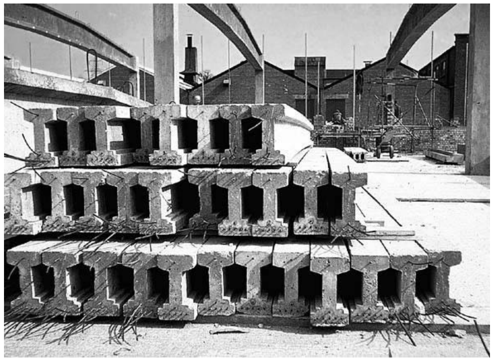
APPLIED ACHITECTURAL STRUCTURES:
STRUCTURAL ANALYSIS AND SYSTEMS
ARCH 631
DR. ANNE NICHOLS
FALL 2012

lecture
ten



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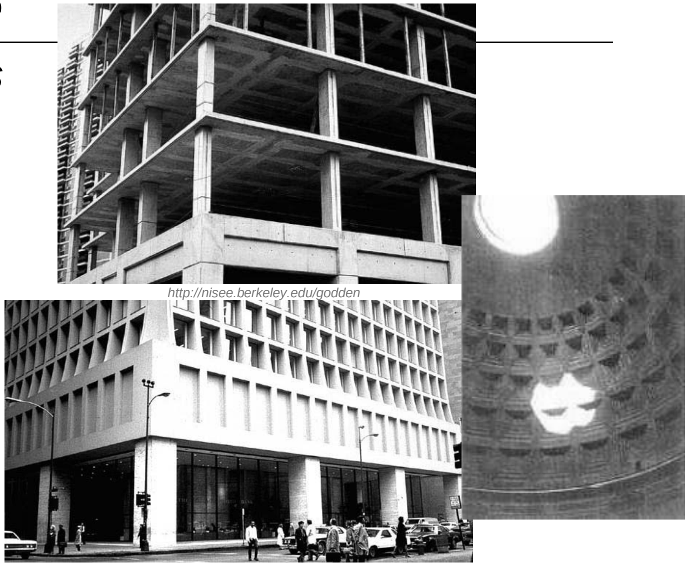
reinforced concrete construction

Concrete

- columns
- beams
- slabs
- domes
- footings



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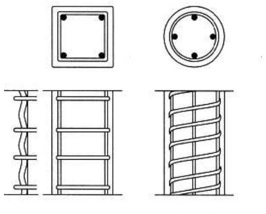
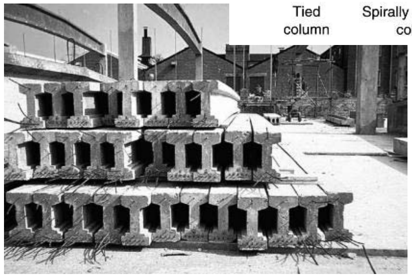
Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning





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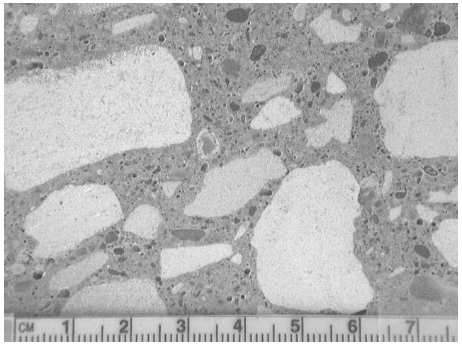
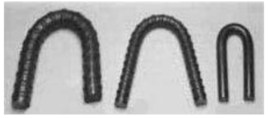
Concrete Materials

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water



Concrete Materials

- *reinforcement*
  - *deformed bars*
  - *prestressing strand*
  - *stirrups*
  - *development length*
  - *anchorage*
  - *splices*



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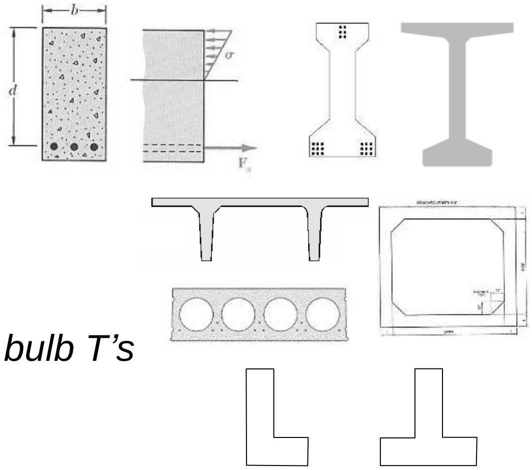
Concrete Materials

- *fire resistance*
  - *most fire-resistive structural material*
  - *low rate of penetration*
  - *retains strength if exposure not too long*
    - *stable to 900 – 1200 °F internally*
    - *loses 50% after that*
  - *no toxic fumes*
  - *cover necessary to protect steel*

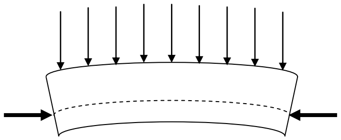
Concrete Beams

- *types*
  - *reinforced*
  - *precast*
  - *prestressed*
- *shapes*
  - *rectangular, I*
  - *T, double T's, bulb T's*
  - *box*
  - *spandrel*



Concrete Beams

- *deformation*
  - *camber (elastic)*
    - *hogging* ↑
    - *sagging* ↓
  - *shrinkage strain*
    - *200-400 x 10^{-6}*
    - *about 2-3 years*
  - *creep strain*
    - *2~3 times elastic strain*
    - *about 2-3 years*



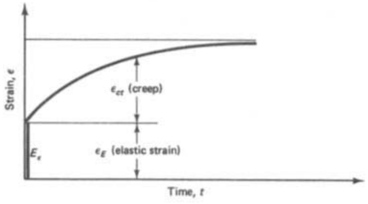
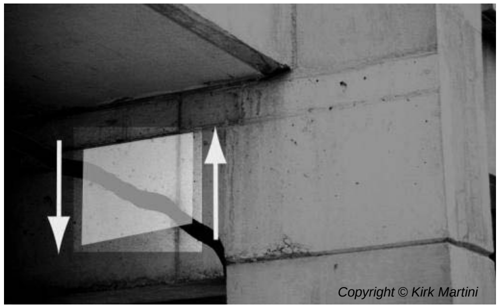


Figure 2.7 Strain-time curve.

