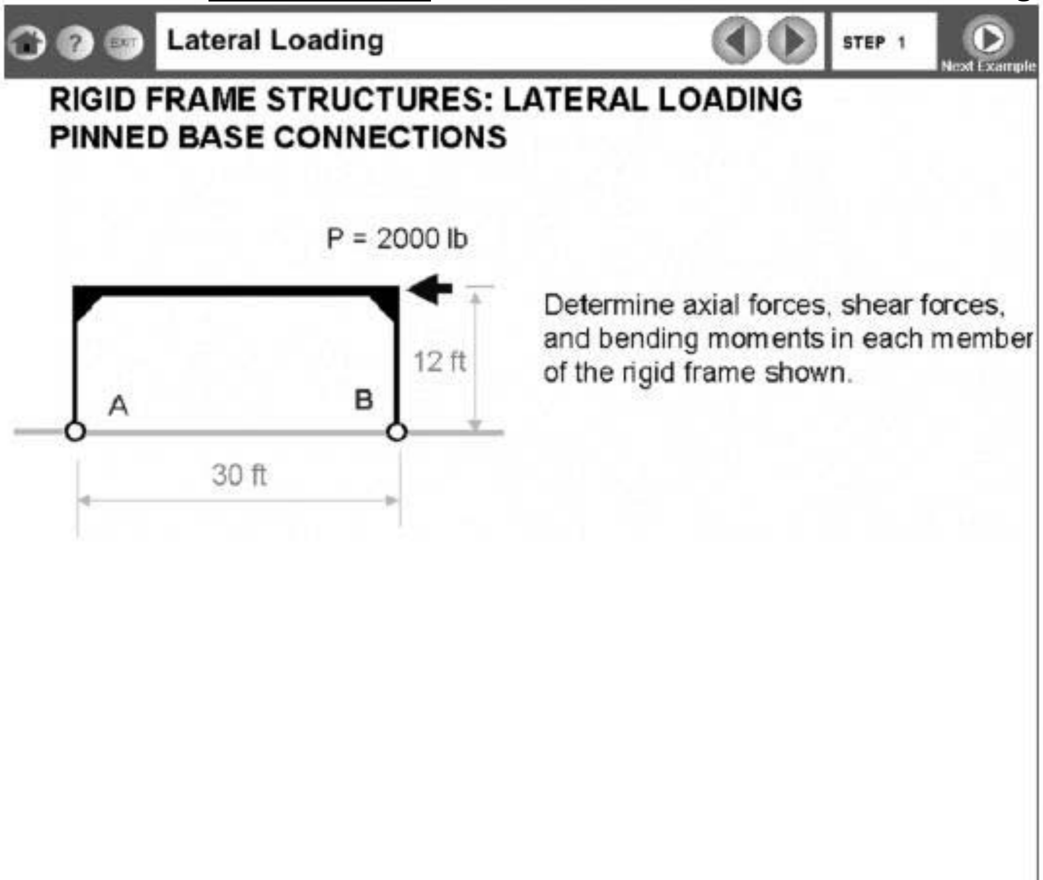
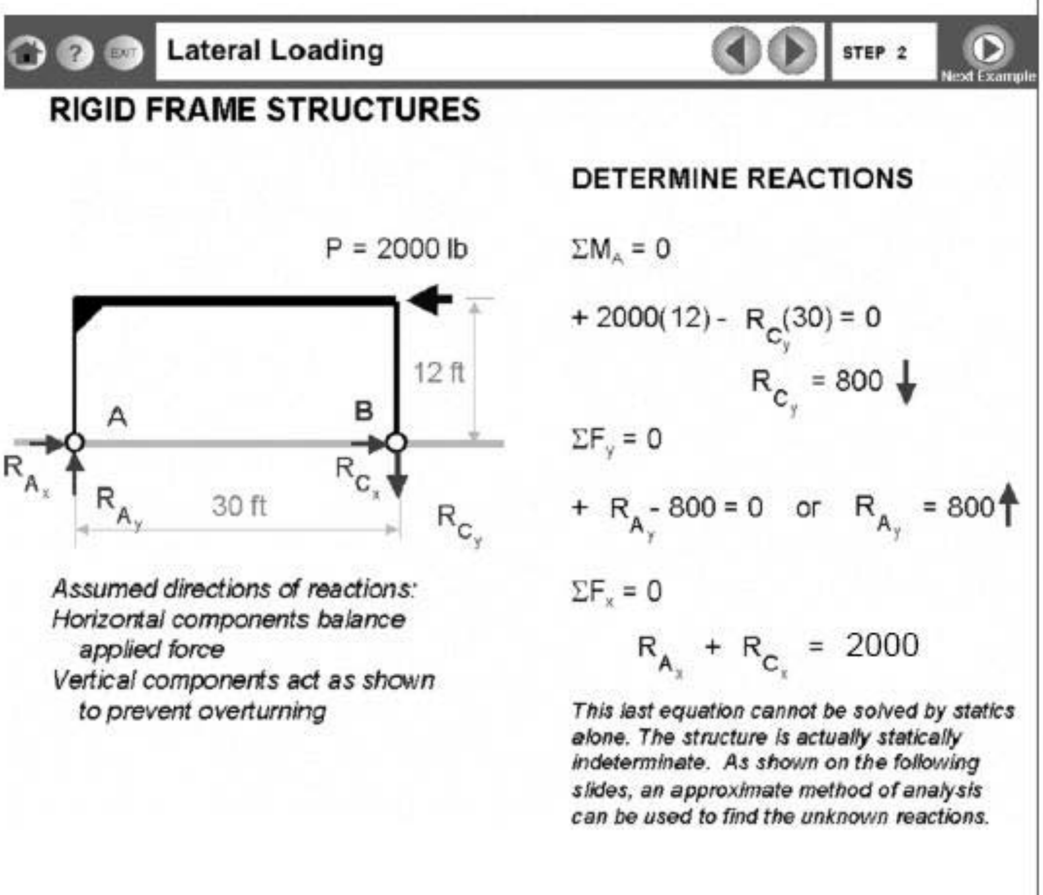
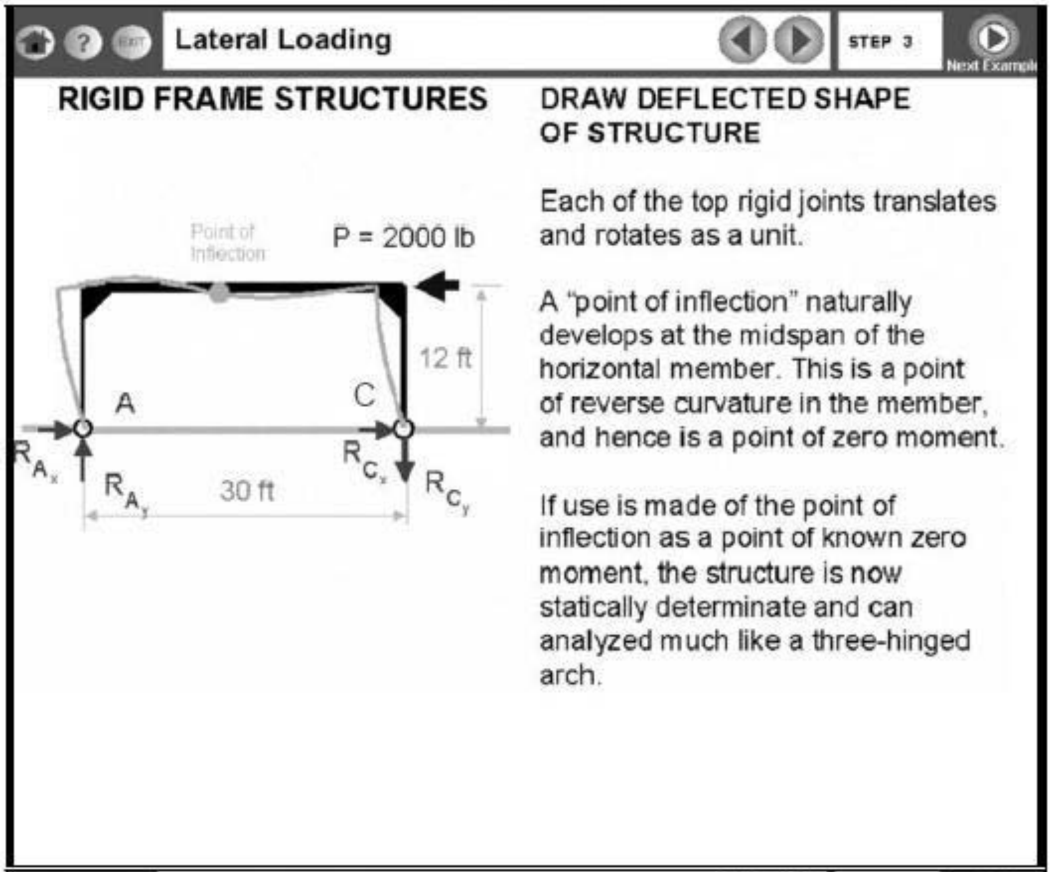
Examples:
Rigid Frames

