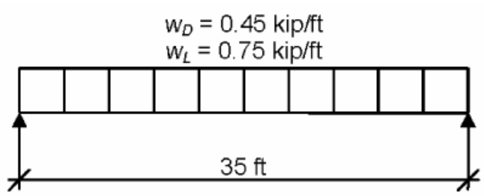
Examples: Steel

Example 1 (AISC Design Examples vV13.0)

Example F.1-1a W-Shape Flexural Member Design in Strong-Axis Bending, Continuously Braced.

Given:

Select an ASTM A992 W-shape beam with a simple span of 35 feet. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to $L/360$. The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced.



Beam Loading & Bracing Diagram
(full lateral support)

Solution:

Material Properties:

ASTM A992 $F_y = 50$ ksi $F_u = 65$ ksi — Manual Table 2-3

Calculate the required flexural strength

LRFD	ASD
$w_u = 1.2(0.450\text{ kip/ft}) + 1.6\,(0.750\text{ kip/ft}) = 1.74\text{ kip/ft}$ $M_u = \dfrac{1.74\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8} = 266\text{ kip-ft}$	$w_a = 0.450\text{ kip/ft} + 0.750\text{ kip/ft} = 1.20\text{ kip/ft}$ $M_a = \dfrac{1.20\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8} = 184\text{ kip-ft}$

Calculate the required moment of inertia for live-load deflection criterion of L/360

$$\Delta_{max} = \frac{L}{360} = \frac{35.0\text{ ft}(12\text{ in./ft})}{360} = 1.17\text{ in.}$$

$$I_{x(reqd)} = \frac{5wl^4}{384E\Delta_{max}} = \frac{5(0.750\text{ kip/ft})(35.0\text{ ft})^4(12\text{ in./ft})^3}{384\,(29{,}000\text{ ksi})(1.17\text{in.})} = 748\text{ in.}^4$$

Manual Table 3-23 Diagram 1

Select a W18×50 from Table 3-2

Per the User Note in Section F2, the section is compact. Since the beam is continuously braced and compact, only the yielding limit state applies.

LRFD	ASD
$\phi_b M_n = \phi_b M_{px} = 379\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b} = 252\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

Manual Table 3-2

$I_x = 800\text{ in.}^4 > 748\text{ in.}^4$ **o.k.**

Manual Table 3-2

Example 1 (continued)

F_y = 50 ksi

Table 3–2 (continued)
W Shapes
Selection by Z_x

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF		L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W21×48[f]	**107**	**265**	**398**	**162**	**244**	**9.78**	**14.7**	**6.09**	**16.6**	**959**	**144**	**217**
W16×57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212
W14×61	102	254	383	161	242	4.96	7.46	8.65	27.5	640	104	156
W18×50	101	252	379	155	233	8.69	13.1	5.83	17.0	800	128	192
W10×77	97.6	244	366	150	225	2.59	3.90	9.18	45.2	455	112	169
W12×65[f]	96.8	237	356	154	231	3.60	5.41	11.9	35.1	533	94.5	142
W21×44	**95.4**	**238**	**358**	**143**	**214**	**11.2**	**16.8**	**4.45**	**13.0**	**843**	**145**	**217**
W16×50	92.0	230	345	141	213	7.59	11.4	5.62	17.2	659	124	185
W18×46	90.7	226	340	138	207	9.71	14.6	4.56	13.7	712	130	195
W14×53	87.1	217	327	136	204	5.27	7.93	6.78	22.2	541	103	155
W12×58	86.4	216	324	136	205	3.76	5.66	8.87	29.9	475	87.8	132
W10×68	85.3	213	320	132	199	2.57	3.86	9.15	40.6	394	97.8	147
W16×45	82.3	205	309	127	191	7.16	10.8	5.55	16.5	586	111	167
W18×40	**78.4**	**196**	**294**	**119**	**180**	**8.86**	**13.3**	**4.49**	**13.1**	**612**	**113**	**169**
W14×48	78.4	196	294	123	184	5.10	7.66	6.75	21.1	484	93.8	141
W12×53	77.9	194	292	123	185	3.65	5.48	8.76	28.2	425	83.2	125
W10×60	74.6	186	280	116	175	2.53	3.80	9.08	36.6	341	85.8	129

ASD	LRFD
$\Omega_b = 1.67$	$\phi_b = 0.90$
$\Omega_v = 1.50$	$\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.

