ( $\mathrm{C}=\underline{1.25 \mathrm{~S}}$ |  | $\left(C_{s}=\frac{1.2 \mathrm{~A}_{v} \mathrm{~S}}{\left.\mathrm{RT}^{2 / 3}\right)}\right.$ |  |
    |  |  |  |  |  |
    | Zone |  | $\begin{aligned} & 5 \text { Zones } \\ & 0.075,0.15,0.20,0.30,0.40 \end{aligned}$ | $\begin{aligned} & 6 \text { Zones } \\ & 0.05,0.10,0.15,0.20,0.30,0.40 \end{aligned}$ |  |
    | Importance |  | Building Occupancy $(1.0,1.25)$ |  | Exposure Groups (3 categories) <br> PC Performance Categories (5 categories) |
    | Struct. <br> Response | $\mathrm{R}_{\mathrm{w}}$ | Response Modifications based on 5 basic Structural types | R | Response Modifications based on 6 basic Structural types |
    | Soil | S | $\begin{aligned} & 4 \text { Soil Profiles } \\ & (1.0,1.2,1.5,2.0) \end{aligned}$ | S | $\begin{aligned} & 4 \text { Soil Profiles } \\ & (1.0,1.2,1.5,2.0) \end{aligned}$ |
    | Mass | W | Building Weight |  | Building Weight |
    | Period |  | Building Period |  | Building Period |

    Table 5.A

    Table 5.A shows a comparison between the basic provisions of the 1991 UBC and the 1991 NEHRP Provisions. This summary shows that these two codes are very similar in concept and in the factors that are included.

    Prior to 1988, the UBC and the NEHRP Provisions tended to pursue somewhat diverging approaches to code development and modification. However, in the 1988 edition of the UBC and the NEHRP Provisions, a notable merging of some concepts in the two documents occurred. While updating these documents continues independently, the concepts within them are subject to constant mutual review. Taken together, the SEAOC and NEHRP efforts represent probably the most influential and consistent effort in the world to provide a technical basis for seismic code development.

    The UBC represents only one of the commonly used model codes in the U.S. The BOCA model code, developed by the Building Officials and Code Administrators organization is used extensively in the East and Midwest, and the Standard Building Code, developed by the Southern Building Code Congress International, is used extensively in the Southern states.

    Until recently, the two latter model building codes groups have lagged behind in the development of seismic codes, primarily because these model codes were used in areas of little perceived seismic hazard. Concern for the seismic hazard present in other states in the U.S. besides California has resulted in a new interest in the development and adoption of appropriate codes, an interest which the development of the NEHRP Provisions was intended to support. Consequently, both the BOCA model code and the Standard Building Code now incorporate slightly modified versions of the NEHRP Provisions in their model building codes. (Figure 5.2)

    Thus, on a national basis, the seismic code issue is basically accommodated by variations of the two main technical documents; the NEHRP Provisions and the UBC (or, more precisely, the SEAOC provisions upon which it is based.)

    ## APPLYING CODES

    The primary purpose of seismic building codes is to provide a simple uniform method to determine the seismic forces for any location with enough accuracy to ensure a safe and economical design. The code needs to provide for approximate uniformity of results so that no building owner, building type, or materials supplier is unfairly discriminated against.

    In Chapter 1 it was shown that the earthquake forces on a building can be referred back to the basic formula for inertial forces $-F$ equals MA. $M$ is easy to obtain by calculating the weight of the building. How about $A$, the acceleration?

    The NEHRP Provisions provide a number of sets of maps of the United States: these provide contour lines, or color codes of the counties in each state, so that the entire country is divided into seven areas. (Figure 5.3 shows a small scale reproduction of one of the maps provided with the Provisions.) Each area in turn is equivalent to a number from 0.05 to 0.40 in steps of $0.05,0.10,0.15$, $0.20,0.30$, and 0.40 . These represent accelerations in percentages of G -so that 0.40 represents $40 \%$ of G . This is the $A$ for the $F=M A$ formula. It's not quite as simple as that, but nevertheless the relationship of the maps to the fundamental formula is quite direct and clear.

    These maps reflect a number of assumptions. The general criterion is that the risk at any location has only a 10 percent probability of being exceeded in 50 years, which translates into a mean recurrence interval of 475 years. This is a statistical number and not a prediction: the important thing is that the map is expressing a uniform risk, so that by looking at the different numbers you get an approximation of the relative risk among different regions of the country.

    The Provisions state that, for most instances the horizontal force on the building can be represented as a horizontal shear force trying to push the base of the building across the ground where the building is attached to its foundation. This force is called the base shear, and a formula is provided for its estimation. Application of this formula is a key part of the code methodology and is called
    

    Figure 5.2: Seismic code provisions have undergone continuous development since the 1950's in response to both damaging earthquakes and to advances in engineering science.
    

    Figure 5.3: Contour map for coefficient $A_{a}{ }^{*}$ for the intinental United States.
    the equivalent lateral force procedure. This general methodology is characteristic of all seismic codes throughout the world.

    In the Provisions this formula is $\mathbf{V}=\mathbf{C}_{\mathbf{i}} \mathbf{W}$ where:
    $C_{8}=$ the seismic design coefficient, which is related to an "expansion" of A, the acceleration number. The expansion adds other coefficients, or multipliers, which represent some of the other factors discussed in Chapter 1.
    $\mathbf{W}=$ the building weight, which can easily be calculated.
    $\mathrm{C}_{\mathrm{s}}=1.2 \mathrm{~A}, \mathrm{~S} / \mathrm{RT}^{2 / 3}$
    where $\mathrm{A}_{\mathrm{v}}$ is obtained from the contour map.*
    S is the Coefficient for soil profile type (i.e. relating to soil amplification). This is obtained from a table in the Provisions. The coefficient varies from 1 to 2.0 . For poor ground, where the coefficient is 2.0 , the acceleration number is multiplied by two, thus increasing the design force - the forces for which the building must be designed.
    $\mathbf{R}$ is a response modification coefficient, relating to the type and ductility of the chosen structural system. R factors are also obtained from a table in the Provisions. This is a number from 1.25 to 8 : it is a divisor, so it has the


    effect of reducing the design forces, and the higher the number, the higher the reduction.

    T is the period of the building (simple formulae for estimating this are provided in the Provisions.)

    It can be seen that these coefficients, $\mathbf{A}, \mathbf{W}, \mathbf{S}, \mathbf{R}$, and $\mathbf{T}$ encompass most of the characteristics discussed in Chapters 1 and 3 that affect the building's earthquake performance.

