Frame Analysis by Coefficients and Live Load Reduction

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2.2.2 Live Load Reduction for Columns, Beams, and Slabs

Most general building codes permit a reduction in live load for design of columns, beams and slabs to account for the probability that the total floor area "influencing" the load on a member may not be fully loaded simultaneously. Traditionally, the reduced amount of live load for which a member must be designed has been based on a tributary floor area supported by that member. According to ASCE 7-02, the magnitude of live load reduction is based on an influence area rather than a tributary area. The influence area is a function of the tributary area for the structural member. The influence area for different structural members is calculated by multiplying the tributary area for the member A_T, by the coefficients K_{LL} given in Table 2-3, see ASCE 4.8.

The reduced live load **L** per square foot of floor area supported by columns, beams, and two-way slabs having an influence area ($K_{LL}A_T$) of more than 400 sq ft is:

$$L = L_0\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \qquad \text{ASCE (Eq. 4-1)}$$

where L_o is the unreduced design live load per square foot. The reduced live load cannot be taken less than 50% for members supporting one floor, or less than 40% of the unit live load L_o otherwise. For other limitations on live load reduction, see ASCE 4.8.

Using the above expression for reduced live load, values of the reduction multiplier as a function of influence area are given in Table 2-4.

Table 2-3 Live Load Element Factor K_{LL}

Element	K_{LL}
Interior columns	4
Exterior column without cantilever slabs	4
Edge column with cantilever slabs	3
Corner columns with cantilever slabs	2
Edge beams without cantilever slabs	2
Interior beams	2
All other members not identified above including: Edge beams with cantilever slabs Cantilever beams Two-way slabs	1

The above limitations on permissible reduction of live loads are based on ASCE 4.8. The governing general building code should be consulted for any difference in amount of reduction and type of members that may be designed for a reduced live load.

2.3 FRAME ANALYSIS BY COEFFICIENTS

The ACI Code provides a simplified method of analysis for both one-way construction (ACI 8.3.3) and two-way construction (ACI 13.6). Both simplified methods yield moments and shears based on coefficients. Each method will give satisfactory results within the span and loading limitations stated in Chapter 1. The direct design method for two-way slabs is discussed in Chapter 4.

Table 2-4 Reduction Multiplier (RM) for Live Load = $\left(0.25+\frac{15}{\sqrt{K_{LL}A_T}}\right)$

Influence Area $K_{LL}A_T$	RM	Influence Area $K_{LL}A_T$	RM
400[a]	1.000	5600	**0.450**
800	0.780	6000	**0.444**
1200	0.683	6400	**0.438**
1600	0.625	6800	**0.432**
2000	0.585	7200	**0.427**
2400	0.556	7600	**0.422**
2800	0.533	8000	**0.418**
3200	0.515	8400	**0.414**
3600	0.500[b]	8800	**0.410**
4000	0.487	9200	**0.406**
4800	0.467	10000	**0.400[c]**
5200	0.458		

[a]No live load reduction is permitted for influence area less than 400 sq ft.
[b]Maximum reduction permitted for members supporting one floor only.
[c]Maximum absolute reduction.

2.3.1 Continuous Beams and One-Way Slabs

When beams and one-way slabs are part of a frame or continuous construction, ACI 8.3.3 provides approximate moment and shear coefficients for gravity load analysis. The approximate coefficients may be used as long as all of the conditions illustrated in Fig. 2-2 are satisfied: (1) There must be two or more spans, approximately equal in length, with the longer of two adjacent spans not exceeding the shorter by more than 20 percent; (2) loads must be uniformly distributed, with the service live load not more than 3 times the dead load ($L/D \leq 3$); and (3) members must have uniform cross section throughout the span. Also, no redistribution of moments is permitted (ACI 8.4). The moment coefficients defined in ACI 8.3.3 are shown in Figs. 2-3 through 2-6. In all cases, the shear in end span members at the interior support is taken equal to $1.15w_u\ell_n/2$. The shear at all other supports is $w_u/2$ (see Fig. 2-7). $w_u\ell_n$ is the combined factored load for dead and live loads, $w_u = 1.2w_d + 1.6w_\ell$. For beams, w_u is the uniformly distributed load per unit length of beam (plf), and the coefficients yield total moments and shears on the beam. For one-way slabs, w_u is the uniformly distributed load per unit area of slab (psf), and the moments and shears are for slab strips one foot in width. The span length ℓ_n is defined as the clear span of the beam or slab. For negative moment at a support with unequal adjacent spans, ℓ_n is the average of the adjacent clear spans. Support moments and shears are at the faces of supports.