Concrete Beams

- *shear*
  - – *vertical*
  - – *horizontal*
  - – *combination:*
    - *tensile stresses at 45°*
- *bearing*
  - – *crushing*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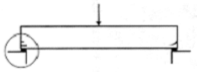
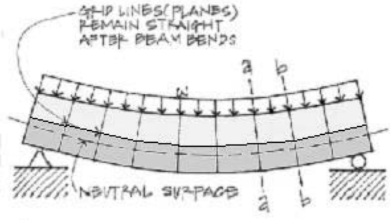


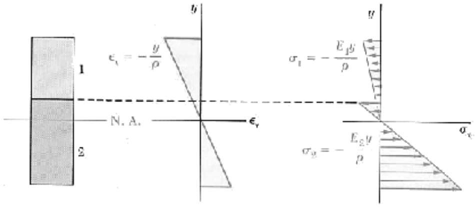
Concrete Beam Design

- *composite of concrete and steel*
- *American Concrete Institute (ACI)*
  - – *design for failure*
  - – *strength design (LRFD)*
    - *service loads x load factors*
    - *concrete holds no tension*
    - *failure criteria is yield of reinforcement*
    - *failure capacity x reduction factor*
    - *factored loads < reduced capacity*
  - – *concrete strength = f'_c*

Behavior of Composite Members

- *plane sections remain plane*
- *stress distribution changes*

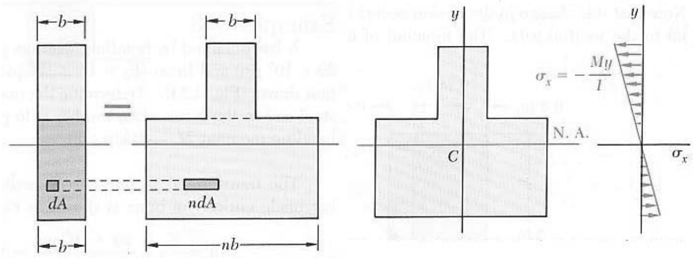




$$f_1 = E_1\varepsilon = -\frac{E_1 y}{\rho} \qquad f_2 = E_2\varepsilon = -\frac{E_2 y}{\rho}$$

Transformation of Material

- *n is the ratio of E's* $$n = \frac{E_2}{E_1}$$
- *effectively widens a material to get same stress distribution*



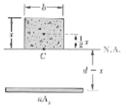
Stresses in Composite Section

- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by n

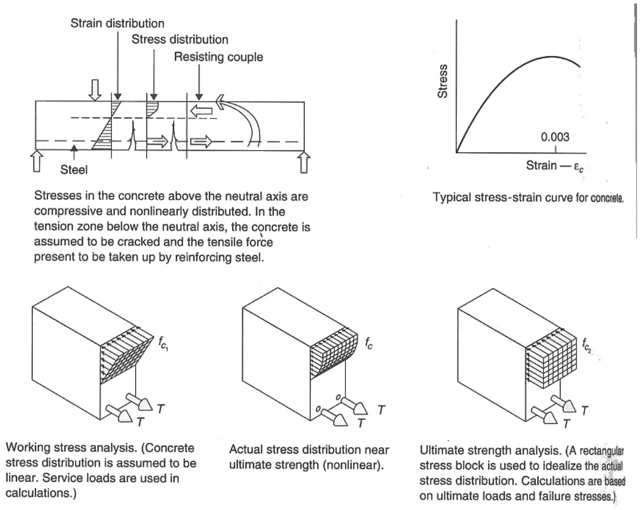
$$n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$$

$$f_c = -\frac{My}{I_{transformed}}$$

$$f_s = -\frac{Myn}{I_{transformed}}$$



Reinforced Concrete - stress/strain



Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.

Typical stress-strain curve for concrete.

Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)

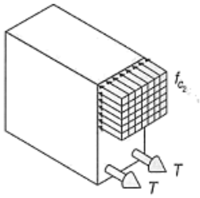
Actual stress distribution near ultimate strength (nonlinear).

Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

FIGURE 6–37 Reinforced concrete beams.

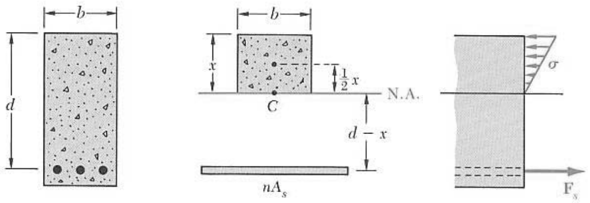
Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent <u>stress block</u>
  - concrete takes <u>no tension</u>
  - steel takes tension
  - force <u>ductile</u> failure



Location of n.a.

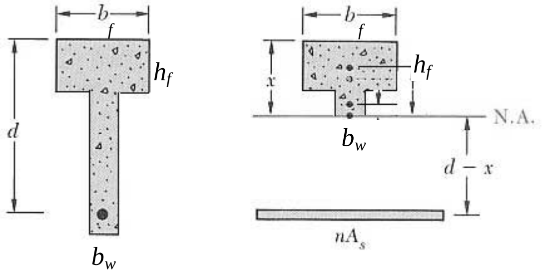
- ignore concrete below n.a.
- transform steel
- same area moments, solve for x



$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

T sections

- *n.a. equation is different if n.a. below flange*



$$b_f h_f\left(x - \frac{h_f}{2}\right) + (x - h_f)b_w\frac{(x - h_f)}{2} - nA_s(d - x) = 0$$

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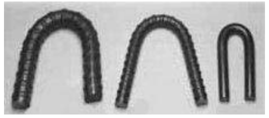
*ACI Load Combinations**