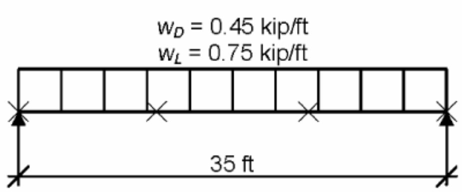
I required is in this grouping, with the W21x44 (bold) the most economical. But this section must be 18 inches maximum, and the W18 x 46 does not have enough (even though it has enough moment capacity of 340 k-ft ($\phi_b M_{px}$))

Look to the next section above for a W18 with I > 748 in^4.

Example F.1-2a W-Shape Flexural Member Design in Strong-Axis Bending, Braced at Third Points

Given:

Verify the strength of the W18×50 beam selected in Example F.1-1a if the beam is braced at the ends and third points rather than continuously braced.



Beam Loading & Bracing Diagram
(bracing at ends and third points)

Solution:

Required flexural strength at midspan from Example F.1-1a

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Example 1 (continued)

$$L_b = \frac{35.0\text{ ft}}{3} = 11.7\text{ ft}$$

By inspection, the middle segment will govern. For a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

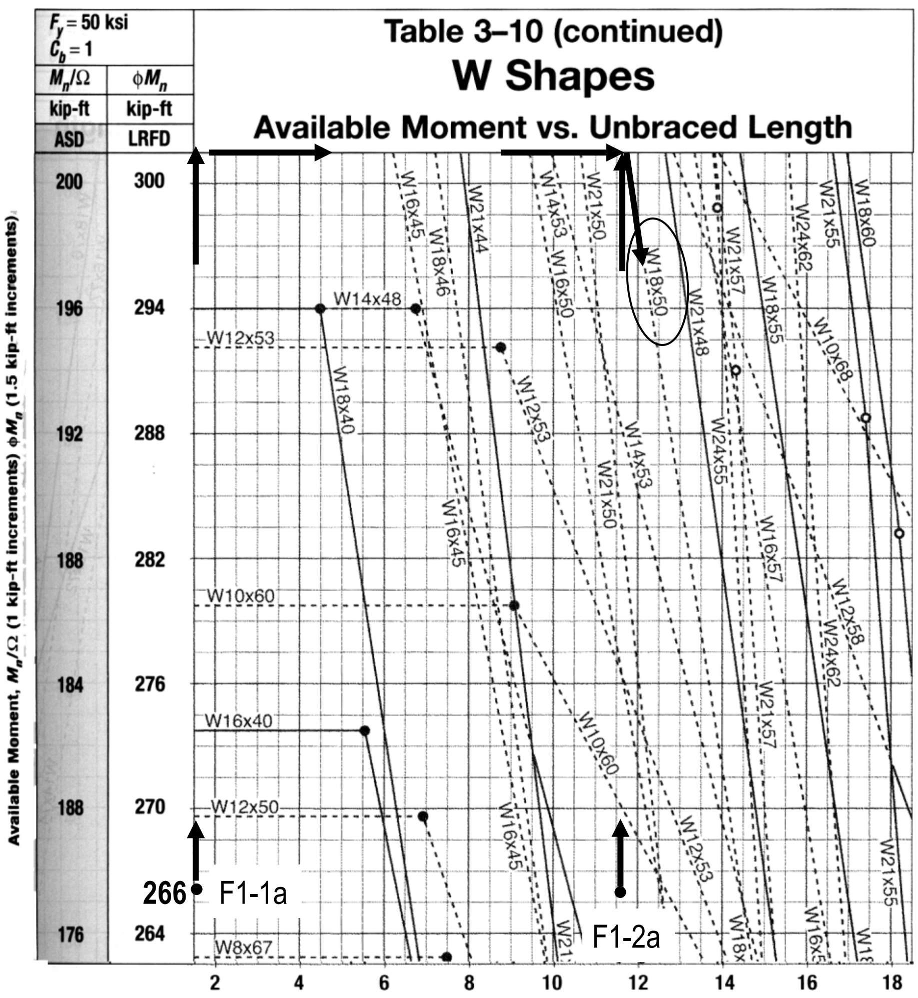
Manual Table 3-1

Obtain the available strength from Table 3-10

Enter Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 302$ kip-ft > 266 kip-ft **o.k.**	$\frac{M_n}{\Omega_b} \approx 201$ kip-ft > 184 kip-ft **o.k.**

Manual Table 3-10



For F1-1a, the unbraced length is zero. There is no zero on the chart, so the far left is used starting at the moment required of 266 k-ft. When a W18 is not encountered with a greater moment capacity going up on the page, going to the right will intersect with a W18 line.