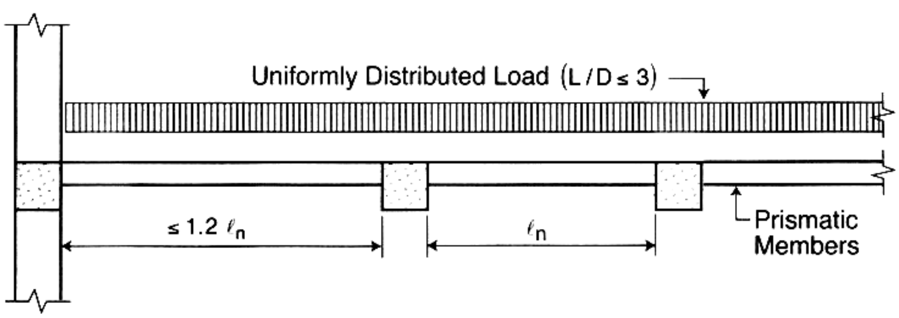


Figure 2-2 Conditions for Analysis by Coefficients (ACI 8.3.3)

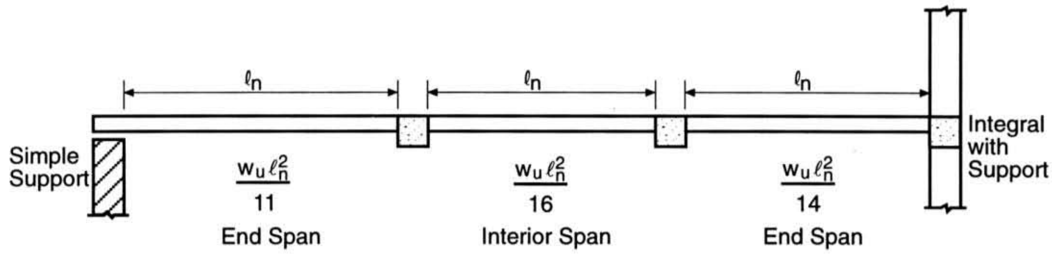


Figure 2-3 Positive Moments—All Cases

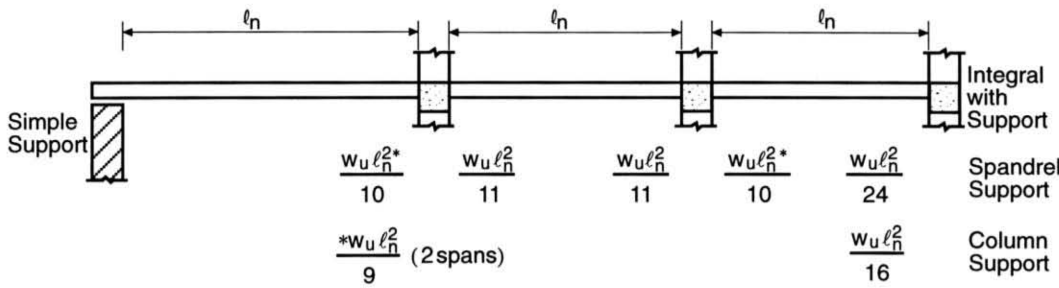


Figure 2-4 Negative Moments—Beams and Slabs

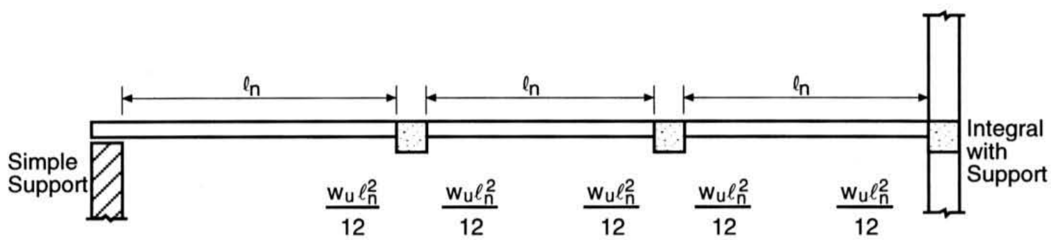


Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft

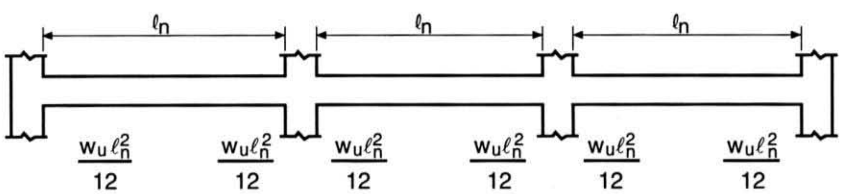


Figure 2-6 Negative Moments—Beams with Stiff Columns ($\Sigma K_c/\Sigma K_b > 8$)

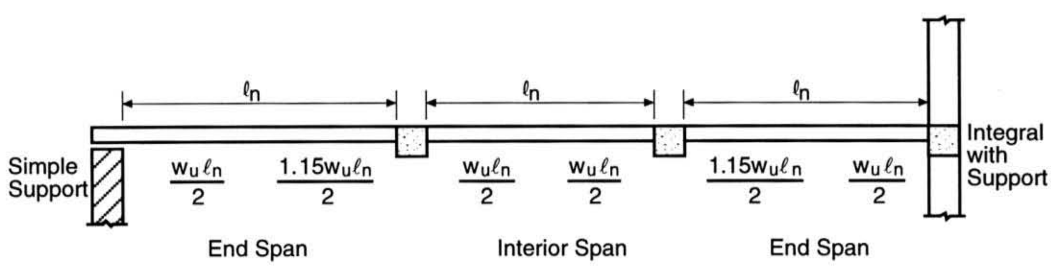


Figure 2-7 End Shears—All Cases

4.3 TWO-WAY SLAB ANALYSIS BY COEFFICIENTS

For gravity loads, ACI Chapter 13 provides two analysis methods for two-way slab systems: 1) the Direct Design Method (ACI 13.6) and the Equivalent Frame Method (ACI 13.7). The Equivalent Frame Method, using member stiffnesses and complex analytical procedures, is not suitable for hand calculations. Only the Direct Design Method, using moment coefficients, will be presented in this Chapter.

Table 4-1 Minimum Thickness for Two-Way Slab Systems

Two-Way Slab System		α_m	β	Minimum h
Flat Plate		—	≤ 2	$\ell_n/30$
Flat Plate with Spandrel Beams [1]	[Min. h = 5 in.]	—	≤ 2	$\ell_n/33$
Flat Slab [2]		—	≤ 2	$\ell_n/33$
Flat Slab [2] with Spandrel Beams[1]	[Min. h = 4 in.]	—	≤ 2	$\ell_n/36$
Two-Way Beam-Supported Slab[3]		≤ 0.2	≤ 2	$\ell_n/30$
		1.0	1	$\ell_n/33$
			2	$\ell_n/36$
		≥ 2.0	1	$\ell_n/37$
			2	$\ell_n/44$
Two-Way Beam-Supported Slab[1,3]		≤ 0.2	≤ 2	$\ell_n/33$
		1.0	1	$\ell_n/36$
			2	$\ell_n/40$
		≥ 2.0	1	$\ell_n/41$
			2	$\ell_n/49$