- *1.4D*
- *1.2D + 1.6L + 0.5(L_r or S or R)*
- *1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)*
- *1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)*
- *1.2D + 1.0E + 1.0L + 0.2S*
- *0.9D + 1.0W*
- *0.9D + 1.0E*

**can also use old ACI factors*

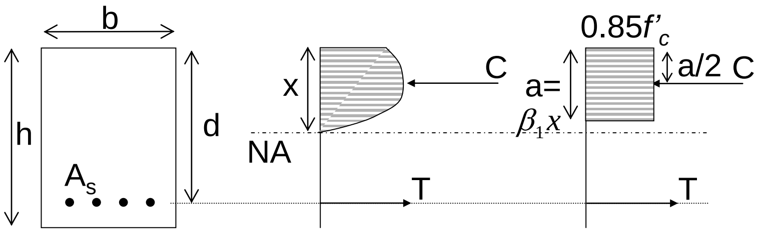
Reinforcement

- *deformed steel bars (rebar)*
  - *Grade 40, F_y = 40 ksi*
  - *Grade 60, F_y = 60 ksi - most common*
  - *Grade 75, F_y = 75 ksi*
  - *US customary in # of 1/8” ϕ*
- *longitudinally placed*
  - *bottom*
  - *top for compression reinforcement*
  - *spliced, hooked, terminated...*



Reinforced Concrete Design

- *stress distribution in bending*

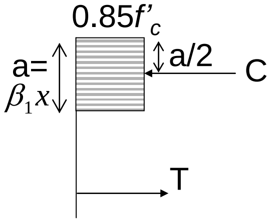


actual stress

Whitney stress block

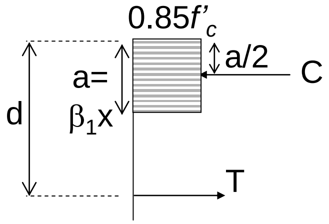
Wang & Salmon, Chapter 3

Force Equations



- $C = 0.85\, f'_c ba$
- $T = A_s f_y$
- *where*
  - f'_c = *concrete compressive strength*
  - *a = height of stress block*
  - *b = width of stress block*
  - f_y = *steel yield strength*
  - A_s = *area of steel reinforcement*

Equilibrium



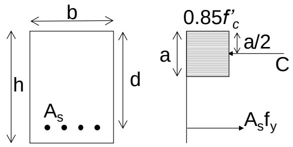
- $T = C$
- $M_n = T(d-a/2)$
  - *d = depth to the steel n.a.*
- *with* A_s
  - $a = \dfrac{A_s f_y}{0.85 f'_c b}$
  - $M_u \leq \phi M_n$ $\quad \phi = 0.9$ *for flexure*
  - $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

Over and Under-reinforcement

www.thfrc.gov

- *over-reinforced*
  - *steel won't yield*
- *under-reinforced*
  - *steel will yield*
- *reinforcement ratio*
  - $\rho = \dfrac{A_s}{bd}$
  - *use as a design estimate to find* A_s, *b, d*
  - *max* ρ *is found with* $\varepsilon_{steel} \geq 0.004$ *(not* ρ_{bal}*)*

A_s for a given Section



- *several methods*
  - *guess a and iterate*
    1. *guess a (less than n.a.)*
    2. $A_s = \dfrac{0.85 f'_c ba}{f_y}$
    3. *solve for a from* $M_u = \phi A_s f_y \left(d - a/2\right)$

       $a = 2\left(d - \dfrac{M_u}{\phi A_s f_y}\right)$
    4. *repeat from 2. until a from 3. matches a in 2.*

A_s For Given Section (cont)

- chart method
  - Wang & Salmon Fig. 3.8.1 R_n vs. ρ

1. calculate $R_n = \dfrac{M_n}{bd^2}$
2. find curve for f'_c and f_y to get ρ
3. calculate A_s and a

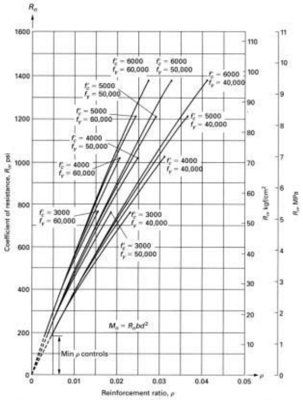
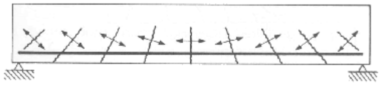


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max}.

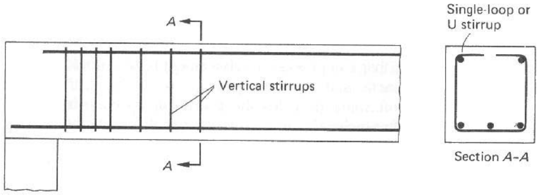
- simplify by setting h = 1.1d

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks



- horizontal reinforcement doesn't help
- stirrups = vertical reinforcement



ACI Shear Values

- V_u is at distance d from face of support
- shear capacity: $V_c = \upsilon_c \times b_w d$
  - where b_w means thickness of <u>web</u> at n.a.
- shear stress (beams)
  - $\upsilon_c = 2\sqrt{f'_c}$

$$\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$$

ϕ = 0.75 for shear

f'_c is in <u>psi</u>

- shear strength: $V_u \le \phi V_c + \phi V_s$
  - V_s is strength from stirrup reinforcement

ACI 11.3, 11.12

Stirrup Reinforcement

- shear capacity:

$$V_s = \frac{A_v f_y d}{s}$$

  - A_v = area in all legs of stirrups
  - s = spacing of stirrup

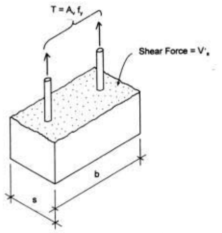


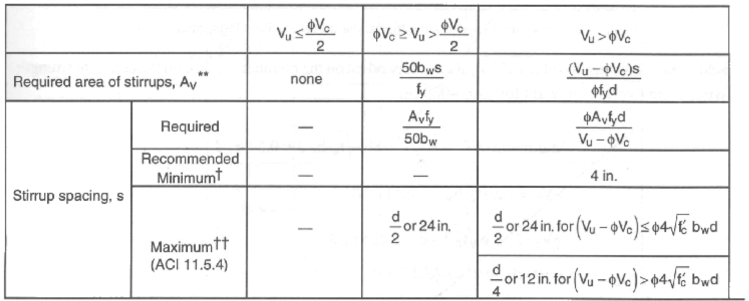
Figure 13.17 Consideration for spacing of a single stirrup.