For F1-2a, the unbraced length is 11.7 ft. The same procedure applies, starting at a moment required of 266 k-ft. If no match is close to the

Example 2 (LRFD)

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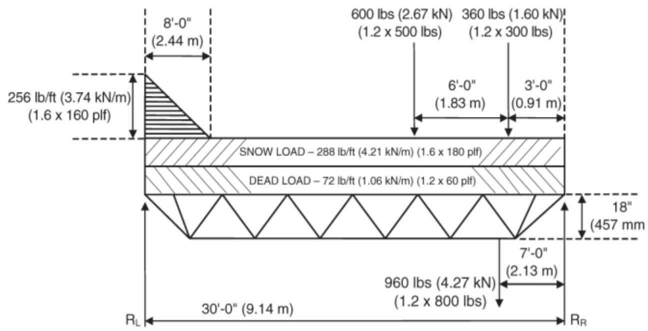
Factored Load diagram per ASCE 7 2.3.2(3) 1.2D + 1.6S



18K SP

Joist manufacturer to design joist to support **factored loads** as shown.

Joist Supplier to design joist to support loads as shown above.

$$\text{Total Load} = \frac{256}{2}(8) + (288+72)30 + 600 + 960 + 360 = \mathbf{13{,}744\ lbs.}$$

$$R_L = \frac{256(8)}{2}\left[\frac{30-\frac{8}{3}}{30}\right] + \frac{(288+72)(30)}{2} + 600\left[\frac{9}{30}\right] + 960\left[\frac{7}{30}\right] + 660\left[\frac{3}{30}\right] =$$

$R_L = 6773$ lbs.

$R_R = 6971$ lbs.

$$\text{Assume } R_R = \frac{W_{e1}(L)}{2}, \quad W_{e1} = \frac{2(6971)}{30} = 465 \text{ lbs/ft}$$

Point of Max. Mom. = Point of Zero Shear(V) = L_1

(dist. from rt. end of Jst)

$V = \text{Zero} = 6971 - (360+600+960) - (288+72)(L_1)$

$L_1 = \mathbf{14.03}$ **ft**.

$$M @ L_1 = 6971(14.03) - 360(11.03) - 960(7.03) - 600(5.03) - \frac{(288+72)(14.03)^2}{2}$$

$M = 48{,}634$ ft. lbs.

$$\text{Assume } M = \frac{W_{e2}(L)^2}{8}, \; W_{e2} = \frac{8(48.634)}{(30)^2} = 432.3 \text{ lbs./ft.}$$

Using W_{e1} = 465 LB/ft. @ SPAN = 30', and D = 18"

Select 18K7 for total load (502) and live load (180) and call it: **18K9SP**

(c) Special Considerations

The **specifying professional** shall indicate on the construction documents special considerations including:

a) Profiles for non-standard joist and Joist Girder configurations (Standard joist and Joist Girder configurations are as indicated in the Steel Joist Institute Standard Specifications Load Tables & Weight Tables of latest adoption).

b) Oversized or other non-standard web openings

c) Extended ends

d) Deflection criteria for live and total loads for non-SJI standard joists

e) Non-SJI standard bridging

LRFD

STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES

Based on a 50 ksi Maximum Yield Strength – Loads shown in Pounds per Linear Foot (plf)

Joist Designation	18K3	18K4	18K5	18K6	18K7	18K9	18K10
Depth (In.)	18	18	18	18	18	18	18
Approx. Wt. (lbs./ft.)	6.6	7.2	7.7	8.5	9	10.2	11.7
Span (ft.) ↓							
18	825	825	825	825	825	825	825
	550	550	550	550	550	550	550
19	771	825	825	825	825	825	825
	494	523	523	523	523	523	523
20	694	825	825	825	825	825	825
	423	490	490	490	490	490	490
21	630	759	825	825	825	825	825
	364	426	460	460	460	460	460
22	573	690	777	825	825	825	825
	316	370	414	438	438	438	438
23	523	630	709	774	825	825	825
	276	323	362	393	418	418	418
24	480	577	651	709	789	825	825
	242	284	318	345	382	396	396
25	441	532	600	652	727	825	825
	214	250	281	305	337	377	377
26	408	492	553	603	672	807	825
	190	222	249	271	299	354	361
27	378	454	513	558	622	747	825
	169	198	222	241	267	315	347
28	351	423	477	519	577	694	822
	151	177	199	216	239	282	331
29	327	394	444	483	538	646	766
	136	159	179	194	215	254	298
30	304	367	414	451	502	603	715
	123	144	161	175	194	229	269
31	285	343	387	421	469	564	669
	111	130	146	158	175	207	243

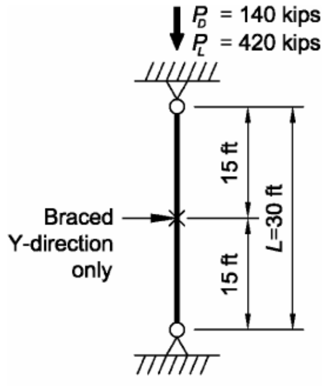
- Top values are total factored distributed load from strength and deflection criteria.
- Values below in gray are for live load deflection limit (unfactored).