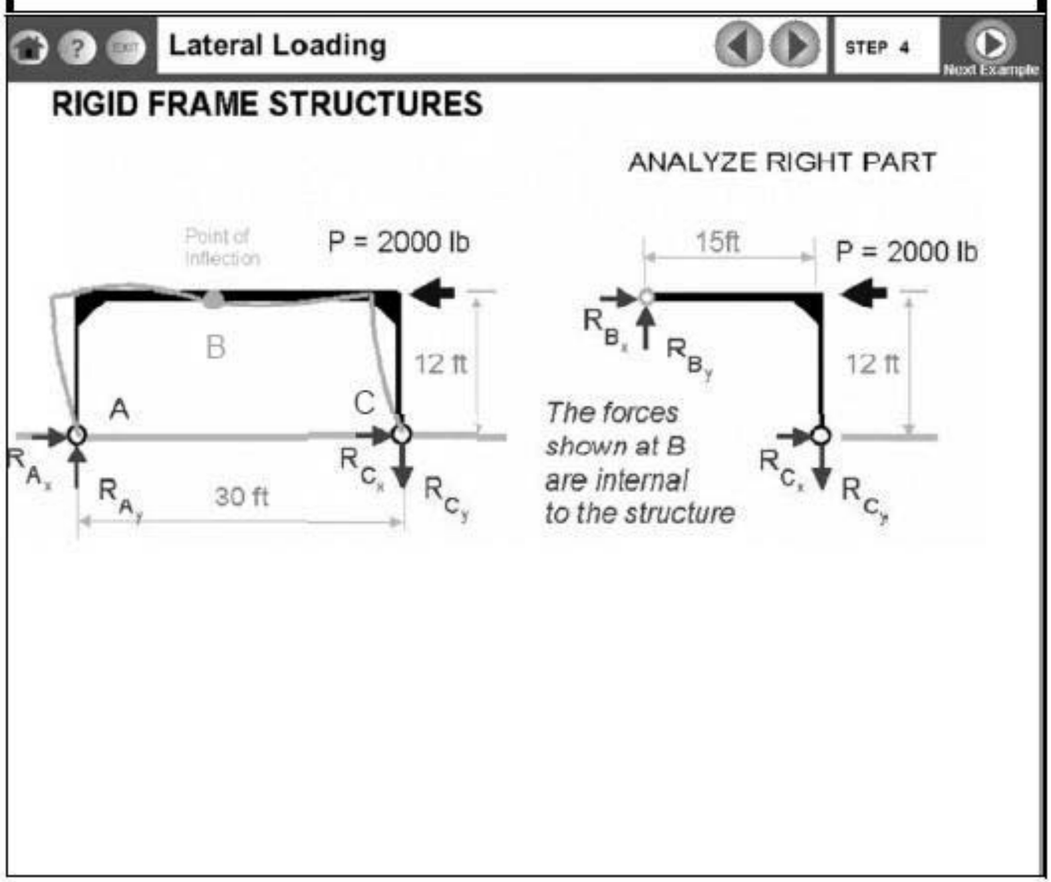
Example 1 **From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College**