- may need stirrups when concrete has enough strength!

Required Stirrup Reinforcement

- spacing limits

Table 3-8 ACI Provisions for Shear Design*



		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v**		none	$\frac{50 b_w s}{f_y}$	$\frac{(V_u - \phi V_c)s}{\phi f_y d}$
Stirrup spacing, s	Required	—	$\frac{A_v f_y}{50 b_w}$	$\frac{\phi A_v f_y d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $\left(V_u - \phi V_c\right) \le \phi 4\sqrt{f'_c}\, b_w d$
				$\frac{d}{4}$ or 12 in. for $\left(V_u - \phi V_c\right) > \phi 4\sqrt{f'_c}\, b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2\sqrt{f'_c}\, b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is d/4

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

Concrete Deflections

- elastic range
  - I transformed
  - E_c (with f'_c in psi)
    - normal weight concrete (~ 145 lb/ft^3)

$$E_c = 57{,}000\sqrt{f'_c}$$

    - concrete between 90 and 155 lb/ft^3

$$E_c = w_c^{1.5}\,33\sqrt{f'_c}$$

- cracked
  - I cracked
  - E adjusted

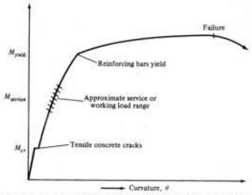


Figure 2.4 Moment–curvature diagram for reinforced concrete beam with tensile reinforcing only

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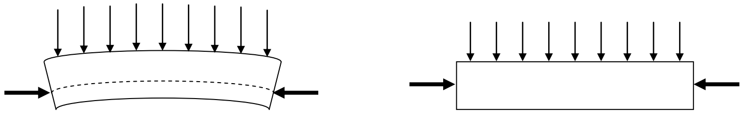
Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

L/180	roof systems (typical) – live
L/240	floor systems (typical) – live + long term
L/360	supporting plaster – live
L/480	supporting masonry – live + long term

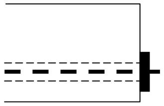
Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit



Prestressed Concrete

- *pretensioned*
  - *reinforcement bonded*
- *post-tensioned*
  - *bonded or unbonded*
  - *end bearing*



- *precast*
  - *concrete premade in a position other than its final position in the structure*

Prestressed Concrete

- *high strength tendons*
  - *grade 250*
  - *grade 270*

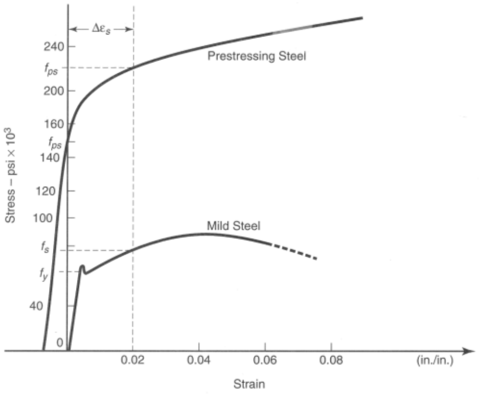
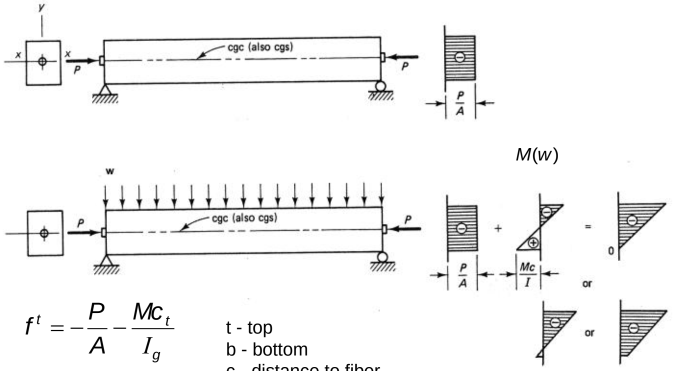


Figure 2.18(b) Stress-Strain Diagram for Prestressing Steel Strands in Comparison with Mild Steel Bar Reinforcement.

Prestressed Concrete

- *axial prestress (e=0)*



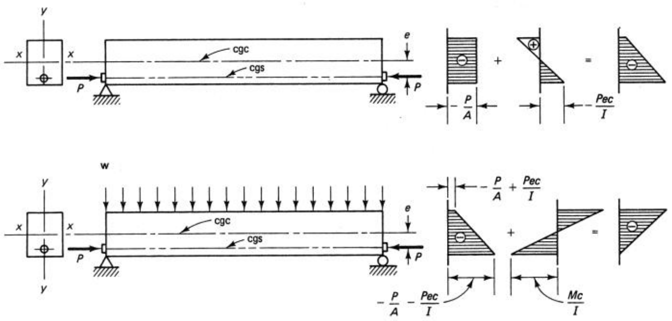
$$f^t = -\frac{P}{A} - \frac{Mc_t}{I_g}$$

$$f_b = -\frac{P}{A} + \frac{Mc_b}{I_g}$$

t - top
b - bottom
c - distance to fiber
I_g - gross cross section inertia

Prestressed Concrete

- *axial prestress (e≠0)*



$$f^t = -\frac{P}{A} + \frac{Pec_t}{I_g} - \frac{Mc_t}{I_g} = -\frac{P}{A}\left(1 - \frac{ec_t}{r^2}\right) - \frac{Mc_t}{I_g}$$

$$f_b = -\frac{P}{A} - \frac{Pec_b}{I_g} + \frac{Mc_b}{I_g} = -\frac{P}{A}\left(1 + \frac{ec_b}{r^2}\right) + \frac{Mc_b}{I_g}$$