    For a really simple way of establishing the seismic force, the Equivalent Lateral Force method provides an alternative equation that can be used at the designer's option. This is:

    $$
    \mathrm{C}_{\mathrm{t}}=\frac{2.5 \mathrm{~A}_{\mathrm{a}}}{\mathrm{R}}
    $$

    Note that to use this equation it is not necessary to calculate the building period or estimate the soil type. Use of this equation will generally result in a larger force factor, for a small structure, such as a house, this is not usually significant.

    In addition to the equivalent lateral force equation, a formula is provided for calculating the vertical distribution of forces that makes some allowance for possible amplification, and allocates a higher proportion of the forces to the upper floors of the building.

    Application of the equivalent lateral force formula to locations of maximum shaking (i.e: $\mathrm{A}_{\mathrm{a}}=0.40$ on the map) produces a coefficient $\mathrm{C}_{s}$ that varies approximately from 0.125 for a steel moment resistant frame building to 0.80 for an unreinforced masonry building. (Figure 5.4)

    In other words, an unreinforced masonry building, which is a very poor seismic force resisting structure, would have to be designed to resist a base shear force equal to $80 \%$ of its weight - a very high acceleration. (In fact, unreinforced masonry structures are not permitted to be constructed in California, and it would be very difficult, if not impossible, to design an unreinforced masonry structure for these forces). On the other hand, a moment resisting frame would only have to be designed to resist lateral forces equal to $121 / 2 \%$ of its weight.

    So the equivalent lateral force equation provides a simple mathematical formula by which most of the factors that determine the lateral force on the building can be accounted for in a uniform way. Moreover, since the code defines a minimum force level, any of these coefficients can be revised upwards if the owner wishes to obtain a higher level of protection. This is a common practice.

    Other parts of the Provisions set limits on drift, require the design to be checked for overturning, and require calculations for torsion. If severe configuration irregularities are present, the Provisions require that a more complex analysis be used instead of the simple equivalent lateral force procedure. There are, of

    ## SAMPLE CALCULATION

    (simple equation):
    $\mathrm{V}=\mathrm{C}_{\mathbf{3}} \mathrm{W}$ and $\mathrm{C}_{\mathbf{4}}=2.5 \mathrm{~A}_{\mathbf{a}} / \mathrm{R}$

    ## For San Francisco:

    $\mathrm{A}_{\mathrm{a}}=0.40$ (map)
    For steel moment frame:
    $\mathrm{R}=8.0$ (Table 3.3)

    ## For URM:

    $\mathrm{R}=1.25$ (Table 3.3)
    Then:
    For steel moment frame:

    $$
    \mathrm{C}_{\mathrm{s}}=2.5 \times 0.40 / 8=0.0125\left(12.5 \% \text { " } \mathrm{G}^{\prime}\right)
    $$

    ## For URM:

    $$
    \mathrm{C}_{s}=2.5 \times 0.40 / 1.25=0.80\left(80 \% \text { " } \mathrm{G}^{\prime}\right)
    $$

    Figure 5.4
    

    Figure 5.5: Executive Order 12699 requires adoption of seismic standards in the design of all new buildings used, purchased or constructed with Federal assistance. The purpose is to avoid failures, such as that pictured above and opposite, and to reduce risks to occupants.

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    course many other issues presented in the Provisions that are not reflected in this simplified presentation. Nevertheless, the essence of any seismic code philosophy resides in the equivalent lateral force formula, and its relationship to the basic principles that have been discussed can readily be seen.

    ## PERFORMANCE OBJECTIVES

    One issue currently the focus of considerable effort is that of attempting to define performance objectives for seismic design, and ultimately to embody these in guidelines and codes. Performance objectives are statements of the limits of damage which a structure will be expected to sustain when subjected to specified earthquake demands, expressed in terms of defined ground motion. Performance objectives are expressed in terms of the performance of both the structural and nonstructural components.

    The Guidelines for the Seismic Rehabilitation of Buildings, now under development by the Building Seismic Safety Council, defines three performance levels. Collapse Prevention requires that all significant components of the gravity load-resisting system must continue to carry their demands, although significant risk of injury due to falling hazards may exist. Life Safety requires that, while considerable structural damage may have occurred, major structural and nonstructural components have not become dislodged, creating a threat to life: the risk of life-threatening injury is very low. Immediate Occupancy is a damage state in which only very limited damage may have occurred. Nonstructural damage is minimized such that basic access and life safety systems including doors, elevators, emergency lighting, fire alarms, and suppression systems remain operable if power is available. Minor clean-up could be required.

    While the specific terms for these damage states, and others, may change as work on this document proceeds, the philosophy of recognizing the inevitability of damage is characteristic of all the current focus on performance.

    ## PRESIDENTIAL EXECUTIVE ORDER

    An important development in the nationwide regulation of seismic building standards was the enactment into law in January 1990 of Executive Order 12699. This order requires that methods be taken to reduce risks to the lives of occupants of buildingsleasedfor federal uses or purchased or constructed with federal assistance, to reduce risks to the lives of persons who would be affected by engineering failures of federally assisted or regulated buildings, and to protect public investments, all in a cost-effective manner.

    The order directed federal agencies to issue regulations or procedures by February 1993 that incorporate seismic safety measures for all federal buildings that are owned, leased, assisted, or regulated by the federal government.

    The link between seismic safety requirements and the availability of federal funds for new building construction was expected to encourage local governments and private sector building designers and contractors to update their codes and practices. (Figures 5.5, 5.6 and 5.7)

    The order applies to any building located worldwide which is federally owned, lease constructed, leased ( $15 \%$ or more of total space), regulated or financially assisted. This includes new construction financed with federal grants or loans, or federally insured or guaranteed loans or mortgages.

    Individual federal agencies must ensure that building construction under their programs complies with the Executive Order. The Interagency Committee on Seismic Safety in Construction (ICSSC), which is a committee of federal agencies, recommends the use of seismic design and construction standards and practices equivalent to or exceeding those in the most recent (or immediately preceding) edition of the NEHRP Provisions.

    The ICSSC determined that the following model building codes, including local codes that adopt and enforce these model codes in their entirety, are substantially equivalent to the NEHRP Provisions, and thus are appropriate for implementing the Executive Order.