[1]Spandrel beam-to-slab stiffness ratio $\alpha \geq 0.8$ (ACI 9.5.3.3)

[2]Drop panel length $\geq \ell/3$, depth $\geq 1.25h$ (ACI 13.4.7)

[3]Min. h = 5 in. for $\alpha_m \leq 2.0$; min. h = 3.5 in. for $\alpha_m > 2.0$ (ACI 9.5.3.3)

α is the ratio of flexural stiffness of a beam section ti the slab; α_m is the average α for all beams on edges of a panel

β is the ratio of clear spans in long to short direction

The Direct Design Method applies when all of the conditions illustrated in Fig. 4-4 are satisfied (ACI 13.6.1):

- There must be three or more continuous spans in each direction.
- Slab panels must be rectangular with a ratio of longer to shorter span (c/c of supports) not greater than 2.
- Successive span lengths (c/c of supports) in each direction must not differ by more than one-third of the longer span.
- Columns must not be offset more than 10% of the span (in direction of offset) from either axis between centerlines of successive columns.
- Loads must be due to gravity only and must be uniformly distributed over the entire panel. The live load must not be more than 3 times the dead load (L/D ≤ 3). Note that if the live load exceeds one-half the dead load (L/D > 0.5), column-to-slab stiffness ratios must exceed the applicable values given in ACI Table 13.6.10, so that the effects of pattern loading can be neglected. The positive factored moments in panels supported by columns not meeting such minimum stiffness requirements must be magnified by a coefficient computed by ACI Eq. (13-5).
- For two-way slabs, relative stiffnesses of beams in two perpendicular directions must satisfy the minimum and maximum requirements given in ACI 13.6.1.6.
- Redistribution of moments by ACI 8.4 shall not be permitted.

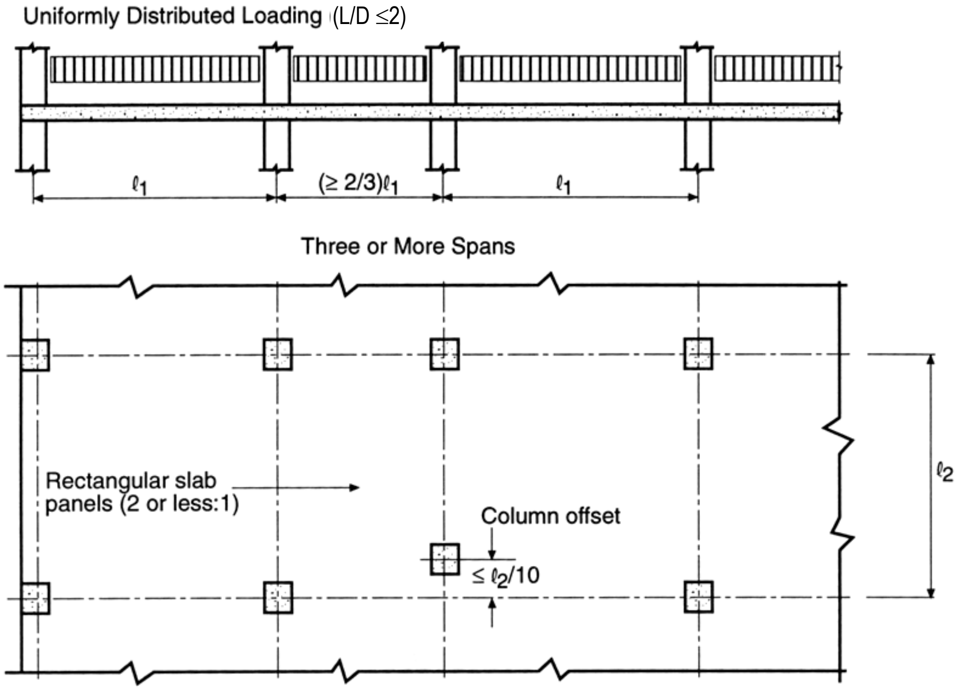


Figure 4-4 Conditions for Analysis by Coefficients

In essence, the Direct Design Method is a three-step analysis procedure. The first step is the calculation of the total design moment M_o for a given panel. The second step involves the distribution of the total moment to the negative and positive moment sections. The third step involves the assignment of the negative and positive moments to the column strips and middle strips.