$(remember - r = \sqrt{\frac{I}{A}})$

Prestressed Concrete

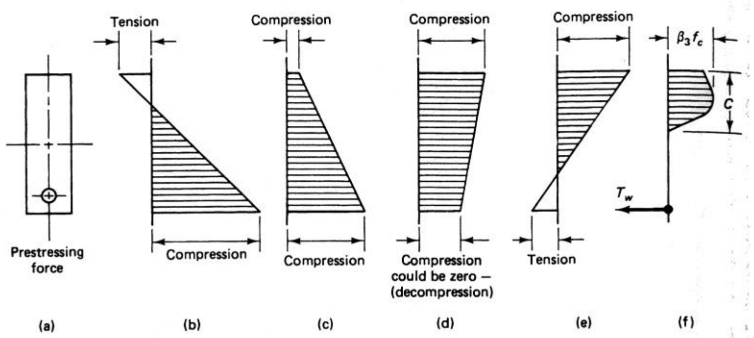
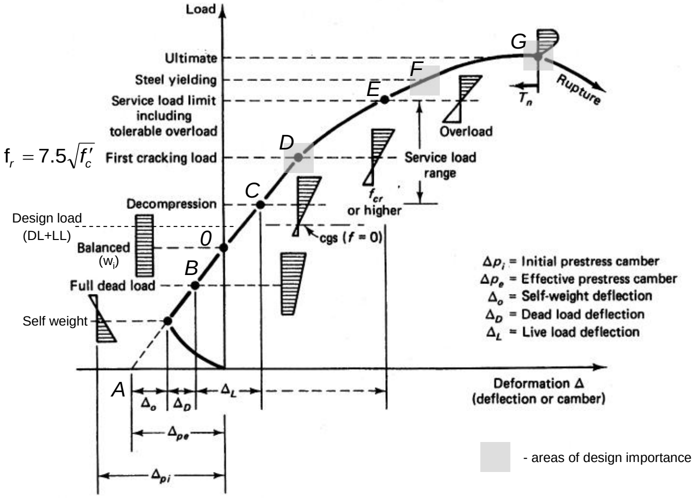


Figure 4.2 Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

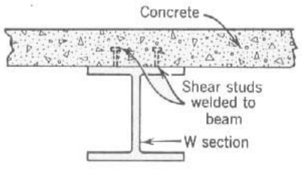
Prestressed Concrete



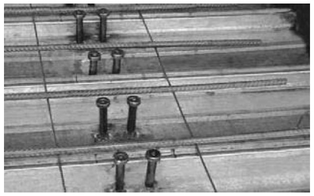
▨ - areas of design importance

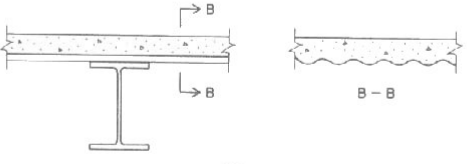
Composite Beams

- *concrete*
  - *– in compression*
- *steel*
  - *– in tension*
- *shear studs*



(c) Composite beam.

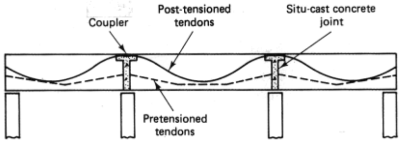




(b)

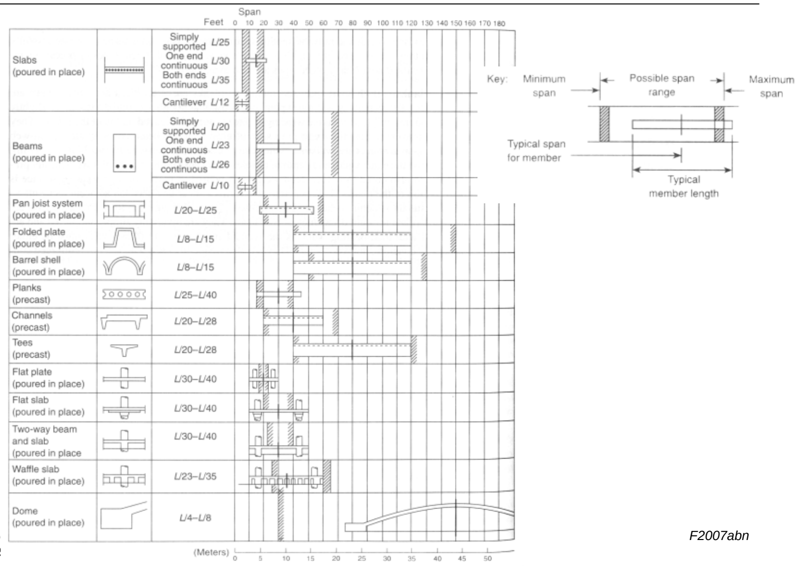
Continuous Beams

- *reduced size*
- *reduced moments*



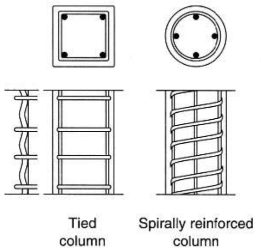
- *moments can reverse with loading patterns*
- *need top & bottom reinforcement*
- *<u>sensitive to settlement</u>*

Approximate Depths



Concrete Columns

- *columns require*
  - *ties or spiral reinforcement to “confine” concrete (#3 bars minimum)*

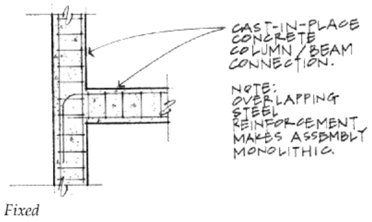


  - *minimum amount of longitudinal steel (#5 bars minimum: 4 with ties, 5 with spiral)*

Concrete Columns

- *effective length in monolithic casts must be found with respect to stiffness of joint*
- *not slender when*

$$\frac{kL_u}{r} < 22$$



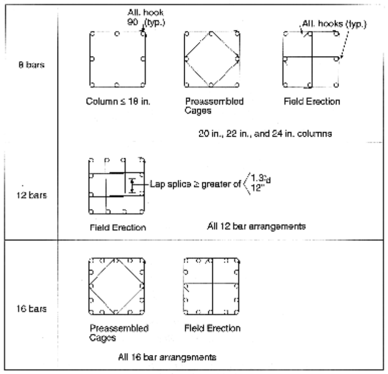


Figure 5-7 Column Tie Details

Concrete Columns

Figure 13.3.2 Spirally reinforced column behavior. (Courtesy of Portland Cement Association.)