    - 1991 Uniform Building Code of the International Congress of Building Officials (ICBO)
    - 1992 Supplement to the Building Officials and Code Administrators (BOCA) National Building Code, and
    - 1992 Amendments to the Southern Building Code Congress (SBCCI) Standard Building Code.

    Revisions of these model codes are considered appropriate for order implementation, as long as they are substantially equivalent to the latest version of the triennially published NEHRP Provisions. The order allows federal agencies to use local building codes if they, or the ICSSC, determine that the local codes provide adequately for seismic safety. Each federal agency must determine the steps that participants in its program must take to comply with the provisions of the Executive Order. FEMA has the responsibility of reporting every two years to the President and Congress on the execution of the order.

    The implications of this Executive Order are far-reaching. In effect, the federal government is taking a leadership role in earthquake hazard mitigation by insisting that its own buildings, whether owned, leased or assisted, meet appropriate seismic standards. The results of the Executive Order will be watched with interest. Under normal rates of construction and retirement of buildings, a large proportion of federal buildings will be seismically resistant in 25 years.
    

    Figures 5.6 and 5.7

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    Earthquake Hazards Program

    ## Earthquake History of Texas

    The October 22, 1882 earthquake felt, in Texas, was probably centered in Oklahoma or Arkansas; the total felt area covered about 375,000 square kilometers. At Sherman, Texas, heavy machinery vibrated, bricks were thrown from chimneys, and movable objects overturned. A May 3, 1887, earthquake in Sonora, Mexico, caused damage at Bavispe and was felt strongly in parts or Arizona, New Mexico and Texas. The epicenter was in the Sierra Madre Occidental Range.

    On January 8, 1891, violent shaking of buildings and a few toppled chimneys were reported from Rusk, Texas. These effects were evaluated as intensity VII, although other towns in eastern Texas along a northeast- southwest line through Rusk experienced tornadoes and sudden, violent wind storms producing effects similar to, and in some cases more damaging than, those in Rusk.

    A locally damaging earthquake occurred at Panhandle, Texas, on March 28, 1917. Some cracked plaster was reported, and children were evacuated from a school building (VI).

    Another disturbance occurred in the area on July 30, 1925. There were three distinct shocks over a period of 15 seconds. Major problems were the shaking of dishes from shelves and rattling and creaking of furniture (V). The shocks were felt over an area of approximately 518,000 square kilometers including distant points such as Roswell, New Mexico, 350 kilometers away; Tulsa, Oklahoma, 480 kilometers away; and Leavenworth, Kansas, 640 kilometers away.

    The 1931 western Texas earthquake heavily damaged many buildings at Valentine. Also, many chimneys fell (VIII). The shock occurred at 5:40 a.m. on August 16; although people were panic stricken, there were no fatalities and only a few minor injuries from falling adobe. Adobe buildings suffered most, and cement and brick walls in many places were badly cracked. Even though Valentine bore the brunt of the shock, damage was reported from widely scattered points in Brewster, Culberson, Jeff Davis, and Presidio Counties. Cracked walls and damaged chimneys were reported from several towns. The total felt area covered about 647,000 square kilometers in Texas and New Mexico and an estimated 518,000 square kilometers in Mexico. The earthquake was accompanied by rumbling subterranean sounds heard over practically the entire affected area. The shock, measured at magnitude 6.4, was strongly recorded on all seismographs in North America and at stations all over the world. Numerous aftershocks were felt in the epicentral region; the strongest, on August 18, was intensity V at Alpine, Lobo, Pecos, and Valentine and intensity IV at Carlsbad, New Mexico. A minor aftershock was felt at Valentine on November 3.

    Slight damage resulted from an earthquake in the Mexia - Wortham area on April 9, 1932. Loose bricks were thrown down, and some plaster cracked (V-VI). The shock was also felt at Coolidge, Currie, Groesbeck, Hillsboro, Teague, and Richland. A moderate earthquake affected an area of about 7700 square kilometers in northeastern Texas and an adjoining portion of Oklahoma on April 11, 1934. The tremor was most distinctly felt at Arthur City, Caviness, Chicota, Powderly, and Trout Switch (intensity V). Many persons who felt the shock reported having heard a roaring or rumbling noise. Two shocks were recognized by many observers.

    A widely felt earthquake with an epicenter in the Panhandle region occurred on June 19, 1936. Intensity V effects were noted at Gruver, White Deer, and Whittenberg, Texas, Kenton, Oklahoma, and Elkhart, Kansas. The area of perceptibility covered about 103,000 square kilometers. On March 11, 1948, another shock in the Panhandle area caused minor damage, consisting mainly of cracked plaster, in northern Texas, a few places in northeastern New Mexico and northwestern Oklahoma, and one place in southeastern Colorado. The strongest effects (VI) were reported from Amarillo, Channing, Dalhart, Electric City, Panhandle, Perico, and Perryton. The felt area, which was slightly larger than that of the preceding earthquake, covered about 129,000 square kilometers. The Texas Panhandle area was the center for another moderate shock on June 20, 1951. Damage to plaster (VI) occurred at Amarillo and Hereford. The felt region extended from Lubbock to Borger.

    Four shocks over 6 hours affected an area of about 26,000 square kilometers in northeastern Texas and bordering portions of Arkansas and Louisiana on March 19, 1957. Press reports noted that a few objects were upset and at least one or two windows were broken. Newspaper office and police station switchboards were swamped with calls from alarmed residents. Intensity V effects were felt at Diana, Elkhart, Gladewater, Marshall, Nacogdoches, and Troup, Texas, and Magnolia, Arkansas.

    A series of moderate earthquakes in the Texas - Louisiana border region near Hemphill started on April 23, 1964. Epicenters were determined on April 23, 24, 27, and 28. There were numerous additional shocks reported felt at Pineland, Hemphill, and Milam. The only damage reported was from the magnitude 4.4 earthquake on April 28 - wall paper and plaster cracked at Hemphill (V). The magnitude of the other epicenters changed from 3.4 to 3.7. Shocks were also felt at Pineland on April 30 and May 7. On June 2, three more shocks were reported in the same area. The strongest was measured at magnitude 4.2 ; intensities did not exceed IV. Another moderate earthquake on August 16 awakened several people at Hemphill and there were some reports of cracked plaster (V). The shock was also felt at Bronson, Geneva, Milam, and Pineland.