For uniform loading, the total design moment M_o for a panel is calculated by the simple static moment expression, ACI Eq. (13-3):

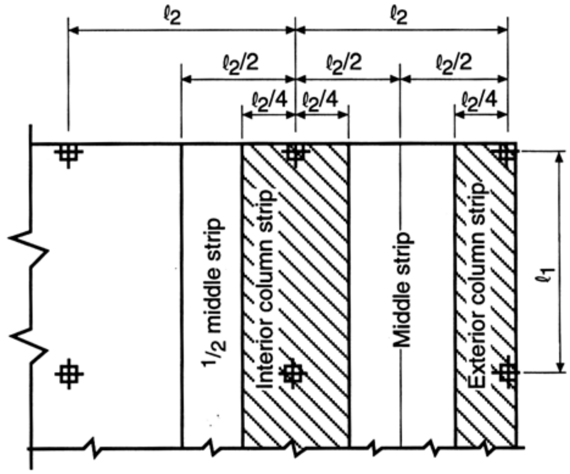
$$M_o = w_u \ell_2 \ell_n^2/8$$

where w_u is the factored combination of dead and live loads (psf), $w_u = 1.2w_d + 1.6w_\ell$. The clear span ℓ_n is defined in a straightforward manner for columns or other supporting elements of rectangular cross section (ACI 12.6.2.5). Note that circular or regular polygon shaped supports shall be treated as square supports with the same area (see ACI Fig. R13.6.2.5). The clear span starts at the face of support. One limitation requires that the clear span never be taken less than 65% of the span center-to-center of supports (ACI 13.6.2.5). The span ℓ_2 is simply the span transverse to ℓ_n; however, when the span adjacent and parallel to an edge is being considered, the distance from edge of slab to panel centerline is used for ℓ_2 in calculation of M_o (ACI 13.6.2.4).

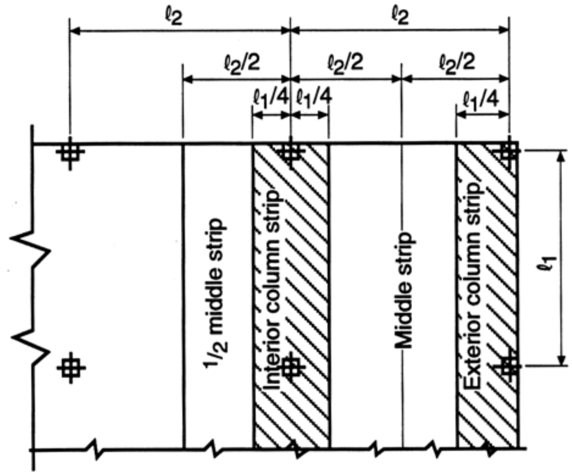
Division of the total panel moment M_o into negative and positive moments, and then into column and middle strip moments, involves direct application of moment coefficients to the total moment M_o. The moment coefficients are a function of span (interior or exterior) and slab support conditions (type of two-way slab system). For design convenience, moment coefficients for typical two-way slab systems are given in Tables 4-2 through 4-6. Tables 4-2 through 4-5 apply to flat plates or flat slabs with various end support conditions. Table 4-6 applies to two-way slabs supported on beams on all four sides. Final moments for the column strip and middle strip are computed directly using the tabulated values. All coefficients were determined using the appropriate distribution factors in ACI 13.6.3 through 13.6.6.