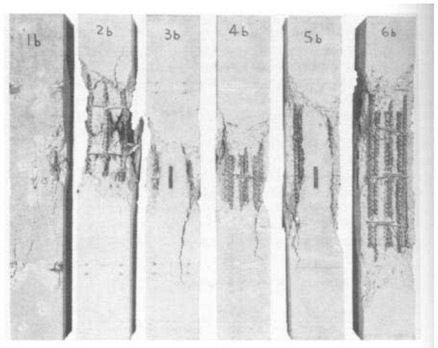


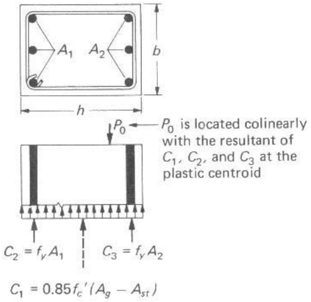
Figure 13.3.3 Tied column behavior. (Courtesy of Portland Cement Association.)

Concrete Columns

- P_o – no bending

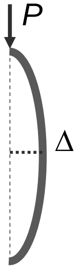
$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$

- ϕ_c = 0.65 for ties with $P_n = 0.8P_o$
- ϕ_c = 0.70 for spirals with $P_n = 0.85P_o$
- $P_u \le \phi_c P_n$
- nominal axial capacity:
  - presumes steel yields
  - concrete at ultimate stress



Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection (P- Δ)



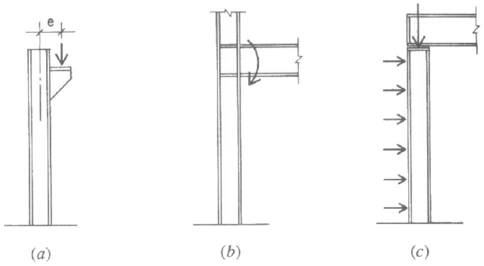


Figure 10.6 Considerations for development of bending in steel columns; (*a*) bending induced by eccentric load, (*b*) bending transferred to column in a rigid frame, and (*c*) combined loading condition, separately producing axial compression and bending.

Columns with Bending

- for ultimate strength behavior, ultimate strains can't be exceeded
  - concrete 0.003
  - steel $\dfrac{f_y}{E_s}$
- P reduces with M

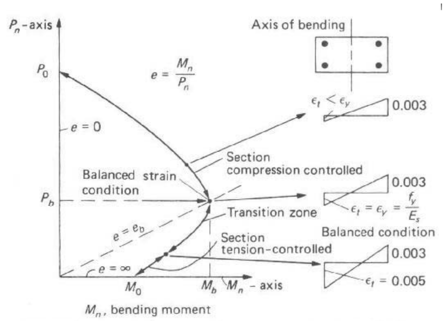


Figure 13.6.1 Typical strength interaction diagram for axial compression and bending moment about one axis. Transition zone is where $\epsilon_y \le \epsilon_t \le 0.005$.

Columns with Bending

- need to consider combined stresses

$$\frac{P_n}{P_o} + \frac{M_n}{M_o} \le 1$$

- plot <u>interaction</u> diagram

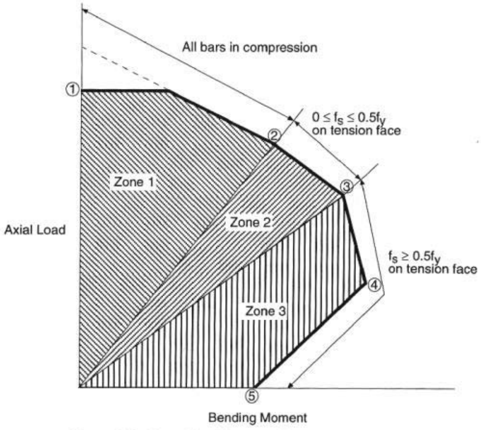
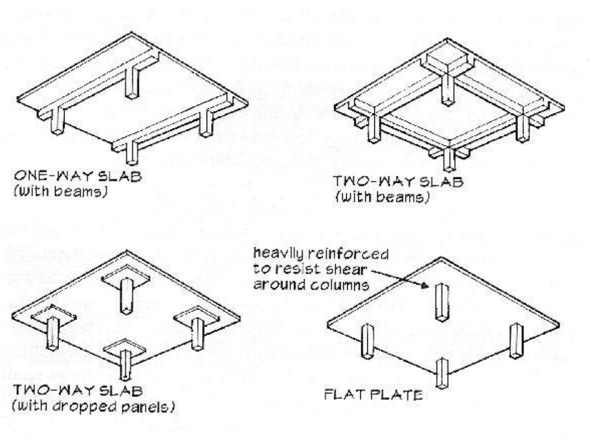


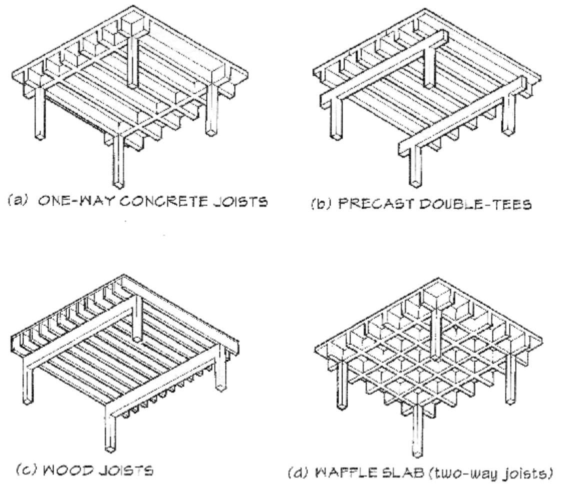
Figure 5-3 Transition Stages on Interaction Diagram