    The Texas Panhandle region experienced another tremor on July 20, 1966. The magnitude 4.8 earthquake knocked books from a shelf in one home and was felt by nearly all (V) in Borger. At Amarillo, an observer in the courthouse reported a chair moved 4 or 5 inches. A similar effect was noted at the Federal Aviation Administration control tower at the Municipal Airport; observers thought a truck had hit the tower. Several street signs were knocked down and windows were broken (VI) at Kermit from a magnitude 3.4 earthquake on August 14, 1966. The shock was also felt at Wink, Texas, and Loco Hill, New Mexico.

    Four small earthquakes occurred near El Paso on May 12, 1969. The first two shocks, 23 minutes apart, were measured at magnitude 3.3 and 3.4. One house in El Paso had hairline cracks in the ceiling and cracks in the cement driveway (VI). These earthquakes were also felt at Newman.

    On February 15, 1974, an earthquake in the Texas Panhandle caused plaster cracks (V) at Booker, Darrovzett, and Perryton. Similar effects were noted at Liberal, Kansas, and Texhoma and Woodward, Oklahoma. The magnitude 4.5 shock was felt over an area of about 37,000 square kilometers.

    Earthquake Information Bulletin, Volume 9, Number 3, May - June 1977, by Carl A. von Hake.

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    ## Earthquake Ground Motion, 0.2 Second Spectral Response International Building Code 2003:

    

    ## US Geological Survey, Earthquake Hazards Program, ShakeMap Scientific Background at http://earthquake.usgs.gov/eqcenter/shakemap/background.php

    ## Spectral Response Maps

    Following earthquakes larger than magnitude 5.5, spectral response maps are made. Response spectra portray the response of a damped, single-degree-of-freedom oscillator to the recorded ground motions. This data representation is useful for engineers determining how a structure will react to ground motions. The response is calculated for a range of periods. Within that range, the Uniform Building Code (UBC) refers to particular reference periods that help define the shape of the "design spectra" that reflects the building code.

    ## Checklists for Seismic Inspection and Design <br> Applied Technology Council

    ## Job Aid:

    Inspection Checklist for Wood Frame Shear Walls

    1. Verify from the structural framing plans and architectural floor plans the location and length of all shear walls
    2. Verify the nailing of the sheathing agrees with the shear wall schedule

    O Nail Type (common, galvanized box);
    O Nail Diameter (8d or 10d);
    O Nail Length (minimum penetration into framing 12 times nail diameter)

    Spacing Along Each Edge of Each Piece of Sheathing ( 6 " $4^{\prime \prime}, 3^{\prime \prime}$ etc.)

    O Nail Head Shape (clipped heads not permitted)

    O Nail Placement
    _ Driven flush but not overdriven
    _ Minimum $3 / 8^{\prime \prime}$ from sheathing edge to center of nail
    _ View the stud side to check for nails that missed framing
    _ Staggered along edges where spacing is 3 inches 0.c. or less
    _ Edge nails into hold-down post
    3. Verify sheathing material agrees with the structural notes

    O Type (Plywood or OSB);
    O Grade (APA Rated Panel or APA Rated Panel - Structural I) and

    O Thickness ( $3 / 8^{\prime \prime}, 15 / 32^{\prime \prime}$ )
    O Number of Plys (If specified for plywood)
    4. Verify lumber size and grade agrees with the structural notes

    O Framing Grade of Studs \& Posts (Stud, Construction, No. 2, No. 1);

    O Lumber Species (Douglas Fir Larch, HemFir)

    O Framing Size ( $3 x$ studs, sill at heavily nailed edges, 2-2x, 4x or 6x at HD posts)
    5. Verify bottom of wall shear transfer (sill/ sole plate) connection is based on the structural notes or specific sections and details

    Nailing size and spacing of wall sole plate to floor framing below from shear wall schedule; verify nails penetrate framing below

    O Foundation sill bolt diameter and spacing from shear wall schedule or notes

    Bolts not less than 7 bolt diameters from ends of sill piece; not more than 12 inches from ends; not less than 1 inch from edge of sill plate; not less than $11 / 2$ inches to edge of concrete foundation.

    Verify square plate washer is used on bolts.
    O Verify bolt hole in sill plate is not more than $1 / 16^{"}$ larger than bolt diameter.

    ## Job Aid:

    Inspection Checklist for Wood Frame Shear Walls (continued)
    6.

    Verify top of wall shear transfer connection by looking at the shear wall schedule and typical sections at roof and floor level

    Location of edge nail row along top plate of lower wall and sole plate of upper wall, and if required, along the rim joist or blocking

    Size and spacing of framing clips, when required, from top plate to floor or roof framing, with all nail holes filled

    Where 10 d nails are required for the sheathing, and when edge nailing is required into the rim member, the minimum rim member thickness is $13 / 4$ inch. Therefore a nominal $2 x$ is NOT sufficient.
    7. Verify top plate splice connections along shear wall lines, not only those occurring directly above the shear wall

    Check for a detail or note on framing plans calling for typical or special plate splices.

    Verify the strap size (gage thickness and length) number of rows of nails, and total number of nails per the product manufacturer's catalogue
    Verify straps are centered on the splice and have all nail holes filled.

    Splices are needed anywhere that top plates are interrupted (by perpendicular beams or headers in the plane of the wall)
    8. Verify Hold-Down Installation

    Confirm locations per Framing and Foundation Plans (usually, but not always, are holddowns required at each end of a shear wall)

    Verify minimum Post Size and Lumber Grade

    Verify equal number of nails to upper and lower wall framing for Nailed Strap Type Hold-downs Spanning Floor Framing

    Verify bolt hole diameter through posts is not more than $1 / 16$ inch larger than the actual bolt diameter.

    Verify bolts heads or nuts are not countersunk into the post, unless specifically permitted

    Verify a washer is installed under the nut on side of the post opposite the HD
    Verify nuts are tight on all bolts, including the anchor bolt into the foundation and the ends of threaded rods spanning between floor levels.

    Anchor bolts and threaded rods should not be bent. HD location should be installed to minimize the length of threaded rods.

    Verify all bolt diameters are as specified either by the hold-down product manufacturer's catalogue or as specified on the drawings.

    Verify prior to concrete pour the length of embedment of anchor bolts and the embedded end condition (e.g., L-hook, J-hook, nut and square plate washer, hex headed bolt) match the drawings
    Verify anchor bolt clearance from edges and ends of footings as specified on the drawings.