NOTE: The interior column strip is defined by one quarter of the smaller of ℓ_1 and ℓ_2 each side of the column centerline. The exterior column strip is bound by the slab edge and one quarter of the smaller of ℓ_1 and ℓ_2 from the column centerline. The middle strip is the remaining width between column strips.

The column strip and middle strip moments are distributed over an effective slab width as illustrated in Fig. 4-9. The column strip is defined as having a width equal to one-half the transverse or longitudinal span, whichever is smaller (ACI 13.2.1). The middle strip is bounded by two column strips.



(a) Column strip for $\ell_2 \leq \ell_1$

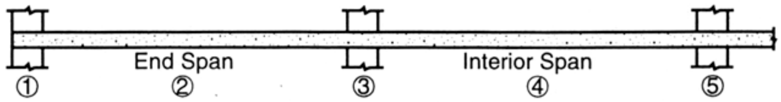


(b) Column strip for $\ell_2 > \ell_1$

Figure 4-9 Definition of Design Strips

Once the slab and beam (if any) moments are determined, design of the slab and beam sections follows the simplified design approach presented in Chapter 3. Slab reinforcement must not be less than that given in Table 3-5, with a maximum spacing of 2h or 18 in. (ACI 13.4).

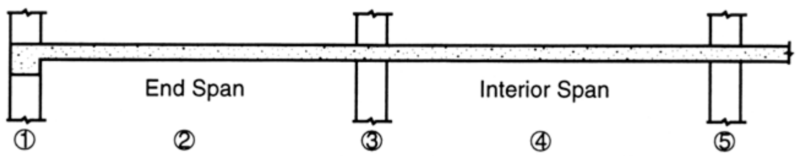
Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span: 1 Exterior Negative	End Span: 2 Positive	End Span: 3 First Interior Negative	Interior Span: 4 Positive	Interior Span: 5 Interior Negative
Total Moment	0.26 M_o	0.52 M_o	0.70 M_o	0.35 M_o	0.65 M_o
Column Strip	0.26 M_o	0.31 M_o	0.53 M_o	0.21 M_o	0.49 M_o
Middle Strip	0	0.21 M_o	0.17 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

Table 4-3 Flat Plate or Flat Slab with Spandrel Beams

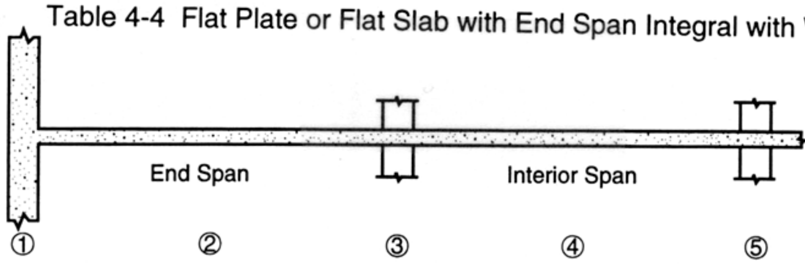


Slab Moments	End Span: 1 Exterior Negative	End Span: 2 Positive	End Span: 3 First Interior Negative	Interior Span: 4 Positive	Interior Span: 5 Interior Negative
Total Moment	0.30 M_o	0.50 M_o	0.70 M_o	0.35 M_o	0.65 M_o
Column Strip	0.23 M_o	0.30 M_o	0.53 M_o	0.21 M_o	0.49 M_o
Middle Strip	0.07 M_o	0.20 M_o	0.17 M_o	0.14 M_o	0.16 M_o

Notes: (1) All negative moments are at face of support.

(2) Torsional stiffness of spandrel beams $\beta_t \geq 2.5$. For values of β_t less than 2.5, exterior negative column strip moment increases to $(0.30 - 0.03\beta_t)\ M_o$.