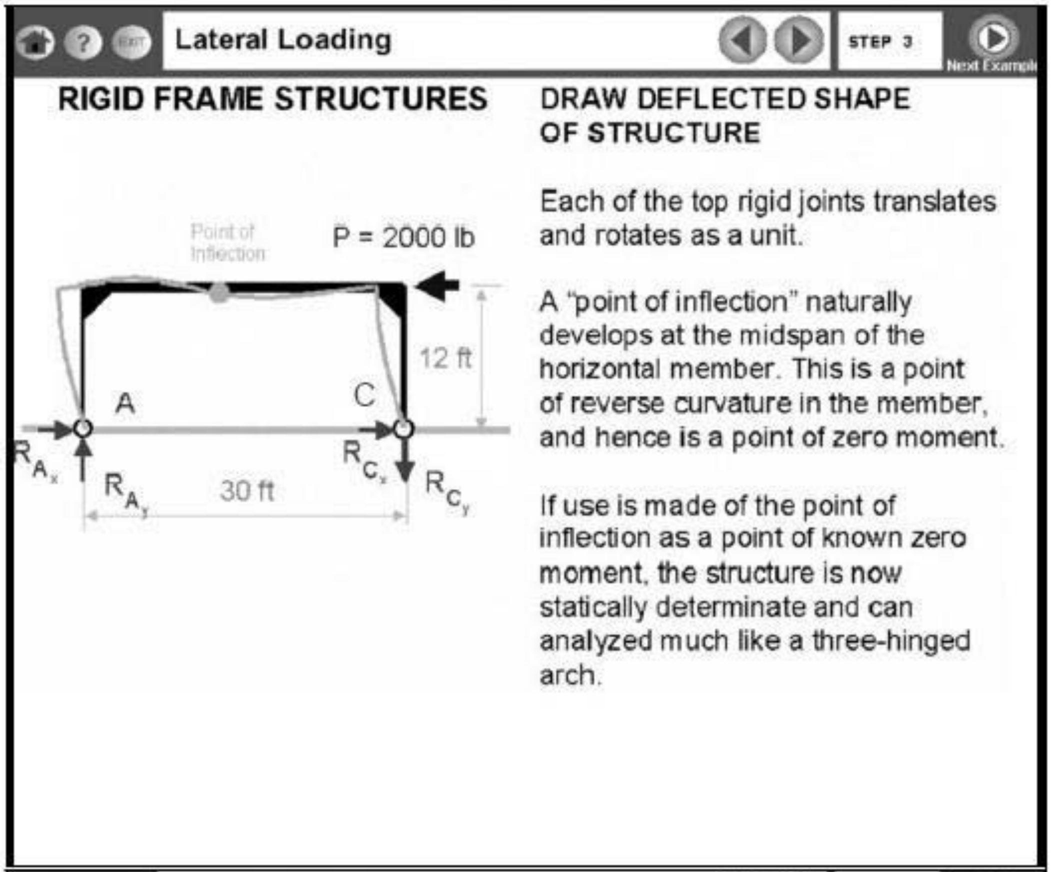


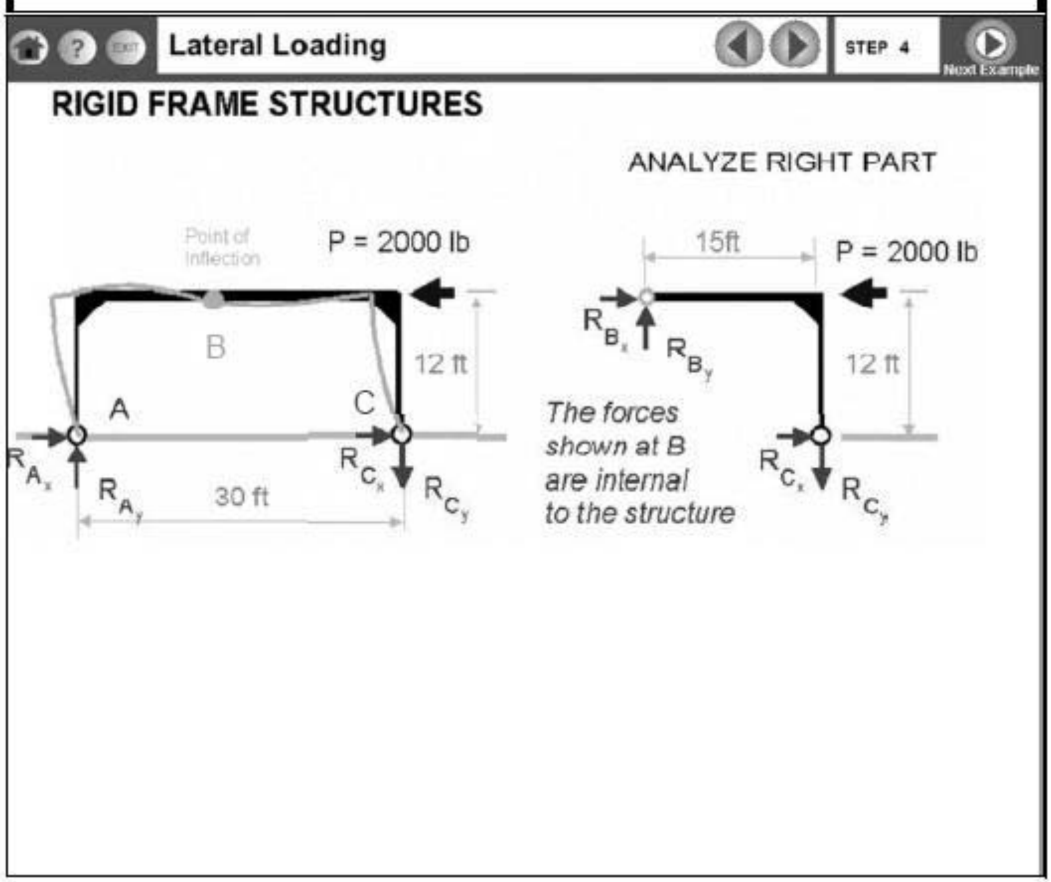
Example 1 (continued)



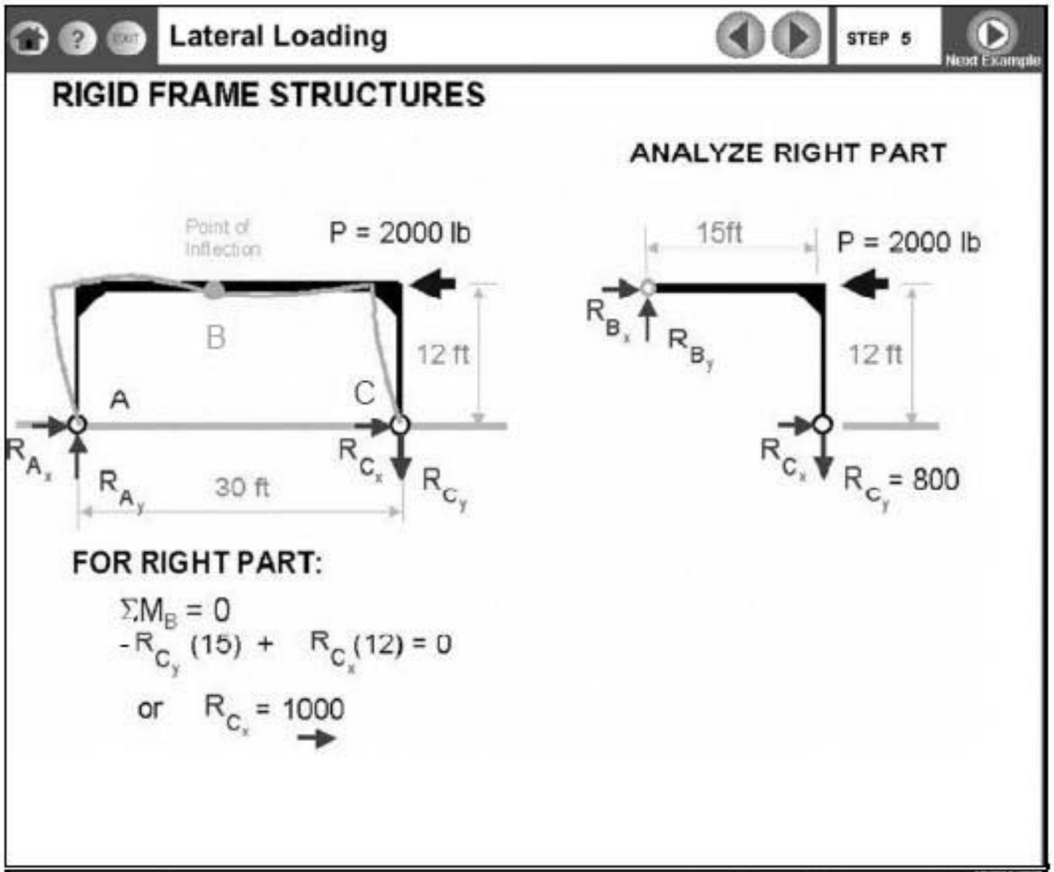


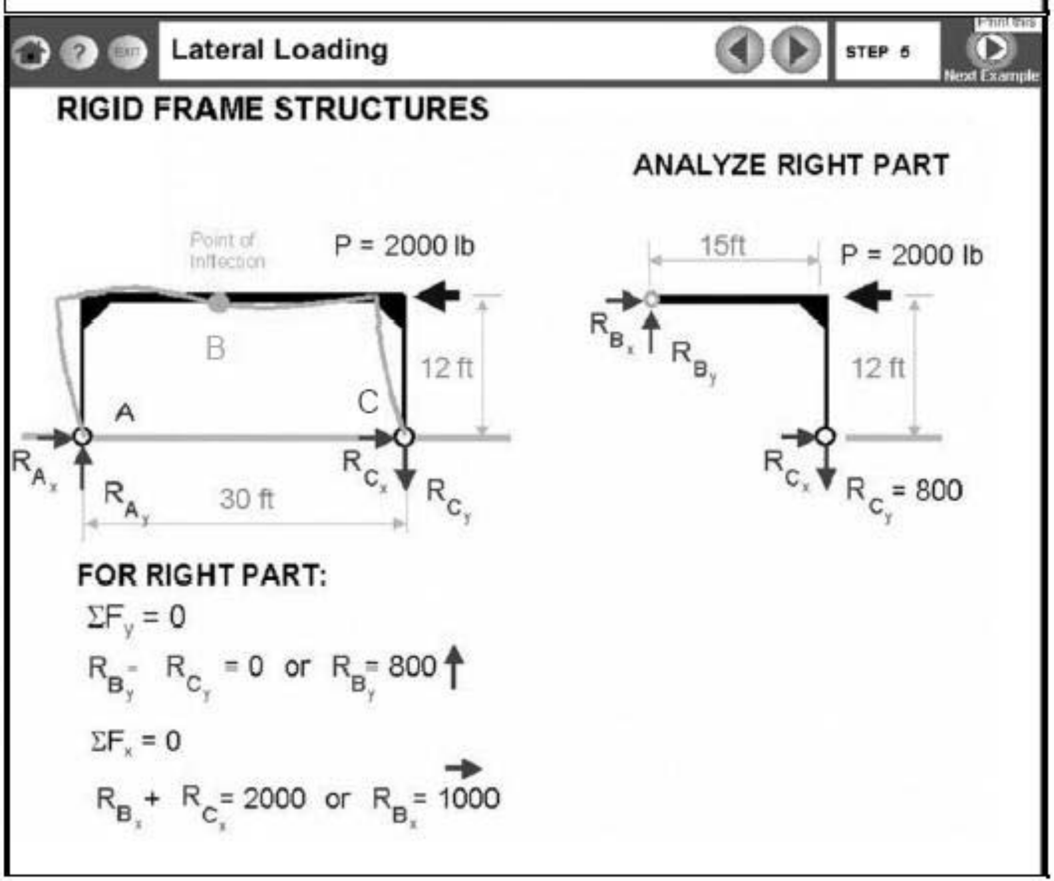
Example 1 (continued)