    ## Job Aid:

    ## Checklist for Design of Masonry

    ## Structural Notes

    ## Design

    O Is h/t less than 30 ? If not, verify calculations.
    O Is the wall laterally supported with straps or other methods capable of resisting at least 420 $\mathrm{lb} / \mathrm{ft}$ ?
    O Does the bar fit in the cell?
    O Are locations of laps shown (Min. 48 dia. )? Are they in locations were stresses are less than $80 \%$ of the allowable?
    O Are dowel laps sufficient (Min. 48 dia.)?
    O Is there continuous horizontal reinforcement at the window, and door head?
    O Is there continuous horizontal reinforcement at the floor?
    O Are window and door connections designed and shown on the drawings?
    O Are there expansion joints at the corners?
    O Are there provisions made in connections to accommodate thermal movement? (Steel roof rigidly attached at a masonry corner)?
    O Is the brick masonry confined between other materials without expansion joints?

    ## Specifications

    ```
    O Is a color, pattern and workmanship panel required?
    O Is a grouting demonstration panel required?
    O Are materials specified in accordance with the correct standards? O Brick?
    O Is the Hollow clay brick of sufficient strength? O Cement? O Lime? O Sand?
    O Grout? O Mortar? O Is the mortar specified by proportions?
    O Reinforcement? O Is weldable steel required?
    O Are there requirements for handling and storage of materials?
    O Is there a requirement for a preconstruction meeting?
    O Are shop drawings required?
    O Are control joint size and materials specified?
    O Are sealant compatibility tests required?
    O Are the cleaning methods included?
    O Does the specification require wetting of the brick?
    O Are the joint finished specified? If raked joints are used is this in the analysis?
    O Are weep holes and fill materials specified?
    O Is the sealing procedures and materials specified?
    O Are cold weather and hot weather construction provisions included?
    O Are requirements for protecting the work included?
    O Is it required to verify dimensions prior to laying the masonry?
    O Is a written quality control procedure required?
    O Are prism test requirements included both prior to construction and during construction?
    ```


    ## Job Aid:

    Inspection Checklist for Masonry Construction

    ## Plans

    1. Is continuous inspection necessary?

    O Are called inspections necessary?

    ## Materials

    2. Concrete masonry units:

    O Type and quality
    O Strength of the masonry complies with plans
    O Is a laboratory test required?
    O Correct size and type, (per UBC Standard Nos. 21-4, 21-5).
    O Curing (UBC Standard Nos. 214, 21-5)
    O Cleanliness.
    O Soundness (UBC Standard Nos. 21-4, 21-5)
    O Are required inspection holes provided?
    3. Sand:

    O Cleanliness
    O Quality and fineness
    O Compliance with code requirements (ASTM C144)
    4. Cement:

    O Meets requirements of the UBC Standards (UBC Standard No. 21-15).
    5. Aggregates:

    O Meet the requirements of UBC Standards (ASTM C144 and C404).
    6. Lime:

    O Conforms to the UBC Standards (UBC Standard No. 21-13).
    7. Water:

    O Is clean and free from harmful substances.
    8. Plasticizing agents:

    O Bonform to Standards.
    9. Admixtures conform to the following requirements:

    O Have been approved.
    O Are of right quantity.
    O Are not used with plastic cement.
    10. Reinforcing steel:

    O Kind and grade.
    O Max. Size (UBC No. 2107.2.2. 1)

    ## Workmanship

    11. O Sample panels have been provided and approved, if required.
    12. 

    Mortar:
    O Proportions of the mortar mix and time Of mixing.
    O Consistency of mortar.
    O Clean water is used
    O Mortar is properly handled in mixing
    O Mortar is not excessively retempered.
    O Work is kept dry at all times.
    O Mortar classified by type and use (UBC Table No. 21-A)
    13. Grout:

    O Proportions (UBCTableNo.21-B).
    O Consistency.
    O Compressive strength (UBC Standard No. 2119).

    O Handling.
    O Segregation.

    ## Construction

    14. Bearing on solid masonry:

    O Suitability of bearing masonry
    O Size of bearing masonry
    O Location of bolt ties (UBC No. 2106.3.7)
    O Size, length, placement and embedment of connectors.
    15. Masonry on concrete:

    O Width and depth of footing excavations.
    O Anchorage around main steel
    O Grouting and metal inserts
    O Type, spacing and material of ties.
    O Embedment of ties or connection to main steel.
    16.

    O Proper sill material and anchorage of supporting members to footings.
    17. Head, bed, end and wall joints:

    O Correct size and type
    O Buttered where required
    O Joints where fresh masonry is joined to set masonry.
    O Properly filled with mortar. (Exception: UBC No. 2104.4.4).
    O Watertight (bug holes filled).

    ## Job Aid:

    Inspection Checklist for Masonry Construction (continued)

    Construction (continued)
    18. Reinforced hollow unit masonry:

    O Vertical alignment and continuity of cells
    O Requirements when work is stopped for one hour or longer.
    O Leakage of grout.
    O Cleanout openings for pours over 5 ft . (15 m) (UBC No. 2104.6. 1).

    O Overhanging mortar.
    O Sealing of cleanout cells.
    O Position of reinforcement.
    O Reinforcing hooks and splices (UBC Nos. 2107.2.2.5, 2107.2.2.6).
    19. O Racking and toothing at wall intersections.
    20. O Corners and returns.
    21. Reinforcing steel:

    O Clearances.
    O Deformation.
    O Additional steel around openings (UBC No. 2106.1.12.3 Item 3)

    O Placed within allowable tolerances (UBC No. 2104.5).
    22.

    Connections:
    O Size and location of joist anchors.
    O Size, location and number of bolts
    O Size and location of dowels
    O Location of stirrups.
    O Veneer ties (if any)
    23. O Separation between buildings.
    24. O Thickness of the walls.
    25. O Size of bond beams.
    26. O Placement of headers and lintels of material other than masonry.
    27. ○ Wall ties.
    28. Unprotected steel supporting members:

    O Correct location of mechanical installation supports.
    O Size and location of bolts and connections.
    O Size and spacing of bracing connections.
    O Size and alignment of connection holes.
    O Shims and dry packing.
    O Location and size of stiffeners.
    O Size and alignment of base plates.
    29. Anchoring of wood floor joists to supporting masonry members:
    O Required size of ledges.
    O Required size, spacing and length of bolts and joist anchors.
    30. Where floor joists are parallel to the wall:

    O Placing of required blocking.
    O Type of anchors required.
    O Use of proper connections to anchors.
    31. Floor joists tying to a masonry wall:

    O Required size, spacing and bearing of joists
    O Required air space around joists
    O Required anchors
    O Required bridging and/or blocking
    O Connection to ledger
    O Required connectors for anchors
    32. Where fire-resistant floors are required.