Example 3 (AISC Design Examples vV13.0)

Example E.1b W-Shape Column Design with Intermediate Bracing

Given:

Redesign the column from Example E.1a assuming the column is laterally braced about the y-y axis and torsionally braced at the midpoint.



Solution:

Calculate the required strength

LRFD	ASD
P_u = 1.2(140 kips) + 1.6(420 kips) = 840 kips	P_a = 140 kips + 420 kips = 560 kips

Select a column using Manual Table 4-1.

For a pinned-pinned condition, K = 1.0

Commentary Table C-C2.2

Since the unbraced lengths differ in the two axes, select the member using the y-y axis then verify the strength in the x-x axis.

Enter Table 4-1 with a y-y axis effective length, KL_y, of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a W14×90. A 15 ft long W14×90 provides an available strength in the y-y direction of

LRFD	ASD
ϕP_n = 1000 kips	P_n/Ω = 667 kips

The r_x /r_y ratio for this column, shown at the bottom of Manual Table 4-1, is 1.66. The equivalent y-y axis effective length for strong axis buckling is computed as

$$KL = \frac{30.0\text{ ft}}{1.66} = 18\text{ ft}$$

From the table, the available strength of a W14×90 with an effective length of 18 ft is

LRFD	ASD
$\phi_c P_n$ = 928 kips > 840 kips **o.k.**	P_n/Ω_c = 618 kips > 560 kips **o.k.**

Manual Table 4-1

The available compression strength is governed by the *x-x* axis flexural buckling limit state.

Example 3 (continued)

F_y = 50 ksi

Table 4–1 (continued) Available Strength in Axial Compression, kips

W Shapes

W14

Shape	W14×											
Wt/ft	145		132		120		109		99		90	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length *KL* (ft) with respect to least radius of gyration r_y												
0	1280	1920	1160	1740	1060	1590	959	1440	872	1310	792	1190
6	1250	1870	1130	1700	1030	1550	934	1400	849	1280	771	1160
7	1240	1860	1120	1680	1020	1530	924	1390	840	1260	763	1150
8	1220	1840	1110	1660	1010	1510	914	1370	831	1250	754	1130
9	1210	1820	1090	1640	995	1500	902	1360	820	1230	745	1120
10	1200	1800	1080	1620	981	1470	889	1340	808	1210	734	1100
11	1180	1770	1060	1590	965	1450	875	1320	795	1200	722	1090
12	1160	1740	1040	1570	949	1430	860	1290	781	1170	709	1070
13	1140	1720	1020	1540	931	1400	844	1270	767	1150	696	1050
14	1120	1690	1000	1510	912	1370	827	1240	751	1130	682	1020
15	1100	1650	982	1480	893	1340	809	1220	734	1100	667	1000
16	1080	1620	959	1440	872	1310	790	1190	717	1080	651	978
17	1050	1580	936	1410	851	1280	771	1160	699	1050	635	954
18	1030	1550	912	1370	829	1250	751	1130	681	1020	618	928
19	1000	1510	887	1330	806	1210	730	1100	662	995	600	902
20	979	1470	862	1300	783	1180	709	1070	642	966	583	876
22	926	1390	809	1220	735	1100	665	1000	602	906	546	821
24	871	1310	756	1140	685	1030	620	932	562	844	509	765
26	815	1230	702	1050	636	956	575	864	520	782	471	708
28	759	1140	647	973	586	881	530	797	479	720	434	652
30	702	1060	594	892	537	807	485	730	438	659	397	596
32	647	972	541	814	489	735	442	664	399	599	361	542
34	592	890	491	738	443	666	400	601	360	542	326	490
36	540	811	441	663	398	598	359	540	323	486	292	439
38	489	734	396	595	357	537	322	484	290	436	262	394
40	441	663	358	537	322	484	291	437	262	393	236	355
Properties												
P_{wo} (kips)	191	287	175	263	151	227	128	191	111	167	95.9	144
P_{wi} (kips/in.)	22.7	34.0	21.5	32.3	19.7	29.5	17.5	26.3	16.2	24.3	14.7	22.0
P_{wb} (kips)	477	717	407	612	312	468	220	330	173	260	129	194
P_{fb} (kips)	222	334	199	298	165	249	138	208	114	171	94.3	142
L_p (ft)	14.1		13.3		13.2		13.2		13.5		15.2	
L_r (ft)	61.7		56.0		52.0		48.4		45.3		42.6	
A_g (in.^{2})	42.7		38.8		35.3		32.0		29.1		26.5	
I_x (in.^{4})	1710		1530		1380		1240		1110		999	
I_y (in.^{4})	677		548		495		447		402		362	
r_y (in.)	3.98		3.76		3.74		3.73		3.71		3.70	
Ratio r_x/r_y	1.59		1.67		1.67		1.67		1.66		1.66	
$P_{ex}(KL^2)/10^4$ (k-in.^{2})	48900		43800		39500		35500		31800		28600	
$P_{ey}(KL^2)/10^4$ (k-in.^{2})	19400		15700		14200		12800		11500		10400	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