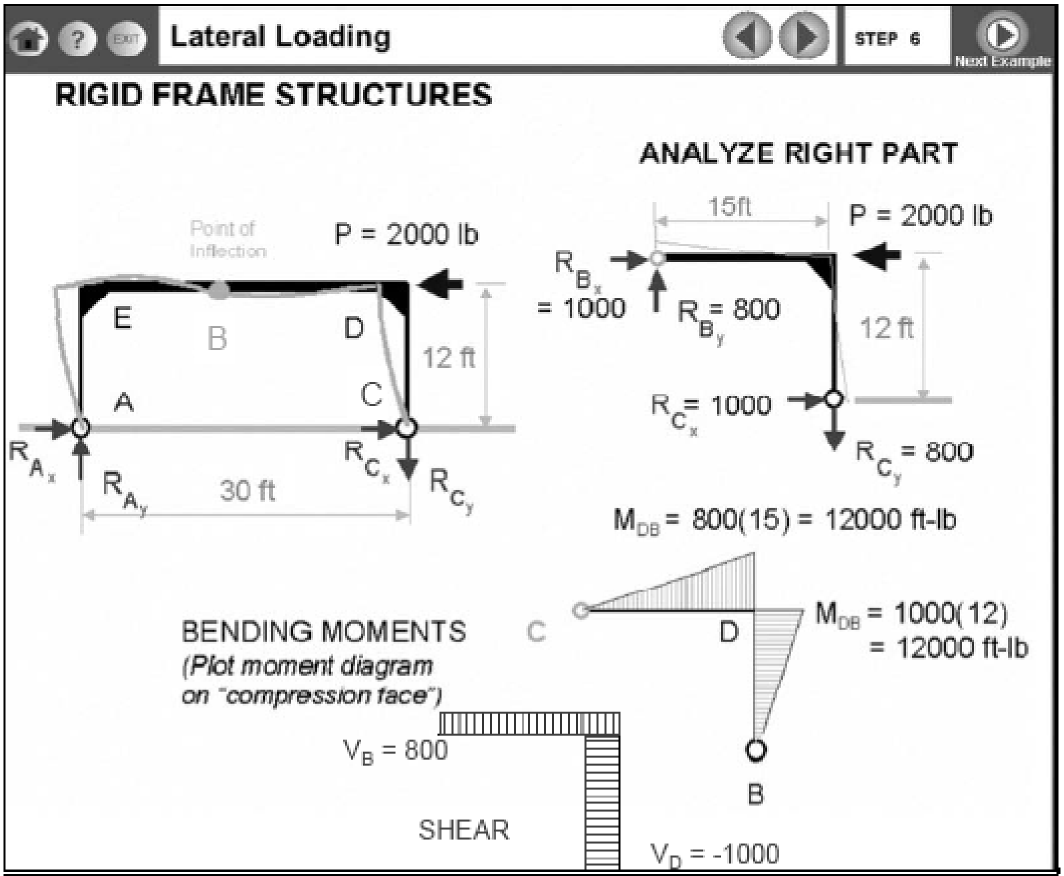


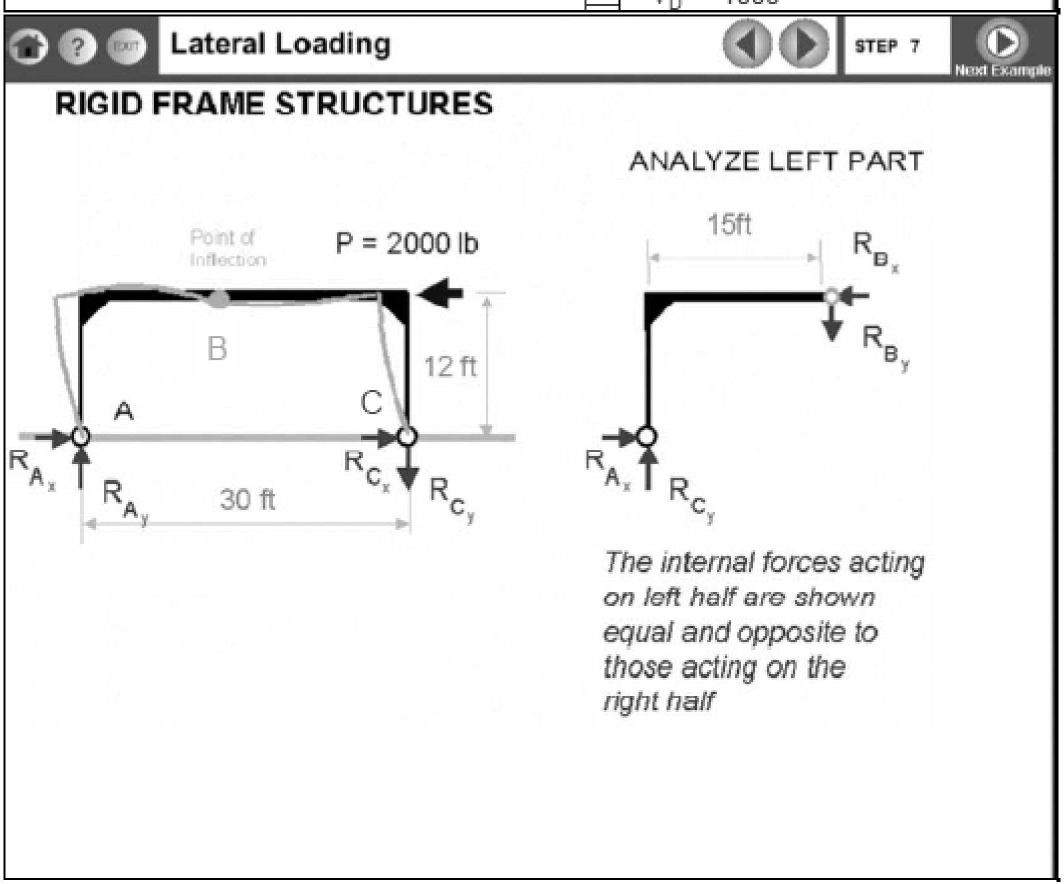
Example 1 (continued)