    O Proper material for fire resistance
    O Required thickness of floor slab
    O Required supports
    O Required reinforcing
    O Required time for supports and forms to remain in place for concrete floors
    33. O Contraction joints and control joints are located and provided as indicated or required.
    34. O Weepholes are provided if required.
    35. Chases.

    O Location and spacing on approved plans.
    O Purpose.
    O Maximum permitted depth.
    O No reduction of the required strength and fire resistance of the wall.
    36. Where there is a change of thickness in non-bearing walls
    O Locate the position on plans.
    O Required top plates comply
    O location of ties, anchors, bolts and blocking.
    37. Corbeling:

    O Maximum projections
    O Bonding and anchorage
    O Required temporary supports
    O Required reinforcing.
    38. O Pointing, replacement of defective units, and repair of other defects are promptly performed.
    39. O Waterproofing of walls is performed as required.
    40. O Methods of final cleaning are as required.

    ## Example: <br> Seismic Loading

    ## Example 1

    ## Example 5

    The floor plan of a single story commercial building located in Seismic Zone 3 is shown in Fig. 5-22. The 14 -foot high masonry shear walls are load bearing and have a weight of 70 pounds per square foot. The weight of the roof is 50 pounds per square foot and all other weights may be neglected. Determine the seismic base shear.

    ## Solution

    The relevant dead loads are given by:
    Roof $=W_{R}=0.05 \times 40 \times 20=40 \mathrm{kips}$
    North wall $=W_{3}=0.07 \times 12 \times 14=11.76 \mathrm{kips}$
    South wall $=W_{1}=11.76 \mathrm{kips}$
    East wall $=W_{2}=0.07 \times 10 \times 14=9.80 \mathrm{kips}$
    West wall $=W_{4}=9.80 \mathrm{kips}$
    Total seismic dead load is then
    $W=W_{R}+W_{1}+W_{2}+W_{3}+W_{4}$
    

    Fig. 5-22

    The seismic base shear is given by Formula (28-1) as
    $V=\left(Z I C / R_{W}\right) W$ where
    $Z=0.3$ for Zone 3 from Table 16-I
    $I=1.0$ for a standard occupancy structure as defined in Table 16-K
    $C=2.75$, the maximum value specified by UBC Section 1628.2.1
    $R_{W}=6$ from Table $16-\mathrm{N}$ for a bearing wall system
    $W=83.12 \mathrm{kips}$, as calculated
    Then the seismic base shear is

    $$
    \begin{aligned}
    V & =(0.3 \times 1 \times 2.75 / 6) \mathrm{W} \\
    & =0.1375 \mathrm{~W} \\
    & =11.43 \mathrm{kips} .
    \end{aligned}
    $$

    ## Connection and Tension Member Design

    | Notation: |  |
    | :---: | :---: |
    | $\begin{aligned} A= & \text { area }(\text { net }=\text { with holes, bearing }=\text { in } \\ & \text { contact, etc...) } \end{aligned}$ | $L^{\prime} \quad=$ length of an angle in a connector with staggered holes |
    | $A_{e} \quad=$ effective net area found from the product of the net area $A_{n}$ by the shear lag factor $U$ | $\begin{aligned} & L R F D=\text { load and resistance factor design } \\ & n \\ & =\text { number of connectors across a joint } \\ & N \quad=\text { bearing length on a wide flange } \end{aligned}$ |
    | $A_{b} \quad=$ area of a bolt | eel section |
    | $\begin{aligned} & A_{g}= \text { gross area, equal to the total area } \\ & \text { ignoring any holes } \end{aligned}$ | $=$ bearing type connection with threads included in shear plane |
    | $\begin{aligned} A_{g v}= & \text { gross area subjected to shear for } \\ & \text { block shear rupture } \end{aligned}$ | $\begin{array}{ll} p & =\text { pitch of connector spacing } \\ P & =\text { name for axial force vector, as is } T \end{array}$ |
    | $A_{n} \quad=$ net area, equal to the gross area subtracting any holes, as is $A_{\text {net }}$ | $R \quad=$ generic load quantity (force, shear, moment, etc.) for LRFD design |
    | $\begin{aligned} A_{n t}= & \text { net area subjected to tension for } \\ & \text { block shear rupture } \end{aligned}$ | $\begin{array}{ll} R_{a} & =\text { required strength (ASD) } \\ R_{n} & =\text { nominal value (capacity) to be } \end{array}$ |
    | $\begin{aligned} & A_{n v}= \text { net area subjected to shear for block } \\ & \text { shear rupture } \end{aligned}$ | $\begin{aligned} & \text { multiplied by } \phi \\ R_{u}= & \text { factored design value for LRFD } \end{aligned}$ |
    | $A S D=$ allowable stress design | design |
    | $d \quad=$ diameter of a hole | $=$ longitudinal center-to-center spacing |
    | $f_{p} \quad=$ bearing stress (see P) | of any two consecutive holes |
    | $f_{t} \quad=$ tensile stress | $S \quad=$ allowable strength per length of a |
    | $f_{v} \quad=$ shear stress | weld for a given size |
    | $F_{\text {connector }}=\text { shear force }$ | $\begin{array}{ll} S C & =\text { slip critical bolted connection } \\ t & =\text { thickness of a hole or member } \end{array}$ |
    | $\begin{aligned} F_{n}= & \text { nominal tension or shear strength of } \\ & \text { a bolt } \end{aligned}$ | $\begin{array}{ll} t_{w} & =\text { thickness of web of wide flange } \\ T & =\text { throat size of a weld } \end{array}$ |
    | $F_{u} \quad=$ ultimate stress prior to failure | $V=$ internal shear force |
    | $F_{E X X}=$ yield strength of weld material | $V_{\text {longitudinal }}=$ longitudinal shear force |
    | $F_{y}=$ yield strength | $U \quad=$ shear lag factor for steel tension |
    | $F_{y w}=$ yield strength of web material | member design |
    | $g \quad=$ gage spacing of staggered bolt holes | $U_{b s}=$ reduction coefficient for block |
    | $=$ moment of inertia with respect to neutral axis bending | $X=\begin{aligned} & \text { shear rupture } \\ & =\text { bearing type connection with } \end{aligned}$ |
    | $k \quad=$ distance from outer face of W flange to the web toe of fillet | threads excluded from the shear plane |
    | $l=$ name for length | = vertical distance |
    | $L \quad=$ name for length | $=\mathrm{pi}\left(3.1415\right.$ radians or $\left.180^{\circ}\right)$ |
    | $L_{c} \quad=$ clear distance between the edge of a hole and edge of next hole or edge | $\phi \quad=$ resistance factor |
    | of the connected steel plate in the | = diameter symbol $=$ load factor in LRFD design |
    | d | $\begin{array}{cl} \Omega & =\text { safety factor for ASD } \\ \Sigma & =\text { summation symbol } \end{array}$ |

    ## Connections

    Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column. Steel construction accomplishes this with bolt and welds. Wood construction uses nails, bolts, shear plates, and split-ring connectors.