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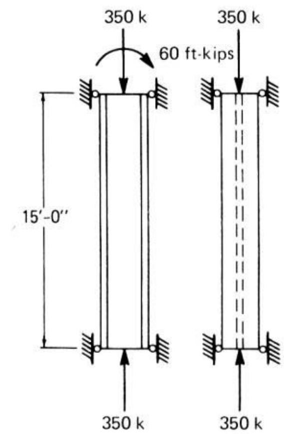
<u>Example 4 (LRFD)</u>

Investigate the accepatbility of a W16 x 67 used as a beam-column under the unfactored loading shown in the figure. It is A992 steel (F_y = 50 ksi). Assume 25% of the load is dead load with 75% live load.

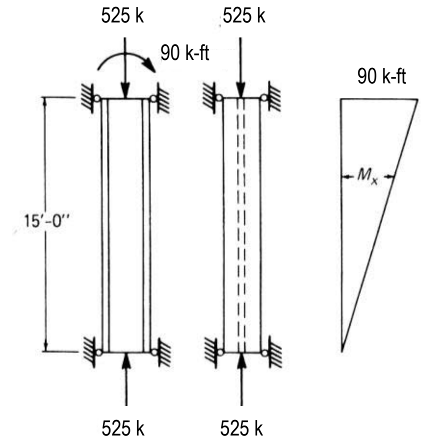


SOLUTION:

DESIGN LOADS (shown on figure):

Axial load = 1.2(0.25)(350k)+1.6(0.75)(350k)=525k

Moment at joint = 1.2(0.25)(60 $^{k\text{-}ft}$) + 1.6(0.75)(60 $^{k\text{-}ft}$) = 90 $^{k\text{-}ft}$



Determine column capacity and fraction to choose the appropriate interaction equation:

$$\frac{kL}{r_x} = \frac{15\,ft(12\,{}^{in}\!/\!_{ft})}{6.96in} = 25.9 \quad \text{and} \quad \frac{kL}{r_y} = \frac{15\,ft(12\,{}^{in}\!/\!_{ft})}{2.46in} = 73 \quad \text{(governs)}$$

$$P_c = \phi_c P_n = \phi_c F_{cr} A_g = (30.5ksi)19.7in^2 = 600.85k$$

$$\frac{P_r}{P_c} = \frac{525k}{600.85k} = 0.87 > 0.2 \quad \text{so use} \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \le 1.0$$

There is no bending about the y axis, so that term will not have any values.

Determine the bending moment capacity in the x direction:

The unbraced length to use the full plastic moment (L_p) is listed as 8.69 ft, and we are over that so of we don't want to determine it from formula, we can find the beam in the Available Moment vs. Unbraced Length tables. The value of ϕM_n at L_b =15 ft is 422 k-ft.

Determine the magnification factor when M_1 = 0, M_2 = 90 k-ft:

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} = 0.6 - \frac{0^{k-ft}}{90^{k-ft}} = 0.6 \le 1.0 \qquad P_{e1} = \frac{\pi^2 EA}{\left(Kl/r\right)^2} = \frac{\pi^2(30x10^3 ksi)19.7in^2}{(25.9)^2} = 8{,}695.4k$$

$$B_1 = \frac{C_m}{1-(P_u/P_{e1})} = \frac{0.6}{1-(525k/8695.4k)} = 0.64 \ge 1.0 \quad \text{USE 1.0} \qquad M_u = (1)90 \text{ k-ft}$$

Finally, determine the interaction value:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.87 + \frac{8}{9}\left(\frac{90^{k-ft}}{422^{k-ft}}\right) = 1.06 \not\le 1.0$$

This is **NOT OK.** (and outside error tolerance). The section should be larger.