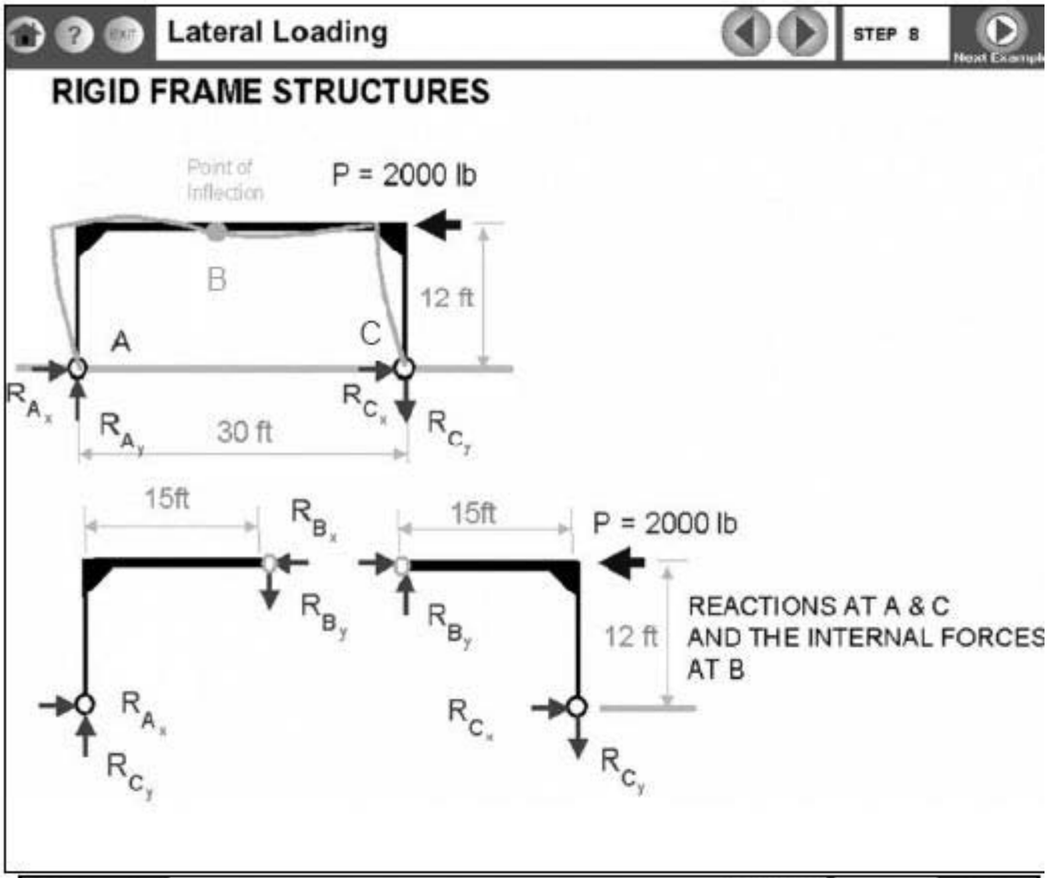


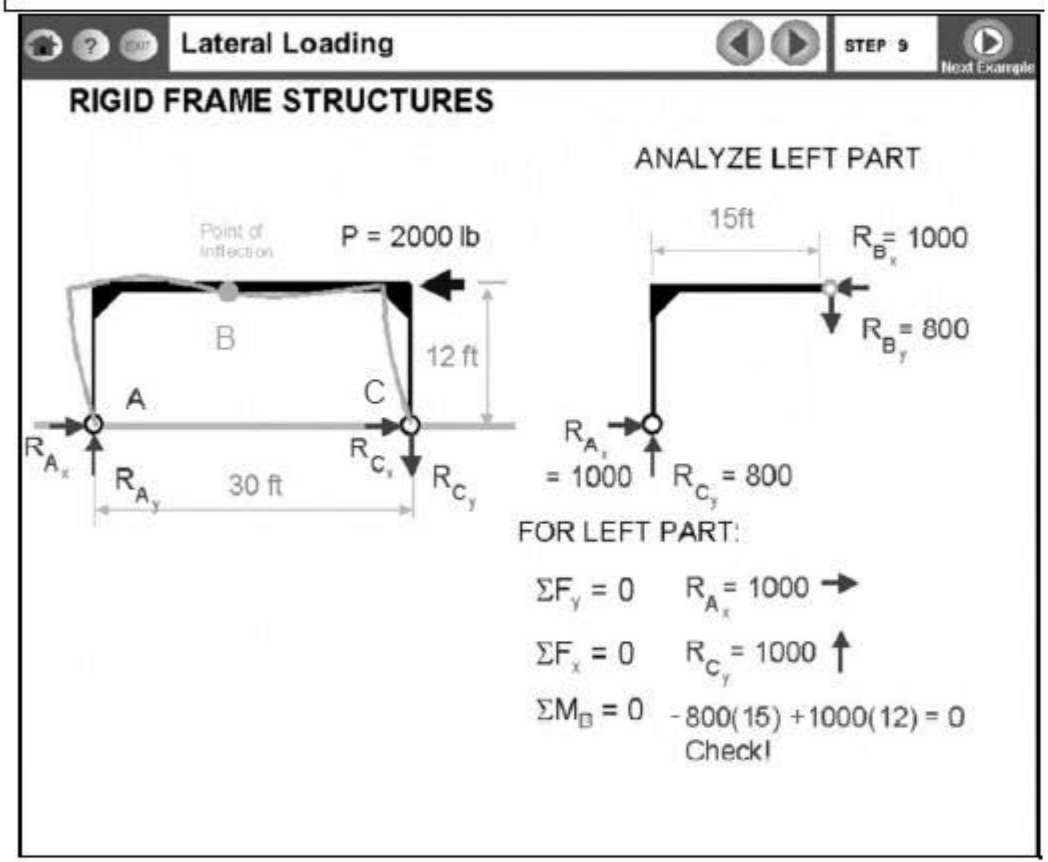
Example 1 (continued)