    Single Shear - forces cause only one shear "drop" across the bolt.
    
    (a) Two steet phates bolted using one boil.
    
    (b) Elevation showing the boit in shear.

    $$
    \begin{aligned}
    & f_{v}=\text { Average shear stress through bolt cross } \\
    & \text { section } \\
    & A=\text { Bolt cross-sectional area } \\
    & f_{v}=\frac{P}{A}=\frac{P}{\pi r^{2}}
    \end{aligned}
    $$

    (d)

    Figure 5.11 Abolled comection-single shatar:

    Double Shear - forces cause two shear changes across the bolt.

    $$
    f_{v}=\frac{P}{2 A}=\frac{P}{2 \pi r^{2}}
    $$

    (two shear planes)
    

    Free-body diagram of middlle section of the bolt in shear.
    Figure 5.12 A bolted comection in double shear.

    Bearing of a Bolt on a Bolt Hole - The bearing surface can be represented by projecting the cross section of the bolt hole on a plane (into a rectangle).

    $$
    f_{p}=\frac{P}{A}=\frac{P}{t d}
    $$

    

    Bearing stress on plate,

    ## Horizontal Shear in Composite Beams

    Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices.

    The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear
     flow over the spacing interval, p.

    $$
    \frac{V_{\text {longitudiall }}}{p}=\frac{V Q}{I} \quad V_{\text {lonsitudiall }}=\frac{V Q}{I} \cdot p
    $$

    where
    $\mathrm{p}=$ pitch length

    $$
    n F_{\text {connector }} \geq \frac{V Q_{\text {connectedarea }}}{I} \cdot p
    $$

    $\mathrm{n}=$ number of connectors connecting the connected area to the rest of the cross section
    $\mathrm{F}=$ force capacity in one connector
    $\mathrm{Q}_{\text {connected area }}=\mathrm{A}_{\text {connected area }} \times \mathrm{y}_{\text {connected area }}$
    $y_{\text {connected area }}=$ distance from the centroid of the connected area to the neutral axis

    ## Connectors to Resist Horizontal Shear in Composite Beams

    Even vertical connectors have shear flow across them.
    The spacing can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, p .

    $$
    p \leq \frac{n F_{\text {connector }} I}{V Q_{\text {connected area }}}
    $$

    

    ## Tension Member Design

    In tension members, there may be bolt holes that reduce the size of the cross section.

    ## Effective Net Area:

    The smallest effective are must be determined by subtracting the bolt hole areas. With staggered holes, the shortest length must be evaluated.

    $$
    f_{t}=\frac{P}{A_{e}} \text { or } \frac{T}{A_{e}}
    $$

    

    A series of bolts can also transfer a portion of the tensile force, and some of the effective net areas see reduced stress.
    

    ## Connections in Wood

    Connections for wood are typically mechanical fasteners. Shear plates and split ring connectors are common in trusses. Bolts of metal bear on holes in wood, and nails rely on shear resistance transverse and parallel to the nail shaft.

    ## Bolted Joints

    Stress must be evaluated in the member being connected using the load being transferred and the reduced cross section area called net area. Bolt capacities are usually provided in tables and take into account the allowable shearing stress across the diameter for single and double shear, and the allowable bearing stress of the connected material based on the direction of the load with respect to the grain (parallel or perpendicular). Problems, such as ripping of the bolt hole at the end of the member, are avoided by following code guidelines on minimum edge distances and spacing.

    ## Nailed Joints

    Because nails rely on shear resistance, a common problem when nailing is splitting of the wood at the end of the member, which is a shear failure. Tables list the shear force capacity per unit length of embedment per nail. Jointed members used for beams will have shear stress across the connector, and the pitch spacing, $p$, can be determined from the shear stress equation when the capacity, $F$, is known.

    ## Other Connectors

    Screws - Range in sizes from \#6 (0.138 in. shank diameter) to \#24 (0.372 in. shank diameter) in lengths up to five inches. Like nails, they are best used laterally loaded in side grain rather than in withdrawal from side grain. Withdrawal from end is not permitted.

    Lag screws (or bolts) - Similar to wood screw, but has a head like a bolt. It must have a load hole drilled and inserted along with a washer.

    Split ring and shear plate connectors - Grooves are cut in each piece of the wood members to be joined so that half the ring is in each section. The members are held together with a bolt concentric with the ring. Shear plate connectors have a central plate within the ring.

    Splice plates - These are common in pre-manufactured joists and consist of a sheet of metal with punched spikes.

    Framing seats \& anchors - for instance, joist hangers and post bases...

    ## Connections in Steel

    The limit state for connections depends on the loads:

    1. tension yielding
    2. shear yielding
    3. bearing yielding
    4. bending yielding due to eccentric loads
    
    5. rupture

    Fig. C-J4.1. Failure for block shear rupture limit state.
    High strength bolts resist shear (primarily), while the connected part must resist yielding and rupture.

    Welds must resist shear stress. The design strengths depend on the weld materials.
    

    ## Bolted Connection Design

    Bolt designations signify material and type of connection where
    

    SC: slip critical
    N : bearing-type connection with bolt threads included in shear plane
    X : bearing-type connection with bolt threads excluded from shear plane

    A307: similar in strength to A36 steel (also known as ordinary, common or unfinished bolts)
    A325: high strength bolts (Group A)
    A490: high strength bolts (higher than A325, Group B)
    

    Bearing-type connection: no frictional resistance in the contact surfaces is assumed and slip between members occurs as the load is applied. (Load transfer through bolt only).
    Slip-critical connections: bolts are torqued to a high tensile stress in the shank, resulting in a clamping force on the connected parts. (Shear resisted by clamping force). Requires inspections and is useful for structures seeing dynamic or fatigue loading.