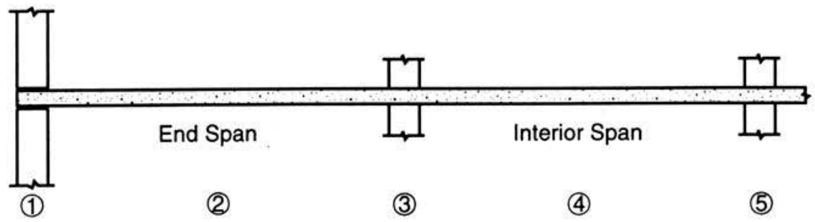
Table 4-4 Flat Plate or Flat Slab with End Span Integral with Wall



Slab Moments	End Span: 1 Exterior Negative	End Span: 2 Positive	End Span: 3 First Interior Negative	Interior Span: 4 Positive	Interior Span: 5 Interior Negative
Total Moment	0.65 M_o	0.35 M_o	0.65 M_o	0.35 M_o	0.65 M_o
Column Strip	0.49 M_o	0.21 M_o	0.49 M_o	0.21 M_o	0.49 M_o
Middle Strip	0.16 M_o	0.14 M_o	0.16 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

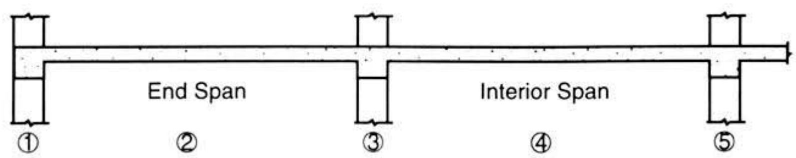
Table 4-5 Flat Plate or Flat Slab with End Span Simply Supported on Wall



Slab Moments	End Span: 1 Exterior Negative	End Span: 2 Positive	End Span: 3 First Interior Negative	Interior Span: 4 Positive	Interior Span: 5 Interior Negative
Total Moment	0	0.63 M_o	0.75 M_o	0.35 M_o	0.65 M_o
Column Strip	0	0.38 M_o	0.56 M_o	0.21 M_o	0.49 M_o
Middle Strip	0	0.25 M_o	0.19 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

Table 4-6 Two-Way Beam-Supported Slab



Span ratio	Slab Moments	End Span: 1 Exterior Negative	End Span: 2 Positive	End Span: 3 First Interior Negative	Interior Span: 4 Positive	Interior Span: 5 Interior Negative
ℓ_2/ℓ_1	Total Moment	0.16 M_o	0.57 M_o	0.70 M_o	0.35 M_o	0.65 M_o
0.5	Column Strip Beam	0.12 M_o	0.43 M_o	0.54 M_o	0.27 M_o	0.50 M_o
	Slab	0.02 M_o	0.08 M_o	0.09 M_o	0.05 M_o	0.09 M_o
	Middle Strip	0.02 M_o	0.06 M_o	0.07 M_o	0.03 M_o	0.06 M_o
1.0	Column Strip Beam	0.10 M_o	0.37 M_o	0.45 M_o	0.22 M_o	0.42 M_o
	Slab	0.02 M_o	0.06 M_o	0.08 M_o	0.04 M_o	0.07 M_o
	Middle Strip	0.04 M_o	0.14 M_o	0.17 M_o	0.09 M_o	0.16 M_o
2.0	Column Strip Beam	0.06 M_o	0.22 M_o	0.27 M_o	0.14 M_o	0.25 M_o
	Slab	0.01 M_o	0.04 M_o	0.05 M_o	0.02 M_o	0.04 M_o
	Middle Strip	0.09 M_o	0.31 M_o	0.38 M_o	0.19 M_o	0.36 M_o

Notes:
(1) Beams and slab satisfy stiffness criteria: $\alpha_1\ell_2/\ell_1 \geq 1.0$ and $\beta_t \geq 2.5$.
(2) Interpolate between values shown for different ℓ_2/ℓ_1 ratios.
(3) All negative moments are at face of support.
(4) Concentrated loads applied directly to beams must be accounted for separately.