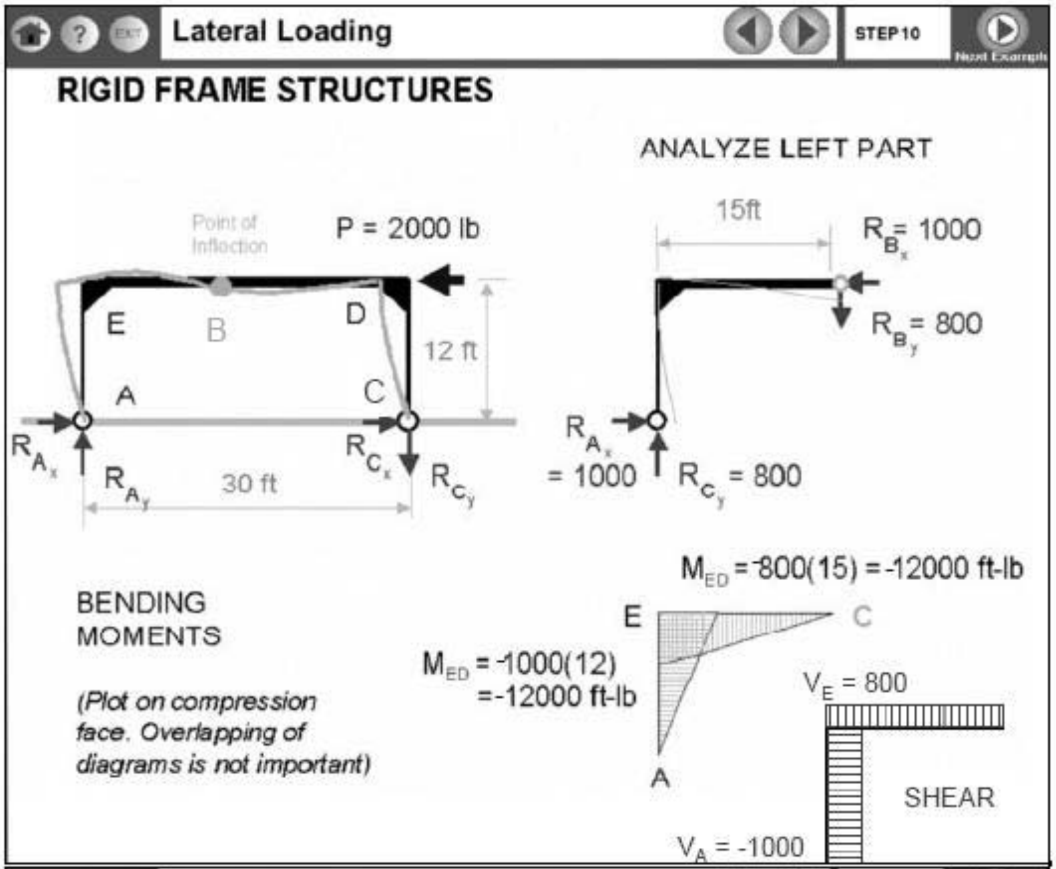


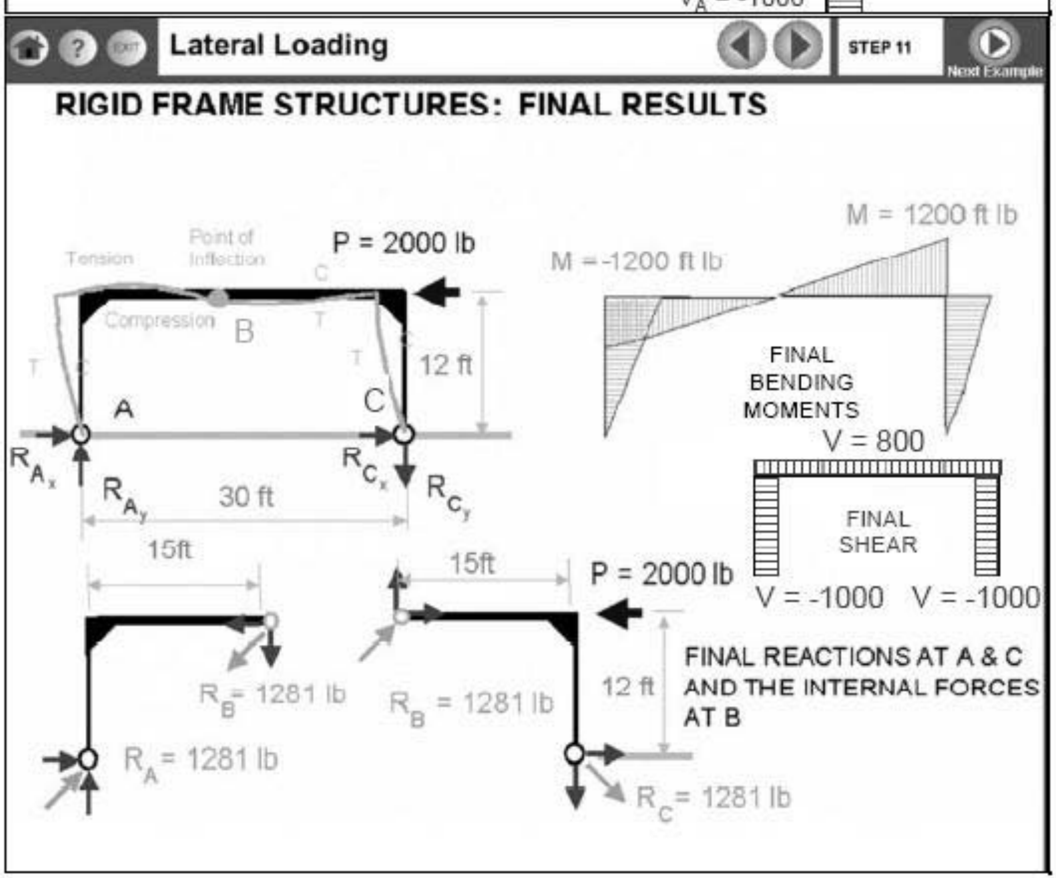
Example 1 (continued)



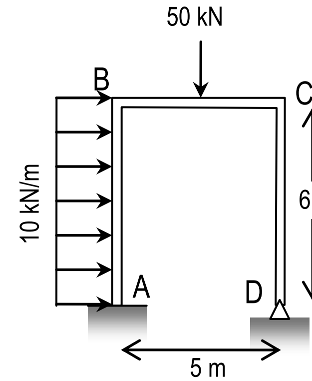


Example 1 (continued)





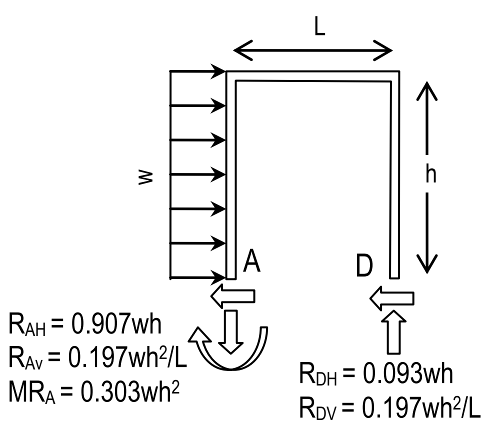
Example 2

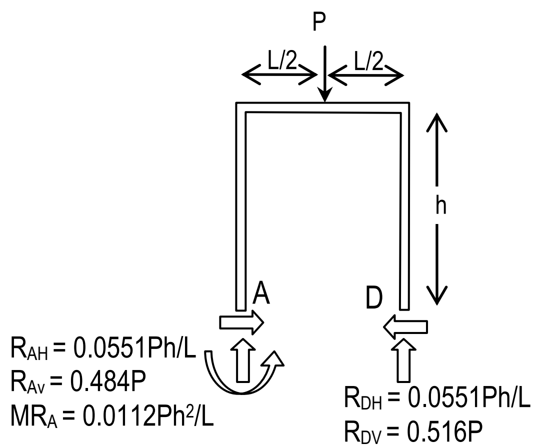


The rigid frame shown at the right has the loading and supports as show. Using superpositioning from approximate analysis methods, draw the shear and bending moment diagrams.

Solution:

Reactions The two loading situations for which approximate reaction values are available are shown below. These values must be calculated *and added together* (allowed by superpositioning).





R_{AH} = -0.907wh + 0.0551Ph/L = $-0.907(10^{kN/m})(6m)+\dfrac{0.0551(50kN)(6m)}{5m}$ = -51.11 kN

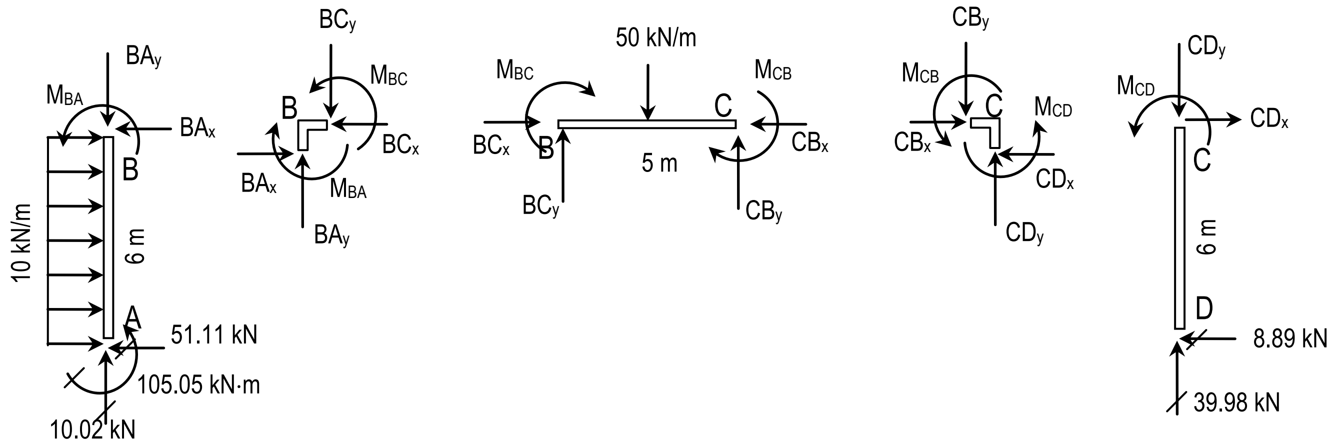
R_{AV} = -0.197wh^2/L + 0.484P = $\dfrac{-0.197(10^{kN/m})(6m)^2}{5m}+0.484(50kN)$ = 10.02 kN