Concrete Floor Systems

- *types & spanning direction*



Concrete Floor Systems



Concrete Floor Systems

- *flexure design as T-beams (+/- M)*
- *increase of 10% V_c permitted*
- *one-way and two-way moments*
- *slabs need steel*
- *effective width is*
  - L/4
  - $b_w + 16t$
  - center-to-center of beams

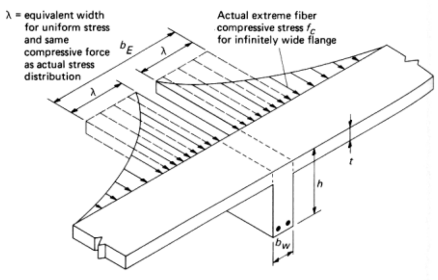
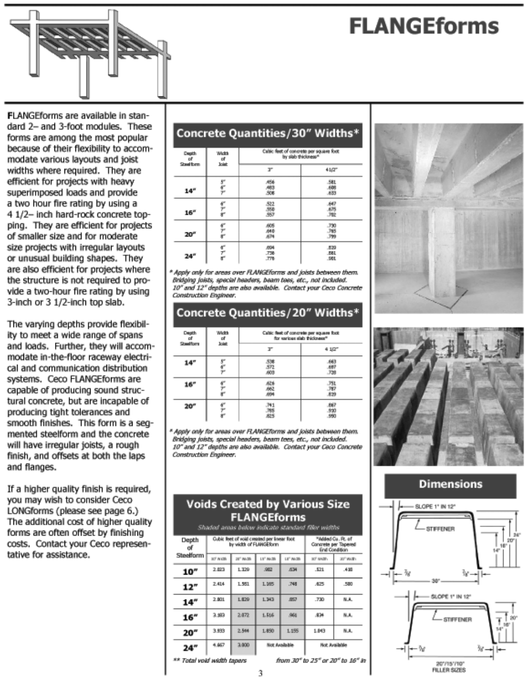


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

One-way

Joists

- standard stems
- 2.5” to 4.5” slab
- ~30” widths
- reusable forms



FLANGEforms

FLANGEforms are available in standard 2– and 3-foot modules. These forms are among the most popular because of their flexibility to accommodate various layouts and joist widths where required. They are efficient for projects with heavy superimposed loads and provide a two hour fire rating by using a 4 1/2– inch hard-rock concrete topping. They are efficient for projects of smaller size and for moderate size projects with irregular layouts or unusual building shapes. They are also efficient for projects where the structure is not required to provide a two-hour fire rating by using 3-inch or 3 1/2-inch top slab.

The varying depths provide flexibility to meet a wide range of spans and loads. Further, they will accommodate in-the-floor raceway electrical and communication distribution systems. Ceco FLANGEforms are capable of producing sound structural concrete, but are incapable of producing tight tolerances and smooth finishes. This form is a segmented steelform and the concrete will have irregular joists, a rough finish, and offsets at both the laps and flanges.

If a higher quality finish is required, you may wish to consider Ceco LONGforms (please see page 6.) The additional cost of higher quality forms are often offset by finishing costs. Contact your Ceco representative for assistance.

Concrete Quantities/30” Widths*

Depth of Steelform	Width of Joist	Cubic feet of concrete per square foot by slab thickness*	
		3”	4 1/2”
14”	5” 6” 7”	.456 .483 .508	.581 .608 .633
16”	6” 7” 8”	.522 .550 .557	.647 .675 .702
20”	6” 7” 8”	.605 .640 .674	.730 .765 .799
24”	6” 7” 8”	.694 .736 .778	.839 .881 .901

* Apply only for areas over FLANGEforms and joists between them. Bridging joists, special headers, beam tees, etc., not included. 10” and 12” depths are also available. Contact your Ceco Concrete Construction Engineer.

Concrete Quantities/20” Widths*

Depth of Steelform	Width of Joist	Cubic feet of concrete per square foot for various slab thickness*	
		3”	4 1/2”
14”	5” 6” 7”	.538 .571 .603	.663 .697 .728
16”	6” 7” 8”	.626 .662 .694	.751 .787 .839
20”	6” 7” 8”	.741 .765 .825	.867 .930 .950

* Apply only for areas over FLANGEforms and joists between them. Bridging joists, special headers, beam tees, etc., not included. 10” and 12” depths are also available. Contact your Ceco Concrete Construction Engineer.

Voids Created by Various Size FLANGEforms

Shaded areas below indicate standard filler widths

Depth of Steelform	Cubic feet of void created per linear foot by width of FLANGEform				*Added Cu. Ft. of Concrete per Tapered End Condition	
	30” Width	20” Width	15” Width	10” Width	30” Width	20” Width
10”	2.023	1.329	.982	.634	.521	.418
12”	2.414	1.581	1.165	.748	.625	.500
14”	2.801	1.829	1.343	.857	.730	N.A.
16”	3.183	2.072	1.516	.961	.834	N.A.
20”	3.933	2.544	1.850	1.155	1.043	N.A.
24”	4.667	3.000	Not Available		Not Available	