    Bolts rarely fail in bearing. The material with the hole will more likely yield first.
    For the determination of the net area of a bolt hole the width is taken as $1 / 16$ " greater than the nominal dimension of the hole. Standard diameters for bolt holes are $1 / 16^{\prime \prime}$ larger than the bolt diameter. (This means the net width will be $1 / 8$ " larger than the bolt.)

    ## Design for Bolts in Bearing, Shear and Tension

    Available shear values are given by bolt type, diameter, and loading (Single or Double shear) in AISC manual tables. Available shear value for slip-critical connections are given for limit states of serviceability or strength by bolt type, hole type (standard, short-slotted, long-slotted or oversized), diameter, and loading. Available tension values are given by bolt type and diameter in AISC manual tables.

    Available bearing force values are given by bolt diameter, ultimate tensile strength, $\mathrm{F}_{\mathrm{u}}$, of the connected part, and thickness of the connected part in AISC manual tables.

    For shear OR tension (same equation) in bolts:

    $$
    \begin{gathered}
    R_{a} \leq R_{n} / \Omega \text { or } R_{u} \leq \phi R_{n} \\
    \text { where } R_{u}=\Sigma \gamma_{i} R_{i}
    \end{gathered}
    $$

    - single shear (or tension) $\quad R_{n}=F_{n} A_{b}$
    - double shear $\quad R_{n}=F_{n} 2 A_{b}$
    where $\phi=$ the resistance factor
    $\mathrm{F}_{\mathrm{n}}=$ the nominal tension or shear strength of the bolt
    $\mathrm{A}_{\mathrm{b}}=$ the cross section area of the bolt
    $\phi=0.75(\mathrm{LRFD}) \quad \Omega=2.00(\mathrm{ASD})$

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    |  | $\stackrel{\square}{4}$ | 先 | \％ | 号 | $\stackrel{\substack{9 \\ \rightleftharpoons}}{\infty}$ | 命 | Nّ | $\stackrel{\infty}{\sim} \stackrel{N}{n}$ | $\begin{array}{ll} \infty & 0 \\ \infty & \stackrel{1}{=} \end{array}$ | $\stackrel{\sim}{\square}$ | $\stackrel{\text { ¢ }}{\sim}$ | \％ | 号 |  | กั | へั่ ®్ర | $\stackrel{\sim}{\stackrel{n}{\sim}}$ | 움 | 若 |  |
    |  |  |  | $\frac{G}{5}$ | \％ | $\left\|\begin{array}{ll} 9 \\ = & 0 \\ \hline \end{array}\right\|$ | 응 | 응 | $\infty$ | $\stackrel{\leftrightarrow 9}{6}$ |  |  | c | 安 | N్ల్లు | $\left\|\begin{array}{cc} \infty \\ \underset{\sim}{\infty} \\ \mathscr{\infty} \end{array}\right\|$ | $\underset{\sim}{\infty}$ | 눙 | ¢～～ | $\begin{aligned} & 0 \\ & \hline \\ & \hline \end{aligned}$ |  |
    |  | $\pm$ | 僉 | $\stackrel{\Sigma}{\circ}$ | 号 | $\left\|\begin{array}{c} \underset{む}{\circ} \\ \underset{\sim}{む} \end{array}\right\|$ | $\stackrel{N}{\dot{m}} \underset{m}{2}$ | $\stackrel{n}{\stackrel{m}{m}}$ | $\left\|\begin{array}{l\|l\|} \underset{\sim}{\infty} \\ \dot{\sigma} \end{array}\right\|$ | $\underset{\sim}{\cong}$ | $\stackrel{\infty}{\rightleftharpoons}$ | 茳 | \％ | $\frac{\stackrel{0}{4}}{4}$ | $\left\lvert\, \begin{array}{ll} m \\ \dot{8} \\ \dot{8} \\ \hline \end{array}\right.$ | ก－ | î̀ | Ợ | N |  |  |
    |  |  |  | $\underset{\mathrm{c}}{\mathrm{~g}}$ | 8 | $\begin{array}{\|cc\|} \hline \infty & 0 \\ \infty & 0 \\ \hline \end{array}$ | 무 | 는 ヘi | かi in | $\left\lvert\, \begin{array}{l\|} \hline \frac{\pi}{4} \\ \hline \infty \end{array}\right.$ |  |  | G | 安 | ¢i̊ |  | $\prod_{0}^{\infty}$ |  | ¢ |  |  |
    |  |  |  | 髟 을 |  | $\cdots 0$ | $\infty 0$ | $\infty 0$ | $\infty 0$ | $\infty 0$ |  |  | 产 |  | の 0 | $\infty 0$ | $\infty 0$ | $\cdots 0$ | $\infty 0$ |  |  |
    |  | $\frac{\vdots}{5}$ |  |  | 只 | ஜ | $\frac{0}{i n}$ | $\frac{0}{i}$ | Oi | $\stackrel{\text { ® }}{\sim}$ |  |  |  | 号 | Ơ | $\frac{0}{i}$ | 은 | ®ి | ๗ |  |  |
    |  | 产 |  | $\frac{G}{4} \hat{\vec{B}}$ | 安 | $\stackrel{\stackrel{\mathrm{N}}{\mathrm{~N}}}{ }$ |  | 울 | 묵 | $\stackrel{\sim}{\text { ® }}$ |  |  |  | 安 | $\stackrel{\circ}{\mathrm{N}}$ | 号 | 号 | 우ํ | $\stackrel{\sim}{\text { ¢ }}$ |  |  |
    |  |  |  |  |  | $z$ | $\times$ | $z$ | $\times$ | 1 |  |  |  |  | $z$ | $\times$ | $z$ | $\times$ | 1 | 号 | nor |
    |  |  |  | 든 |  | O |  | 응 |  | ్ָల్ర |  |  | 츤 |  |  |  |  |  | へ－（\％） | \％ |  |

    \begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
    \hline Group A Bolts \& \multicolumn{9}{|l|}{\multirow[t]{2}{*}{\begin{tabular}{l}
    Table 7－3 \\
    Slip－Critical Connections \\
    Available Shear Strength，kips \\
    （Class A Faying Surface，\(\mu=0.30\) ）
    \end{tabular}}} \\
    \hline ```
    A325, A325M Available Shear Strength, kips
    F1858 (Class A Faying Surface, 