MR_A = -0.303wh^2 + 0.0112Ph^2/L = $-0.303(10^{kN/m})(6m)^2+\dfrac{0.0112(50kN)(6m)^2}{5m}$ = -105.05 kN-m

R_{DH} = -0.093wh - 0.0551Ph/L = $-0.093(10^{kN/m})(6m)-\dfrac{0.0551(50kN)(6m)}{5m}$ = -8.89 kN

R_{DV} = 0.197wh^2/L + 0.516P = $\dfrac{0.197(10^{kN/m})(6m)^2}{5m}+0.516(50kN)$ = 39.98 kN

Member End Forces The free-body diagrams of all the members and joints of the frame are shown below. The unknowns on the members are drawn as anticipated, and the opposite directions are drawn on the joint. We can begin the computation of internal forces with either member AB or CD, both of which have only three unknowns.



Member AB With the magnitudes of reaction forces at A know, the unknowns are at end B of BA_x, BA_y, and M_{BA}, which can get determined by applying $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_B = 0$. Thus,

$$\sum F_x = -51.11kN + 10kN(6m) - BA_x = 0 \quad BA_x = 8.89 \text{ kN}, \quad \sum F_y = 10.02kN - BA_y = 0 \quad BAy = 10.02 \text{ kN}$$

$$\sum M_A = 105.05^{kN-m} - 10^{kN/m}(6m)(3m) + 8.89kN(6m) + M_{BA} = 0 \quad M_{BA} = 21.16\text{kN-m}$$

Joint B Because the forces and moments must be equal and opposite, BC_x = 8.89 kN, BC_y = 10.02 kN and M_{BC} = 21.16 kN·m

Member CD With the magnitudes of reaction forces at D know, the unknowns are at end C of CD_x, CD_y, and M_{CD}, which can get determined by applying $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_B = 0$. Thus,

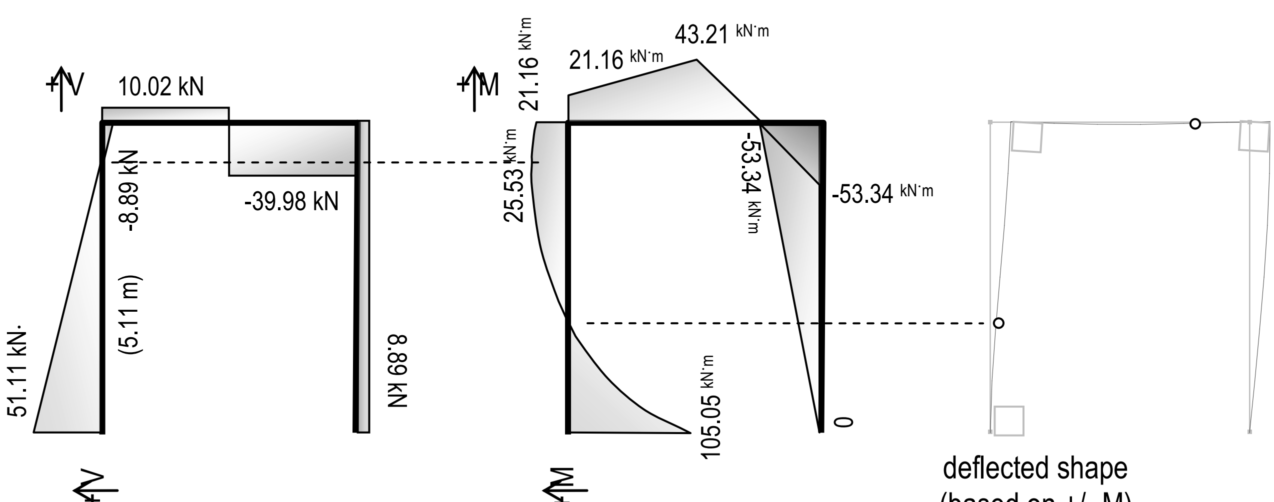
$$\sum F_x = -8.89kN + CD_x = 0 \quad CD_x = 8.89 \text{ kN}, \quad \sum F_y = 39.98kN - CD_y = 0 \quad CD_y = 39.98 \text{ kN}$$

$$\sum M_D = -8.89kN(6m) + M_{CD} = 0 \quad M_{DC} = 53.34 \text{ kN-m}$$

Joint C Because the forces and moments must be equal and opposite, CB_x = 8.89 kN, CB_y = 39.98 kN and M_{CB} = 53.34 kN-m

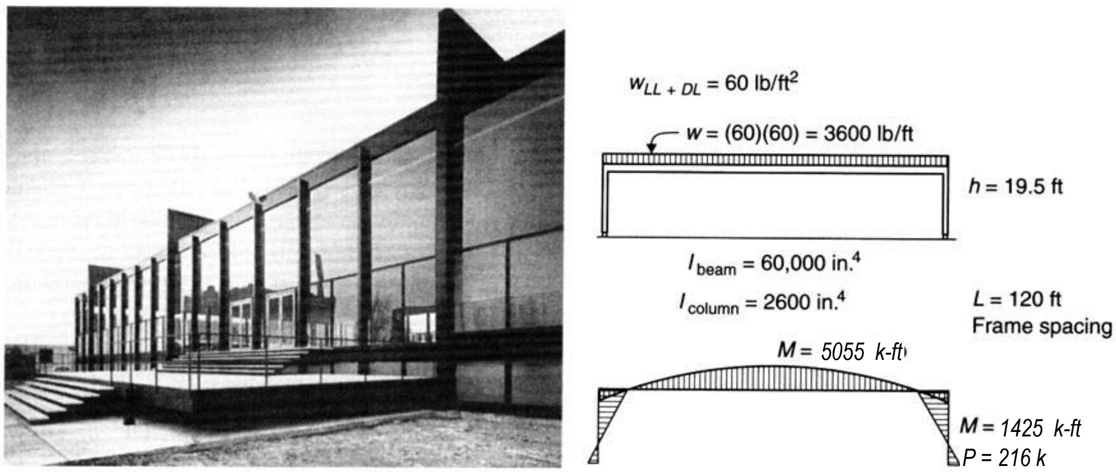
Member BC All forces are known, so equilibrium can be checked.

***(Remember*:** To find the point of zero shear with a distributed load, divide the peak {triangle} shear by the distributed load; ex. 51.11kN/($10^{kN/m}$) = 5.11 m)



Example 3

Using Multiframe4D, verify the bending moment diagram for the example in Figure 9.9:



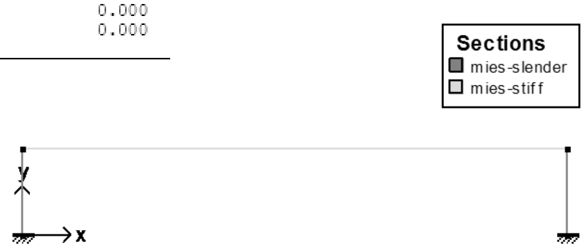
(a) Crown Hall (b) Results of structural analysis

Figure 9.9 The moment distribution illustrates the importance of relative stiffness considerations. The values obtained are quite different from those obtained by estimating points of inflection and using hand calculations.