** Total void width tapers from 30” to 25” or 20” to 16” in

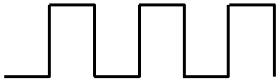
3

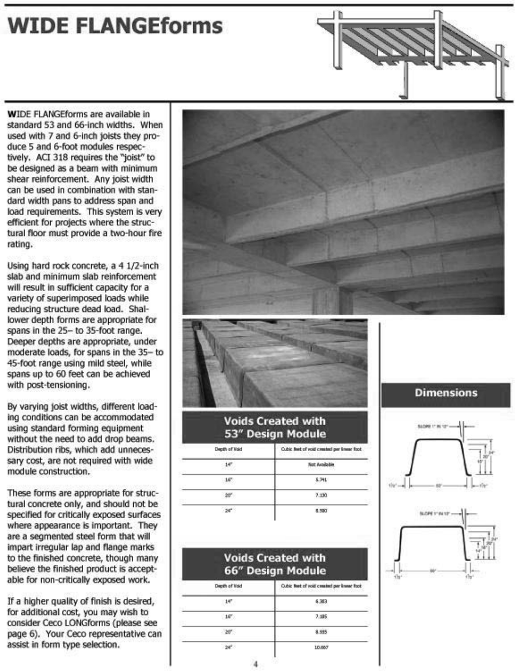
Dimensions

One-way

Joists

  - wide pans
  - 5’, 6’ up
  - light loads & long spans
  - one-leg stirrups





WIDE FLANGEforms

WIDE FLANGEforms are available in standard 53 and 66-inch widths. When used with 7 and 6-inch joists they produce 5 and 6-foot modules respectively. ACI 318 requires the "joist" to be designed as a beam with minimum shear reinforcement. Any joist width can be used in combination with standard width pans to address span and load requirements. This system is very efficient for projects where the structural floor must provide a two-hour fire rating.

Using hard rock concrete, a 4 1/2-inch slab and minimum slab reinforcement will result in sufficient capacity for a variety of superimposed loads while reducing structure dead load. Shallower depth forms are appropriate for spans in the 25– to 35-foot range. Deeper depths are appropriate, under moderate loads, for spans in the 35– to 45-foot range using mild steel, while spans up to 60 feet can be achieved with post-tensioning.

By varying joist widths, different loading conditions can be accommodated using standard forming equipment without the need to add drop beams. Distribution ribs, which add unnecessary cost, are not required with wide module construction.

These forms are appropriate for structural concrete only, and should not be specified for critically exposed surfaces where appearance is important. They are a segmented steel form that will impart irregular lap and flange marks to the finished concrete, though many believe the finished product is acceptable for non-critically exposed work.

If a higher quality of finish is desired, for additional cost, you may wish to consider Ceco LONGforms (please see page 6). Your Ceco representative can assist in form type selection.

Voids Created with 53” Design Module

Depth of Void	Cubic feet of void created per linear foot
14"	Not Available
16"	5.741
20"	7.130
24"	8.500

Voids Created with 66” Design Module

Depth of Void	Cubic feet of void created per linear foot
14"	6.383
16"	7.185
20"	8.935
24"	10.687

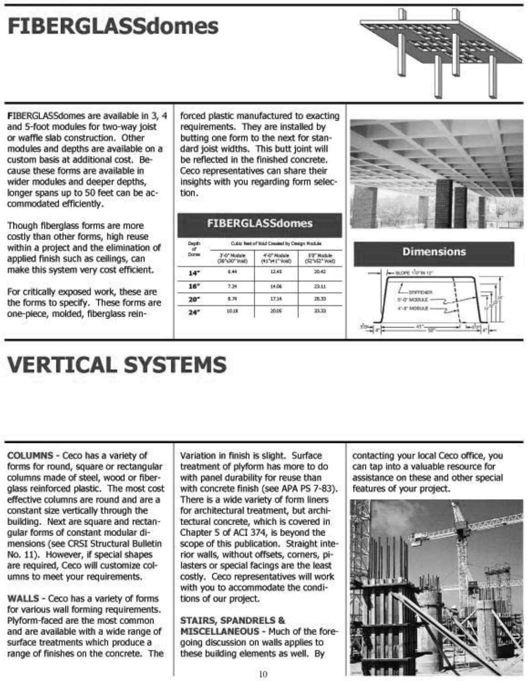
Dimensions

4

Two-way

Joists

  - domed pans
  - 3’, 4’ & 5’



FIBERGLASSdomes

FIBERGLASSdomes are available in 3, 4 and 5-foot modules for two-way joist or waffle slab construction. Other modules and depths are available on a custom basis at additional cost. Because these forms are available in wider modules and deeper depths, longer spans up to 50 feet can be accommodated efficiently.

Though fiberglass forms are more costly than other forms, high reuse within a project and the elimination of applied finish such as ceilings, can make this system very cost efficient.

For critically exposed work, these are the forms to specify. These forms are one-piece, molded, fiberglass reinforced plastic manufactured to exacting requirements. They are installed by butting one form to the next for standard joist widths. This butt joint will be reflected in the finished concrete. Ceco representatives can share their insights with you regarding form selection.

FIBERGLASSdomes

Depth of Dome	Cubic feet of Void Created by Design Module		
	3'-0" Module (30"x30" Void)	4'-0" Module (41"x41" Void)	5'0" Module (52"x52" Void)
14"	6.44	12.45	20.42
16"	7.24	14.06	23.11
20"	8.74	17.14	28.33
24"	10.18	20.06	33.33

Dimensions

VERTICAL SYSTEMS

COLUMNS - Ceco has a variety of forms for round, square or rectangular columns made of steel, wood or fiberglass reinforced plastic. The most cost effective columns are round and are a constant size vertically through the building. Next are square and rectangular forms of constant modular dimensions (see CRSI Structural Bulletin No. 11). However, if special shapes are required, Ceco will customize columns to meet your requirements.

WALLS - Ceco has a variety of forms for various wall forming requirements. Plyform-faced are the most common and are available with a wide range of surface treatments which produce a range of finishes on the concrete. The Variation in finish is slight. Surface treatment of plyform has more to do with panel durability for reuse than with concrete finish (see APA PS 7-83). There is a wide variety of form liners for architectural treatment, but architectural concrete, which is covered in Chapter 5 of ACI 374, is beyond the scope of this publication. Straight interior walls, without offsets, corners, pilasters or special facings are the least costly. Ceco representatives will work with you to accommodate the conditions of our project.

STAIRS, SPANDRELS & MISCELLANEOUS - Much of the foregoing discussion on walls applies to these building elements as well. By contacting your local Ceco office, you can tap into a valuable resource for assistance on these and other special features of your project.

10

Construction Supervision

- proper placement of all reinforcement
  - welding
  - splices
- mix design
  - slump
  - in-situ strength
    - cast cylinders
    - cylinder cores – if needed