Joint Coordinates (ft)

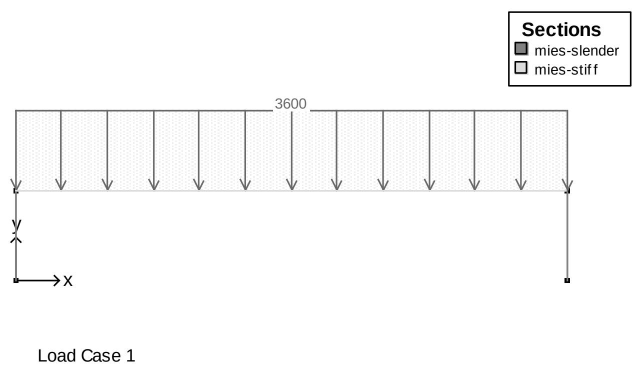
Joint	*Label*	*x*	*y*	*z*
1		0.000	0.000	0.000
2		0.000	19.500	0.000
3		120.000	19.500	0.000
4		120.000	0.000	0.000

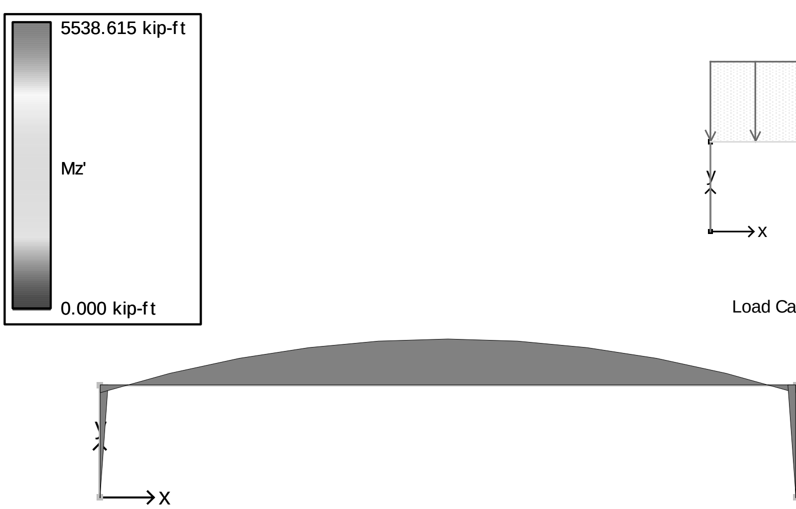
Assuming steel (E = 29,000 ksi)



Section Properties

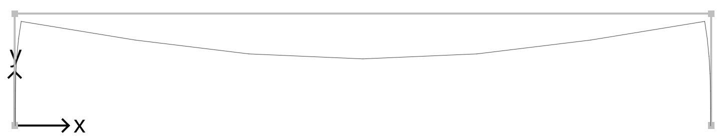
Section	*A*	*Ix*	*Ix*
	in²	in^4	in^4
mies-slender	1.000	2380.000	2380.000
mies-stiff	1.000	58700.001	58700.001





Example 3 (continued)

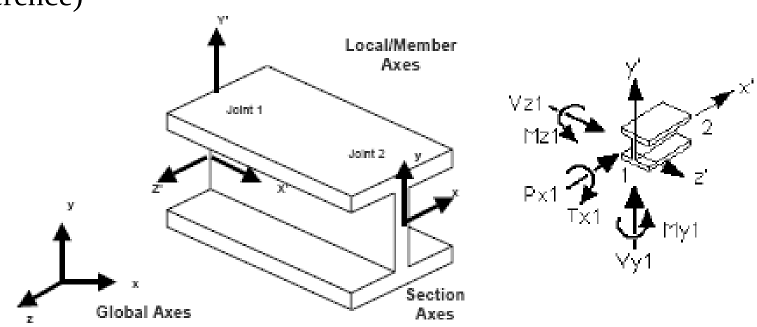
Displacement:



Maximum Actions for all members (column-1, beam-2, column-3):

	Memb	Label	Section	Sign	Px' kip	Vy' kip	Vz' kip	Tx' kip-ft	My' kip-ft	Mz' kip-ft	dy' in	dz' in
1	**1**		**mies-slender**	**+ve**	**216.000**	**0.000**	**0.000**	**0.000**	**0.000**	**1424.716**	**0.486**	**0.000**
2	1		mies-slender	-ve	0.000	-109.079	0.000	0.000	0.000	-702.318	-0.032	0.000
3	1		mies-slender	abs	216.000	109.079	0.000	0.000	0.000	1424.716	0.486	0.000
4	**2**		**mies-stiff**	**+ve**	**109.079**	**216.000**	**0.000**	**0.000**	**0.000**	**1424.716**	**0.000**	**0.000**
5	2		mies-stiff	-ve	0.000	-216.000	0.000	0.000	0.000	-5055.282	-7.326	0.000
6	2		mies-stiff	abs	109.079	216.000	0.000	0.000	0.000	5055.282	7.326	0.000
7	**3**		**mies-slender**	**+ve**	**216.000**	**109.079**	**0.000**	**0.000**	**0.000**	**702.318**	**0.032**	**0.000**
8	3		mies-slender	-ve	0.000	0.000	0.000	0.000	0.000	-1424.716	-0.486	0.000
9	3		mies-slender	abs	216.000	109.079	0.000	0.000	0.000	1424.716	0.486	0.000

(axes orientation reference)



Maximum Stresses for all members (column-1, beam-2, column-3):

	Memb	Label	Section	Sign	Sbz' top ksi	Sbz' bot ksi	Sx' ksi	Sx'+Sbz' top ksi	Sx'+Sbz' bot ksi	dy' in	dz' in
1	**1**		**mies-sl**	**+ve**	**42.494**	**86.203**	**7.714**	**50.208**	**93.917**	**0.486**	**0.000**
2	1		mies-slen	-ve	-86.203	-42.494	0.000	-78.489	-34.780	-0.032	0.000
3	1		mies-slen	abs	86.203	86.203	7.714	78.489	93.917	0.486	0.000
4	**2**		**mies-sti**	**+ve**	**38.237**	**10.776**	**1.283**	**39.521**	**12.060**	**0.000**	**0.000**
5	2		mies-stiff	-ve	-10.776	-38.237	0.000	-9.493	-36.954	-7.326	0.000
6	2		mies-stiff	abs	38.237	38.237	1.283	39.521	36.954	7.326	0.000
7	**3**		**mies-sl**	**+ve**	**86.203**	**42.494**	**7.714**	**93.917**	**50.208**	**0.032**	**0.000**
8	3		mies-slen	-ve	-42.494	-86.203	0.000	-34.780	-78.489	-0.486	0.000
9	3		mies-slen	abs	86.203	86.203	7.714	93.917	78.489	0.486	0.000

Beam-Column stress verification (combined stresses) when d = 24 in, A = 28 in^2. I_x = 2380 in^4:

$$f_{\max} = \frac{P}{A} + \frac{M}{S} = \frac{P}{A} + \frac{Mc}{I} = \frac{216k}{28in^2} + \frac{1425^{k-ft}\cdot(24in/2)}{2380in^4}\cdot\frac{12in}{ft} = 7.71ksi + 86.22ksi = 93.93ksi$$